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Glueballs and Their Kaluza-Klein Cousins

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Abstract

Spectra of glueball masses in non-supersymmetric Yang-Mills theory in three and four dimensions have recently been computed using the conjectured duality between superstring theory and large N gauge theory. The Kaluza-Klein states of supergravity do not correspond to any states in the Yang-Mills theory and therefore should decouple in the continuum limit. On the other hand, in the supergravity limit $g_{YM}^2 N \to \infty$, we find that the masses of the Kaluza-Klein states are comparable to those of the glueballs. We also show that the leading $(g_{YM}^2 N)^{-1}$ corrections do not make these states heavier than the glueballs. Therefore, the decoupling of the Kaluza-Klein states is not evident to this order.

1 Introduction

Spectra of glueball masses in non-supersymmetric Yang-Mills theory in three and four dimensions have recently been calculated [1] using the conjectured duality between string theory and large N gauge theory [2–5]. The results are apparently in good numerical agreement with available lattice gauge theory data, although a direct comparison may be somewhat subtle, since the supergravity computation is expected to be valid for large ultraviolet coupling $\lambda = g_{YM}^2 N$, whereas we expect that QCD in the continuum limit is realized for $\lambda \to 0$ [5, 6]. As explained in [6, 1], the supergravity computation at $\lambda \gg 1$ gives the glueball masses in units of the fixed ultraviolet cutoff Λ_{UV}. For finite λ, the glueball mass M is expected to be a function of the form

$$M^2 = F(\lambda)\Lambda_{UV}^2. \tag{1.1}$$

In the continuum limit $\Lambda_{UV} \to \infty$, M should remain finite and of order Λ_{QCD}. This would require $F(\lambda) \to 0$ as $\lambda \to 0$. In [1], the leading string theory corrections to the masses were computed and shown to be negative and of order $\lambda^{-3/2}$, in accordance with expectation.

Witten has proposed [5] that three-dimensional pure QCD is dual to type IIB string theory on the product of an AdS_5 black hole and $\mathbf{S}^5$. This proposal requires that certain states in string theory decouple in the continuum limit $\lambda \to 0$. One class of such states are Kaluza-Klein excitations on $\mathbf{S}^5$. The supergravity fields on the AdS_5 black hole $\times$ $\mathbf{S}^5$ can be classified by decomposing them into spherical harmonics (the Kaluza-Klein modes) on $\mathbf{S}^5$ [7, 8]. They fall into irreducible representations of the isometry group $SO(6)$ of $\mathbf{S}^5$, which is the R-symmetry of the four-dimensional $\mathcal{N} = 4$ supersymmetric gauge theory from which QCD_3 is obtained by compactification on a circle. Consequently, only $SO(6)$ singlet states should correspond to physical states in QCD_3 in the continuum limit. These are the glueball states studied in [1]. However, we find that, in the supergravity limit, masses of the $SO(6)$ non-singlet states are of the same order as the $SO(6)$ singlet states. Since these states should decouple in the limit $\lambda \to 0$, it was speculated in [1] that the string theory corrections should make the non-singlet states heavier than the singlet states.

The purpose of this paper is to test this idea. We compute the masses of the $SO(6)$ non-singlet states coming from the Kaluza-Klein excitations of the dilaton in ten dimensions. We find the masses in the supergravity limit to be of the same order as those of the $SO(6)$ singlet states. We then calculate the leading string theory corrections to the masses. We find that the leading corrections do not make the Kaluza-Klein states heavier than the

glueballs. Therefore, the decoupling of the Kaluza-Klein states is not evident to this order. This suggests that the quantitative agreement between the glueball masses from supergravity and the lattice gauge theory data should be taken with a grain of salt.

2 The Supergravity Limit

We calculate the masses of the Kaluza-Klein states following the analysis of [1]. According to [5], QCD_3 is dual to type IIB superstring theory on the AdS_5 black hole $\times$ $\mathbf{S}^5$ geometry given by

$$\frac{dx^2}{l_s^2\sqrt{4\pi g_{YM}^2 N}} = \frac{d\rho^2}{\left(\rho^2 - \frac{b^4}{\rho^2}\right)} + \left(\rho^2 - \frac{b^4}{\rho^2}\right) d\tau^2 + \rho^2 \sum_{i=1}^{3} dx_i^3 + d\Omega_5^2, \tag{2.1}$$

where $d\Omega_5$ is the line element on the unit $\mathbf{S}^5$ and l_s is the string length. The horizon of the black hole is located at $\rho = b$. In order for the geometry to be regular at the horizon, the coordinate τ must be periodic with period $2\pi R$, where $R = (2b)^{-1}$. The inverse radius R^{-1} serves as the ultraviolet cutoff of QCD_3; namely, $\Lambda_{UV} = (2R)^{-1} = b$.

To compute the mass of an $SO(6)$ non-singlet state, we express the dilaton field Φ as

$$\Phi = f_0(\rho) e^{ikx} Y_l(\Omega_5), \tag{2.2}$$

where $Y_l(\Omega_5)$ is the l-th spherical harmonic on $\mathbf{S}^5$, and solve the dilaton equation in the geometry (2.1). This equation reduces to a second-order ordinary differential equation for $f_0(\rho)$; in units in which $b = 1$,

$$\rho^{-1}\frac{d}{d\rho}\left((\rho^4 - 1)\rho\frac{df_0}{d\rho}\right) - (k_0^2 + l(l+4)\rho^2) f_0 = 0. \tag{2.3}$$

The mass in three dimensions is equal to $-k_0^2$ [5]. Since the geometry (2.1) is smooth everywhere, we require that $f_0(\rho)$ be regular everywhere, and in particular at $\rho = \infty$ and at the horizon $\rho = 1$. The equation admits a regular solution $f_0(\rho)$ for discrete values of k_0^2. This determines the mass spectrum.

As in [1], we determine $M^2 = -k_0^2$ numerically by the shooting method. We first solve the differential equation (2.3) as an asymptotic expansion in ρ^{-2} and compute the first few terms in the expansion. We then numerically integrate the equation, with boundary conditions derived from the asymptotic expansion imposed at a sufficiently large value of ρ ($\rho \gg k_0^2$). The solution must be regular at the horizon $\rho = 1$. This requirement determines the spectrum of k_0^2. In the numerical evaluation, we find it convenient to set the boundary condition to be $f_0' = 0$ at the horizon. As we will show in the Appendix,

this shooting method can be used to compute k_0^2 and the wavefunction $f_0(\rho)$ to arbitrarily high precision. The results of the numerical work are listed in Table 1. As expected, the masses are all of the order of the ultraviolet cutoff $\Lambda_{UV} = b$.

l	0	1	2	3	4	5	6	7
M_l^2	11.59	19.43	29.26	41.10	54.93	70.76	88.60	108.4
M_l^{*2}	34.53	48.07	63.60	81.11	100.6	122.1	145.6	171.1
M_l^{**2}	68.98	88.24	109.5	132.7	157.9	185.1	214.3	245.5

Table 1: 3d (Mass)^2 of the l-th Kaluza-Klein modes on $\mathbf{S}^5$ and their excited states, in units of b^2

3 Leading String Theory Corrections

Witten's proposal requires that the Kaluza-Klein states decouple in the continuum limit $\lambda \to 0$. Here we examine whether this efffect is evident from the leading string theory corrections.

According to [9], the leading $\alpha' = (4\pi g_{YM}^2 N)^{-1/2}$ correction to the AdS$_5$ black hole metric is

$$\frac{ds^2}{l_s^2\sqrt{4\pi g_{YM}^2 N}} = (1+\delta_1)\frac{d\rho^2}{\left(\rho^2 - \frac{b^4}{\rho^2}\right)} + (1+\delta_2)\left(\rho^2 - \frac{b^4}{\rho^2}\right)d\tau^2 + \rho^2\sum_{i=1}^{3} dx_i^2 + d\Omega_5^2, \quad (3.1)$$

where

$$\begin{aligned} \delta_1 &= +15\gamma\left(5\frac{b^4}{\rho^4} + 5\frac{b^8}{\rho^8} - 19\frac{b^{12}}{\rho^{12}}\right) \\ \delta_2 &= -15\gamma\left(5\frac{b^4}{\rho^4} + 5\frac{b^8}{\rho^8} - 3\frac{b^{12}}{\rho^{12}}\right), \end{aligned} \quad (3.2)$$

and $\gamma = \frac{1}{8}\zeta(3)\alpha'^3$. In this geometry, the dilaton is no longer constant, but is given by

$$\Phi_0 = -\frac{45}{8}\gamma\left(\frac{b^4}{\rho^4} + \frac{b^8}{2\rho^8} + \frac{b^{12}}{3\rho^{12}}\right). \quad (3.3)$$

There is also a correction to the ten-dimensional dilaton action [10, 11],

$$I_{dilaton} = -\frac{1}{16\pi G_{10}}\int d^{10}x\sqrt{g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \gamma e^{-\frac{3}{2}\Phi}W\right], \quad (3.4)$$

where W is given in terms of the Weyl tensor. In our background and in units where $b = 1$, $W = 180/\rho^{16}$. The relation between the location of the horizon $\rho = b$ and the

periodicity $2\pi R$ of τ is also modified to

$$R = \left(1 - \frac{15}{8}\zeta(3)\alpha'^3 + \cdots\right)\frac{1}{2b}. \tag{3.5}$$

It is the inverse radius R^{-1} that serves as the ultraviolet cutoff of QCD_3.

To solve the dilaton wave equation in the α'-corrected geometry (3.1), we write

$$\Phi = \Phi_0 + f(\rho)e^{ikx}Y_l(\Omega_5), \tag{3.6}$$

where Φ_0 is the dilaton background given by (3.3), and expand $f(\rho)$ and k^2 in γ as

$$f(\rho) = f_0(\rho) + \gamma h(\rho), \quad k^2 = k_0^2 + \gamma\delta k^2. \tag{3.7}$$

Here $f_0(\rho)$ obeys the lowest order equation (2.3) and is a numerically given function, and k_0^2 is likewise determined from (2.3). The second-order differential equation obtained from the action (3.4) in the background metric (3.1) and dilaton field (3.3) is, in units in which $b = 1$,

$$\begin{aligned}
&\rho^{-1}\frac{d}{d\rho}\left((\rho^4 - 1)\rho\frac{dh}{d\rho}\right) - (k_0^2 + l(l+4)\rho^2)h = \\
&= (75 - 240\rho^{-8} + 165\rho^{-12})\frac{d^2 f_0}{d\rho^2} \\
&+(75 + 1680\rho^{-8} - 1815\rho^{-12})\rho^{-1}\frac{df_0}{d\rho} \\
&+(\delta k^2 - 120(k_0^2 + l(l+4)\rho^2)\rho^{-12} - 405\rho^{-14})f_0(\rho).
\end{aligned} \tag{3.8}$$

With $f_0(\rho)$ and k_0^2 given, one may regard this as an inhomogeneous version of the equation (2.3). We solve this equation for $h(\rho)$ and δk^2.

We are now ready to present our results. Let us denote the lowest mass of the l-th Kaluza-Klein state by M_l. In units of the ultraviolet cutoff $\Lambda_{UV} = (2R)^{-1}$, with R given by (3.5), we find

$$\begin{aligned}
M_0^2 &= 11.59 \times (1 - 2.78\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_1^2 &= 19.43 \times (1 - 2.66\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_2^2 &= 29.26 \times (1 - 2.62\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_3^2 &= 41.10 \times (1 - 2.61\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_4^2 &= 54.93 \times (1 - 2.63\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_5^2 &= 70.76 \times (1 - 2.66\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_6^2 &= 88.60 \times (1 - 2.69\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2 \\
M_7^2 &= 108.4 \times (1 - 2.72\zeta(3)\alpha'^3 + \cdots)\Lambda_{UV}^2.
\end{aligned} \tag{3.9}$$

Similar behavior is observed for the excited levels of each Kaluza-Klein state.

Thus the corrections do not make the Kaluza-Klein states heavier than the glueballs, and the decoupling of the Kaluza-Klein states is not evident to this order. According to Maldacena's duality, the $\lambda^{-1/2}$ expansion of the gauge theory corresponds to the α'-expansion of the two-dimensional sigma model with the AdS_5 black hole $\times$ $\mathbf{S}^5$ as its target space. It is possible that the decoupling of the Kaluza-Klein states takes place only non-perturbatively in the sigma model.

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Appendix: The Boundary Condition at the Horizon

In this appendix, we show that the boundary condition at the horizon $\rho = b$ used in the shooting method [1] is consistent, and that the eigenvalue k^2 and the wavefunction $f(\rho)$ can be evaluated to an arbitrarily high precision using this method.

In the neighborhood of $\rho = b$, the dilaton wave equation takes the form

$$\partial_\rho(\rho - b)\partial_\rho f(\rho) + \cdots = 0. \tag{3.10}$$

Its general solution is of the form

$$f(\rho) = c_1\left[1 + \alpha(\rho - b) + \cdots\right] + c_2\left[\log(\rho - b) + \cdots\right] \tag{3.11}$$

with arbitrary coefficients $c_{1,2}$ (the constant α is determined by the wave equation and is in general non-zero). The regularity of the dilaton field requires $c_2 = 0$. In the shooting method, we numerically integrate the differential equation starting from a sufficiently large value of ρ down to the horizon. For generic k^2, the function thus obtained, when expanded as in (3.11), would have $c_2 \neq 0$. The task is to adjust k^2 so that $c_2 = 0$.

Since $f(\rho)$ is divergent at $\rho = b$ for generic k^2, it is numerically difficult to impose the boundary condition directly at $\rho = b$. Instead, in [1] and in this paper, we required $f' = 0$ at $\rho = b + \epsilon$ for a given small ϵ (for example, $\epsilon = 0.0000001b$ in this paper). By (3.11), this condition implies

$$c_2 = -c_1 \alpha \epsilon + \cdots . \tag{3.12}$$

Therefore, c_2 can be made arbitrarily small by adjusting ϵ. This justifies the numerical method used in [1] and in this paper.

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