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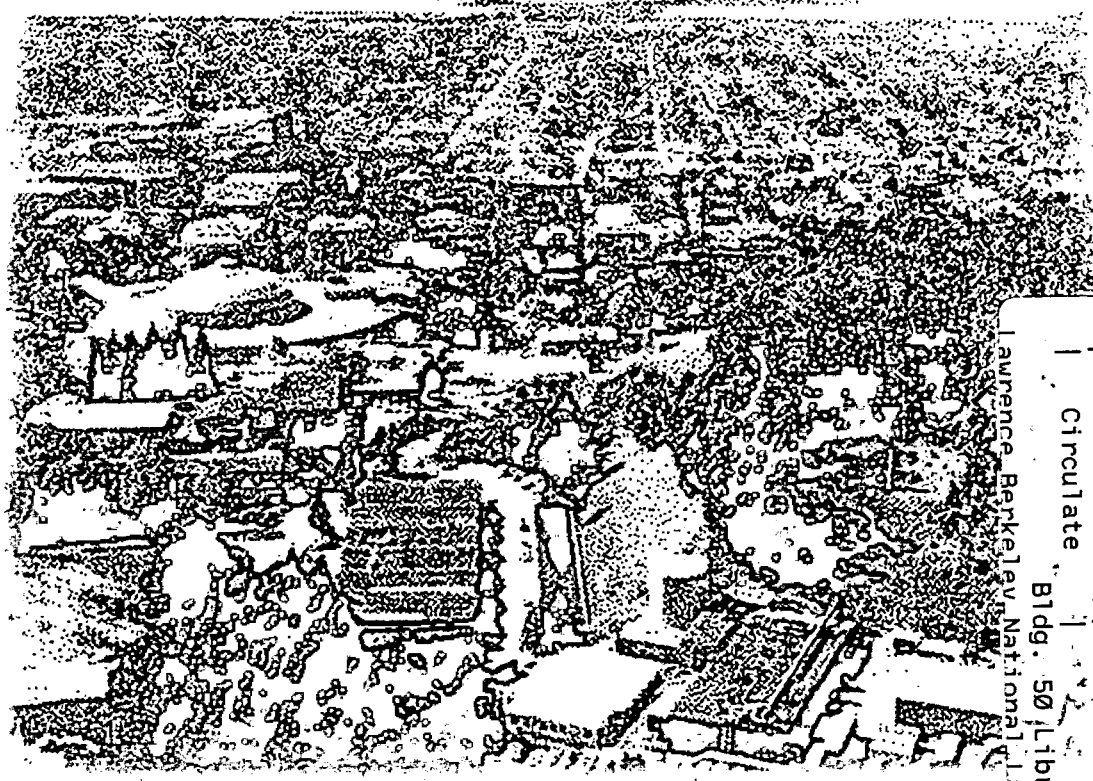

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Getting the Supersymmetric Unification Scale from Quantum Confinement with Chiral Symmetry Breaking [1]

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Abstract

Two models which generate the supersymmetric Grand Unification Scale from the strong dynamics of an additional gauge group are presented. The particle content is chosen such that this group confines with chiral symmetry breaking. Fields that are usually introduced to break the Grand Unified group appear instead as composite degrees of freedom and can acquire vacuum expectation values due to the confining dynamics. The models implement known solutions to the doublet-triplet splitting problem. The $SO(10)$ model

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only requires one higher dimensional representation, an adjoint. The dangerous coloured Higgsino-mediated proton decay operator is naturally suppressed in this model to a phenomenologically interesting level. Neither model requires the presence of gauge singlets. Both models are only technically natural.

1 Introduction

One of the most beautiful ideas for physics beyond the Standard Model (SM) is the idea [1] that the gauge groups of the Standard Model (SM) unify into a single gauge group, the Grand Unified Theory (GUT). This would provide some common understanding for the diversity of particle content and parameters that constitute the Standard Model. That one generation of fermions can be accommodated by a single **16** of $SO(10)$ is too remarkable to be a coincidence! More indirect evidence for this framework is provided by the precision electroweak data. These suggest that the gauge couplings of the Standard Model unify at a high energy scale. In fact, a very good agreement with the data is obtained if softly-broken supersymmetry is realised close to the weak scale.

This naturally leads to a consideration of supersymmetric GUTs [2]. The scale of supersymmetric unification inferred from the data is $M_{GUT} \sim 2 \times 10^{16}$ GeV. Above this scale Nature may be described by a supersymmetric GUT. The value of this scale given by the data does not appear to be directly related to any other mass scale in Nature. The closest scale is the reduced Planck mass, $M = 1/\sqrt{8\pi G_N}$, which is about a factor of 100 larger than the GUT scale. Most attempts at supersymmetric model building remain agnostic about the origin of the GUT scale, and simply input both the scale and pattern of symmetry breaking into the theory by hand. While this is technically natural in supersymmetric theories, it completely avoids the issues of the origin of the GUT symmetry breaking and the small value of M_{GUT}/M. This issue is particularly relevant if the scale M is representative of a fundamental scale of new physics. If this is the case, then the small value of the supersymmetric Grand Unification scale compared to the Planck scale is perplexing.

Some of these issues can be addressed by applying some of the recent developments in the

strong dynamics of supersymmetric gauge theories [3]. In particular, the strong dynamics of an additional gauge group that confines with chiral symmetry breaking at a scale close to the GUT scale is considered. The idea of using strong dynamics to generate the supersymmetric GUT scale has only recently been explored [4, 5, 6]. This was first explored in Reference [4], where a dynamically generated superpotential with a runaway behavior is used to generate M_{GUT}/M. In Reference [6] the confining dynamics without chiral symmetry breaking is used in a novel manner to solve the doublet-triplet splitting problem. In that model though, a large top quark Yukawa coupling is only possible if the unification scale is uncomfortably close to the Planck scale. In Reference [5] the quantum confinement with chiral symmetry breaking is used to generate the GUT scale.

The idea of using strong supersymmetric dynamics to generate ratios of symmetry breaking scales has also been applied to flavour symmetries [7, 8]. The first phenomenological application of quantum confinement with chiral symmetry breaking in this context is given in Reference [8].

The outline of this paper is as follows. Section 2 describes some features that are common to the models presented in Section 3 and 4. Section 3 introduces a model with an $SU(6)$ GUT group. Section 4 introduces the preferred model which has an $SO(10)$ GUT group.

2 Overview

In the models presented in this paper an extra gauge group G_C is introduced and assumed to become strong at a scale $\Lambda \sim M_{GUT}$. The particle content of G_C is chosen so that it confines with chiral symmetry breaking. This sector of the theory will be called the 'confining sector'. By identifying the GUT group, G_{GUT}, with a global symmetry of the confining sector, the composite fields of the confining sector are charged under the GUT group. For example, in

the first model presented below, an adjoint of $SU(6)_{GUT}$ is composite. In the second model, a symmetric and antisymmetric tensor of $SO(10)_{GUT}$ is composite. This differs from the model of Reference [5], where the confining sector in that model does not contain particles charged under the GUT group. Below the scale of confinement, some of the composite fields will acquire vacuum expectation values (vevs) as a consequence of the dynamics of confinement. In the models presented here there is a discrete set of supersymmetric vacua. In one of these vacua the vevs of the composite fields break the GUT group; this together with some superpotential interactions lead to a phenomenologically acceptable vacuum. The small value of M_{GUT}/M_{PL} is then understood as naturally arising from the dimensional transmutation of the small gauge coupling of G_C at the Planck scale.

The simplest example of a supersymmetric gauge theory that exhibits confinement with chiral symmetry breaking is $SU(N)$ with N flavours $Q + \overline{Q}$ and no superpotential [3]. This will be the model for the confining sector. It is conjectured that below the scale of strong dynamics, Λ, of the $SU(N)$ group, the appropriate degrees of freedom are the confined "baryons" B, $\overline{B}$, and "mesons" M of the $SU(N)$ group, where

$$M_i^j \sim \overline{Q}_a^j Q_i^a \sim (\overline{\square}, \square, 0) \tag{1}$$

$$B \sim \epsilon_{a_1 \cdots a_N} Q_{i_1}^{a_1} \cdots Q_{i_N}^{a_N} \sim (\mathbf{1}, \mathbf{1}, 1) \tag{2}$$

$$\overline{B} \sim \epsilon^{a_1 \cdots a_N} \overline{Q}_{a_1}^{i_1} \cdots \overline{Q}_{a_N}^{i_N} \sim (\mathbf{1}, \mathbf{1}, -1). \tag{3}$$

The charges of the baryons and mesons under the global $SU(N) \times SU(N) \times U(1)_{B'}$ are indicated in parantheses. The space of supersymmetric vacua for the baryons and mesons is described by [3]

$$\det M - B\overline{B} = \Lambda^{2N}. \tag{4}$$

The left-hand-side of this equation vanishes at the classical level as a consequence of the Bose statistics of the superfields Q and $\overline{Q}$. Quantum corrections result in a non-vanishing value for the right-hand-side. The important point is that along the supersymmetric vacua, some of the confined fields necessarily acquire vevs, breaking the global symmetry down to a subgoup. This conjecture satisfies two nontrivial consistency tests [3]: holomorphic decoupling of one flavour; and t'Hooft anomaly matching of the unbroken global symmetries.

In this paper a diagonal subgroup of the global symmetry of the confining sector is gauged and identified with the GUT group. The mesons of the confining sector therefore transform under the GUT group. I will make the dynamical assumption that weakly gauging a global symmetry of the confining sector does not affect the confining dynamics of G_C, and does not ruin the quantum modification with chiral symmetry breaking. This is a reasonable assumption since the GUT group is weakly gauged at the scale $\Lambda \sim M_{GUT} \sim 2\times10^{16}$ GeV.

Perhaps the most difficult problem in GUT model building is the origin of the doublet-triplet mass splitting. The excellent agreement between the measured and theoretically predicted value of $\sin^2\theta_W$ assumes that the particle content below the unification scale contains the (supersymmetric) SM chiral matter content plus two electroweak Higgs doublets. In a minimal $SU(5)$ GUT, the Higgs fields fit into a $\mathbf{5}$ and $\overline{\mathbf{5}}$ of $SU(5)$. The presence of the remaining particle content of these representations-the two coloured Higgs triplets- much further than a few decades below the GUT scale completely ruins this agreement. More generally, requiring that there exists one large split $SU(5)$ representation is a strong constraint on model building. The models presented in this paper implement two known solutions to this problem: the Higgs as "pseudo-Goldstone bosons" [9] and the "Dimopoulos-Wilzcek" [10] missing vevs mechanism. The latter solution is implemented in an $SO(10)$ GUT gauge group, whereas the former is based upon an $SU(6)$ GUT

group.

In the models presented here the quantum confinement is therefore not directly responsible for the doublet-triplet splitting. The structure outlined above must be supplemented with a non-vanishing superpotential in order to implement the doublet-triplet splitting. A non-vanishing superpotential must be added in any case: a generic point on the quantum modified constraint breaks $SU(N) \times U(1)_{B'}$ down to $U(1)^{N-1}$. This provides too much symmetry breaking. A point that only breaks to a larger subgroup is therefore an enhanced symmetry point, corresponding to a particular choice of the vevs of M and B. At the enhanced symmetry point, there are many massless particles in addition to the Nambu-Goldstone multiplets. These correspond to the would-be Goldstone bosons of the more generic symmetry breaking pattern, and at the enhanced symmetry point, transform as adjoints under the unbroken gauge group. These particles must acquire masses from additional superpotential interactions. These superpotential interactions then explicitly break the global symmetry of the confining sector down to $G_{GUT} \times U(1)_{B'}$.

It is then a concern whether the presence of this superpotential might destabilise the confinement and chiral symmetry breaking. The form of the superpotential for the fundamental fields of the group G_C, Q, $\overline{Q}$, and any fields ψ_M not charged under G_C, in the two models presented here is

$$W = W_C(Q, \overline{Q}, \psi_M) + W_M(\psi_M). \tag{5}$$

The superpotential W_C involving the confining fields will by *fiat* contain only non-renormalizable operators, suppressed by a scale assumed to be either the Planck mass or reduced Planck mass. If confinement occurs, the coefficient c of an operator with mass dimension d in the low-energy theory that arose from an operator with N $(\overline{Q}Q)$s in the high energy theory is expected to be

$$c \sim \lambda \times \Lambda^N / M^{N-d}, \tag{6}$$

where λ is a constant that is expected to be of order unity. For the models considered below, $d = -1, 0$ or 1, N is 1 or 2, and $N - d$ is postive. Since these coefficients are suppressed by powers of Λ/M, the presence of these terms in the superpotential is a small perturbation to the quantum confinement. It is then reasonable to expect that these operators do not destroy the quantum confinement with chiral symmetry breaking. This assumption will be made for the remainder of the paper.

In the usual GUT model building framework, the unification of the gauge couplings can be significantly affected by the presence of M^{-1} suppressed operators [11]. In an $SU(5)$ model, for example, the gauge field-strength tensor F can have non-renormalizable interactions with an adjoint Σ. The operator $c\Sigma FF/4M$ results in a tree-level relative shift of the gauge couplings $1/g_i^2$ that is approximately cM_{GUT}/M. This translates into a shift in the low-energy value of $\sin\theta_W^2$ that for $M/M_{GUT} = 20$ is $\Delta \sin\theta_W^2(M_Z) \sim \pm\text{few} \times c \times 10^{-3}$. In the GUT models presented in this paper, some of the higher dimensional representations are composite. For the composite fields, the gravitational smearing operator arises from a higher dimension operator in the fundamental theory. The coefficient of this operator below the confinement scale then contains an additional suppression of Λ/M. This extra factor completely suppresses the smearing effect unless the coefficient of the operator in the fundamental theory is unnaturally large-of $O(M/M_{GUT})$-and M_{GUT}/M is $\sim 1/20$. Non-composite higher dimensional fields can contribute to the gravitational smearing. In the $SO(10)$ model, it turns out that these contributions are completely negligible.

I conclude this Section with a discussion of some technical issues that occur throughout the paper. Implicit in the discussion that follows will be the assumptions that (global) supersymmetry is unbroken, and that the non-trivial Kahler potential has a strictly positive definite Kahler metric [8].

To find supersymmetric minima I will look for solutions to the $F-$flatness equations $0 = F = \partial_{\phi_i} W$ for the confined and ψ_M fields. This is rather naive, since the vevs of the fields will typically be $O(\Lambda)$ and the Kahler potential is non-calculable for these field values. It is not clear then that the "baryons" and "mesons" are the correct degrees of freedom. For the purposes of determining the existence of supersymmetric vacua with a particular pattern of symmetry breaking, however, the last assumption of the previous paragraph is suffucient [8]. With these assumptions, a supersymmetric vacuum found using a trivial Kahler potential will remain supersymmetric for the non-trivial Kahler potential.

The spectrum of the particle masses is also important for phenomenology. For this, knowledge of the Kahler potential is required. Despite the absence of this information, a few important points about the mass spectrum can be extracted from the superpotential [8]. For example, a particle that is massless (zero eigenvector of $F_{i,k}$) in the case of a canonical Kahler potential for the confined fields will remain massless in the case of a non-trivial Kahler potential. Similarly, a massive particle in the trivial Kahler potential will remain massive for a non-trivial Kahler potential. So I will use the mass spectrum computed by assuming a trivial Kahler potential to check that the superpotential with a non-trivial Kahler potential results in superheavy masses to all the particles that should have superheavy masses.

In the models presented here, the superpotential interactions that involve the confining fields occur from higher dimension operators, so that after confinement the superpotential coupling of those operators is $\tilde{\lambda} \sim \lambda(\Lambda/M)^n \ll \lambda$, with $\lambda \sim O(1)$. Particles that acquire their mass from these operators will then have masses somewhat below the GUT scale. These masses remain uncalculable though, since they should be computed at a scale that is comparable to the *vev* that is generating the mass, which in this case is $O(\Lambda)$.

The one-loop prediction for $\sin^2\theta_W$ is modified by the presence of these light states below the GUT scale since they do not in general form complete $SU(5)$ representations. An attempt at quantifying this correction is made by assuming that the naive calculation-i.e. assuming a canonical Kahler potential-of the spectrum gives the correct mass spectrum to within a factor of a few times unity, and further, that the correction to $\sin^2\theta_W$ from particles with masses much smaller than the confinement scale is well-approximated by the usual one-loop computation. The corrections from particles with masses near the confinement scale are not calculable and not discussed.

Finally, in the two models presented here certain operators allowed by the gauge symmetries of the theory must be absent from the superpotential in order not to ruin the doublet-triplet splitting mechanisms. All the dangerous operators cannot be forbidden by any global symmetries, since some of them will have the same quantum numbers as other operators that are required to be present in the superpotential. If these models were only the effective theory of some more fundamental field theory, then the dangerous operators could perhaps be generated at the tree-level by integrating out some heavy particles at the scale M. In this case however, the full theory above the Planck scale is not known and probably not a field theory. It is then possible that the full theory could be responsible for the absence of these dangerous operators, even though from the low-energy theory they cannot be forbidden by any symmetries.

3 $SU(6) \times SU(6)$

The gauge group is $SU(6)_C \times SU(6)_{GUT}$ where one factor of $SU(6)$ is the confining group G_C, and the other factor is the SM unified gauge group. I introduce six flavours, $Q+\overline{Q}$ of $SU(6)$ that are also charged under the $SU(6)_{GUT}$. I further introduce two Higgs fields H, $\overline{H}$, and an adjoint

Σ_N that are charged under only the $SU(6)_{GUT}$. The particle content under $SU(6)_C \times SU(6)_{GUT}$ is then

$$\begin{aligned} Q &\sim (\mathbf{6},\ \overline{\mathbf{6}}), \\ \overline{Q} &\sim (\overline{\mathbf{6}},\ \mathbf{6}), \\ H &\sim (\mathbf{1},\ \mathbf{6}), \\ \overline{H} &\sim (\mathbf{1},\ \overline{\mathbf{6}}), \\ \Sigma_N &\sim (\mathbf{1},\ \mathbf{35}). \end{aligned}$$

I assume that the $SU(6)_C$ group confines at a scale $\Lambda \sim M_{GUT}$ with a quantum modified constraint. In this case the confined "meson" $M_i^j \sim \overline{Q}_i^a Q_a^j \sim \mathbf{35}+\mathbf{1}$ under the $SU(6)$ GUT symmetry. The "baryons" $B \sim \epsilon Q^6$ and $\overline{B} \sim \epsilon \overline{Q}^6$ are singlets under the $SU(6)_{GUT}$ group. No gauge singlets are required in the fundamental theory.

The superpotential in terms of the fundamental fields is chosen to be

$$W_0 = \frac{1}{2}\lambda_1 \mathrm{tr}(Q\overline{Q})^2/M + \lambda_3 H\overline{H}\mathrm{tr}(Q\overline{Q})/M + \lambda \mathrm{tr}(\Sigma_N^2 Q\overline{Q})/M + \bar{g}(H\overline{H})\mathrm{tr}\Sigma_N^2/M. \qquad (7)$$

The scale M is assumed to be the reduced Planck mass $\sim 2\times 10^{18}$ GeV. The trace sums over the $SU(6)_{GUT}$ indices. All the dimensionless parameters are assumed to be of order unity. This superpotential is the most minimal, in the sense that (as shown below) it successfully implements in the phenomenologically preferred vacuum the doublet-triplet splitting and gives GUT scale masses to all the other particles. A more general superpotential is allowed provided that: (1) Only non-renormalizable operators involving Q, $\overline{Q}$ are allowed. (2) To keep the Higgs doublets light, the superpotential that only involves the **35**s and the H, $\bar{H}$ fields must preserve a $SU(6) \times SU(6)$ global symmetry [9]. The operators $\overline{H}(\overline{Q}Q)^n H$ and $\overline{H}(\Sigma_N)^n H$, for example, must be absent. (3) Supersymmetry is not spontaneously broken.

After confinement occurs, the superpotential written in terms of the confined fields $\Sigma \sim \mathbf{35}$ and $\sigma \sim \mathbf{1}$, i.e. $\overline{Q}_i^a Q_a^j \sim \Lambda \Sigma_i^j + \Lambda \sigma \delta_i^j/\sqrt{6}$, is

$$\begin{aligned} W_0 &= \mathcal{A}\left(\det(\Sigma + \sigma/\sqrt{6}) - B\overline{B} - \Lambda^6\right) + \frac{1}{2}\tilde{\lambda}_1 \Lambda \mathrm{tr}\Sigma^2 + \frac{1}{2}\tilde{\lambda}_2 \Lambda \sigma^2 \\ &+ \tilde{\lambda}_4 \mathrm{tr}\Sigma_N^2 \Sigma + \tilde{\lambda}_5 \sigma \mathrm{tr}\Sigma_N^2 + (H\overline{H})(\tilde{\lambda}_3 \sigma + \bar{g}\mathrm{tr}\Sigma_N^2/M). \end{aligned}$$

I expect that

$$\tilde{\lambda}_{1,2} \sim \lambda_1 \Lambda/M, \quad \tilde{\lambda}_3 \sim \lambda_3 \Lambda/M, \quad \tilde{\lambda}_{4,5} \sim \lambda \Lambda/M \tag{8}$$

as an estimate of the size of the couplings in the confined description. The quantum modified constraint has been added using a Lagrange multiplier $\mathcal{A}$. This superpotential contains all the non-perturbative (superpotential) information from the strong $SU(6)_C$ dynamics. It is interesting that in this case a term in the superpotential for $Q\overline{Q}$ that generates a cubic term $\mathrm{tr}\Sigma^3$ is not required. In most supersymmetric GUT models, the cubic term is required to obtain a non-trivial vacuum. In this case, it is the interaction $\mathcal{A}\det(\Sigma + \sigma)$ from the quantum modified constraint that balances the mass terms to obtain a non-trivial supersymmetric vacuum.

The $F-$flatness equations are

$$\det(\Sigma + \sigma/\sqrt{6}) - B\overline{B} = \Lambda^6, \tag{9}$$

$$0 = F_{\overline{B}} = \mathcal{A}B, \; 0 = F_B = \mathcal{A}\overline{B}, \tag{10}$$

$$0 = F_{\overline{H}} = (\tilde{\lambda}_3 \sigma + g\mathrm{tr}\Sigma_N^2/M)H, \tag{11}$$

$$0 = F_H = (\tilde{\lambda}_3 \sigma + g\mathrm{tr}\Sigma_N^2/M)\overline{H}, \tag{12}$$

$$0 = F_\sigma = \tilde{\lambda}_3 H\overline{H} + \tilde{\lambda}_2 \Lambda \sigma + \frac{\mathcal{A}}{\sqrt{6}}\det(\Sigma + \sigma/\sqrt{6})\mathrm{tr}(\Sigma + \sigma/\sqrt{6})^{-1} + \tilde{\lambda}_5 \mathrm{tr}\Sigma_N^2, \tag{13}$$

$$0 = F_\Sigma = \tilde{\lambda}_1 \Lambda \Sigma + \mathcal{A}\det(\Sigma + \sigma/\sqrt{6})((\Sigma + \sigma/\sqrt{6})^{-1} - \frac{1}{6}\mathrm{tr}(\Sigma + \sigma/\sqrt{6})^{-1}) + \tilde{\lambda}_4(\Sigma_N^2 - \frac{1}{6}\mathrm{tr}\Sigma_N^2), \tag{14}$$

$$0 = F_{\Sigma_N} = \tilde{\lambda}_4(\Sigma_N \Sigma - \frac{1}{6}\mathrm{tr}\Sigma_N \Sigma) + \tilde{\lambda}_5 \sigma \Sigma_N + g(H\overline{H})\Sigma_N/M. \tag{15}$$

In addition to the phenomenologically preferred vacuum, these equations include other discrete solutions. In some of these solutions $SU(6)_{GUT}$ is unbroken. For example, a solution with σ and $\mathcal{A}$ non-zero, and all other vevs equal to zero, exists. So although the preferred vacuum is discrete, I must assume that it was selected in the early history of the universe. This could occur if, for example, the preferred vacuum is a global minimum of the scalar potential after supersymmetry breaking effects are included.

To break $SU(6)$ down to the SM gauge group, I look for vevs of the form [3]

$$H = \overline{H} = v_H \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Sigma(\Sigma_N) = v_\Sigma(v_N) \begin{pmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}. \tag{16}$$

The vevs $\mathcal{A}$, σ, v_Σ, v_N and v_H are the solution to

$$0 = (\tilde{\lambda}_3\sigma + 12gv_N^2/M)v_H, \tag{17}$$

$$0 = (\tilde{\lambda}_5\sigma + gv_H^2/M - \tilde{\lambda}_4 v_\Sigma)v_N, \tag{18}$$

$$0 = \frac{1}{3}\mathcal{A}K(a-b) + \tilde{\lambda}_1\Lambda v_\Sigma - \tilde{\lambda}_4 v_N^2, \tag{19}$$

$$0 = 2\frac{\mathcal{A}K}{\sqrt{6}}(2a+b) + \tilde{\lambda}_2\Lambda\sigma + \tilde{\lambda}_3 v_H^2 + 12\tilde{\lambda}_5 v_N^2, \tag{20}$$

and for $\mathcal{A} \neq 0$, $\det(\Sigma+\sigma/\sqrt{6}) = \Lambda^6$. The quantities a, b and K are defined to be $a^{-1} \equiv v_\Sigma+\sigma/\sqrt{6}$, $b^{-1} \equiv -2v_\Sigma + \sigma/\sqrt{6}$ and $K \equiv \det(\Sigma + \sigma/\sqrt{6}) = a^{-4}b^{-2}$. In Appendix A it is demonstrated that

[3] $H = \overline{H}$ is required by $SU(6)_{GUT}$ $D-$flatness.

a discrete solution exists with $\mathcal{A} \sim (\Lambda/M)\Lambda^{-3}$ and with all vevs non-zero and of $O(\Lambda)$. Thus at this vacuum the vevs of the baryon fields are forced to the origin.

This vacuum implements the Higgs as "pseudo-Goldstone bosons" solution to the doublet-triplet splitting problem [9]. This mechanism is now briefly described. Firstly, the scalar potential for H, $\overline{H}$ and Σ, Σ_N has a $U(6) \times SU(6)$ global symmetry. The $U(6)$ acts on H and $\overline{H}$, whereas the $SU(6)$ symmetry acts on Σ and Σ_N. For the vacuum in Equation 16, the global $U(6) \times SU(6)$ symmetry is broken to $[SU(5)] \times [SU(4) \times SU(2) \times U(1)]$ by the vevs of H, Σ and Σ_N. The unbroken gauge group is then $SU(3)_C \times SU(2) \times U(1)_Y$. The breaking of the gauge symmetry results in 23 Nambu-Goldstone boson multiplets; the breaking of the $SU(6) \times U(6)$ results in 27 Goldstone boson multiplets. So all but 4 of the Goldstone bosons acquire mass of $O(M_{GUT})$ from the super-Higgs mechanism.

To see that these four pseudo-Goldstone bosons carry the quantum number charges of two electroweak doublets, first note that under $SU(4) \times SU(2)$, $\mathbf{35} = (\mathbf{4},\mathbf{2}) + (\overline{\mathbf{4}},\mathbf{2}) + (\mathbf{15},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{1},\mathbf{1})$. Inspecting the vevs of Σ and Σ_N, the combination $\tilde{v}_\Sigma \tilde{\Sigma} \equiv v_\Sigma \Sigma + v_N \Sigma_N$ of the fields $(\mathbf{4},\mathbf{2})$, and of the fields $(\overline{\mathbf{4}},\mathbf{2})$, in Σ and Σ_N are the Goldstone bosons of the breaking of one global $SU(6)$ symmetry. Since $SU(3)_C$ is embedded in $SU(4)$, these Goldstone bosons contain two electroweak doublets. The Goldstone bosons of the $SU(6) \to SU(5)$ breaking are $\mathbf{5} + \overline{\mathbf{5}} + \mathbf{1}$ of $SU(5)$, and also contain two electroweak doublets. The combination $3\tilde{v}_\Sigma \tilde{\Sigma} + v_H H$ of electroweak Higgs doublets are the fields eaten by the super-Higgs mechanism. The orthogonal combination remain massless and are the two Higgs doublets of the SM. The non-renormalization theorems of supersymmetry guarantee that these fields remain massless to all orders in perturbation theory.

The fields in the adjoint $(\mathbf{15},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ of both Σ and Σ_N, as well as the remaining combination of $(\mathbf{4},\mathbf{2})$, and of $(\overline{\mathbf{4}},\mathbf{2})$, in Σ and Σ_N orthogonal to $\tilde{\Sigma}$, do not correspond to any

broken generators and must acquire their masses from the superpotential interactions. It is conveinent to express the $SU(5)$ or SM charge assignments of this particle content: one complete $\mathbf{24}$ and $\mathbf{5}+\overline{\mathbf{5}}$ of $SU(5)$; 4 singlets; and one $(\mathbf{8},\mathbf{1},0)+(\mathbf{1},\mathbf{3},0)+(\mathbf{3},\mathbf{1},-1/3)+(\overline{\mathbf{3}},\mathbf{1},1/3)$. A naive estimate for the masses of the physical fields is obtained by computing the fermion mass matrix assuming a canonical Kahler potential. The results are presented in Appendix A, and are summarized here. All the fields have a mass $m \sim \Lambda^2/M$, a consequence of the suppression of the superpotential couplings for the confined theory.

These light fields affect the unification of the gauge couplings and may in principle also mediate proton decay. I first discuss the corrections to $\sin^2\theta_W$. These corrections occur from two sources. There could be large threshold corrections from the strong dynamics occuring at Λ. These are non-calculable and will not be considered. The other is from the light states $(\mathbf{8},\mathbf{1},0)$, $(\mathbf{3},\mathbf{1},0)$, $(\overline{\mathbf{3}},\mathbf{1},1/3)$ and $(\mathbf{3},\mathbf{1},-1/3)$ which have a mass $m \sim \Lambda^2/M$. The correction to $\sin^2\theta_W$ from these light states, using a naive one-loop running approximation from M_{GUT} to their masses is

$$\Delta\sin^2\theta_W = -\frac{\alpha_{em}}{5\pi}\ln M_{GUT}/m \sim -0.003 \times \frac{\ln(M_{GUT}/m)}{\ln 200}. \tag{21}$$

The reason[4] for the small correction is that the shift in $\sin^2\theta_W$ is dominated by the light $(\overline{\mathbf{3}},\mathbf{1},1/3)$ and $(\mathbf{3},\mathbf{1},-1/3)$ states. This is because the shift from the $(\mathbf{8},\mathbf{1})$ and $(\mathbf{1},\mathbf{3})$ states almost cancel. Recall that a sufficient condition for the prediction for $\sin^2\theta_W$ to be unchanged by the presence of some extra matter at a scale m is that $(\delta b_3-\delta b_2)/(\delta b_2-\delta b_1)=(b_3-b_2)/(b_2-b_1)$, independent of m. For an adjoint of $SU(3)$ and $SU(2)$, $\delta b_3=3$, $\delta b_2=2$ and $\delta b_1=0$. In this case the LHS of this condition is 1 and the RHS is $\frac{5}{7}\times 2$, which is close to 1. The other light states form approximate complete $SU(5)$ representations and do not significantly affect the gauge-coupling

[4]The author thanks N. Arkani-Hamed for this observation.

unification. The theoretical prediction without the light fields, $\sin^2\theta_W \sim .233 \pm O(10^{-3})$ [12], is a little larger than the measured value of 0.231[13]. The effect of these light states is to shift the prediction in the correct direction. The uncertainty in the uncalculable corrections to $\sin^2\theta_W$, however, are probably of the same order, with an unknown sign.

I next discuss the problem of forbidding operators of the form $\mathcal{O}_n \sim \overline{H}(Q\overline{Q})^n H$. These operators explicitly break the $U(6) \times SU(6)$ symmetry of the scalar potential. Consequently, if these operators are present they could give too large of a mass to the electroweak Higgs doublets. In this model, the term $\overline{H}H\mathrm{tr}\overline{Q}Q$ occurs in the superpotential. Any symmetry that allows this term also allows the term $\overline{H}(\overline{Q}Q)H$ in the superpotential. This operator ruins the doublet-triplet splitting, so I must assume that this term is absent. Higher dimensional operators must also be forbidden. Since the confinement introduces additional suppressions of $O(\Lambda^n/M^n)$, only a few of the first higher dimensional operators must be absent. More concretely, if I require that $\mathcal{O}_n$ not result in a mass for the Higgs superfields that is larger than a TeV and assume that $\Lambda/M_{PL} \sim 1/200$, then only the first three (n =1,2 and 3) higher dimensional operators must be forbidden. Operators of the type $\overline{H}(\Sigma_N)^n H$ are also dangerous and must be absent.

At this point it is probably not clear what role the extra adjoint plays in this model. In fact, this field is not needed to obtain an acceptable spectrum for the massive fields. It is introduced instead to obtain a large top quark Yukawa coupling. In order for the top quark not to have an irrelevant Yukawa coupling, it is necessary that the Yukawa interactions between the top quark and the Higgs doublet explicitly break the global $SU(6) \times U(6)$ symmetry. The top quark must therefore couple to both H and Σ. If Σ is composite, then such a coupling cannot be of order unity; rather, it will be suppressed by Λ/M. The top quark must therefore interact with a fundamental Σ.

The large top quark Yukawa coupling arises from considering the following embedding of the SM chiral fields [14] . The chiral matter content is one $\mathbf{20}$, $3 \times \mathbf{15}$ and $6 \times \overline{\mathbf{6}}$. The $SU(5)$ decomposition of these fields is, $\mathbf{20} = \mathbf{10} + \overline{\mathbf{10}}$, $\mathbf{15} = \mathbf{10} + \mathbf{5}$ and $\overline{\mathbf{6}} = \overline{\mathbf{5}} + \mathbf{1}$. The three $\overline{\mathbf{5}}$s of the SM are contained in three of the $\overline{\mathbf{6}}$s, and the other 3, call them $\overline{\mathbf{6}}'$, acquire mass at the GUT scale. The first two generation $\mathbf{10}$s are contained in two of the $\mathbf{15}$s, and the third generation $\mathbf{10}$ is a linear combination of the $\mathbf{10}$ in the $\mathbf{20}$ and the $\mathbf{10}$ in the remaining $\mathbf{15} \equiv \mathbf{15}_3$. This spectrum is obtained from the superpotential [14]

$$W_{top} = \lambda \mathbf{20} \Sigma_N \mathbf{20} + \lambda' \mathbf{20} H \mathbf{15}_3 + \lambda''_{ij} \overline{H} \mathbf{15}_i \overline{\mathbf{6}}'_j. \tag{22}$$

The vev of $\overline{H}$ gives GUT-sized Dirac masses to the $\mathbf{5}$ and $\overline{\mathbf{5}}$ fields in the 3 $\mathbf{15}$s and 3 $\overline{\mathbf{6}}'$s. From the vevs of Σ_N and H, a linear combination of the $\mathbf{10}$ in the $\mathbf{20}$ and the $\mathbf{10}$ in $\mathbf{15}_3$ acquires a GUT-sized Dirac mass with the $\overline{\mathbf{10}}$ in the $\mathbf{20}$. The orthogonal combination is the third generation $\mathbf{10}$ and remains massless. In sum, this superpotential leaves 3 $(\mathbf{10} + \overline{\mathbf{5}})$s massless. The large top quark Yukawa coupling arises from the first two interactions.

The $(\mathbf{3}, \mathbf{1}, 1/3)$ and $(\overline{\mathbf{3}}, \mathbf{1}, -1/3)$ fields have a Dirac mass somewhat below the GUT scale. Whether they may mediate proton decay at too large of a rate is then a concern. Since the top quark couples to these fields through the $\mathbf{20} \Sigma_N \mathbf{20}$ interaction, it naively appears that a dangerous proton decay operator is generated by integrating out these heavy fields, and then rotating the top quark to the mass basis. For this operator to be generated, however, a coupling of Σ_N or Σ to a $\overline{\mathbf{5}}$ of $SU(5)$ ($\overline{\mathbf{6}}$ of $SU(6)$) is required. Such a coupling is not present in the superpotential of Equation 22. So this issue depends crucially on the origin of the other fermion masses. For example, if all the fermion masses arise from interactions with H and $\overline{H}$, then a dangerous proton decay operator is not generated by the exchange of these states [14].

An upper bound on M is determined by the value of the Landau pole of the $SU(6)_{GUT}$ gauge

coupling. The $SU(6)$ coupling at the scale M is then

$$\alpha^{-1}_{SU(6)_{GUT}}(M) = 24 - \frac{43}{11\pi}\ln \Lambda/m_I - \frac{7}{2\pi}\ln M/\Lambda. \tag{23}$$

The first logarithm is the contribution to the GUT gauge coupling at the GUT scale from the particle content with mass m; the second logarithm is the contribution of the full $SU(6)$ particle content to the running of the gauge coupling above Λ. Inserting $m \sim \Lambda^2/M$ and requiring that $\alpha^{-1}_{GUT}(M) \geq 1$ implies $\ln M/\Lambda \leq 10$.

4 $SU(10) \times SO(10)$

The gauge group is $SU(10)_C \times SO(10)$. The $SU(10)_C$ group is the confining gauge group, and the Grand Unified group is $SO(10)$. The particle content is

$$\begin{aligned}
Q &\sim (\mathbf{10},\ \mathbf{10}),\\
\overline{Q} &\sim (\overline{\mathbf{10}},\ \mathbf{10}),\\
A &\sim (\mathbf{1},\ \mathbf{45}),\\
\mathbf{16} &\sim (\mathbf{1},\ \mathbf{16}),\\
\overline{\mathbf{16}} &\sim (\mathbf{1},\ \overline{\mathbf{16}}),\\
T_1 &\sim (\mathbf{1},\ \mathbf{10}),\\
T_2 &\sim (\mathbf{1},\ \mathbf{10}).
\end{aligned}$$

This particle content is rather economical as it requires only one higher dimensional representation, an adjoint, and no gauge singlets [5] . I assume that the $SU(10)_C$ group confines at

[5] Also see Reference [15] for an economical model. In this model though, the origin of the unification scale is not addressed.

a scale $\Lambda \sim M_{GUT}$ with a quantum modified constraint. In this case the confined "meson" $M_i^j \sim \overline{Q}_i^a Q_a^j \sim \mathbf{45} + \mathbf{54} + \mathbf{1}$ under the $SO(10)$ GUT symmetry. I label $S \sim \mathbf{54}$, $A'' \sim \mathbf{45}$ and $\sigma \sim \mathbf{1}$. The "baryons" $B \sim \epsilon Q^6$ and $\overline{B} \sim \epsilon \overline{Q}^6$ are singlets under the $SO(10)_{GUT}$ group.

The superpotential in the fundamental theory is chosen to be

$$\begin{aligned} W &= \lambda_1 T_1 A T_2 + \lambda_2 T_2(\overline{Q}Q)T_2/M + \lambda_3 \overline{\mathbf{16}}(\overline{Q}Q)\Sigma\mathbf{16}/M + \lambda_4 \overline{\mathbf{16}}\mathbf{16}\mathrm{tr}(\overline{Q}Q)/M \\ &\quad + \lambda_5 \mathrm{tr}(\overline{Q}Q)^2/M + \lambda_7 A^2(\overline{Q}Q)/M + \lambda_{11}\overline{\mathbf{16}}(Q\overline{Q})_{AS} A\Sigma\mathbf{16}/M^2, \end{aligned} \tag{24}$$

where $\Sigma_{ij} = [\Gamma_j, \Gamma_i]/4i$ are the generators of $SO(10)$ in the spinorial representation. The subscript "AS" indicates that only the anti-symmetric contribution of $Q\overline{Q}$ is allowed to be present; the symmetric contribution spoils the doublet-triplet splitting. It is technically natural for only the anti-symmetric contribution to be present; the full theory above the Planck scale must be responsible for the absence of the symmetric operator. The operators $T_1(\overline{Q}Q)^n T_1$ must also be absent.

The renormalizable and M^{-1} suppressed operators appearing in W are all required: (i) The operators $\propto \lambda_1$, λ_2 are required for the doublet-triplet splitting. (ii) The operator $\propto \lambda_7$ arranges the vev of A to be in the "Dimopoulos-Wilzcek" (DW) form [10], required to perform the doublet-triplet splitting. (iii) The operators $\propto \lambda_3$ and λ_4 are necessary to break the rank of the group. (iv) The operator $\propto \lambda_5$ is necessary to fix all the vevs. This point is made clear later. (v) The operator $\propto \lambda_{11}$ is required to give mass to some fields charged under the SM. This point is also discussed later. Although this operator is linear in A, the DW for A is not ruined because this operator does not contribute to the $F_i = 0$ equations [6]. The choice for this operator is not

[6]This interesting feature is also used in Reference [17, 18] to give mass (in a different context) to some charged particles. This is accomplished by a cubic term in the superpotential that is a product of three different antisymmetric tensors.

unique; other operators that are linear in A^2 are possible, but they are higher dimensional. It is non-trivial that with this choice for W, the low-energy particle content only contains the SM fields and their superpartners.

After confinement occurs the superpotential is

$$W = W_H + W_{DW} + W_{mix}, \tag{25}$$

with

$$W_H = \lambda_1 T_1 A T_2 + \tilde{\lambda}_2 T_2 S T_2 + \tilde{\lambda}_3 \sigma T_2 T_2, \tag{26}$$

$$W_{DW} = \frac{1}{2}\tilde{\lambda}_9 A^2 S + \frac{1}{2}\tilde{\lambda}_{10}\sigma A^2, \tag{27}$$

$$\begin{aligned} W_{mix} &= \mathcal{A}\left(\det(S + A'' + \sigma/\sqrt{10}) - B\overline{B} - \Lambda^{10}\right) + \frac{1}{2}\tilde{\lambda}_5 \Lambda \sigma^2 + \frac{1}{2}\tilde{\lambda}_6 \Lambda S^2 + \frac{1}{2}\tilde{\lambda}_7 \Lambda A''^2 \\ &+ \tilde{\lambda}_{16}\sigma \overline{\mathbf{16}}\mathbf{16} + \tilde{\lambda}_4 A''_{ij}\overline{\mathbf{16}}\Sigma_{ij}\mathbf{16} + \tilde{\lambda}_{11}(AA'')_{ij}\overline{\mathbf{16}}\Sigma_{ij}\mathbf{16}/M. \end{aligned} \tag{28}$$

The naive expectation for the couplings is $\tilde{\lambda}_{2,3} \sim \lambda_2\Lambda/M$, $\tilde{\lambda}_4 \sim \lambda_3\Lambda/M$, $\tilde{\lambda}_{16} \sim \lambda_4\Lambda/M$, $\tilde{\lambda}_{5,6,7} \sim \lambda_5\Lambda/M$, $\tilde{\lambda}_{9,10} \sim \lambda_7\Lambda/M$, and $\tilde{\lambda}_{11} \sim \lambda_{11}\Lambda/M$.

I assume that S, A'', and A acquire the vevs

$$S = s(1,1,1,-\frac{3}{2},-\frac{3}{2}) \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A'' = (a'', a'', a'', b'', b'') \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{29}$$

$$A = (a, a, a, b, b) \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{30}$$

These vevs break $SO(10) \to SU(3) \times SU(2) \times U(1)_Y \times U(1)_X$. The spinor field $\mathbf{16}$ is assumed to acquire a vev χ in the $SU(5)$-singlet direction[7]. The unbroken gauge group is then $SU(3) \times SU(2) \times U(1)_Y$. It is argued below that the superpotential guarantees that the vevs of A, σ, A'',

[7]The $D-$flatness condition for $SO(10)$ requires the vevs of $\mathbf{16}$ and $\overline{\mathbf{16}}$ to be equal.

S, **16** and $\mathcal{A}$ are naturally on the order of $\Lambda \sim M_{GUT}$ and $(\Lambda/M)\Lambda^{-7}$, respectively. Other vacua exist, but they are not continously connected to the vaccum considered here.

The doublets and triplets in T_1 are split using the DW mechanism [10]. The F_A equations $(\tilde{\lambda}_9 s + \tilde{\lambda}_{10}\sigma)a = 0$ and $(-\frac{3}{2}\tilde{\lambda}_9 s + \tilde{\lambda}_{10}\sigma)b = 0$ with $s \neq 0$ forces either a or b to vanish; it is a discrete choice. The DW mechanism for giving the triplets in the $\mathbf{5}_{1,2}$ and $\overline{\mathbf{5}}_{1,2}$ Higgs fields GUT-sized masses requires that $b = 0$. I assume that this minimum was selected in the early history of the universe. With this choice, the mass matrix for the coloured triplets in the $\mathbf{5}_{1,2}$ and $\overline{\mathbf{5}}_{1,2}$ Higgs fields is

$$\mathcal{M} = \begin{pmatrix} 0 & -i\lambda_1 a \\ i\lambda_1 a & \tilde{\lambda}_2\sigma + \tilde{\lambda}_3 s \end{pmatrix} \tag{31}$$

in the (T_1, T_2) basis. Since the diagonal element is suppressed by a factor of $O(\Lambda/M)$ relative to the off-diagonal element, the coloured triplets form two Dirac particles with masses $M_{H_C} \sim \lambda_1 a \sim \lambda_1\Lambda \sim \Lambda$. The mass matrix for the 4 electroweak doublets in T_1 and T_2 only has an entry for $T_2(\mathbf{2})T_2(\bar{\mathbf{2}})$ since $b = 0$. The mass of the Dirac heavy doublet is $\tilde{\lambda}_2\sigma - 3\tilde{\lambda}_3 s/2 \sim \Lambda^2/M$. The two electroweak doublets in T_1 are massless, and are identified as the Higgs fields responsible for giving mass to the up-type and down-type quarks of the SM.

I note that the magnitude of the elements of $\mathcal{M}$ has a structure that is favourable for the suppression of the proton decay rate. It in fact provides a natural realisation of the "weak suppression" of the decay rate that is advocated by Babu and Barr [17]. This is seen as follows. First note that the diagonal element is suppressed by a factor of $O(\Lambda/M)$ relative to the off-diagonal element, reflecting the fact that the diagonal entry arises from a non-renormalizable operator in the fundamental theory. If the SM fermions only couple to T_1, then the proton decay amplitude from the exchange of the heavy coloured Higgsinos is proportional to $\mathcal{M}_{11}^{-1}$. In this case the matrix element is $(\tilde{\lambda}_2\sigma + \tilde{\lambda}_3 s)/(\lambda_1 a)^2 \sim \Lambda^2/M$. This results in a decay rate that is

approximately $(\Lambda/M)^2 \sim 10^{-3}$ times the unsuppressed rate.

This is suffucient to suppress the dangerous Higgsino-exchange proton decay operator to a level that may be observable at SuperKamiokande. To obtain the four-fermion operator responsible for the nucleon decay, the operator gotten by integrating out the coloured triplet Higgsinos must be dressed with a vertex function involving either internal wino or gluino propagators. As emphasized in Reference [16], the gluino-dressed amplitude is comparable to the wino-dressed amplitude if $v_u/v_d \equiv \tan\beta$ is large. Since $\tan\beta \sim m_t/m_b \sim 40$ is naturally predicted within an $SO(10)$ GUT, the decay mode $\mathrm{p} \to K^0\mu^+$ may be competitive with the (wino-dressed) neutrino decay modes [16].

The dominant decay modes for the wino-dressed operator are $\mathrm{p} \to K^+\bar{\nu}_u$ and $\mathrm{n} \to K^0\bar{\nu}_u$ [19]. To obtain an estimate for the nucleon lifetime in this model, I rescale their result for the lifetime of the nucleon by a factor of $(M/\Lambda)^2$. The result is

$$\tau(\mathrm{n} \to K^0\bar{\nu}_u) \sim 10^{32} \times \left(\frac{M}{31\Lambda}\frac{0.0058\mathrm{GeV}^3}{\beta}\frac{M_{H_c}}{10^{16}\mathrm{GeV}}\frac{\mathrm{TeV}^{-1}}{f(\tilde{u},\tilde{d}) + f(\tilde{u},\tilde{e})}\right)^2 \mathrm{yrs}. \tag{32}$$

The function f is obtained by dressing the external squarks with wino propagators to obtain a four-fermion operator. It is computed in Reference [20], and depends on the sparticle spectrum. In the limit that the squark mass, $m_{\tilde{Q}}$, and slepton mass, $m_{\tilde{L}}$, are much larger than the wino mass, $m_{\tilde{w}}$, $f \sim m_{\tilde{w}}/m_{\tilde{X}}^2$, with $m_{\tilde{X}}$ the larger of $m_{\tilde{Q}}$ and $m_{\tilde{L}}$. The hadronic matrix element β is defined in Reference [19]. Requiring that M not exceed the Landau pole of the $SO(10)_{GUT}$ group implies that $M/\Lambda \lesssim 30 - 70$. (This constraint is discussed below.) This requirement of consistency also strongly constrains the presence of any additional matter content (this is also discussed below). This suggests that the Yukawa couplings of the SM fermions to the Higgs doublets are generated close to the GUT scale, a crucial assumption required to obtained the limit quoted in Equation 32. To obtain realistic quark and lepton masses in an $SO(10)$ model

though, these Yukawa couplings probably arise from higher-dimensional operators [21]. In this case the flavour structure of the coloured-triplet Higgs to matter may differ from the electroweak doublet couplings to matter, thereby altering the predicted lifetime [16]. For this reason, the result quoted in Equation 32 should be treated as an estimate. This estimate is to be compared with the existing experimental limit of $\tau(\mathrm{n} \to K^0\bar{\nu}_u) > .86 \times 10^{32}$ years [13]. So the nucleon lifetime is naturally suppressed to a phenomenologically interesting level.

Next I discuss the expected size of the vevs and the mass spectrum. The $F-$flatness equations are (setting $b = 0$)

$$\det(S + A'' + \sigma/\sqrt{10}) - B\overline{B} = \Lambda^{10}, \tag{33}$$

$$\mathcal{A}B = 0\ ,\ \mathcal{A}\overline{B} = 0, \tag{34}$$

$$0 = F_A = (\tilde{\lambda}_9 s + \tilde{\lambda}_{10}\sigma)a, \tag{35}$$

$$0 = F_{16} = \left(\tilde{\lambda}_{16}\sigma + \tilde{\lambda}_4(3a'' + 2b'')\right)\chi, \tag{36}$$

$$0 = F_\sigma = \tilde{\lambda}_{16}\chi^2 - 3\tilde{\lambda}_{10}a^2 + \tilde{\lambda}_5\Lambda\sigma + \frac{2}{\sqrt{10}}\mathcal{A}K\,(3u + 2v)\,, \tag{37}$$

$$0 = F_{A''_3} = \tilde{\lambda}_4\chi^2 - 2\tilde{\lambda}_7\Lambda a'' + 2\mathcal{A}KA, \tag{38}$$

$$0 = F_{A''_2} = \tilde{\lambda}_4\chi^2 - 2\tilde{\lambda}_7\Lambda b'' + 2\mathcal{A}KB, \tag{39}$$

$$0 = F_S = \tilde{\lambda}_6\Lambda s - \frac{2}{5}\left(\frac{1}{2}\tilde{\lambda}_9 a^2 - \mathcal{A}K(u - v)\right), \tag{40}$$

where $K \equiv \det(S + A'' + \sigma/\sqrt{10}) = (u^2 + A^2)^{-3}(v^2 + B^2)^{-2}$. The functions u, v, A and B are

$$u = \frac{\sigma/\sqrt{10} + s}{(\sigma/\sqrt{10} + s)^2 + a''^2}\ ,\ v = \frac{\sigma/\sqrt{10} - 3s/2}{(\sigma/\sqrt{10} - 3s/2)^2 + b''^2}, \tag{41}$$

$$A = \frac{a''}{(\sigma/\sqrt{10} + s)^2 + a''^2}\ ,\ B = \frac{b''}{(\sigma/\sqrt{10} - 3s/2)^2 + b''^2}. \tag{42}$$

An inspection of these equations also indicates that without the operators ΛS^2, $\Lambda\sigma^2$ and $\Lambda A''A''$, the $F-$flatness equations would only constrain the values of $\mathcal{A}$, χ^2 and a^2 in the combination

$\chi^2/\mathcal{A}$ and $a^2/\mathcal{A}$. Thus one of these vevs would be unconstrained. As a result, not all the particle masses would be fixed by the input parameters. This problem is avoided by including the $(Q\overline{Q})^2$ operator, *i.e.* the operator $\propto \lambda_5$, in the fundamental theory. In this case, a new solution cannot be gotten by rescaling $\mathcal{A}$, with the $\tilde{\lambda}_i$ and Λ fixed, and rescaling the vevs of any of the fields, thus indicating that a^2, χ^2 and $\mathcal{A}$ are fixed by the input parameters.

I now argue that these equations fix the vevs of S, A, σ and A'' to be on the order of Λ, without any fine tuning of the couplings in the fundamental theory. By redefining $\mathcal{A} = (\Lambda/M)\tilde{\mathcal{A}}$ the $F_i = 0$ equations now contain an overall factor of Λ/M if the expected relation between the superpotential couplings in the fundamental and confined theories is valid. As a result the F_i equations no longer contain any small dimensionless couplings. The expected solution to this new set of equations is then χ, σ, a'', b'', $a \sim s$ and $\tilde{\mathcal{A}} \sim \Lambda^{-7}$. The confinement equation fixes $s \sim \Lambda$. Therefore all the vevs are $v \sim \Lambda$ and $\mathcal{A} \sim (\Lambda/M)\Lambda^{-7}$. This result is not obvious *a priori*, since the superpotential couplings appearing in the F equations are suppressed by powers of Λ/M. A slightly more rigorous argument, also showing that $\mathcal{A} \neq 0$, is presented in Appendix B. This implies that the baryons vevs are forced to zero at this minimum. Two numerical solutions which supports these arguments are also given in Appendix B. These expectations for the size of the couplings, $\mathcal{A}$, and vevs will be important below in estimating the mass spectrum.

The superpotential for this model contains enough operators to give superheavy masses to all the particles that should be heavy. The results of computing the mass matrices assuming a canonical Kahler potential are given in Appendix B, and are summarized here. The particles have masses at one of three scales: $m_L \equiv \Lambda^4/M^3$; $m_I \equiv \Lambda^2/M$; and Λ. The naive expectation is that all the particles have a mass $m \sim m_I$. This is because all the vevs are $O(\Lambda)$, and the mass matrices are linear in the superpotential couplings which contain a factor Λ/M, and in the

parameter $\mathcal{A}$ which also contains a factor of Λ/M.

This expectation turns out to be correct except for a $u_L \sim (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$ and $\overline{u}_L \sim u_L^\dagger$, which acquire a Dirac mass from the superpotential operator $(A''A)_{ij}\overline{\mathbf{16}}\Sigma_{ij}\mathbf{16}$. These fields are massless in the absence of this operator for the following reason. The $SU(5)$ decomposition of $A = \mathbf{24} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{1}$. This clearly contains a $u \in \mathbf{10}$ and $\overline{u} \in \overline{\mathbf{10}}$. The only possible source for a mass term for these fields is given by W_{DW}. Further, since S does not contain a u and $\overline{u}$, this mass term must occur from setting S and σ to their vevs. The resulting mass is proportional to $\tilde{\lambda}_9 s + \tilde{\lambda}_{10}\sigma$. The DW form for A and $F_A = 0$, however, forces this quantity to vanish [8]. The addition of the operator $\mathrm{tr}A^4/M$ does not change the conclusion of this argument. The mass of these fields is gotten therefore from the M^{-2} suppressed operator. The result of a computation of the mass spectrum, presented in Appendix B, implies that the naive expectation for their mass is $m \sim m_L$.

The particle content of the fields with mass $m \sim m_I$ is now enumerated. The $SU(5)$ quantum numbers of the representations at this scale are: $1\times(\mathbf{10}+\overline{\mathbf{5}})+1\times(\overline{\mathbf{10}}+\mathbf{5})+2\times\mathbf{24}+1\times(\mathbf{15}+\overline{\mathbf{15}})$. At the scale m_I there is also a split [9] $\mathbf{24}$, with SM quantum numbers $\mathbf{8} \equiv (\mathbf{8}, \mathbf{1}, 0)$ and $\mathbf{3} \equiv (\mathbf{1}, \mathbf{3}, 0)$. There are also some leftover fields, that together with u_L and $\overline{u}_L$ which have a mass $m \sim m_L$, form a complete $\mathbf{10}+\overline{\mathbf{10}}$ of $SU(5)$. These leftover fields have a mass $m \sim m_I$. The representations in the $SO(10)$ $\mathbf{10_1} + \mathbf{10_2}$ are split by the DW mechanism. One pair of electroweak doublets is massless and are the Higgs fields responsible for giving mass to the up-type quarks, down-type quarks, and leptons. The other doublet fields, $h \equiv (\mathbf{1}, \mathbf{2}, -1/2)$ and $\overline{h} \equiv (\mathbf{1}, \mathbf{2}, 1/2)$, acquire a Dirac mass $m_h \sim m_I$. There are also a number of gauge singlets which acquire masses $m \sim m_I$.

[8] The same argument also implies that the Majorana mass term for the $\mathbf{8}$ in A vanishes. These fields, however, acquire a Dirac mass with the $\mathbf{8} \in S$.

[9] The missing partners are the Nambu-Goldstone bosons of the $SU(5) \to SU(3)_C \times SU(2) \times U(1)_Y$ breaking.

The triplets in the $SO(10)$ $\mathbf{10_1} + \mathbf{10_2}$, $2 \times (\overline{\mathbf{3}}, \mathbf{1}, 1/3) + 2 \times (\mathbf{3}, \mathbf{1}, -1/3)$, acquire masses $O(\Lambda)$. The 33 Nambu-Goldstone bosons multiplets acquire a mass $m \sim \Lambda$ from the super-Higgs mechanism.

The incomplete $SU(5)$ representations affect the prediction for $\sin^2\theta_W$, which I now discuss. I first approximate that all the charged particles at each of the three scales m_L, m_I and M_{GUT} are degenerate. In this approximation the contribution to $\Delta\sin^2\theta_W$ occurs from splitting between the scales. I find using the usual one-loop computation that the light particles shift the prediction for $\sin^2\theta_W$ by an amount

$$\Delta\sin^2\theta_W = -\frac{\alpha_{em}}{2\pi}\left(\ln\frac{m_I}{m_L} - \frac{4}{5}\ln\frac{\Lambda}{m_I}\right). \tag{43}$$

The first term is the contribution from u_L and $\overline{u}_L$; these fields only contribute between m_L and m_I, since above the mass scale m_I they fit into a complete $\mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)$. The second term is the sum of the contributions from $\mathbf{8}$, $\mathbf{3}$, h and $\overline{h}$. As is evident, for $m_L < m_I$ there is an $O(1)$ cancellation between the two contributions. Since m_L arises from a higher dimensional operator than does m_I, $m_L < m_I$ applies for this model. It is then reasonable to expect that the $O(1)$ cancellation occurs. Inserting the naive expectation $m_L \sim \Lambda^4/M^3$ and $m_I \sim \Lambda^2/M$, gives

$$\Delta\sin^2\theta_W \sim -5\times10^{-3}\times\frac{\ln M/\Lambda}{\ln 30}. \tag{44}$$

As is shown below, requiring that the $SO(10)_{GUT}$ not have a Landau pole below M restricts $M/\Lambda \lesssim 30 - 70$. With this constraint, the shift in $\sin^2\theta_W$ is consistent with the measured value, once other theoretical uncertainties are considered. The largest of these are uncalculable threshold corrections from the light (approximately) complete $SU(5)$ representations. Since the splitting within each multiplet gives a contribution that is naively $\alpha_{em}/2\pi \times O(1)$, the large size of the light representations could result in a correction that is comparable or larger than the

correction given in Equation 44.

I now argue that any "gravitational smearing" [11] of the couplings at the GUT scale is small in this model. First, the only possible dimension-4 operator in the superpotential involving the $SO(10)_{GUT}$ chiral gauge multiplet W_{ij} is $A_{ij}W_{jk}W_{ki}/M$. This, however, vanishes due to the anti-symmetry of A. Next, the operators $g_S SWW/M$ and $g_\sigma \sigma WW/M$ are allowed. The vev of σ does not break $SU(5)$, so it only results in a common shift of the gauge couplings. The shift is tiny since $g_\sigma \sim \Lambda/M$. The vev of S does break $SU(5)$, so this operator results in a tree-level correction to the unification of the couplings. An estimate for the shift in $\sin^2\theta_W$ that this incurs is

$$\Delta \sin^2\theta_W \sim \pm 10^{-3} g_S \frac{30s}{M}. \tag{45}$$

It is expected that $g_S \sim \Lambda/M$ since this operator occurs from a dimension-4 operator in the superpotential of the fundamental theory. So this results in a tiny shift to $\sin^2\theta_W$. Finally, operators only involving $\mathbf{16}, \overline{\mathbf{16}}$ and WW are also suppressed by an extra factor of Λ/M. The vev of $\mathbf{16}$ does not break $SU(5)$, so this operator only results in a tiny common shift to the gauge couplings.

An upper limit to M is given by the value of the Landau pole of the $SO(10)$ GUT gauge coupling. This model is not asymptotically-free above the GUT scale since it contains a large particle content. More problematic though, is the fact that most of the particle masses are a factor of Λ/M below the GUT scale. While this particle content does not result in a large shift to $\sin^2\theta_W$ since they mostly form complete $SU(5)$ representations, the matter content does increase the value of α_{GUT}. The value of $\alpha_{SO(10)}(M)$, using naive one-loop running and with tree-level matching, and including the contribution of 3 $\mathbf{16}$s of the SM, is

$$\alpha^{-1}_{SO(10)}(M) = 24 - \frac{3}{22\pi}\left((2+\frac{5}{3})\ln\frac{\Lambda}{m_L} + (93-\frac{5}{3})\ln\frac{\Lambda}{m_I}\right) - \frac{16}{2\pi}\ln\frac{M}{\Lambda}. \tag{46}$$

The second term is the contribution from $u_L + \overline{u}_L$, the third term is the contribution from the particles with mass m_I, and the last term is the contribution from the $SO(10)$ particle content above Λ. Inserting $m_L \sim \Lambda^4/M^3$ and $m_I \sim \Lambda^2/M$, the limit is

$$\frac{M}{\Lambda} \lesssim 31. \tag{47}$$

This implies $M \sim .6 - 1 \times 10^{18}$ GeV. I note, however, that this limit is sensitive to the actual spectrum. For example, if the naive expectation underestimates the spectrum by a factor of 4, then the limit increases to $M/\Lambda \lesssim 75$. This corresponds to $M \sim 1 - 2 \times 10^{18}$ GeV.

The Landau pole limit also strongly constrains any modifications to the model. For example, adding to the model either an extra adjoint A' which acquires a mass at $2 \times M_{GUT}$, or an extra $\mathbf{16}' + \overline{\mathbf{16}}' + \mathbf{10}' + \mathbf{10}''$ which all acquire a mass M_{GUT} restricts $M/\Lambda \lesssim 20$. The presence of N_5 additional $SU(5)$ $\mathbf{5} + \overline{\mathbf{5}}$ multiplets is also strongly constrained by this requirement of consistency. These fields would be required, for example, in any low-energy physics that is responsible for the origin of supersymmetry or flavour symmetry breaking. Requiring $M/\Lambda > 20$ implies that the mass M_5 of these multiplets satisfies

$$N_{\mathbf{5}+\overline{\mathbf{5}}} \ln M/M_5 \lesssim 18. \tag{48}$$

In particular: $N_{\mathbf{5}+\overline{\mathbf{5}}} = 1$ is marginally allowed if $M_5 = 10^{10}$ GeV; $N_{\mathbf{5}+\overline{\mathbf{5}}} = 2$ is marginally allowed if $M_5 = 10^{14}$ GeV. These constraints are weakened if the naive estimate, Λ^2/M, for the chiral GUT spectrum underestimates the spectrum by a factor of 4. In this case,

$$N_{\mathbf{5}+\overline{\mathbf{5}}} \ln M/M_5 \lesssim 45, \tag{49}$$

for $M/\Lambda > 20$. In particular: $N_{\mathbf{5}+\overline{\mathbf{5}}} = 2$ is allowed for $M_5 = 10^{10}$ GeV; $N_{\mathbf{5}+\overline{\mathbf{5}}} < 5$ is required for $M_5 = 10^{14}$ GeV. Either direct or indirect evidence for additional chiral content that does not satisfy Equations 48 or Equations 49 would strongly disfavour this model.

I conclude this Section with a few comments about the consistency of neglecting certain operators in the superpotential. The superpotential terms $\sigma A_{ij}\overline{\mathbf{16}}\Sigma_{ij}\mathbf{16}$ or $S_{ik}A_{kj}\overline{\mathbf{16}}\Sigma_{ij}\mathbf{16}$ must be absent to avoid ruining the DW form for A. These operators would contribute to $F_A(2)$, forcing a non-vanishing value for b. These operators are present in the low-energy theory if the operators $\mathrm{tr}(\overline{Q}Q)A\overline{\mathbf{16}}\Sigma\mathbf{16}$ or $(\overline{Q}Q)_S A\overline{\mathbf{16}}\Sigma\mathbf{16}$ are present in the superpotential of the fundamental theory. Any symmetry which forbids these dangerous operators also forbids the operator $(A''A)_{ij}\overline{\mathbf{16}}\Sigma_{ij}\mathbf{16}$. This option is not viable since this operator is required to give mass to a $(\overline{\mathbf{3}},\mathbf{1},-2/3)$+h.c. fields. (The DW form for A, however, is unaffected by the presence of this operator since it does not contribute to the F_i equations.) So I must assume that the dangerous operators are not present in the fundamental theory. The perturbative non-renormalisation theorems then guarantee that these operators will not be generated, at least in perturbation theory. This argument does not exclude the possibility that these dangerous operators could be generated by the non-perturbative dynamics of the $SU(10)_S$ or $SO(10)_{GUT}$ groups. By combining the requirement of holomorphy of the superpotential with some anomalous fake $U(1)$ symmetries [3] it is possible to exactly show, however, that if these operators are initially absent in the high-energy theory they will not be generated as the cutoff is lowered. In particular, it can be shown that the coefficient of a dangerous operator at a lower cutoff is only proportional to its initial value; i.e. it is independent of $\Lambda_{SU(10)}/M$, $\Lambda_{SO(10)}/M$ and all the other superpotential couplings. I then see no reason for these dangerous operators to be generated by the confining dynamics.

5 Conclusions

In this paper two models are presented that generate the Grand Unification scale from the strong dynamics of a confining group. The particle content of the confining group is chosen so that this sector confines with chiral symmetry breaking. The particles in this sector are also charged under the Grand Unified group. It follows that the composite fields which arise from the confining dynamics transform under the GUT group as either higher dimensional representations or singlets. Below the scale of confinement these composite fields acquire vevs. In each of the models presented here, there is a locally isolated supersymmetric vacuum in which the GUT group is broken to the SM group, and the resulting spectrum provides an acceptable phenomenology. Two GUT models are considered: $SU(6)$ and $SO(10)$. Known solutions to the doublet-triplet splitting problem are incorporated in each model. Proton decay in both models is at an acceptable rate, and in particular, the dangerous dimension-5 proton decay operator is suppressed in the $SO(10)$ model to an interesting level. This suppression is a natural consequence of the confining dynamics. Each model requires no fine tuning of any non-vanishing superpotential couplings. The fundamental theory in both models also contains an economical particle content, requiring no gauge singlets and only one higher dimensional representation.

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7 Appendix A: $SU(6) \times SU(6)_{GUT}$

First I discuss the existence of a solution to the $F_i = 0$ equations with all vevs of $O(\Lambda)$ and $\mathcal{A} \sim (\Lambda/M)\Lambda^{-3}$. The second part of this Appendix contains the results of calculating the mass spectrum, assuming a canonical Kahler potential.

Since the $F_i = 0$ equations are linear in v_H^2 and v_N^2, it is straightforward to solve for them in terms of σ and v_Σ. The remaining two equations determine $\mathcal{A} \neq 0$ and $x \equiv \sigma/v_\Sigma$. In particular,

x is the solution to

$$\beta x^2 - (\sqrt{6}\beta - \alpha - \gamma)x - \sqrt{6}\alpha + 12\beta = 0, \tag{50}$$

where $\alpha \equiv -\tilde{\lambda}_1$, $\beta \equiv -\tilde{\lambda}_3\tilde{\lambda}_4 M/(12g\Lambda)$, and $\gamma \equiv -\tilde{\lambda}_2 - 24\tilde{\lambda}_5\beta/\tilde{\lambda}_4$. Note that $\beta \sim \alpha \sim \gamma \sim \Lambda/M$. Since each term appearing in Equation 50 is linear in Λ/M, it follows that $x \sim O(1)$, *i.e.* $\sigma \sim v_\Sigma$ is expected. The quantum constraint then fixes $v_\Sigma \sim \Lambda$. It follows from $F_H = 0$ that $v_N^2 = -\tilde{\lambda}_3 M\sigma/(12g)$ is $O(\Lambda^2)$. Next, $v_H^2 = -(Mv_\Sigma/g)(\tilde{\lambda}_5 x - \tilde{\lambda}_4)$ is also $O(\Lambda^2)$. Finally, either $F_\Sigma = 0$ or $F_\sigma = 0$ determines $\mathcal{A} \sim (\Lambda/M)\Lambda^{-3}$.

The non-Nambu-Goldstone multiplet fields charged under the SM, with the exception of the SM Higgs doublets, are all contained in Σ and Σ_N. Since these fields acquire their mass from the $SU(4) \times SU(2)$ preserving vevs of Σ, Σ_N or $(H\overline{H})$, it is conveinent to classify the mass spectrum according to the $SU(4) \times SU(2)$, rather than the $SU(3) \times SU(2) \times U(1)_Y$, charge assignments.

The mass matrix for the $Q \sim (\mathbf{4}, \mathbf{2})$ and $\overline{Q} \sim (\overline{\mathbf{4}}, \mathbf{2})$ fields (after some algebra using the $F_i = 0$ equations) in the (Σ, Σ_N) basis is

$$M_{\overline{Q}Q} = \begin{pmatrix} -\mathcal{A}Kab + \tilde{\lambda}_1\Lambda & -\tilde{\lambda}_4 v_N \\ -\tilde{\lambda}_4 v_N & \tilde{\lambda}_4 v_\Sigma \end{pmatrix}. \tag{51}$$

By using the $F_i = 0$ equations it can be verified that this matrix annihilates the state (v_Σ, v_N), which is a Nambu-Goldstone boson of the gauge symmetry breaking. The massive eigenvalue is non-zero and naively $m_Q \sim \Lambda^2/M$.

The mass matrix for the $(\mathbf{15}, \mathbf{1})$ fields (after some algebra using the $F_i = 0$ equations) in the (Σ, Σ_N) basis is

$$M_{\mathbf{15}} = \begin{pmatrix} -\mathcal{A}Ka^2 + \tilde{\lambda}_1\Lambda & 2\tilde{\lambda}_4 v_N \\ 2\tilde{\lambda}_4 v_N & 4\tilde{\lambda}_4 v_\Sigma \end{pmatrix}. \tag{52}$$

It can be shown after some algebra that the determinant of this matrix is $-4\tilde{\lambda}_4\mathcal{A}Kav_\Sigma(a-b)$.

This is non-zero since $v_\Sigma \neq 0$ implies that $a \neq b$. The expected masses for the two eigenvalues is then $m_{\mathbf{15}} \sim \Lambda^2/M$.

The mass matrix for the $(\mathbf{1},\mathbf{3})$ fields (after some algebra using the $F_i = 0$ equations) in the (Σ, Σ_N) basis is

$$M_{\mathbf{3}} = \begin{pmatrix} -\mathcal{A}Kb^2 + \tilde{\lambda}_1\Lambda & -4\tilde{\lambda}_4 v_N \\ -4\tilde{\lambda}_4 v_N & -2\tilde{\lambda}_4 v_N \end{pmatrix}. \tag{53}$$

It can be shown that the determinant of this matrix is $-\sqrt{6}\tilde{\lambda}_4\Lambda b v_\Sigma^2(3\beta x^2 - 5\beta\sqrt{6}x + \sqrt{6}\alpha)$. A comparison of this result with Equation 50 indicates that it is non-vanishing for generic values of the $\tilde{\lambda}_i$s. The expected masses for the two eigenvalues is then $m_{\mathbf{3}} \sim \Lambda^2/M$.

8 Appendix B: $SU(10) \times SO(10)_{GUT}$

<u>*Arguing that all the vevs are of order* Λ</u> ; <u>*Numerical solution*</u>

In this case I am only concerned about whether a discrete solution with all $\mathcal{A}$, a, a'', b'', σ, s and χ non-zero exists. This result is obtained by showing that if $s \neq 0$, then $\mathcal{A} \neq 0$ and all other vevs are comparable to s. Then the non-vanishing of $\mathcal{A}$ implies that $B = \overline{B} = 0$. The confinement condition then fixes $s \sim \Lambda$. To begin, first note that F_A fixes $\sigma \sim s$. The F_{16} equation implies that $3a'' + 2b'' \sim \sigma \sim s$. Thus either $a'' \sim b'' \sim s$, or $b'' \ll a'' \sim s$ (or $a'' \ll b'' \sim s$). I next argue that the last two cases do not occur. In the first case, $b'' \ll a''$, so that $B \ll A$. Next, the two $F_{A''}$ equations are inconsistent if either $\mathcal{A}KA \ll \tilde{\lambda}_7 a''$ or $\mathcal{A}KA \gg \tilde{\lambda}_7 a''$. So $\mathcal{A}KA \sim \tilde{\lambda}_7 a''$ and $\tilde{\lambda}_7 b'' \sim \chi^2 \ll \tilde{\lambda}_7 a''$ is the only consistent solution to the two $F_{A''}$ equations. Thus if $b'' \ll a''$, F_{16} fixes $a'' \sim s$ up to small corrections of $O(b'')$. Similarly, the first $F_{A''}$ fixes $\mathcal{A}$ up to small corrections. But now the two equations F_σ and F_S each determine $a \sim s$; these two equations for a cannot in general be simultaneously satisfied. Therefore, $b'' \ll a''$ is not a viable

(supersymmetric) solution. The argument against $a'' \ll b''$ is similar. Therefore $a'' \sim b''$. Next suppose that $\mathcal{A} = 0$. Then $F_{A''}$ fixes $a'' = b''$, and together with F_{16} and F_A, determines $\chi \sim s$. But now there are two remaining equations, F_S and F_σ, for one unknown, a. More concretely, $a^2 = 5(\tilde{\lambda}_6/\tilde{\lambda}_9)\Lambda s$ and $a^2 = (\tilde{\lambda}_5 - \frac{2}{5}\tilde{\lambda}_7\tilde{\lambda}_{16}^2/\tilde{\lambda}_4^2)\Lambda\sigma/3\tilde{\lambda}_{10}$. In general, these two equations will not be satisfied; therefore $\mathcal{A} \neq 0$. The vev a can be eliminated from F_S and F_σ; the remaining equation, together with $F_{A''}$ and F_{16} may be used in principle to determine $\chi, a'', b'' \sim s$ and also fix $\mathcal{A}$. ($\chi^2 \ll \Lambda a''$ is not possible; F_A, F_S, $F_{A''}$, F_{16} and F_σ are 6 equations in only 5 unknowns: σ, a'', b'', a and $\mathcal{A}$.) The F_S equation will not in general be satisfied with $a^2 \ll \Lambda s$ or $a^2 \gg \Lambda s$; since $\mathcal{A}K(u-v)$ is $O(\Lambda^2 s/M)$ and $\neq \tilde{\lambda}_6\Lambda s$ in general, F_S determines $a \sim s$.

Two numerical solutions to the $F_i = 0$ equations supports these arguments. In the first (I) solution, the input parameters are chosen to be : $\tilde{\lambda}_4 = 0.01$, $\tilde{\lambda}_5 = 0.02$, $\tilde{\lambda}_6 = 0.03$, $\tilde{\lambda}_7 = 0.04$, $\tilde{\lambda}_9 = 0.05$, $\tilde{\lambda}_{10} = 0.06$ and $\tilde{\lambda}_{16} = 0.045$. The solution, in units of $\Lambda = 1$, is

$$\sigma = -0.64,\ s = 0.77,\ a'' = 0.50,\ b'' = 0.70,\ a = 1.2,\ \chi = 2.5,\ \mathcal{A} = -0.01. \tag{54}$$

In the second (II) solution, the input parameters are chosen to be : $\tilde{\lambda}_4 = 0.0134$, $\tilde{\lambda}_5 = 0.0123$, $\tilde{\lambda}_6 = -0.03$, $\tilde{\lambda}_7 = 0.0225$, $\tilde{\lambda}_9 = 0.045$, $\tilde{\lambda}_{10} = 0.0623$ and $\tilde{\lambda}_{16} = 0.03657$. The solution, in units of $\Lambda = 1$, is

$$\sigma = -0.62,\ s = 0.85,\ a'' = -0.14,\ b'' = 1.1,\ a = -0.87,\ \chi = 1.2,\ \mathcal{A} = 0.04. \tag{55}$$

These parameters are chosen to be small since $\tilde{\lambda} \sim \lambda\Lambda/M \sim 0.03\lambda$ for $\Lambda/M \sim 1/30$. Aside from this feature, there is nothing special about this choice of superpotential couplings. As expected, all the vevs are $O(\Lambda)$ and $\mathcal{A} \sim (\Lambda/M)\Lambda^{-7}$.

Detailed Mass Spectrum

The mass matrices presented here were computed assuming a canonical Kahler potential;

this is suffucient to determine the rank of the matrix.

For future purposes it will be useful to note that the F_i equations are invariant under the following rescaling of couplings and fields:

$$(\tilde{\lambda}_4, \tilde{\lambda}_9, \tilde{\lambda}_{10}, \tilde{\lambda}_{16}) \to (g^{-2}\tilde{\lambda}_4, g^{-2}\tilde{\lambda}_9, g^{-2}\tilde{\lambda}_{10}, g^{-2}\tilde{\lambda}_{16}), \tag{56}$$

$$(\chi, a) \to (g\chi, ga)\ , (a'', b'', s, \sigma, K) \to (a'', b'', s, \sigma, K). \tag{57}$$

Any coupling not listed is left invariant. This mapping relates the solutions to the $F_i = 0$ equations in two theories with different superpotential couplings which are related by this scale transformation.

The $u^c \sim (\overline{\mathbf{3}}, 1, -2/3)$+h.c. mass matrix in the $(A'', \mathbf{16}(\overline{\mathbf{16}}), A)$ basis is, with $\bar{\lambda} \equiv \tilde{\lambda}_{11}/M$,

$$M_{u^c,\overline{u}^c} = \begin{pmatrix} 2\mathcal{A}K(u^2 + A^2) - 2\tilde{\lambda}_7\Lambda & 2i\tilde{\lambda}_4\chi - 2\bar{\lambda}a\chi & i\bar{\lambda}\chi^2 \\ -2i\tilde{\lambda}_4\chi - 2\bar{\lambda}a\chi & -4\tilde{\lambda}_4 a'' & -2\bar{\lambda}a''\chi \\ -i\bar{\lambda}\chi^2 & -2\bar{\lambda}a''\chi & 0 \end{pmatrix}. \tag{58}$$

Using the $F_i = 0$ equations the reader can verify that this matrix has only one zero eigenvalue. The product of the two non-zero eigenvalues is given by the coefficient of $O(e)$ in the expansion of $\det(M_u - e\mathbf{1})$. This coefficient is $\overline{\lambda}^2\chi^2(4a^2 + 4a''^2 + \chi^2)$. Therefore, this matrix contains an extra massless particle in the limit $\overline{\lambda} \to 0$. With $\overline{\lambda} \neq 0$, the naive expectation for this product of eigenvalues is $(\Lambda/M)^4\Lambda^2$. The larger eigenvalue is $m_{u_H} = \tilde{\lambda}_4(4a'' + \chi^2/a'')$, and is approximately Λ^2/M. So the smaller eigenvalue is $m_{u_L} = \bar{\lambda}^2\chi^2(4a^2 + 4a''^2 + \chi^2)/m_{u_H}$. The naive expectation for this quantity is $(\Lambda/M)^3\Lambda$.

The mass matrix for $E^c \sim (\mathbf{1}, \mathbf{1}, 1)$ + h.c., in the $(A'', \mathbf{16}(\overline{\mathbf{16}}), A)$ basis is

$$M_{E^c,\overline{E^c}} = \begin{pmatrix} 2\mathcal{A}K(v^2 + B^2) - 2\tilde{\lambda}_7\Lambda & 2i\tilde{\lambda}_4\chi & i\bar{\lambda}\chi^2 \\ -2i\tilde{\lambda}_4\chi & -4\tilde{\lambda}_4 b'' & -2\bar{\lambda}b''\chi \\ -i\bar{\lambda}\chi^2 & -2\bar{\lambda}b''\chi & 5\tilde{\lambda}_9 s \end{pmatrix}. \tag{59}$$

Using the $F_i = 0$ equations it can be verified that this mass matrix has one zero eigenvalue. The masses of the other two states are $5\tilde{\lambda}_9 s$ and $-\tilde{\lambda}_4(4b'' + \chi^2/b'')$, to lowest order in $\bar{\lambda}\Lambda$.

The mass matrix for the $Y \sim (\mathbf{3}, \mathbf{2}, -5/6)$ and $X \sim (\bar{\mathbf{3}}, \mathbf{2}, 5/6)$ fields is given in the (A'', S, A) basis by

$$M_{YX} = \begin{pmatrix} -2\mathcal{A}K(uv - AB) + 2\tilde{\lambda}_7\Lambda & -2i\mathcal{A}K(uB + vA) & 0 \\ -2i\mathcal{A}K(uB + vA) & -2\mathcal{A}K(uv - AB) + 2\tilde{\lambda}_6\Lambda & i\tilde{\lambda}_9 a \\ 0 & i\tilde{\lambda}_9 a & -\frac{5}{2}\tilde{\lambda}_9 s \end{pmatrix}. \tag{60}$$

It can be verified, after some tedious algebra, that this matrix has one zero eigenvalue. This matrix is therefore rank 2. The masses of the other two states are $O(\Lambda^2/M)$.

The $Q \sim (\mathbf{3}, \mathbf{2}, 1/6)$ and $\overline{Q} \sim (\bar{\mathbf{3}}, \mathbf{2}, -1/6)$ mass matrix, in the $(A'', S, \mathbf{16}(\overline{\mathbf{16}}), A)$ basis, is

$$M_{Q\overline{Q}} = \begin{pmatrix} 2\mathcal{A}K(uv + AB) - 2\tilde{\lambda}_7\Lambda & -2i\mathcal{A}K(Av - Bu) & 2i\tilde{\lambda}_4\chi - \bar{\lambda}a\chi & i\bar{\lambda}\chi^2 \\ 2i\mathcal{A}K(Av - Bu) & -2\mathcal{A}K(uv + AB) + 2\tilde{\lambda}_6\Lambda & 0 & -i\tilde{\lambda}_9 a \\ -2i\tilde{\lambda}_4\chi - \bar{\lambda}a\chi & 0 & -2\tilde{\lambda}_4(a'' + b'') & -\bar{\lambda}(a'' + b'')\chi \\ -i\bar{\lambda}\chi^2 & i\tilde{\lambda}_9 a & -\bar{\lambda}(a'' + b'')\chi & \frac{5}{2}\tilde{\lambda}_9 s \end{pmatrix}. \tag{61}$$

It can be verified that this matrix has at least one zero eigenvalue. To verify that it has only one zero eigenvalue, it is suffucient to verify that the coefficient of $O(e)$ in the expansion of $\det(M_{Q\overline{Q}} - e\mathbf{1})$ is non-vanishing. Since the entries proportional to $\bar{\lambda}$ result in a tiny perturbation to the spectrum of $M_{Q\overline{Q}}$, it is suffucient to compute the $O(e)$ coefficient, call it p, while setting $\bar{\lambda} = 0$. In this case it is

$$\begin{aligned} p \;=\; & 4\mathcal{A}K\frac{(Bu - Av)}{(u^2 + A^2)(v^2 + B^2)}(-2\tilde{\lambda}_4\tilde{\lambda}_6(uB^2 - (u - v)uv - A^2 v)\Lambda \\ & -\tilde{\lambda}_7\tilde{\lambda}_9(B(u^2 + A^2) + A(v^2 + B^2))\Lambda \\ & -\tilde{\lambda}_4\tilde{\lambda}_9((A + B)^2 + (u - v)^2) + \mathcal{A}K(\tilde{\lambda}_9(A + B) - 2\tilde{\lambda}_4(u - v))(u^2 + A^2)(v^2 + B^2)). \end{aligned} \tag{62}$$

If this vanishes at generic values for the couplings constants, then it must, in particular, vanish for two solutions and sets of couplings constants that are related by Equations 56 and 57. Under this scaling, however, $p \propto C \times (c_1 g^{-2} + c_2 g^{-4})$, with C, c_1 and c_2 functions of the initial vevs and couplings. This vanishes only if either $C = 0$ or $c_1 = 0$ *and* $c_2 = 0$. The first condition implies $Av = Bu$, and the second implies that $A + B = 0$ *and* $u - v = 0$. These conditions over-constrain the vevs, so they will not be satisfied at a generic solution. In particular, $p = (0.07)^3$ for the numerical solution (I) given by Equation 54. The expected mass for the three massive eigenvalues is therefore $O(\Lambda^2/M)$.

The mass matrix for the coloured adjoints $(\mathbf{8}, \mathbf{1}, 0)$ in the (A'', S, A) basis is

$$M_{\mathbf{88}} = \begin{pmatrix} -\mathcal{A}K(u^2 - A^2) + \tilde{\lambda}_7\Lambda & -2i\mathcal{A}KuA & 0 \\ -2i\mathcal{A}KuA & -\mathcal{A}K(u^2 - A^2) + \tilde{\lambda}_6\Lambda & i\tilde{\lambda}_9 a \\ 0 & i\tilde{\lambda}_9 a & 0 \end{pmatrix} \tag{63}$$

The determinant is $(\tilde{\lambda}_9 a)^2(\tilde{\lambda}_7\Lambda - \mathcal{A}K(u^2 - A^2))$ and is non-vanishing. The size of the three masses is expected to be $m_8 \sim \Lambda^2/M$. For the numerical solution (I) in Equation 54, this determinant is $(0.05)^3$.

The mass matrix for the $SU(2)$ adjoints $(\mathbf{1}, \mathbf{3}, 0)$ in the (A'', S, A) basis is

$$M_{\mathbf{33}} = \begin{pmatrix} -\mathcal{A}K(v^2 - B^2) + \tilde{\lambda}_7\Lambda & -2i\mathcal{A}KvB & 0 \\ -2i\mathcal{A}KvB & -\mathcal{A}K(v^2 - B^2) + \tilde{\lambda}_6\Lambda & 0 \\ 0 & 0 & -\frac{3}{2}\tilde{\lambda}_9 s \end{pmatrix} \tag{64}$$

The determinant is $-3\tilde{\lambda}_9 s \left(\mathcal{A}K\left(\mathcal{A}K(v^2 + B^2)^2 - (\tilde{\lambda}_6 + \tilde{\lambda}_7)(v^2 - B^2)\Lambda\right) + \tilde{\lambda}_6\tilde{\lambda}_7\Lambda^2\right)/2$ and is non-vanishing. The size of the three masses is expected to be $m_3 \sim \Lambda^2/M$. For the numerical solution (I) in Equation 54, this determinant is $-(0.04)^3$.

The S field contains $(\mathbf{6}, \mathbf{1}, 2/3)$+h.c. and $(\mathbf{1}, \mathbf{3}, -1)$+h.c.. These fields acquire Dirac masses

$-\mathcal{A}K(u^2 + A^2)$ and $-\mathcal{A}K(v^2 + B^2)$, respectively. The $(\overline{\mathbf{3}}, \mathbf{1}, 1/3)$+h.c. and $(\mathbf{1}, \mathbf{2}, -1/2)$+h.c. fields in the $\mathbf{16} + \overline{\mathbf{16}}$ acquire Dirac masses $-4\tilde{\lambda}_4(a'' + b'')$ and $-2\tilde{\lambda}_4(3a'' + b'')$, respectively.

Finally, there are 8 gauge singlets in this model. The quantum modified constraint implies that only 7 of these are independent. The quantum modified constraint can be used to solve for one of the gauge singlets. Of the remaining 7, one of these is the Nambu-Goldstone boson multiplet of the $SO(10) \to SU(5)$ symmetry breaking. The mass matrix for the remaining 6 gauge singlets is rather cumbersome and is not presented here. For the numerical solution (I) presented at the start of this Appendix, I have checked that the determinant of this matrix is -6×10^{-7} (in units of $\Lambda = 1.$); the typical mass of each singlet is then $\sim 0.09\Lambda$.

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