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't HOOFT ANOMALY MATCHING FOR DISCRETE SYMMETRIES*

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Abstract

We show how to extend the 't Hooft anomaly matching conditions to discrete symmetries. We check these discrete anomaly matching conditions on several proposed low-energy spectra of certain strongly interacting gauge theories. The excluded examples include the proposed chirally symmetric vacuum of pure $N = 1$ supersymmetric Yang-Mills theories, certain non-supersymmetric confining theories and some self-dual $N = 1$ supersymmetric theories based on exceptional groups.

1 't Hooft Anomaly Matching for Continuous Global Symmetries

't Hooft anomaly matching [1] is a powerful tool to constrain the massless fermionic bound-state spectrum. Finding the massless spectrum is very important, since it is the first step towards establishing an effective low-energy Lagrangian (an analog of the chiral Lagrangian for QCD) of a given strongly interacting theory. 't Hooft was arguing that the global symmetries can be used to severely restrict the massless fermion spectrum. Below we briefly summarize 't Hooft's original argument. Assume we have a strongly interacting gauge theory based on the gauge group G_c, and that the theory has in addition a G_F flavor symmetry. In order for the theory to be consistent, the gauge anomalies G_c^3 have to cancel. In order for the G_F to be an unbroken global symmetry, the mixed $G_c^2 G_F$ anomalies have to vanish as well. However, a priori there is no reason for the G_F^3 anomalies, the anomalies calculated solely with respect to the global symmetries themselves, to vanish. It turns out that these G_F^3 anomalies, instead of being vanishing, will put a non-trivial constraint on the massless spectrum of the theory.

To see this, introduce *spectator* fields, which *do not* transform under the strong gauge group G_c, only under the flavor symmetry G_F, such that all G_F^3 anomalies vanish. In this enlarged theory we can weakly gauge the G_F flavor symmetry, and consider the low-energy limit of this modified theory. At low energies, the G_c gauge group will confine the original degrees of freedom

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into bound states. However, since we can take the gauge coupling of G_F to be arbitrarily small, we expect G_F to be still weak at low energies. Thus the low-energy effective theory should be a weakly interacting G_F gauge theory of the composite G_c bound states. In order for this effective theory to be consistent, the G_F^3 anomalies still have to be vanishing in the effective theory. Now let us compare the extended theory to our original one. We notice, that since the spectators do not transform under G_c, they do not participate in forming the bound states. Thus they are included both into the high-energy and low-energy theories as elementary fields, therefore their contribution to the G_F^3 anomalies is identical in the low-energy and in the high-energy theories. Thus the remaining degrees of freedom also have to have matching G_F^3 anomalies: *the global anomalies of the elementary degrees of freedom have to match those of the massless bound states, if G_F is not spontaneously broken.* This statement is 't Hooft anomaly matching. It is a set of necessary conditions which the correct low-energy spectrum has to satisfy, and which played a central role in establishing exact results in $N = 1$ supersymmetric gauge theories [2]. Explicitly, the expressions for the anomalies which have to be matched for a G_c gauge theory with global symmetry $G_F = G_1 \times G_2 \times \ldots \times U(1)_1 \times U(1)_2 \times \ldots$ are: $G_i^3 : \sum_R A_R^i$, $G_i^2 U(1)_j : \sum_R \mu_R^i q_R^j$, $U(1)_i U(1)_j U(1)_k : \sum_R q_R^i q_R^j q_R^k$, $U(1)(\text{gravity})^2 : \sum_R q_R^i$, where A is the cubic anomaly coefficient defined by the relation $\mathrm{Tr}_R \{T^a, T^b\} T^c = A_R d^{abc}$ (the T's being the generators of the group G_i in a given representation R), μ_R is the Dynkin index $\mathrm{Tr}_R T^a T^b = \mu_R \delta^{ab}$, and q^i's are the $U(1)_i$ charges. The sum over R denotes the summation over all representations of fermions present in the high-energy or the low-energy descriptions.

2 Discrete Anomaly Matching

Following the logic of the previous section, the following question arises naturally: can we extend the 't Hooft anomaly matching conditions to discrete global symmetries as well? We will show, that the answer is yes, however, these conditions are weaker than those for continuous symmetries. We consider only abelian Z_N discrete symmetries. Since the Z_N charges are defined only mod N, the best we can hope for are matching conditions that have to be satisfied mod N. We will see, that indeed, some of the discrete anomaly matching conditions will be mod N, but some of them slightly weaker.

Consider first the case of the $G_F^2 Z_N$ anomalies, where G_F is a simple Lie group, and where G_F and Z_N are assumed to be global symmetries of our gauge theory with gauge group G_c. As in 't Hooft's argument, we can include spectators which do not transform under the gauge group, only under G_F and Z_N, such that the G_F^3 and the $G_F^2 Z_N$ anomalies vanish. Now we can weakly gauge the G_F global symmetry. Since the $G_F^2 Z_N$ anomaly vanishes, that is $\sum_i \mu_i q_i = 0$ mod N where μ is the Dynkin index and q is the Z_N charge, the Z_N discrete symmetry is unbroken even in the background of G_F instantons. Now we consider the low-energy effective theory. Since the Z_N is a good symmetry of the full extended theory, the $G_F^2 Z_N$ anomalies have to vanish in the low-energy effective theory as well, thus $\sum_i \mu_i q_i$ still vanishes mod N. Since the spectators do not transform under the strong gauge group G_c, their contribution to $\sum_i \mu_i q_i$ is identical in the high-energy and the low-energy theories. Thus we conclude that the $G_F^2 Z_N$ anomalies have to be matched mod N. Note, that in this argument, one never had to promote the discrete symmetry to a continuous one, contrary to the criticism of Ref. [9]. Therefore, the objection raised in Ref. [9] has no basis.

Similarly, one can consider the $Z_N(\text{gravity})^2$ anomaly, which constrains the quantity $\sum_i q_i$.

Considering correlators in gravitational instanton backgrounds, we conclude that $\sum_i q_i$ has to be matched mod $N/2$. The origin of the (weaker) mod $N/2$ matching is that every fermion has at least two zero modes in gravitational instanton backgrounds. An alternative explanation for the possible additional $N/2$ contribution to this this anomaly is to note that a heavy Majorana fermion with Z_N charge $N/2$ does not have a vanishing $Z_N(\text{gravity})^2$ anomaly, instead it exactly contributes $N/2$.

Thus we conclude, that the $G_F^2 Z_N$ anomaly has to be matched mod N, and the $Z_N(\text{gravity})^2$ anomaly mod $N/2$. We call these the Type I anomalies. For the remaining (Type II) anomalies (Z_N^3, $U(1)^2 Z_N$, $U(1)Z_N^2$, $Z_N^2 Z_M$) there is no similar argument in favor of anomaly matching based on instantons. However, by promoting certain parameters of the Lagrangian to background fields one can extend the discrete symmetry to a continuous global $U(1)$ symmetry, from which one can argue that the Type II anomalies still have to be matched mod N (for more details see Ref. [3]). There are however some important subtleties which one has to consider for these Type II anomalies:

- Decoupling of Majorana fermions can weaken the Z_N^3 matching condition, just like in the case of the $Z_N(\text{gravity})^2$ anomaly
- One has to choose a normalization where all $U(1)$ charges are integers in order to have a mod N matching condition
- Existence of fractionally charged massive states can invalidate the matching of Type II anomalies (but not that of Type I anomalies)

Thus, we conclude that the discrete anomalies have to be matched in the following way: Type I: $G^2 Z_N$: mN, $Z_N(\text{gravity})^2$: $mN + \frac{m'}{2}N$; Type II: Z_N^3: $mN + \frac{m'}{8}N^3$, $U(1)^2 Z_N$: mN, $U(1)_i U(1)_j Z_N$: mN, $U(1)Z_N^2$: mN, $U(1)Z_N Z_M$: mK, where after each anomaly we have indicated the possible difference. Here m, m' are integers, and m' can be non-vanishing only if N, M are even. K is the GCD of N and M. Type I anomaly matching constraints have to be satisfied regardless of the details of the massive spectrum. Type II anomalies have to be also matched except if there are fractionally charged massive states.

3 An Example

We have checked, that all Seiberg dualities [2], including Kutasov type dualities [4] (at least the ones we have checked from the long list of theories in [4]) satisfy the discrete anomaly matching conditions presented in the previous section, even though some of these matching conditions are very non-trivial. Here we present only one simple example, which is based on an s-confining $N = 1$ supersymmetric theory [5]. The theory together with the confining spectrum is given in the table below.

	$SO(7)$	$SU(6)$	$U(1)_R$	Z_{12}
S	8	$\square$	$\frac{1}{6}$	1
S^2		$\square\square$	$\frac{1}{3}$	2
S^4		$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{2}{3}$	4

$SO(7)$ is the gauge group and $SU(6) \times U(1)_R \times Z_{12}$ are the global symmetries. The anomaly

matching conditions are:

	UV	IR
$SU(6)^2 Z_{12}$	8	$2\times 8+4\times 4=8+2\times 12$
$Z_{12}(\text{gravity})^2$	48	$2\times 12+4\times 15=8\times 12+6$
Z_{12}^3	48	$2^3\times 21+4^3\times 15=94\times 12$
$U(1)_R^2 Z_{12}$	1200	76×12
$U(1)_R Z_{12}^2$	$-5\times 8\times 6$	-68×12

where the contributions to the first three anomalies in the magnetic theory are quoted in the order S^2, S^4. The $U(1)_R$ charges are multiplied by a factor of 6 to make all the charges integers.

All anomalies match mod 12 except the $Z_{12}(\text{gravity})^2$ anomaly, which is matched mod 6, and signals the presence of massive Majorana fermions with charge 6. But we do not see the corresponding contribution to Z_{12}^3 anomaly because $12^3/8=216$ is a multiple of 12.

4 Excluded Examples

Kovner and Shifman argued recently [6], that there might be an additional, chirally symmetric phase of $N=1$ pure Yang-Mills theory, with vanishing gaugino condensate. In this vacuum, the Z_{2N} (for the case of $SU(N)$ gauge groups) discrete R-symmetry is not broken, therefore the discrete anomalies have to be matched by the massless fields of the low-energy effective theory. The most natural candidate for the massless field is $\Phi=(W_\alpha W^\alpha)^{\frac{1}{3}}$, since this is the basic variable of the Veneziano-Yankielowicz Lagrangian [7], based on which Kovner and Shifman concluded that there might be an additional vacuum. In this case, the R-charge of the fermionic component of Φ is $-\frac{1}{3}$, which signals the fractionalization of the Z_{2N} charges. Therefore, it is convenient to rescale the discrete charges such that the gaugino of the high-energy theory has charge 3, and check the anomaly matching conditions for the resulting Z_{6N} symmetry. The discrete anomalies for $SU(N)$ are:

	UV	IR
$Z_{6N}(\text{gravity})^2$	$3(N^2-1)$	-1
Z_{6N}^3	$27(N^2-1)$	-1

The difference in the $Z_{6N}(\text{gravity})^2$ anomalies of the UV and the IR descriptions is 2 mod $3N$, which means that the discrete anomalies can not be matched for any value of N. Recall that the $Z_{6N}(\text{gravity})^2$ anomaly is Type I and must be matched irrespective of charge fractionalization. Therefore, this low-energy description of the pure $SU(N)$ YM theories is excluded. One can show in an analogous way, that the Kovner-Shifman vacua are excluded by discrete anomaly matching for the other simple groups as well.

However, this does not completely exclude the idea of a chirally symmetric phase of $N=1$ pure Yang-Mills theories. It excludes only a specific realization of it described above. One could, for example, try to match anomalies with the operator $S=W_\alpha W^\alpha$ instead of Φ. Here no charge fractionalization occurs, and hence anomalies should be matched mod $2N$. The anomalies for $SU(N)$ are

	UV	IR
$Z_{2N}(\text{gravity})^2$	N^2-1	1
Z_{2N}^3	N^2-1	1

The differences in the anomalies are both N^2-2, which is divisible by N only for $N=1,2$. Performing a similar analysis we find that the field S matches the discrete anomalies for $SO(N)$

only if N is odd, while it matches always for $Sp(2N)$. None of the discrete anomalies for the exceptional groups are matched by S. Even though anomalies are matched for some special cases by S, generically it does not match the discrete anomalies and therefore we conclude that it is not a likely candidate for a low-energy solution.

In addition to the Kovner-Shifman vacuum, the excluded examples include several non-supersymmetric theories which were conjectured to be confining in the early 80's [8]. An example of such a theory with the conjectured low-energy spectrum is given in the table below.

	$SU(4)$	$SU(2)$	$U(1)$	Z_{12}
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	2	1
X	$\begin{smallmatrix}\square\square\\\square\end{smallmatrix}$	1	-1	1
(A^2X)	1	$\square\square$	3	3

All the continuous global anomalies ($SU(2)^2U(1)$, $U(1)(\text{gravity})^2$ and $U(1)^3$) are matched between the high-energy and the confining spectrum. The discrete anomalies are:

	UV	IR
$SU(2)^2Z_{12}$	6	12
$Z_{12}(\text{gravity})^2$	27	9
Z_{12}^3	27	81
$U(1)^2Z_{12}$	63	81
$U(1)Z_{12}^2$	9	81

The $U(1)^2Z_{12}$ anomaly matching is satisfied mod 12 and the $Z_{12}(\text{gravity})^2$ anomaly matching is satisfied mod 6. However, while the $SU(2)^2Z_{12}$, the $U(1)^2Z_{12}$ and the Z_{12}^3 anomalies must match mod 12, they match only mod 6, and hence the discrete anomaly matching conditions are violated. In the absence of any dynamical explanation of spontaneous breaking of Z_{12}, and since $SU(2)^2Z_{12}$ is a Type I anomaly, one has to consider this model excluded based on discrete anomaly matching.

Similarly, certain $N = 1$ supersymmetric dualities based on exceptional groups can be excluded as well using the discrete anomaly matching conditions (see [3] for details).

Finally, we comment on an interesting example, where continuous anomaly matching can lead to misleading conclusions [10]. The theory is $N = 1$ $SO(N)$ with a symmetric tensor. All continuous anomalies are matched by the set of independent gauge invariant operators, and the Type I discrete anomalies match as well in this example. However, the Type II conditions are not satisfied. As explained before, the failure of the matching of Type II anomalies does not automatically exclude a given low-energy spectrum due to the possibility of charge fractionalization. However, we have to emphasize that all established theories satisfy the Type II conditions as well, and that the charge fractionalization of the heavy states is quite unlikely. Thus this raises the suspicion that this theory is not confining. Indeed, it was noted in [10] that there are several reasons to believe that the theory is not confining at the origin. This is a good example where the failure of Type II discrete anomaly matching is the first sign of the incorrect guess on the low-energy dynamics.

5 Conclusions

We have shown how to extend the 't Hooft anomaly matching conditions to discrete global symmetries. There are two types of discrete anomalies. Type I anomaly matching conditions

($G_F^2 Z_N$ and $Z_N(\text{gravity})^2$) have to be satisfied regardless of assumptions on the massive bound states. Type II constraints have to be satisfied except if there are fractionally charged massive states. We have tested several conjectured low-energy solutions using discrete anomaly matching. The excluded examples are: the chirally symmetric phase of $N = 1$ pure Yang-Mills theories, certain non-supersymmetric theories conjectured to be confining, and $N = 1$ supersymmetric self-dualities based on exceptional groups.

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References

[1] G. 't Hooft, in "Recent Developments in Gauge Theories" eds. G. 't Hooft et. al. (Plenum Press, New York, 1980), 135.

[2] N. Seiberg, *Phys. Rev.* **D49**, 6857 (1994), hep-th/9402044; *Nucl. Phys.* **B435**, 129 (1995), hep-th/9411149; K. Intriligator and N. Seiberg, *Nucl. Phys.* **B444**, 125 (1995), hep-th/9503179.

[3] C. Csáki and H. Murayama, *Nucl. Phys.* **B515**, 114 (1998), hep-th/9710105.

[4] D. Kutasov, *Phys. Lett.* **351B**, 230 (1995), hep-th/9503086; K. Intriligator, R. Leigh and M. Strassler, *Nucl. Phys.* **B456**, 567 (1995), hep-th/9506148.

[5] C. Csáki, M. Schmaltz and W. Skiba, *Phys. Rev. Lett.* **78**, 799 (1997), hep-th/9610139; *Phys. Rev.* **D55**, 7840 (1997), hep-th/9612207.

[6] A. Kovner and M. Shifman, *Phys. Rev.* **D56**, 2396 (1997), hep-th/9702174.

[7] G. Veneziano and S. Yankielowicz, *Phys. Lett.* **113B**, 231 (1982).

[8] C. Albright, *Phys. Rev.* **D24**, 1969 (1981).

[9] I. Kogan, A. Kovner and M. Shifman, *Phys. Rev.* **D57**, 5195 (1998), hep-th/9712046.

[10] J. Brodie, P. Cho and K. Intriligator, hep-th/9802092.

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