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Cosmological Moduli Problem in a Supersymmetric Model with Direct Gauge Mediation*

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Abstract

Recently, an interesting class of the direct gauge mediation supersymmetry (SUSY) breaking models are proposed, in which the minimum of the potential of the SUSY breaking field is determined by the inverted hierarchy mechanism. We consider their cosmological implications. In this class of models, SUSY breaking field has a very flat potential, which may have a cosmological importance. Assuming the initial amplitude of the SUSY breaking field to be of the order of the Planck scale, it can be a source of a large entropy production. A special attention is paid to the cosmological moduli problem, and we will see the cosmological mass density of the moduli field can be significantly reduced.

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1 Introduction

Low energy supersymmetry (SUSY) has been regarded as one of the attractive new physics beyond the standard model, since it may provide a natural explanation to the stability of the electroweak scale against radiative corrections. However, contrary to our theoretical interests, any superpartner of the standard model particle has not discovered yet, and hence SUSY has to be broken in nature. Unfortunately, we do not have a clear picture of the SUSY breaking in nature, and the understanding of the origin of the SUSY breaking is one of the most important issues in the study of the supersymmetric models.

In recent years, a new framework for the SUSY breaking, i.e., gauge mediated SUSY breaking (GMSB) [1], has been attracting many interests, and its phenomenological implications have been extensively investigated [2]. In particular, in GMSB, SUSY breaking in the SUSY standard model (SSM) sector is mediated by the gauge interactions which do not distinguish flavors. In this scheme, dangerous off-diagonal elements in the sfermion mass matrices are suppressed, and serious SUSY flavor changing neutral current (FCNC) problem can be evaded.

In spite of the phenomenological interests, however, cosmology of GMSB is not fully satisfactory. In particular, relic abundances of the gravitino [3, 4] and the moduli field [5] have been known to be problematic. These problems are often called "gravitino problem" and "cosmological moduli problem." In some sense, they are more serious than the gravity mediated SUSY breaking case, and they may be crucial weak points of GMSB. (These issues will be reviewed in the next section.) However, one should note that there are rooms to solve or improve these difficulties; since these difficulties are based on a kind of "minimal" assumption, some of them may be solved or relaxed by a new idea. For example, thermal inflation [6] is proposed to dilute the unwanted particles.

In this paper, we would like to propose a new mechanism for a large entropy production, which can be a resolution to the cosmological problems in GMSB. Our scenario is based on a class of models with direct gauge mediation in which the messenger particles have a direct coupling to the original SUSY breaking field. In particular, recently, several direct gauge mediation models are proposed in which SUSY breaking field has exactly flat potential at the tree level [7, 8, 9, 10]. (For other classes of models of direct gauge mediation, see Refs. [11, 12].) In this class of models, minimum of the potential of the SUSY breaking field is determined by the inverted hierarchy mechanism [13], and SUSY breaking field has a very flat potential even after the potential is lifted. In this case, SUSY breaking field may play an important role to dilute unwanted particles; with an assumption that the SUSY breaking field has an initial amplitude of the order of the Planck scale, various cosmological problems can be naturally solved. One virtue of this scenario is that the source of the large entropy production is already in the framework of the SUSY breaking mechanism. Therefore, the scenario is fairly economical, and we do not have to introduce any new field only for the entropy production (like "flaton"), contrary to the case of thermal inflation [6]. In the following sections, we see how this works, and consider if we may have a cosmologically consistent scenario.

The organization of this paper is as follows. In Section 2, we give an overview of the cosmological difficulties in GMSB. Then, in Section 3, we briefly review the important aspects of the direct gauge mediation model with the inverted hierarchy mechanism. In particular, the SUSY breaking field plays a very important role in our discussion, so we see the properties of the potential of the SUSY breaking field in some detail. In Section 4, cosmology based on the direct gauge mediation model with the inverted hierarchy mechanism is discussed. In particular, we concentrate on the cosmological evolution of the SUSY breaking field, and we discuss how it improves the cosmological difficulties. Section 5 is devoted to discussion.

2 Overview of the Cosmology of GMSB

Before discussing the cosmology of the direct gauge mediation model, let us first briefly overview the cosmology of the gauge mediation model.

Cosmologically, one important outcome of the GMSB is the light gravitino; the gravitino mass $m_{3/2}$ in this scheme has to be much lighter than the SSM scale $m_{\rm SSM}$. This is because mass squared matrices of sfermions have off-diagonal elements of $O(m_{3/2}^2)$. If this effect is comparable to the dominant contribution from GMSB, dangerous SUSY FCNC problem arises again, which spoils the important motivation of GMSB. Consequently, the gravitino becomes the lightest superparticle (LSP) in this scenario.

Keeping this feature in mind, one of the most famous cosmological constraint in GMSB is from the mass density of the gravitino in the Universe. If the gravitino is thermalized in the early Universe, and if it is not diluted, its mass density may significantly contribute to the energy density of the Universe. Without dilution, mass density of the gravitino is proportional to $m_{3/2}$, and the Universe is overclosed if the gravitino mass is heavier than about 1 keV [3].

With an enough entropy production after the decoupling of the gravitino from the thermal bath, gravitino mass heavier than 1 keV may be also viable. However, even in this case, the gravitinos are produced in the thermal bath due to scattering and decay processes. These secondary gravitinos also contribute to the mass density of the Universe, and hence gravitino production after the entropy production has to be inefficient. Since the gravitino production is more effective for higher temperature, we obtain an upper bound on the maximal temperature of the Universe after the late entropy production in order not to overclose the Universe [4]. Notice that the interaction of the longitudinal gravitino with matter is proportional to $m_{3/2}^{-1}$, and hence the constraint becomes weaker for a heavier gravitino. In Fig. 1, we show the upper bound on the maximal temperature $T_{\rm max}$ as a function of the gravitino mass; the upper bound is from ~ 100 GeV to $\sim 10^8$ GeV, for the gravitino mass 1 keV – 1 GeV. In particular, the constraint is very strict in the case of 1 keV $\lesssim m_{3/2} \lesssim$ 100 keV, where the gravitinos are mainly produced by the decay processes. If the Universe starts with a temperature higher than this upper bound, the Universe is overclosed by the gravitino, unless there is an enough entropy production below this temperature.

More serious problem arises in the framework of superstring models. In superstring

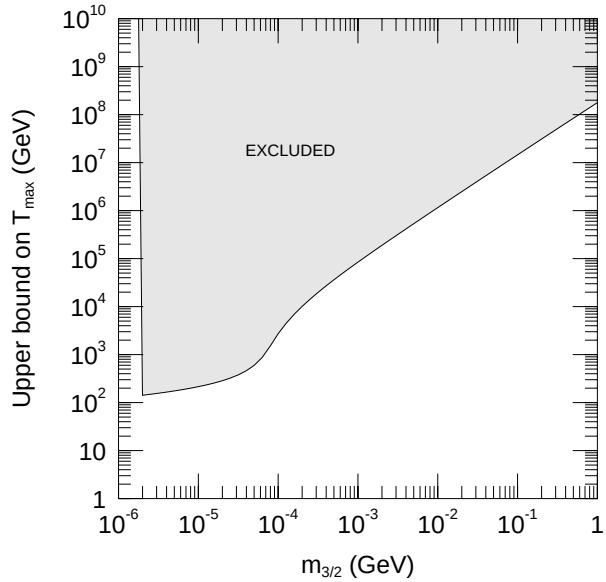

Figure 1: Upper bound on the maximal temperature of the Universe as a function of the gravitino mass $m_{3/2}$.

models, dilaton and moduli fields (which we call "moduli" fields hereafter) exist which are the flat directions related to symmetries in the superstring theory. Mass of the moduli field originates to SUSY breaking effect, and is of the same order of the gravitino mass. (Throughout this paper, we approximate the mass of the moduli field to be the gravitino mass $m_{3/2}$.) Generically, moduli field takes an initial amplitude of the order of the Planck scale, unless our vacuum lies at or near a point of enhanced symmetry [15].[#1] Then, it starts oscillation when the expansion rate of the Universe becomes comparable to the mass of the moduli field. Since the interactions of the moduli field are suppressed by inverse powers of the Planck scale, moduli field lighter than about 100 MeV has a lifetime longer than the present age of the Universe. In GMSB, this is (almost) always the case. In this case, mass density of the moduli field becomes enormous, if there is no dilution. Assuming the radiation dominance before the moduli field starts to move, naive calculation results in the density parameter of the moduli field as

$$\Omega_\phi h^2 \sim 6 \times 10^{14} \times \left(\frac{g_*}{100}\right)^{-1/4} \left(\frac{m_{3/2}}{100\ \mathrm{keV}}\right)^{1/2} \left(\frac{\phi_0}{M_*}\right)^2, \tag{2.1}$$

where h is the Hubble constant in units of 100 km/sec/Mpc, g_* is the effective number of the massless degrees of freedom when the moduli field starts to move, $M_* \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale, and ϕ_0 is the initial amplitude of the moduli field. For example, for $m_{3/2} = 100$ keV, the initial amplitude has to be smaller than $\sim 10^{-7} M_*$ in order not

[#1] However, it is difficult to construct a realistic model which has our vacuum as a symmetry enhanced point.

to overclose the Universe, and this is an extreme fine tuning of the initial condition. In other words, if the initial amplitude takes the natural value (i.e., $\sim M_*$), a large entropy production is inevitable for a viable cosmological scenario.

Even if we adopt a large entropy production to dilute the gravitino and the moduli field away, it is still non-trivial whether a consistent cosmological scenario can be obtained. If there is a large entropy production, it also dilutes the possible baryon asymmetry generated in the early stage of the Universe. Naively speaking, baryon number asymmetry has to be generated after the entropy production. However, in some case, the reheating temperature after the entropy production becomes too low for baryogenesis. In Ref. [5], it has been pointed out that Affleck-Dine mechanism for baryogenesis [16] may be able to generate enough baryon asymmetry even if there is a large entropy production. This topics is reviewed later. Candidates of the cold dark matter (CDM) is another interesting issue in GMSB. In the gravity mediated SUSY breaking scenario, the lightest neutralino [17] or sneutrino [18] can be a promising candidate of the CDM, if it is the LSP. However, in GMSB, they cannot be the CDM, since they can decay into gravitino and their superpartner. Several candidates of the CDM are proposed in the framework of GMSB [19, 20], but these candidates are also diluted by the entropy production. One interesting candidate is the coherent oscillation of the moduli field, if its energy density is diluted enough. Therefore, it is very important to consider the possibility to dilute the energy density of the moduli field down to $\Omega_\phi \lesssim 1$.

Another class of cosmological problems are related to the structure of the scalar potential; the scalar potential may have unwanted minimum which is deeper than the phenomenologically viable local minimum. For example, original low energy gauge mediation model may have a color breaking minimum [21], and the models proposed in Refs. [8, 9, 10] have a SUSY preserving true vacuum at the origin of the potential of the SUSY breaking field.[#2] Usually, the tunneling rate to the true vacuum can be so small that the transition does not happen for the time scale of the age of the Universe. Therefore, it is phenomenologically consistent once the SUSY breaking field is trapped in the minimum we want. However, cosmologically, we have not understood how the SUSY breaking field is trapped in the relevant (local) minimum, not in the unwanted (global) one. In particular, if we assume a naive SUSY breaking phase transition, many horizons choose the unwanted deeper minimum, and the current horizon contains many regions which dropped into the unwanted minimum. Notice that it is unclear whether the thermal inflation could solve this problem, even though the reheating temperature after the thermal inflation is relatively low. This is because the current horizon scale contains many different horizons before the thermal inflation. Therefore, even if the SUSY breaking phase transition occurs before the thermal inflation, current horizon still contains many regions of unwanted minimum.

Keeping these arguments in mind, it is important to develop a cosmologically consistent scenario based on GMSB. Importantly, one should remember that above problems are usually based on "minimal" assumption, and in particular in direct gauge mediation model, the above arguments do not take account of a possible effect from the SUSY breaking field. In the

[#2] However, the structure of the potential is model-dependent, and models without unwanted minimum may be constructed.

following sections, we will see what happens if we include its effects.

3 Model

In this section, we first briefly review the class of models we are interested in. As we mentioned in the introduction, we consider a cosmology of the direct gauge mediation model in which the potential of the SUSY breaking field is stabilized by the inverted hierarchy mechanism. In this section, we discuss the general features of such models [7, 8, 9, 10]. An explicit example of the model is shown in Appendix A.

3.1 Framework

The model is based on the symmetry $\mathrm{G} = \mathrm{G_S} \times \mathrm{G_B} \times \mathrm{G_{SM}}$. Here, $\mathrm{G_S}$ is the strong gauge interaction whose dynamics induces the gaugino condensation. On the other hand, $\mathrm{G_B}$ is introduced to stabilize the minimum of the potential of the SUSY breaking field, and $\mathrm{G_{SM}}$ is the standard $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$ gauge croup. In this framework, we introduce the following class of chiral superfields in order to induce a desired dynamics to break SUSY: the SUSY breaking field Σ which transforms under $\mathrm{G_B}$, "messengers" $q + \bar{q}$ which transform under $\mathrm{G_B}$ and $\mathrm{G_{SM}}$, and "strongly interacting" fields $Q + \bar{Q}$ which transform under $\mathrm{G_S}$ and $\mathrm{G_B}$. These chiral superfields have a superpotential of the form

$$W = y_Q \Sigma \bar{Q} Q + y_q \Sigma \bar{q} q. \tag{3.1}$$

Notice that this superpotential has R-symmetry which is non-anomalous for $\mathrm{G_S}$; under this R-symmetry, Σ has charge $+2$. As we will discuss later, this R-symmetry is explicitly broken by supergravity effect.

We construct a model so that Σ has a flat direction which is parameterized by an invariant made out of Σ. On this flat direction, $\mathrm{G_B}$ is broken by the vacuum expectation value (VEV) of Σ. The first requirement to the model is that, on this flat direction, all but one degrees of freedom in Σ are eaten by Higgs mechanism. We parametrize the remaining degrees of freedom by X.

On this flat direction, chiral superfields coupled to Σ also acquire masses. This effect changes the β-function of $\mathrm{G_S}$ at $\mu \sim y_Q \langle \Sigma \rangle$, since Q and $\bar{Q}$ have mass of $m_Q \sim y_Q \langle \Sigma \rangle$ and decouple at this scale. As a result, the strong scale for the theory above m_Q, Λ, differs from that after Q and $\bar{Q}$ decouples, $\Lambda_{\rm eff}$. These two quantities are related as $\Lambda_{\rm eff}^{3\mu_{\mathrm{G_S}}} = \Lambda^{3\mu_{\mathrm{G_S}} - \mu_Q} m_Q^{\mu_Q}$, where $\mu_{\mathrm{G_S}}$ and μ_Q are Dynkin indices for the adjoint and $Q + \bar{Q}$ representations of $\mathrm{G_S}$, respectively. The second requirement to the model is that these Dynkin indices satisfy the relation $\mu_{\mathrm{G_S}} = \mu_Q$, so that the linear superpotential is induced by the gaugino condensation; below the strong scale, effective superpotential is induced by the gaugino condensation as

$$W_{\rm eff} \sim \Lambda_{\rm eff}^3 \sim (y_Q X)^{\mu_Q/\mu_{\mathrm{G_S}}} \Lambda^{3-\mu_Q/\mu_{\mathrm{G_S}}}. \tag{3.2}$$

Therefore, if $\mu_{\mathrm{G_S}} = \mu_Q$, superpotential is linear in X, and supersymmetry is broken by the VEV of the F-term of the X field; $F_X \sim y_Q \Lambda^2$. (This class of model of SUSY breaking is originally discussed in Ref. [14].)

With the above superpotential, scalar potential is given by

$$V = \frac{|\partial_X W_{\mathrm{eff}}|^2}{\partial_X^* \partial_X K} \simeq \frac{y_Q^2 \Lambda^4}{Z_\Sigma(X^*, X)}, \tag{3.3}$$

where ∂_X represents the derivative with respect to X, K is the Kähler potential, and Z_Σ is the wave function renormalization of Σ. At the tree level, $Z_\Sigma = 1$, and hence V does not depend on X. In this case, VEV of X is undetermined. However, once we include the radiative corrections, the situation changes. Since Σ interacts through gauge and Yukawa interactions, non-trivial Kähler potential is induced by radiative correction. X dependence of Z_Σ is governed by the renormalization group equation (RGE). At the one loop level, RGE for Z_Σ is given by

$$\frac{d \ln Z_\Sigma}{d \ln \mu} = \frac{1}{16\pi^2} \left[C_{\mathrm{B}} g_{\mathrm{B}}^2(\mu) - C_Q y_Q^2(\mu) - C_q y_q^2(\mu) \right], \tag{3.4}$$

where coefficients C_{B}, C_Q, and C_q are all positive. If the β-function for Z_Σ vanishes, we have a minimum (or maximum) of the potential. The important point is that the gauge and the Yukawa contributions have opposite signs in the β-function of Z_Σ. As we can see in Eqs. (3.3) and (3.4), gauge contribution makes Z_Σ larger at higher energy and drives $|X|$ to a larger value, while Yukawa contribution affects in the opposite way. As a result, if the gauge piece dominates the β-function in the low energy, and also if the effect of the Yukawa terms wins in the high energy, X has a minimum at $|X| = v$ where the β-function vanishes:

$$C_{\mathrm{B}} g_{\mathrm{B}}^2(v) = C_Q y_Q^2(v) + C_q y_q^2(v). \tag{3.5}$$

Therefore, in this class of models, potential for X is stabilized by the scale dependence of the wave function renormalization factor Z_Σ. At the minimum, F-component of the X field has a non-vanishing VEV of $O(y_Q \Lambda^2)$, and SUSY is broken. Then, fermionic component of X becomes the goldstino, and in supergravity, it is absorbed in the gravitino. In this case, gravitino mass is related to F_X as

$$m_{3/2} = \frac{F_X}{\sqrt{3} M_*}. \tag{3.6}$$

Once X gets a VEV v, q and $\bar{q}$ acquire a SUSY preserving mass of $\sim y_q v$, as well as SUSY breaking mass squared for the scalar component of $\sim y_q F_X$. Since q and $\bar{q}$ have quantum numbers under the standard model gauge group, the SUSY breaking masses for the SSM superparticles arise by integrating out these messenger particles. As in the case of well-known ordinary gauge mediation model, the ratio,

$$B_Q = \frac{F_X}{v}, \tag{3.7}$$

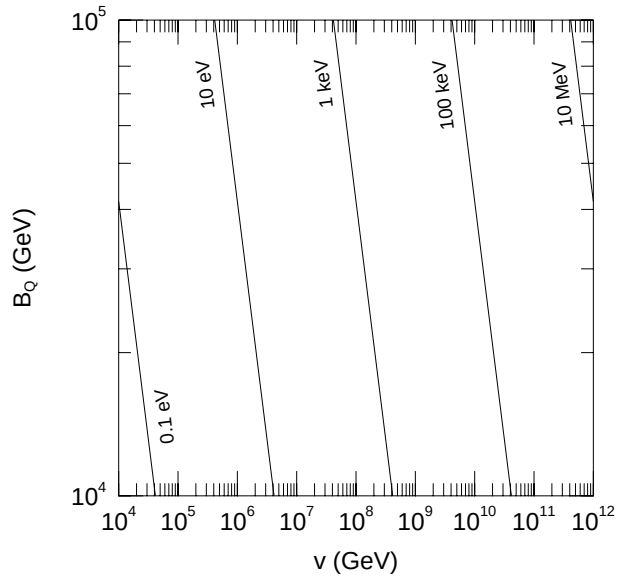

Figure 2: Contours of the constant gravitino mass on the v vs. B_Q plane.

determines the scale of the SUSY breaking masses in the SSM superparticles which are of $O(\alpha_{\rm SM}/4\pi)B_Q$, with $\alpha_{\rm SM}$ being the appropriate gauge coupling of the standard model gauge group. B_Q should be in the range of 10^4 GeV $\lesssim B_Q \lesssim 10^5$ GeV; the lower bound is from experimental lower bounds on the masses of the superparticles, while the upper bound is from the naturalness point of view.

In the following discussions, we use v and B_Q as independent parameters in the model, and rewrite other quantities as functions of them. For example, the gravitino mass is given by

$$m_{3/2} = \frac{vB_Q}{\sqrt{3}M_*}. \tag{3.8}$$

In Fig. 2, we show the contours of the constant gravitino mass on the v vs. B_Q plane.

Before closing this subsection, we discuss the allowed range of v. Recalling that $\sim y_q^2 v^2$ and $\sim y_q F_X$ are the diagonal and off-diagonal elements of the mass squared matrix of the messenger scalars, v has to be larger than $\sim y_q^{-1/2} F_X^{1/2}$ for the positivity of the eigenvalues of the mass squared matrix; otherwise, messenger scalar has a VEV and the standard model gauge groups are broken. Another constraint is from the stability of the SUSY breaking minimum. In some class of models, there is a SUSY preserving true vacuum at the origin ($|X| = 0$). In this case, the tunneling rate from the SUSY breaking vacuum to the true vacuum has to be small enough, so that the lifetime of the SUSY breaking vacuum is longer than the age of the Universe. This issue is discussed in Ref. [8], and it requires $v/\Lambda \gtrsim 10$. Since $F_X \sim y_Q \Lambda^2$, $v \gtrsim 10 y_Q^{-1/2} F_X^{1/2}$ is required in models with true vacuum at the origin. Notice that this constraint is more stringent than the one from the stability of the messenger potential, if $y_Q \sim y_q$.

One important lower bound on v is derived for the validity of our perturbative approach. If v is close to the strong scale Λ, $\mathrm{SU}(2)_\mathrm{S}$ dynamics generates non-perturbative Kähler potential, and our perturbative arguments break down. At the scale Λ_eff, we expect a non-perturbative contribution to the Kähler potential of the form $\delta K \sim [(\mathcal{W}^\alpha \mathcal{W}_\alpha)(\bar{\mathcal{W}}_{\dot\alpha}\bar{\mathcal{W}}^{\dot\alpha})]^{1/3} \sim \Lambda_\mathrm{eff}^* \Lambda_\mathrm{eff}$ [22, 8], where $\mathcal{W}^\alpha$ is the spinor superfield for $\mathrm{SU}(2)_\mathrm{S}$. With the naive dimensional analysis [23], coefficient of this operator is estimated to be of $O((4\pi)^{-2/3})$. Requiring this effect to be smaller than the perturbative one, v has to be larger than $10^9 - 10^{10}$ GeV [8]. However, it may be possible that the minimum of the potential exists even with the non-perturbative effect. (For a model in which the SUSY breaking minimum is stabilized by non-perturbative effects, see Ref. [11].) In this case, we need to make some dynamical assumptions not to change our following discussions, since the non-perturbative effect is not calculable. However, most importantly, strong dynamics does not break the R-symmetry, and hence the properties of the R-axion is unchanged. Consequently, evolution of the R-axion is basically unaffected, even if the non-perturbative effect becomes important at the minimum of the potential. With these caveats in mind, we also consider the case with $v \lesssim 10^9$ GeV in the following discussion. In this case, we make an implicit assumption that the minimum of the potential is stabilized even after taking account of the non-perturbative effect.

Finally, we comment on the upper bound on v. As can be seen from Eq. (3.8), gravitino mass becomes larger than ~ 100 GeV for $v \gtrsim 10^{16}$ GeV with $B_Q \sim 10^4$ GeV. In this case, supergravity contribution to the sfermion masses becomes comparable to the gauge mediation piece, and hence SUSY FCNC problem arises again. Therefore, $v \ll 10^{16}$ GeV is required to solve SUSY FCNC problem.

3.2 Potential of X

In order to discuss the cosmological implication of X, it is important to understand the properties of its potential, which is the subject of this subsection.

We are interested in the models in which the minimum of the X field is determined by the inverted hierarchy mechanism. In this class of models, the potential at the global level is only lifted by the renormalization group effects, and hence it can be expanded by powers of $\ln X^* X$:[#3]

$$\begin{aligned} V_\mathrm{global} &= F_X^2 / Z_\Sigma \\ &= F_X^2 \left[-\frac{1}{16\pi^2} \frac{Z'_\Sigma(\mu)}{Z_\Sigma^2(\mu)} \ln \frac{X^* X}{\mu^2} + \frac{1}{(16\pi^2)^2} \left(\frac{Z'^2_\Sigma(\mu)}{Z_\Sigma^3(\mu)} - \frac{Z''_\Sigma(\mu)}{Z_\Sigma^2(\mu)} \right) \left(\ln \frac{X^* X}{\mu^2} \right)^2 + \cdots \right] \\ &\equiv F_X^2 \left[\frac{\zeta_1(\mu)}{16\pi^2} \ln \frac{X^* X}{\mu^2} + \frac{\zeta_2(\mu)}{(16\pi^2)^2} \left(\ln \frac{X^* X}{\mu^2} \right)^2 + \cdots \right], \end{aligned} \tag{3.9}$$

[#3] After the second line, we omit the constant piece. We assume that the cosmological constant is cancelled out by supergravity effect, and hence the constant term does not matter.

with

$$Z'_\Sigma \equiv 8\pi^2 \frac{dZ_\Sigma}{d\ln\mu}, \quad Z''_\Sigma \equiv (8\pi^2)^2 \frac{d^2 Z_\Sigma}{d(\ln\mu)^2}. \tag{3.10}$$

Here, ζ_n is a $2n$-th polynomial of gauge and Yukawa coupling constants. We factorized relevant powers of $16\pi^2$, so that coefficients in ζ_n become typically of $O(1)$. Without cancellation, ζ_n is close to 1 if some of the gauge or Yukawa coupling is close to 1.

In Eq. (3.9), μ can be an arbitrary scale; μ dependence is cancelled out by the renormalization group effect. If we take $\mu = v$ where Z'_Σ vanishes, $V_{\rm global}$ starts with a term which is quadratic in $\ln X^*X$, and hence the potential has an extremum at $|X| = v$. In particular, if $\zeta_2(v) > 0$, potential has a minimum there, which is what we desired. Hereafter, we assume that the gauge and Yukawa coupling constants are arranged so that $\zeta_2(v) > 0$. Around this minimum, potential starts with $(\ln X^*X)^2$ term which is suppressed by $(16\pi^2)^{-2}$. However, once $|X|$ becomes much larger (or smaller) than v, ζ_1 at that scale may become large so that the potential is approximated to be linear in $\ln X^*X$, with being suppressed only by $(16\pi^2)^{-1}$.

So far, we have discussed the potential in the framework of global SUSY. However, supergravity effect also generates important terms in the potential. First of all, all the scalar fields receive SUSY breaking masses of the order of the gravitino mass. This effect becomes important especially when the amplitude of X becomes large. Another important implication is that the R-symmetry is (explicitly) broken due to the supergravity effect if the cosmological constant is cancelled out [24]. Under $\mathrm{U}(1)_R$ symmetry, superpotential has the charge of $+2$. However, in order to cancel the cosmological constant, a constant term is needed in the superpotential. From the cross term between them, R-symmetry breaking potential is induced:

$$V_{\not{R}} \sim -\frac{F_X^2}{M_*}(X^* + X) \times f(X^*X/M_*^2), \tag{3.11}$$

where f is an unknown function. We expand this function as

$$f(x) = k_0 + k_1 x + \cdots, \tag{3.12}$$

where coefficients k_n are expected to be of $O(1)$. By combining these contributions, supergravity contribution is of the form:

$$V_{\rm SUGRA} \sim m_{3/2}^2 X^*X + V_{\not{R}}. \tag{3.13}$$

In fact, the linear term in $V_{\not{R}}$ may cause a problem. If we neglect the logarithmic terms, non-vanishing k_0 shifts the minimum of the potential from $X = 0$ to $X \sim k_0 M_*$. Thanks to the logarithmic term in $V_{\rm global}$, potential can have a minimum at $|X| = v \ll M_*$. However, with the potential

$$V \sim \frac{\zeta_1}{16\pi^2} F_X^2 \ln X^*X - k_0 \frac{F_X^2}{M_*}(X + X^*) + m_{3/2}^2 X^*X + \cdots, \tag{3.14}$$

another minimum still exists at $|X| \sim M_*$ when $k_0 \sim O(1)$. With this minimum, X is more likely to settle down to this unwanted minimum if X has an initial amplitude of $O(M_*)$. This is because the potential is dominated by the supergravity contribution for such a large amplitude, and hence X does not feel the effect of the logarithmic piece unless $|X|$ becomes small enough. In order to remove this unwanted minimum, we assume k_0 to be small enough,

$$k_0 \lesssim \frac{\zeta_1}{4\pi}. \tag{3.15}$$

If ζ_1 is of $O(1)$, this is a tuning of 10 % level, and we believe accidental cancellation would be enough for this suppression.[#4]

3.3 Properties of the Physical Modes

At around the minimum ($|X| \sim v$), we have two physical scalars from X. In order to discuss the properties of these fields, it is convenient to parametrize the X field as

$$X = \left(v + \frac{1}{\sqrt{2}}\sigma\right) e^{ia/\sqrt{2}v}. \tag{3.16}$$

Expanding the potential around the minimum, we obtain the mass of the σ as

$$m_\sigma^2 = -\frac{Z_\Sigma''(v)}{(16\pi^2)^2}\frac{F_X^2}{v^2} = \frac{\zeta_2(v)}{(16\pi^2)^2}\frac{F_X^2}{v^2}, \tag{3.17}$$

where we normalized as $Z_\Sigma(v) = 1$. Notice that σ is as heavy as the SSM superpartners if all the gauge and Yukawa coupling constants are of the same order. On the contrary, a is the pseudo-Nambu-Goldstone boson for the R-symmetry, which is usually called R-axion. The main source of the R-axion mass is the R-symmetry breaking term from the supergravity effect;

$$V_{\not{R}} = -k_0 \frac{F_X^2}{M_*}(X^* + X), \tag{3.18}$$

where k_0 is (unknown) $O(1)$ constant introduced in Eqs. (3.11) and (3.12). (Around the minimum $|X| = v$, higher order terms are suppressed by powers of v^2/M_*^2.) From this potential, the R-axion mass is given by [24][#5]

$$m_a^2 = k_0 \frac{F_X^2}{vM_*} \simeq \left[6\ \text{GeV} \times k_0^{1/2} \left(\frac{B_Q}{10^5\ \text{GeV}}\right)\left(\frac{v}{10^{10}\ \text{GeV}}\right)^{1/2}\right]^2. \tag{3.19}$$

[#4]The values of k_0 at $|X| \sim M_*$ and at $|X| \sim v$ are supposed to be different. In particular, in our case, moduli field exists which has a very large initial amplitude of $O(M_*)$. The value of k_0 should be affected by the evolution of the moduli field.

[#5]There is another contribution to the R-axion mass from the QCD anomaly, which is, however, much smaller than the supergravity effect. This fact suggests that it is difficult to use this R-axion as a solution to the strong CP problem, unless the supergravity effect is much smaller than the naive expectation.

Next, we consider the decay rate of these fields. The R-axion dominantly decays into gauge boson pairs.[#6] Since the R-axion couples to the messenger multiplets q and $\bar{q}$ which transform under $\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$, the R-axion has a coupling to the standard model gauge bosons at the one-loop level. Then, its decay rate is calculated as

$$\Gamma_a = \frac{n_{\rm G}}{32\pi} \left(\frac{\alpha_{\rm SM} b_q}{4\pi} \right)^2 \frac{m_a^3}{v^2}. \tag{3.20}$$

Here, $n_{\rm G}$ is the number of the final state, b_q is the β-function coefficient of $q + \bar{q}$, and $\alpha_{\rm SM}$ is the corresponding gauge coupling constant. For example, for the decay into the gluon pair ($a \to gg$), $n_{\rm G} = 8$, $b_q = N_5$, and $\alpha_{\rm SM} = \alpha_{\rm s}$, while for the decay into the photon pair ($a \to \gamma\gamma$), $n_{\rm G} = 1$, $b_q = \frac{8}{3} N_5$, and $\alpha_{\rm SM} = \alpha_{\rm em}$, with N_5 being the number of messenger chiral superfields in units of $\mathbf{5} + \bar{\mathbf{5}}$ representations of SU(5). (For the model shown in Appendix A, $N_5 = 2$.)

On the other hand, the σ field dominantly decays into the R-axion pair ($\sigma \to aa$). From the following Lagrangian:

$$\mathcal{L} = \partial_\mu X^* \partial_\mu X \simeq \frac{1}{2} \partial_\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial_\mu a \partial_\mu a + \frac{1}{\sqrt{2} v} \sigma \partial_\mu a \partial_\mu a + \cdots, \tag{3.21}$$

we obtain the decay rate of this process to be

$$\Gamma_\sigma = \frac{1}{64\pi} \frac{m_\sigma^3}{v^2}. \tag{3.22}$$

Notice that σ also decays into the gauge boson pairs, and the decay rate for this process is given by a similar formula as Eq. (3.20) with m_a being replaced by m_σ. Comparing the decay rates for these processes, we can see that $\sigma \to aa$ is the dominant decay mode for σ.

The decay mode into the gravitino pair is potentially significant. Indeed, the SUSY breaking field has an interaction as

$$\mathcal{L} = -\frac{1}{2M_*^2} W \bar{\psi}_\mu \sigma^{\mu\nu} P_L \psi_\nu + \text{h.c.} \simeq -\frac{1}{2M_*^2} F_X X \bar{\psi}_\mu \sigma^{\mu\nu} P_L \psi_\nu + \text{h.c.}, \tag{3.23}$$

with ψ_μ being gravitino. This interaction induces the decay process $X \to \psi_\mu \psi_\mu$, and in particular, the decay into the longitudinal component is the most important. With the Lagrangian given above, we obtain the decay rate as

$$\delta\Gamma_X \equiv \Gamma(X \to \psi_\mu \psi_\mu) = \frac{1}{32\pi} \frac{m_X^5}{F_X^2}, \tag{3.24}$$

[#6] If the ordinary quarks and leptons have non-vanishing R-charge, the R-axion may decay into these fermions. However, it is unknown whether the R-symmetry can be consistently defined in the SSM sector, and the couplings of the R-axion to the quarks and leptons are model-dependent. In particular, these couplings cannot be fixed unless we specify the mechanism to generate so-called μ- and B-parameters. Furthermore, these decay processes are chirality suppressed. In this paper, we assume that these decay modes are negligibly small. The R-axion may also decay into gaugino pairs. However, the R-axion is lighter than the gauginos in most of the parameter region we consider. Therefore, we do not consider this decay mode. Notice that the decay rate for this process is at most comparable to that into gauge bosons. Therefore, qualitatively, the following arguments are unchanged even if the R-axion decays into gaugino pairs.

with $X = a$ and σ. Comparing Eq. (3.24) with Eqs. (3.20) and (3.22), the branching ratio for the process $X \to \psi_\mu \psi_\mu$ is estimated as

$$\mathrm{Br}(a \to \psi_\mu \psi_\mu) \simeq \frac{\delta\Gamma_a}{\Gamma_a} \simeq \frac{1}{n_{\mathrm{G}}} \left(\frac{m_a}{(\alpha_{\mathrm{SM}} b_q / 4\pi) B_Q} \right)^2 , \tag{3.25}$$

$$\mathrm{Br}(\sigma \to \psi_\mu \psi_\mu) \simeq \frac{\delta\Gamma_\sigma}{\Gamma_\sigma} \simeq 2 \left(\frac{m_\sigma}{B_Q} \right)^2 . \tag{3.26}$$

Notice that these branching ratios are much smaller than 1. In the parameter region we consider, the R-axion mass is usually smaller than $O(\alpha_{\mathrm{SM}}/4\pi)B_Q$. Furthermore, as shown in Eq. (3.17), m_σ is smaller than B_Q. Therefore, in both cases, the decay modes into the gravitino pair are suppressed. Even with small branching ratios, however, the decay of the R-axion and σ may overproduce the gravitino, resulting in too much mass density of the Universe. We will come back to this point in the next section.

4 Evolution of the Scalar Fields

Now, we are ready to discuss the evolutions of the scalar fields, i.e., the SUSY breaking field X and the moduli field ϕ. Since the behavior of X changes at $|X| \sim v$, we first consider the evolution when $|X| \gg v$. Then, the behavior of X when $|X| \sim v$ is considered.

4.1 $|X| \sim M_*$

In our discussion, we adopt the picture of the inflationary cosmology; we assume a primordial inflation which solves horizon and flatness problems. During this inflation, X and ϕ are shifted from their minimum, and these fields have very large initial amplitudes. We assume that their initial amplitudes are both of $O(M_*)$. This may happen because of the chaotic assumption on the initial condition of scalar fields [25], due to modification of the scalar potential during the inflation with large expansion rate [15], or in the case of no-scale type supergravity [26]. Importantly, e-folding of this primordial inflation is large enough to solve the horizon problem, and hence X and ϕ have coherent initial states for the scale of the current horizon. After the primordial inflation, reheating happens, and the Universe becomes radiation dominated. At this stage, X and ϕ keep their initial values as far as the expansion rate is larger than their masses.

Once the expansion rate of the Universe becomes comparable to the mass of X and ϕ, these fields start to move. Since these fields (in particular, X) are assumed to have large amplitudes of $O(M_*)$, the potential for these fields are initially dominated by the supergravity contributions. Furthermore, with these large initial amplitudes, energy density of the radiation becomes comparable to those of X and ϕ when these fields start to move. Then, after this stage, energy density of radiation decreases faster than those of X and ϕ, and the Universe is dominated by the coherent oscillation.

Once the moduli field ϕ starts oscillation, its evolution is quite simple. Approximating its potential to be quadratic as $\sim m_{3/2}^2\phi^2$, the energy density of the moduli field scales as R^{-3}, with R being the scale factor.

One may worry about a possible shift of the minimum of the moduli potential, in particular, due to the non-vanishing expansion rate H induced by X. When the Universe is dominated by a condensation of a scalar field (in our case, X), supergravity effect induces extra terms in the moduli potential, which are proportional to H^2 [15]. This effect shifts the minimum of the moduli potential, and ϕ oscillates around the shifted minimum. If the minimum moves more drastically than ϕ, behavior of ϕ is affected by the change of the potential, and our argument breaks down. However, in our case, this is not the case; time scale of the shift of the minimum is H^{-1}, which is much longer than that of the oscillation $m_{3/2}^{-1}$. As a result, ϕ can catch up with the shift of the minimum. Therefore, if we consider the oscillation around the shifted minimum, our argument is unchanged. In particular, this does not affect the calculation of the current energy density of the moduli field since the shifted minimum smoothly approaches to the true minimum as the Universe expands. (For details, see Appendix B.)

Evolution of the SUSY breaking field X is more complicated. When X starts to move, angular component of X (R-axion mode) is excited by the Affleck-Dine mechanism and R-number is generated. Furthermore, when logarithmic potential takes over the supergravity contribution, energy density of X decreases much slower than that of the moduli field.

Let us first discuss the generation of the R-number due to the Affleck-Dine mechanism [16]. For our argument, it is convenient to define the R-number density:

$$n_R = i(X^*\dot{X} - \dot{X}^*X). \tag{4.1}$$

Then, with the R-symmetry breaking terms given in Eq. (3.11), time evolution of n_R is given by

$$\begin{aligned} \dot{n}_R + 3Hn_R &= -i\left(\frac{\partial V}{\partial X}X - \frac{\partial V}{\partial X^*}X^*\right) \\ &= 2F_X^2 \mathrm{Im}(X/M_*) \times f(|X|^2/M_*^2). \end{aligned} \tag{4.2}$$

Notice that, if the potential respects the R-symmetry, the right-hand side of Eq. (4.2) vanishes.

In our case, R-symmetry breaking effect is the largest when X has the maximum amplitude, and R-number asymmetry is generated when X starts to move (see Appendix C). R-number at this stage is estimated as

$$n_R \sim H^{-1} \times 2F_X^2 \mathrm{Im}(X_0/M_*) \times f(|X_0|^2/M_*^2). \tag{4.3}$$

By using $H \sim m_{3/2}$, we obtain

$$n_R \sim \frac{2F_X^2 \xi \sin\theta_0}{m_{3/2}}, \tag{4.4}$$

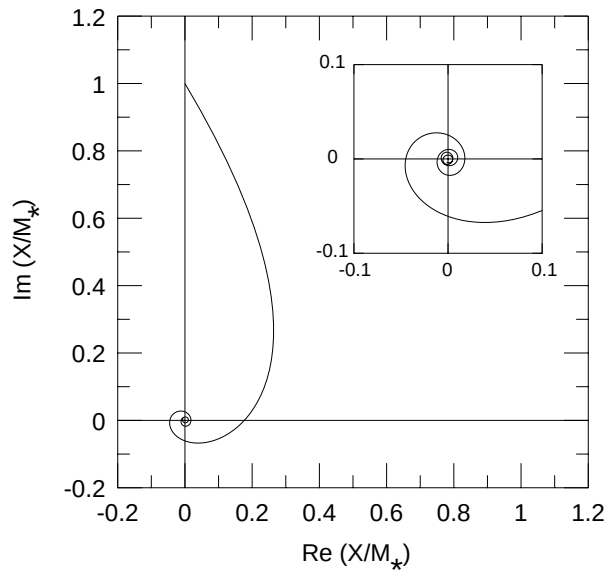

Figure 3: Initial motion of the SUSY breaking field X on the complex X plane. We choose the set of parameters as $X_0 = M_* e^{i\pi/2}$, $\zeta_1 = 0.3$, $k_1 = 0.2$, and other ζ_n's and k_n's are taken to be 0.

where the initial value of X is parameterized as $X_0 = |X_0| e^{i\theta_0}$, and $\xi \sim (|X_0|/M_*) \times f(|X_0|^2/M_*^2)$ is expected to be of $O(1)$ if $|X_0| \sim M_*$.

In Fig. 3, we show a typical behavior of X on the complex X plane. As an example, we choose the set of model parameters as

$$X_0 = M_* e^{i\pi/2}, \quad \zeta_1 = 0.3, \quad k_1 = 0.2, \tag{4.5}$$

and other ζ_n's and k_n's are taken to be 0.

In order to discuss the evolutions of the moduli field and X simultaneously, it is convenient to take the ratio of the energy density of ϕ, ρ_ϕ, to n_R. Since the initial energy density of the moduli field is of the order of $m_{3/2}^2 \phi_0^2 \sim F_X^2 \times (\phi_0/M_*)^2$, we obtain

$$\frac{\rho_\phi}{n_R} \sim \frac{m_{3/2}}{2\xi \sin\theta_0} \left(\frac{\phi_0}{M_*} \right)^2 . \tag{4.6}$$

Notice that ρ_ϕ and n_R are both proportional to R^{-3}, and the ratio ρ_ϕ/n_R remains constant as far as the R-symmetry breaking effects can be neglected.

4.2 $v \lesssim |X| \lesssim M_*$

Once the SUSY breaking field X and the moduli field ϕ start to oscillate, their amplitudes adiabatically decrease with the expansion of the Universe. Their relation is determined by Eq. (4.6), and the final result can be derived without discussing the detail of their evolutions.

However, in this subsection, we discuss how their amplitudes behave when $v \lesssim |X| \lesssim M_*$ for a better understanding of the behaviors of X and ϕ.

As discussed in Appendix D, amplitude of the scalar field φ (corresponding to X and ϕ) obeys the relation

$$m_{\rm eff}\varphi^2 R^3 = \text{const.}, \tag{4.7}$$

where the "effective mass" $m_{\rm eff}$ is defined as

$$m_{\rm eff}^2 = \frac{1}{2}\left(\frac{1}{\varphi^*}\frac{\partial V}{\partial \varphi} + \frac{1}{\varphi}\frac{\partial V}{\partial \varphi^*}\right). \tag{4.8}$$

Effective masses of X and ϕ depend on their amplitudes differently. Potential for the moduli field ϕ is parabolic, and hence $m_{\rm eff}$ is independent of the amplitude. Thus, the amplitude of the moduli field scales as $R^{-3/2}$.

We next discuss the evolution of X. For this purpose, let us remind the structure of the potential of X:[#7]

$$V \sim \frac{\zeta_1}{16\pi^2} F_X^2 \ln X^* X + m_{3/2}^2 X^* X. \tag{4.9}$$

As mentioned in the previous subsection, supergravity contribution wins the global SUSY contribution when $|X|$ is large. Comparing two contributions, supergravity effect is more important above a threshold value $X_{\rm thr}$ which is estimated as

$$X_{\rm thr} \sim \frac{\sqrt{\zeta_1}}{4\pi} M_*. \tag{4.10}$$

For $|X| \gtrsim X_{\rm thr}$, supergravity effect wins the global SUSY contribution, and $m_{\rm eff} \sim m_{3/2}$ for X. In this case, the amplitude of X scales as $R^{-3/2}$. On the other hand, for $|X| \lesssim X_{\rm thr}$, potential is dominated by the logarithmic piece, and $m_{\rm eff} \propto |X|^{-1}$. In this case, the amplitude of X is proportional to R^{-3}.[#8]

Comparing the evolutions of X and ϕ, their amplitudes are related as

$$\phi/|X| \sim \begin{cases} \phi_0/|X_0| & : \quad |X| \gtrsim X_{\rm thr} \\ (X_{\rm thr}/|X|)^{1/2} \times (\phi_0/|X_0|) & : \quad |X| \lesssim X_{\rm thr} \end{cases}, \tag{4.11}$$

with ϕ_0 and X_0 being the initial values of X and ϕ, respectively. For $|X| \gtrsim X_{\rm thr}$, X and ϕ scales in the same way, while for $|X| \lesssim X_{\rm thr}$, the amplitude of X decreases faster than that of ϕ.

The evolution of the energy density of X and ϕ is also non-trivial. When $|X| \gtrsim X_{\rm thr}$, both X and ϕ obey parabolic potential, and their energy density scale as R^{-3}. However, once

[#7] We neglect the R-symmetry breaking part; even if we include its effect, the result is unchanged.

[#8] If $|X|$ becomes close to v, potential is approximately proportional to $(\ln X^* X)^2$. In this case, this relation receives a logarithmic correction; $|X|(\ln|X|)^{1/2} R^3 = \text{const.}$

the amplitude of X becomes smaller than $X_{\rm thr}$, potential for X is lifted only logarithmically, and energy density of X decreases very slowly with the decrease of $|X|$. The important point is that, once the potential of X is dominated by the logarithmic piece, energy density of ϕ decreases much faster than that of X, and hence the energy density of the Universe is dominated by X. Therefore, when X decays, there can be a large entropy production to dilute the moduli field away.

When the amplitude of X becomes of $O(v)$, X is trapped in the minimum of the potential. Then, the amplitude of X is fixed to be v, and we need to consider the excitations around the minimum. They are phase degrees of freedom (i.e., R-axion) and radial degrees of freedom. Effects of these fields are discussed in the following subsections.

Some of the models have a SUSY preserving true vacuum at the origin $X = 0$, and one may worry whether X can be smoothly trapped in the SUSY breaking (false) vacuum of $|X| = v$. If X would overshoot down to the origin, this scenario would not be phenomenologically viable. Since X follows an elliptic trajectory on the complex X plane, as shown in Fig. 3, X can result in the SUSY breaking minimum at $|X| = v$ without being affected by the potential for $|X| \ll v$. Important point is that, thanks to the R-number conservation in the comoving volume, the orbit of X is always elliptic once the R-number is generated. Consequently, even though the amplitude of X decreases with the expansion of the Universe, orbit does not pass by the origin if enough R-number asymmetry is generated. In this case, when $|X| \sim v$, X traces a trajectory which is (approximately) parallel to the minimum of the potential ($|X| = v$), and X is smoothly trapped in the SUSY breaking vacuum. Therefore, once enough R-number asymmetry is generated, X is expected to result in the minimum of the potential $|X| = v$, irrespective of the structure of the potential for $|X| \ll v$. Of course, this scenario depends on the initial value of the R-number asymmetry. Importantly, if the initial amplitude of X is as large as M_*, Affleck-Dine mechanism can generate very large asymmetry, as discussed in the previous subsection. With a reasonable choice of the model parameters, we can easily have an elliptic trajectory of X with eccentricity close to 0. This fact helps us to understand how X can be trapped in the SUSY breaking vacuum at $|X| = v$.

4.3 Effect of the R-axion

Once the amplitude of X becomes of $O(v)$, the X field is trapped in the minimum $|X| = v$. After this stage, the imaginary part (R-axion) and the real part of X behave differently. Thus, we discuss their effects separately.

First, we argue the effect of the R-axion a. Since a is the pseudo-Nambu-Goldstone boson, its potential is approximately flat with slight perturbation due to $V_{\not{R}}$. When we can neglect the R-symmetry breaking effects, motion of a corresponds to the phase rotation of X. At this period, it is convenient to parametrize the X field as

$$X = ve^{i\omega t}. \tag{4.12}$$

Here, we neglect the real part of the X field. Then, the R-number is given by

$$n_R = 2\omega v^2 = 2\rho_a/\omega, \tag{4.13}$$

where ρ_a is the energy density of the R-axion. With the expansion of the Universe, ω decreases adiabatically; since the R-number in the comoving volume is conserved, ω scales as R^{-3}.[#9]

Once the energy density of the R-axion becomes less than the difference of the potential energy ΔV on the circle $|X| = v$, the R-axion starts to oscillate around its minimum. Once this happens, R-symmetry is not a good symmetry any more, and the R-axion approximately obeys the parabolic potential. After this stage, energy density of a scales as R^{-3}. With the potential given in Eq. (3.18), the difference of the potential energy is given by $\Delta V = V(X = -v) - V(X = v) = 4m_a^2 v^2$, and hence two cases should be matched when $\omega \sim O(m_a)$. Connecting two cases at $\omega = \omega_{\rm c}$, we obtain

$$\frac{\rho_\phi}{\rho_a} \sim \frac{\rho_\phi}{(\omega_{\rm c} n_R/2)} \sim \frac{m_{3/2}}{\omega_{\rm c}\xi \sin\theta_0}\left(\frac{\phi_0}{M_*}\right)^2 . \tag{4.14}$$

This ratio remains constant until the R-axion decays.[#10]

Motion of the R-axion at $\rho_a \sim m_a^2 v^2$ is complicated, and analytic estimation of $\omega_{\rm c}$ is difficult. In our analysis, we numerically followed the evolution of the R-axion, and estimated the value of ω for matching. As a result of the numerical calculation, we found that two cases should be connected with

$$\omega = \omega_{\rm c} \simeq 4m_a . \tag{4.15}$$

In the following discussion, we use $\omega_{\rm c} = 4m_a$.

When the expansion rate of the Universe becomes comparable to the decay rate of the R-axion, the R-axion decays and the Universe is reheated. By using the instantaneous decay approximation, the reheating temperature $T_{\rm R}$ is estimated as

$$\begin{aligned} T_{\rm R} &\sim \left(\frac{\pi^2 g_*}{90}\right)^{-1/4} \sqrt{\Gamma_a M_*} \\ &\sim 20\ {\rm MeV} \times k_0^{3/4} \left(\alpha_{\rm SM} b_q n_{\rm G}^{1/2}\right) \left(\frac{g_*}{10}\right)^{-1/4} \left(\frac{B_Q}{10^5\ {\rm GeV}}\right)^{3/2} \left(\frac{v}{10^{10}\ {\rm GeV}}\right)^{-1/4} , \end{aligned} \tag{4.16}$$

where we used the decay rate given in Eq. (3.20) in the second line. (A special case where this may be irrelevant will be discussed later.) At this stage, the energy density of the R-axion is converted to that of the radiation, and large amount of entropy is produced. At the

[#9] For a scalar field φ with flat potential, equation of motion is given by $\ddot{\varphi} + 3H\dot{\varphi} = 0$. By solving this equation, we obtain $\dot{\varphi}R^3 = $ const. Evolution of ω is consistent with this relation, if we re-interpret $e^{i\omega t} \to e^{ia/\sqrt{2}v}$.

[#10] If the amplitude of X is largely fluctuated, domain wall is produced when the R-axion gets trapped in the minimum of the potential. Such a fluctuation is generated during the primordial inflation, and the domain wall production may be effective if the expansion rate during the inflation is larger than of $O(v)$ [27]. Therefore, if we adopt a primordial inflation with relatively small expansion rate, the domain wall production can be evaded. Furthermore, even if the domain wall production is effective, collapse of the domain wall results in semi-relativistic R-axions with averaged energy of $\sim 3m_a$ [28]. Therefore, domain wall production does not modify the ratio ρ_ϕ/ρ_a given in Eq. (4.14) so much, and the dilution factor calculated below is almost unchanged.

decay time, energy density of radiation is given by $\rho_{\rm rad} = \rho_a$, and hence

$$\frac{\rho_\phi}{s} \sim \frac{3}{4} T_{\rm R} \times \frac{m_{3/2}}{4 m_a \xi \sin\theta_0} \left(\frac{\phi_0}{M_*} \right)^2 , \tag{4.17}$$

where s is the entropy density which is related to the energy density of the radiation as $\rho_{\rm rad} = \frac{3}{4} T s$. We compare Eq. (4.17) with the ratio of the critical density to the entropy density in the present Universe:

$$\frac{\rho_{\rm c}}{s} \simeq 3.6 \times 10^{-9} \text{ GeV} \times h^2 , \tag{4.18}$$

and we obtain

$$\Omega_\phi h^2 \sim 40 \times k_0^{1/4} \left(\alpha_{\rm SM} b_q n_{\rm G}^{1/2} \right) \left(\frac{g_*}{10} \right)^{-1/4} \left(\frac{B_Q}{10^5 \text{ GeV}} \right)^{3/2} \left(\frac{v}{10^{10} \text{ GeV}} \right)^{1/4} \left(\frac{\phi_0}{M_*} \right)^2 , \tag{4.19}$$

where we used $\xi \sin\theta_0 \sim 1$.

Now, we can calculate the numerical value of the density parameter, and see if it is cosmologically viable. For this purpose, we first have to specify the dominant decay mode of the R-axion, since the result depends on the decay width Γ_a. If the R-axion mass is large enough (probably, for $m_a \gtrsim 1$ GeV), the R-axion decays into the gluon pair. In this case, we can use the calculation based on the perturbative QCD, and the decay rate is given in the formula in Eq. (3.20) with $\alpha_{\rm SM} = \alpha_{\rm s}$. However, if the R-axion mass becomes as small as (or smaller than) ~ 1 GeV, Eq. (3.20) may not be reliable, since in this case, decay rate into multi-meson final states has to be calculated. However, if the R-axion mass is light enough, decay modes into multi-meson final states are kinematically forbidden. Since the R-axion is a CP-odd particle, its mass has to be larger than at least $3m_\pi$ for the decay into final states without electromagnetic particles. Therefore, if $m_a < 3m_\pi$, $a \to \gamma\gamma$ is expected to be the dominant decay mode. In this case, we can use the Eq. (3.20) again with $\alpha_{\rm SM} = \alpha_{\rm em}$. In the case $3m_\pi \leq m_a \lesssim 1$ GeV, estimation of the decay rate is quite difficult, and we will not discuss this case further in this paper.

In Figs. 4 and 5, we plotted $\Omega_\phi h^2$ on the v vs. B_Q plane with $\phi_0 = M_*$ and $N_5 = 2$. In the calculation, we used Eq. (4.17) with $\xi \sin\theta_0 = 1$, and assumed that $a \to \gamma\gamma$ and $a \to gg$ are the dominant decay modes of the R-axion for $m_a \leq 3m_\pi$ and $m_a \geq 1$ GeV, respectively. In these figures, we shaded the region where the reheating temperature becomes lower than 1 MeV. We also show the contours of the constant $v/F_X^{1/2}$, which has to be larger than $\sim 1 - 10$.

Let us first discuss the case of $m_a \leq 3m_\pi$, where the R-axion decays into the photon pair. This is the case for the smaller value of v (i.e., $v \lesssim 10^8 - 10^9$ GeV). If the R-axion decays into the photon pair, the reheating temperature becomes relatively low. However, if B_Q is large enough, there is still a parameter region where the reheating temperature is high enough ($T_{\rm R} \gtrsim 1$ MeV). At the same time, Ω_ϕ can be of $O(0.1)$ or smaller, and hence the moduli field can be diluted enough by the decay of the R-axion.

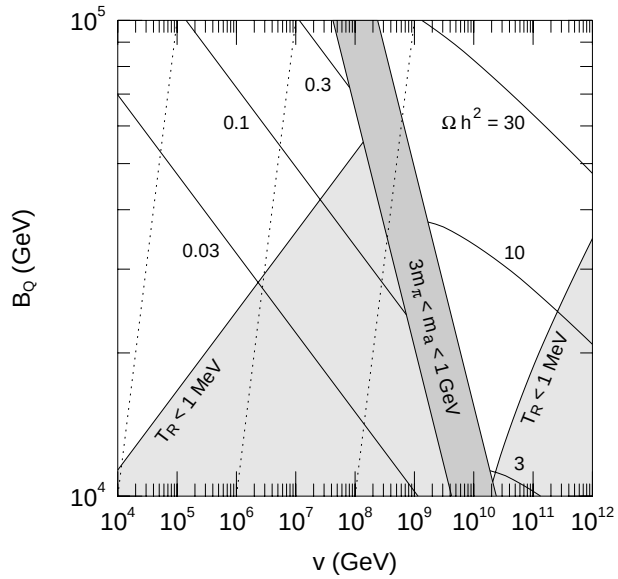

Figure 4: Constant $\Omega_\phi h^2$ contours on the v vs. B_Q plane with $\phi_0 = M_*$, $N_5 = 2$ and $k_0 = 1$. Region with $T_{\rm R} < 1$ MeV is lightly shaded, and darkly shaded region corresponds to $3m_\pi \leq m_a \leq 1$ GeV. Contours of constant $v/F_X^{1/2}$ are also shown in dotted lines (1, 10, and 100, from left to right).

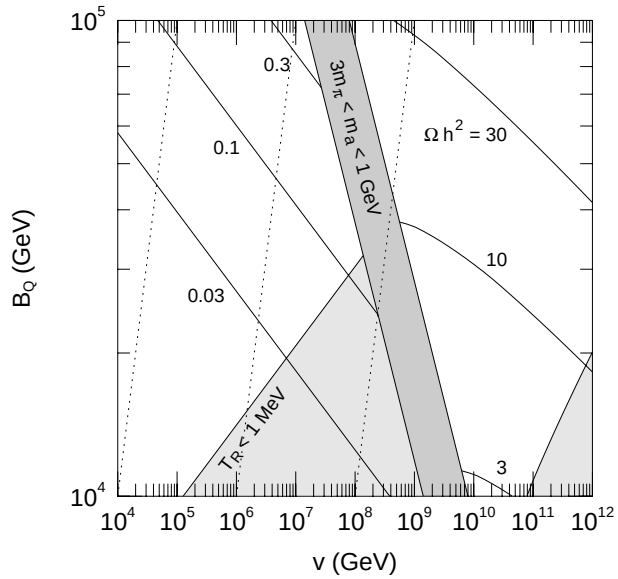

Figure 5: Same as Fig. 4, except for $k_0 = 3$.

One problem of this case may be the non-perturbative effect discussed in Section 3. In the case of $v \lesssim 10^9$ GeV, non-perturbative effect becomes sizable in the Kähler potential, and hence the potential of X cannot be well understood. Therefore, we need to make a dynamical assumption so that the SUSY breaking vacuum exists. However, since the non-perturbative effect respects the R-symmetry, important properties of the R-axion are unchanged. Therefore, once we assume the existence of the SUSY breaking minimum, our arguments are unchanged. One may also worry about the stability of the vacuum if there is a SUSY preserving true minimum at the origin. However, $T_{\rm R} \gtrsim 1$ MeV can be realized even with $v/F_X^{1/2} \gtrsim 10$. Furthermore, this problem can be evaded in models without SUSY preserving minimum.

For a larger value of v (i.e., $v \gtrsim 10^9 - 10^{10}$ GeV), the R-axion becomes heavier than ~ 1 GeV, and decays into the gluon pair. As we can see in Figs. 4 and 5, if the R-axion decays into the gluons, the density parameter of the moduli field becomes relatively larger. This is because the decay rate is more enhanced, resulting in higher reheating temperature. Even in this case, however, $\Omega_\phi \sim 10$ is possible for $\phi_0 \sim M_*$. Reduction of Ω_ϕ by a factor of about 10 may not be a serious problem. For example, suppression of the initial amplitude of the moduli field by a factor of 3 or so is enough for $\Omega_\phi \leq 1$, since Ω_ϕ is proportional to ϕ_0^2. We can naturally imagine an accidental cancellation of this level. Notice that the non-perturbative effect on the Kähler potential is not important in (most of) this case.

4.4 Effect of the Real Part

Next, we discuss the effect of the decay of the real part σ. For this purpose, it is convenient to use the relation $m_{\rm eff}\varphi^2 \propto R^{-3}$ (with $\varphi = X$ and ϕ). Comparing this quantity at $|X| \sim X_0$ and $|X| \sim v$, we obtain

$$\frac{m_{3/2}\phi^2}{m_\sigma \sigma^2} \sim \left(\frac{\phi_0}{X_0}\right)^2 . \tag{4.20}$$

The important point is that the effective mass for the moduli field is always constant of $O(m_{3/2})$, while that for X varies from $\sim m_{3/2}$ to m_σ which is of the order of the SSM scale.

Once X gets trapped in the SUSY breaking minimum, σ obeys the parabolic potential, and hence Eq. (4.20) leads

$$\frac{\rho_\phi}{\rho_\sigma} \sim \frac{m_{3/2}}{m_\sigma}\left(\frac{\phi_0}{X_0}\right)^2 , \tag{4.21}$$

where ρ_σ is the energy density of σ.

When the expansion rate becomes comparable to Γ_σ, σ decays. As discussed in Section 3, σ dominantly decays into the R-axion pair. The number density of σ, n_σ, is related to the energy density as $\rho_\sigma = m_\sigma n_\sigma$. Therefore, by using Eq. (4.21) with $|X_0| \sim M_*$, we obtain

$$\frac{\rho_\phi}{n_a^{\rm dec}} \sim m_{3/2}\left(\frac{\phi_0}{M_*}\right)^2 , \tag{4.22}$$

where $n_a^{\rm dec}$ is the number density of the R-axion produced by the decay of σ. Notice that this ratio is constant even with entropy production.

As the Universe expands, the emitted R-axions are red-shifted and eventually become non-relativistic. Then, when they decay, they also contribute to the entropy production to dilute the moduli field. Comparing Eq. (4.22) with Eq. (4.6), we see that the number of the R-axion produced by the decay of σ is comparable to that of the coherent mode. Therefore, the dilution by this incoherent R-axion is of the same order of that by the coherent mode, and the results given in the previous subsection are almost unchanged.

Finally, we discuss the effect of the decay mode into the gravitino pair. As discussed in Section 3, σ and a may decay into the gravitino. Since the gravitino is stable in GMSB, the mass density of the gravitino, $\rho_{3/2}$, should not exceed the closure limit.

Let us consider the gravitino production due to the decay of σ as an example. In order to discuss the mass density of the gravitino by the decay of σ (and of the R-axion), it is convenient to consider the ratio $m_{3/2}n_{3/2}/\rho_\phi \sim \rho_{3/2}/\rho_\phi$, where $n_{3/2}$ is the number density of the gravitino. This ratio can be easily calculated from Eq. (4.21). When σ decays, the number density of the gravitino is given by

$$n_{3/2} \sim {\rm Br}(\sigma \to \psi_\mu \psi_\mu) \times n_\sigma \sim {\rm Br}(\sigma \to \psi_\mu \psi_\mu) \times \frac{\rho_\sigma}{m_\sigma}. \tag{4.23}$$

Combining this equation with Eq. (4.21) and $|X_0| \sim M_*$, we obtain

$$\frac{m_{3/2}n_{3/2}}{\rho_\phi} \sim {\rm Br}(\sigma \to \psi_\mu \psi_\mu) \times \left(\frac{\phi_0}{M_*}\right)^{-2}. \tag{4.24}$$

Importantly, this ratio remains constant even after a large entropy production. Once the gravitinos are red-shifted to be non-relativistic, the above ratio becomes $\rho_{3/2}/\rho_\phi$. As a result, the density parameter of the gravitino, $\Omega_{3/2}$, is related to Ω_ϕ as

$$\Omega_{3/2} \sim {\rm Br}(\sigma \to \psi_\mu \psi_\mu) \times \Omega_\phi \left(\frac{\phi_0}{M_*}\right)^{-2}. \tag{4.25}$$

It is also straightforward to check that the density parameter of the gravitino from the decay of the R-axion is given by a similar formula (with extra coefficient of $O(1)$) with the relevant branching ratio for the R-axion.

With the branching ratio given in Eqs. (3.25) and (3.26), we can see that the mass density of the gravitino is small enough. By using the formula for m_σ, ${\rm Br}(\sigma \to \psi_\mu \psi_\mu)$ is estimated to be of $O(10^{-4})$. Furthermore, in most of the parameter region we are interested in, the R-axion mass is smaller than about 10 GeV, and hence ${\rm Br}(a \to \psi_\mu \psi_\mu)$ is at most of $O(10^{-2})$. Therefore, even with $\Omega_\phi \times (\phi_0/M_*)^{-2} \sim 10$ for $v \gtrsim 10^9 - 10^{10}$ GeV (i.e., for $m_a \gtrsim 1$ GeV), the energy density of the gravitino is small enough.

4.5 Baryogenesis and the Flat Direction in SSM

So far, we have discussed the dilution of the moduli field by the decay of the R-axion. In this scenario, one may worry about the baryogenesis, since the reheating temperature seems to be too low to generate the baryon asymmetry. In this subsection, we briefly comment that enough baryon asymmetry can be generated by the Affleck-Dine mechanism [16]. (For a detailed discussion, see Ref. [5].)

Important assumptions for the Affleck-Dine baryogenesis are that SSM flat direction $\varphi_{\rm SSM}$ (for Affleck-Dine baryogenesis, we call it Affleck-Dine field) has a large amplitude in the early Universe, and that there is baryon-number breaking term in the potential of $\varphi_{\rm SSM}$. With these assumptions, non-vanishing baryon-number can be generated as the Affleck-Dine field evolves. Physics in this mechanism is basically the same as that in the case of the R-number generation.[#11] Since the potential for the Affleck-Dine field is dominated by the supergravity contribution for $\varphi_{\rm SSM} \sim M_*$, the ratio of the energy density of the moduli field to the baryon number density $n_{\rm B}$ is estimated as [5]

$$\frac{\rho_\phi}{n_{\rm B}} \sim m_{3/2} \left(\frac{\phi_0}{M_*} \right)^2 , \tag{4.26}$$

where we applied a similar argument as in the R-axion case. Here, we assumed that the initial amplitude of $\varphi_{\rm SSM}$ is of $O(M_*)$ and that source of CP violation is of $O(1)$, which are corresponding to $\xi \sim 1$ and $\sin\theta_0 \sim 1$ in the R-axion case, respectively. Notice that this ratio is constant after the moduli and Affleck-Dine fields start to move.

With Eq. (4.26), we can estimate the baryon number density of the present Universe, once we fix the current mass density of the moduli field. As we discussed in the previous subsections, present mass density of the moduli field depends on the magnitude of the entropy production. In this subsection, we just assume the moduli field is diluted enough by the decay of the R-axion. Then, adopting $\Omega_\phi \lesssim 1$, baryon-to-entropy ratio is estimated as [5]

$$\frac{n_{\rm B}}{s} \lesssim 4 \times 10^{-5} \times h^{-2} \left(\frac{m_{3/2}}{100 \text{ keV}} \right)^{-1} \left(\frac{\phi_0}{M_*} \right)^{-2} . \tag{4.27}$$

Comparing the above relation with the baryon number density required from the big-bang nucleosynthesis ($n_{\rm B}/s \sim O(10^{-11})$ [29]), we can see that enough baryon asymmetry can remain even if there is a large entropy production to dilute the moduli field. In fact, as can be seen in Eq. (4.27), baryon-to-entropy ratio may be too large for some value of the gravitino mass, if we adopt a naive initial condition for the Affleck-Dine field. However, this problem may be solved by adopting a smaller value of the source of CP violation, or a smaller value of the initial amplitude of the Affleck-Dine field. Therefore, we do not worry about this issue.

Even apart from the baryogenesis, effect of the SSM flat direction may be interesting, since it may produce a large entropy. The potential for the flat direction has a similarity to

[#11] The structures of the baryon-number breaking terms may be different. However, it does not affect the following discussions, as far as the initial amplitude of $\varphi_{\rm SSM}$ is of $O(M_*)$.

that of the σ field; it is parabolic around the minimum (i.e., around the origin), then the potential is lifted only logarithmically once the amplitude becomes larger than the messenger scale, and finally, supergravity effect dominates when the amplitude is close M_*. Therefore, its effective mass varies from $\sim m_{3/2}$ to the SSM scale, if $\varphi_{\rm SSM}$ changes its amplitude from $\sim M_*$ to ~ 0. By applying the similar argument as in the σ case, we can estimate the density parameter of the moduli field:

$$\Omega_\phi h^2 \sim 2 \times 10^3 \times \left(\frac{m_{\rm SSM}}{1\ {\rm TeV}}\right)^{-1} \left(\frac{m_{3/2}}{1\ {\rm keV}}\right) \left(\frac{T_{\rm SSM}}{10\ {\rm TeV}}\right) \left(\frac{\phi_0}{M_*}\right)^2 , \tag{4.28}$$

where $T_{\rm SSM}$ is the reheating temperature due to the decay of the flat direction. Comparing this result with Eq. (2.1), we can see that a large entropy production is possible even from the flat direction in SSM.

However, usually, the reheating temperature is of $O(10\ {\rm TeV})$, and hence the density parameter becomes typically of $O(10^3)$ even for $m_{3/2} \sim 1$ keV. Therefore, it seems difficult to dilute the moduli density enough in a simple scenario, unless we come up with a model with very light gravitino. However, if the reheating temperature can be somehow lowered down to ~ 10 GeV, dilution due to the decay of the SSM flat direction may be enough to solve the cosmological moduli problem. For example, Pauli blocking may delay the decay of the flat direction, as pointed out in Ref. [30], although the baryon asymmetry may be generated too much if we naively apply the argument in Ref. [30].

5 Discussion

In this paper, we have discussed the cosmology based on the direct gauge mediation model with the inverted hierarchy mechanism. In particular, we have studied the implication of the SUSY breaking field on the cosmology of GMSB.

If the SUSY breaking flat direction X initially has a very large amplitude of $O(M_*)$, it can be a source of the large entropy production. In particular, once the amplitude of X becomes a few orders of magnitude smaller than M_*, potential for X is dominated by the logarithmic piece. Then, the energy density of X decreases more slowly than those of the scalars with quadratic potential, and the SUSY breaking field may play a very important role in cosmology.

In particular, the entropy production by the decay of the R-axion may be so large that the energy density of the moduli field can be diluted enough. Therefore, the direct gauge mediation models with the inverted hierarchy mechanism contain a natural candidate of the large entropy production to solve the serious cosmological moduli problem. We have also seen that enough baryon asymmetry can be generated by the Affleck-Dine mechanism even with this large entropy production.

In our discussion, we mainly focused on the case of the direct gauge mediation model with the inverted hierarchy mechanism. However, the scenario discussed in this paper can be applied to a larger class of models, since the most important building block is just the

logarithmically lifted potential of the SUSY breaking field for $|X| \gtrsim v$. Consequently, if a direct gauge mediation model uses the mechanism of SUSY breaking proposed by Izawa and Yanagida, and by Intriligator and Thomas [14], it automatically contains a reasonable source of large entropy production. Of course, it is non-trivial for such a model to stabilize the potential of the SUSY breaking field. The inverted hierarchy mechanism provides one attractive mechanism for the stabilization [7, 8, 9, 10]. Other approach may be to use a non-perturbative effect on the Kähler potential [11].

One may worry about the astrophysical constraints on the R-axion, as in the case of the QCD axion. However, since the R-axion has a larger mass than the QCD axion, constraints on the R-axion are much weaker. Constraints from the cooling of the horizontal-branch (HB) stars [31] are evaded, since we consider R-axion heavier than 100 keV (see Eq. (3.19)) while the core temperature the HB stars are typically of $O(10\text{ keV})$. Furthermore, the R-axion does not affect the background UV light [32], since it has already decayed away. Constraints from SN1987A are more non-trivial. In order not to affect the cooling of SN1987A, QCD axion with the decay constant from $\sim 10^6$ GeV to $\sim 10^9$ GeV is forbidden [33]. However, for this range of v, the R-axion mass is (almost) always larger than the core temperature of SN1987A (i.e., $O(10\text{ MeV})$), and hence the emission of the R-axion is suppressed. (However, small parameter space around $v \sim 10^6$ GeV and $B_Q \sim 10^4$ GeV may be excluded, though the reheating temperature is too low in this region.) When $v \lesssim 10^6$ GeV, the R-axion is thermalized enough in SN1987A, and it does not affect the cooling process. The QCD axion with small decay constant may be detected in water Čerenkov detectors [34]. However, the R-axion cannot be constrained with this method, since R-axion decays before reaching the earth. This fact suggests another constraint on the R-axion; if the emitted R-axions decay into the photons on the way to the earth, apparent luminosity of SN1987A may be increased. Therefore, for our scenario, light R-axion is potentially dangerous. However, the estimation of the R-axion flux is very complicated, in particular since the R-axion may have a mass comparable to the core temperature of SN1987A. Therefore, it is an open question which parameter region is excluded from this argument. Notice that, if the R-axion mass is heavier than of $O(10\text{ MeV})$, this problem can be evaded thanks to the Boltzmann suppression.

In this paper, we have not paid attention to the primordial inflation, since it is beyond the scope of this paper. Of course, it is important to find a viable candidate of the inflaton for the primordial inflation, and some efforts are made in this issue [35]. Here, we just mention that, in our scenario, inflaton for the primordial inflation does not have to decay into the particles in the SSM sector. Since the background radiation and baryons in the present Universe originate to the decay of the SUSY breaking field X (and probably, to the decay of the Affleck-Dine field), primordial inflation is not required to reheat the SSM sector. In an extreme case, inflaton may decay only into particles in the hidden sector. Even if the energy density of the Universe is once dominated by that of the hidden sector particles, it is eventually diluted by the entropy production by the decay of the SUSY breaking field. This fact relaxes the conventional requirements on the inflaton which is usually required to decay into the SSM particles.

It is interesting to consider candidates of the CDM in this scenario. Because of the low

reheating temperature after the decay of the R-axion, thermal productions of any known candidates are inefficient. In this case, a possible candidate is the coherent oscillation of the moduli field. Indeed, if the energy density of the moduli field can be diluted to be $\Omega_\phi \sim 1$ by the decay of the R-axion, they can be a viable candidate of the CDM. This scenario is constrained by the line spectrum of the background cosmic X-ray emitted from the decay of the moduli field [36]. Even if the lifetime is much longer than the age of the Universe, some fraction of the moduli field has already decayed into the photons, and it contributes to the background X-ray spectrum. Due to the negative observation of such a line spectrum, moduli field heavier than about 200 keV is forbidden, if $\Omega_\phi h^2 = 1$ [36].[#12] In the direct gauge mediation model, the gravitino mass (i.e., the moduli mass) can be lighter than about 100 keV, and hence the moduli can be a good candidate of the CDM.

Cosmological implication of the R-axion in other classes of GMSB is another interesting issue. In general, dynamical SUSY breaking requires spontaneously broken R-symmetry [37]. Therefore, all the dynamical SUSY breaking models contain R-axion in the low energy spectrum. Then, there is a possibility of large R-number generation by the mechanism we discussed, if the SUSY breaking field has a large initial amplitude. However, the interaction of the R-axion is model-dependent, and in some case, its decay rate may be much more suppressed. In this case, it causes a cosmological difficulty, since the reheating temperature after the decay of the R-axion becomes too low for the big-bang nucleosynthesis. Of course, this problem itself can be always evaded by assuming a small initial amplitude of the SUSY breaking field.

Since different models introduce different sets of new particles which have various properties, detailed cosmological scenario is model-dependent. Therefore, one should always keep in mind that the SUSY breaking field (and all the new degrees of freedom) may change the conventional arguments on the cosmology based on supersymmetric models. In some case, it may cause a serious cosmological disaster, but in other case, as we have seen, it may provide a natural and well-motivated solution to several serious cosmological difficulties.

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[#12] However, if the interaction of the moduli field with the photon has extra suppression, lifetime of the moduli field gets longer and this constraint becomes less stringent. In this case, heavier moduli field is still allowed. For example, this may happen if the moduli-photon-photon vertex is induced by loop effects.

A Example of the Model

In this Appendix, we show an example of the direct gauge mediation model in which vacuum is stabilized by the inverted hierarchy mechanism. The model, which is originally proposed in Ref. [8], is based on the symmetry $\mathrm{SU}(2)_{\mathrm{B1}} \times \mathrm{SU}(2)_{\mathrm{B2}} \times \mathrm{SU}(2)_{\mathrm{S}} \times \mathrm{G}_{\mathrm{SM}}$. In this model, $\mathrm{SU}(2)_{\mathrm{B1}}$ is a gauge group which stabilize the minimum of the potential, while $\mathrm{SU}(2)_{\mathrm{S}}$ provides a strong gauge interaction which induces gaugino condensation to break SUSY. Furthermore, G_{SM} is the standard model gauge group. The particle content is shown in the Table 1.

Superpotential in this model is given by

$$W = y_Q \Sigma \bar{Q} Q + y_3 \Sigma \bar{q}_3 q_3 + y_2 \Sigma \bar{q}_2 q_2 + y_1 \Sigma \bar{q}_1 q_1. \tag{A.1}$$

With this superpotential, we concentrate on the flat direction of $X \sim (\det\Sigma)^{1/2}$. Along X, we parametrize

$$\Sigma = \frac{1}{\sqrt{2}} \mathrm{diag}(X, X). \tag{A.2}$$

Once Σ gets this VEV, Q and $\bar{Q}$ acquire a mass of $m_Q \simeq \frac{1}{\sqrt{2}} y_Q X$. For $\mu \lesssim m_Q$, $\mathrm{SU}(2)_{\mathrm{S}}$ is a pure SUSY Yang-Milles theory, and gaugino condensation induces the superpotential of the form $W_{\mathrm{eff}} = 2\Lambda_{\mathrm{eff}}^3$, where Λ_{eff} is the strong scale of $\mathrm{SU}(2)_{\mathrm{S}}$ below the mass scale of Q and $\bar{Q}$. By matching the strong scales for the theory below and above m_Q, we obtain

$$W_{\mathrm{eff}} = \sqrt{2} y_Q X \Lambda^2, \tag{A.3}$$

where Λ is the strong scale for the theory above m_Q. Since the superpotential is linear in X, F-component of X has a VEV of $F_X = \sqrt{2} y_Q \Lambda^2$, and SUSY is broken.

The minimum of the potential is determined by the inverted hierarchy mechanism [13]. At the tree level, potential for X is completely flat, and hence the scale dependence of the wave function normalization of Σ determines the position of the minimum. In this case, the potential for X is given by

$$V = \frac{F_X^2}{Z_\Sigma(X^*, X)}, \tag{A.4}$$

where $Z_\Sigma(X^*, X)$ is the wave function normalization of Σ which is evaluated at $\mu = |X|$. At the 1-loop level, RGE for Z_Σ is given by

$$\frac{d \ln Z_\Sigma}{dt} = \frac{1}{16\pi^2} \left(\frac{3}{2} g_{\mathrm{B1}}^2 + \frac{3}{2} g_{\mathrm{B2}}^2 - 2y_Q^2 - 3y_3^2 - 2y_2^2 - y_1^2 \right), \tag{A.5}$$

where g_{B1} and g_{B2} are the gauge coupling constants for $\mathrm{SU}(2)_{\mathrm{B1}}$ and $\mathrm{SU}(2)_{\mathrm{B2}}$, respectively. Thus, X has an extremum at $X = v$, where

$$\frac{3}{2}(g_{\mathrm{B1}}^2 + g_{\mathrm{B2}}^2) = 2y_Q^2 + 3y_3^2 + 2y_2^2 + y_1^2. \tag{A.6}$$

	$SU(2)_{B1}$	$SU(2)_{B2}$	$SU(2)_S$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Σ	**2**	**2**	**1**	**1**	**1**	0
Q	**2**	**1**	**2**	**1**	**1**	0
$\bar{Q}$	**1**	**2**	**2**	**1**	**1**	0
q_3	**2**	**1**	**1**	**3**	**1**	$-1/3$
$\bar{q}_3$	**1**	**2**	**1**	$\mathbf{3}^*$	**1**	$1/3$
q_2	**2**	**1**	**1**	**1**	**2**	$1/2$
$\bar{q}_2$	**1**	**2**	**1**	**1**	**2**	$-1/2$
q_1	**2**	**1**	**1**	**1**	**1**	0
$\bar{q}_1$	**1**	**2**	**1**	**1**	**1**	0

Table 1: Particle content of a direct gauge mediation model given in Ref. [8].

In order to see whether this is a minimum or a maximum, it is convenient to estimate the mass of the real part of X, which we call σ.[#13] For simplicity, we consider the case where g_{B2}, y_3, and y_2 are small. (Even in the general case, the following discussion is qualitatively correct.) Then, mass of σ is given by

$$m_\sigma^2 \simeq \frac{1}{(16\pi^2)^2}\left(\frac{33}{4}g_{B1}^4 + 24y_Q^4 - 24g_{B1}^2y_Q^2 - 6g_S^2y_Q^2\right)\frac{F_X^2}{v^2}, \tag{A.7}$$

where g_S is the gauge coupling constant of $SU(2)_S$. Importantly, g_S is usually large in order to induces the strong dynamics to break SUSY. Then, for large y_Q, $g_S^2y_Q^2$ term becomes so large that m_σ^2 becomes negative. (For large enough y_Q, m_σ^2 may become positive, but y_Q blows up below the Planck scale.) Numerically, y_Q cannot be larger than $0.2 - 0.3$ for the positivity of m_σ^2. For $v \gtrsim 10^9$ GeV, solution to Eq. (A.6) with positive m_σ^2 can be found with reasonable values of the coupling constants.

For $v \lesssim 10^9$ GeV, we cannot neglect the non-perturbative effects, as we discussed in Section 3. In this case, Kähler potential is dominated by the non-perturbative piece, and it is unclear whether there can be a minimum. However, since the non-perturbative effects are not well understood, there is a possibility to have a stable minimum even with the non-perturbative effect. Therefore, for $v \lesssim 10^9$ GeV, we make a dynamical assumption so that the stable SUSY breaking vacuum exists. Notice that, in this case, upper bound on y_Q is irrelevant.

Once the SUSY is broken and the VEV of X is fixed, SUSY breaking is mediated down to the SSM sector by integrating out messengers, q_i and $\bar{q}_i$ $(i = 3, 2)$. Since these are the only superfields with standard model quantum numbers which couple to the SUSY breaking field, SUSY breaking masses obey the well-known mass formula [38]. Notice that, in this model, $N_5 = 2$ (see Eq. (3.20)).

[#13] Imaginary part is pseudo-Nambu-Goldstone boson, and its mass is from the supergravity effect, as discussed in Section 3.

Several remarks are in order. First, for $\mu \lesssim v$, diagonal SU(2) remains if $\mathrm{SU}(2)_{\mathrm{B2}}$ is gauged. For our cosmological scenario, this may be dangerous, since the R-axion also couples to this gauge field. If R-axion dominantly decays into this gauge field, energy density of the standard model particles cannot be generated enough since the gauge field for this diagonal SU(2) does not couple to the SSM sector. This problem can be evaded if $\mathrm{SU}(2)_{\mathrm{B2}}$ interaction is weak enough (i.e., much weaker than the electromagnetic interaction, or maybe not gauged). Another comment is on the true vacuum of the potential. In fact, this model has a SUSY preserving true vacuum at the origin ($|X| = 0$). However, tunneling rate from the SUSY breaking minimum to the true minimum is small enough, if the SUSY breaking vacuum is far enough away from the confinement scale Λ; numerically, $v/\Lambda \gtrsim 10$ [8].

B Shift of the Minimum of the Moduli Potential

In this Appendix, we consider how the minimum of the moduli potential shifts when the SUSY breaking field X is oscillating. When the Universe is dominated by X, extra terms are induced in the moduli potential by supergravity effects, and they may change the minimum of the moduli potential. If this shift is too large, it may change our argument, since the moduli field may oscillate around a shifted minimum. In this Appendix, we see that this effect is not significant for our case, and that our naive calculations are relevant.

First, let us consider possible modifications of the moduli potential in the presence of X. In supergravity, there can be two effects. One is from the non-vanishing VEV of X; since X and ϕ may have Planck-suppressed interactions, potential may have terms which are proportional to the powers of $(|X|/M_*)$. The other is from the expansion rate H induced by the condensation of X; since the scalar potential contains a term of the form $\sim e^{K/M_*^2} V$, non-vanishing potential energy induces terms proportional to H^2 [15].

With these effects, linear term is induced in the moduli potential, which shifts the minimum of the potential:

$$V(\phi) \sim m_{3/2}^2 \phi^* \phi - m_{3/2}^2 (\bar{\phi}^* \phi + \mathrm{h.c.}), \tag{B.1}$$

where we define the origin of the moduli field so that VEV of ϕ vanishes for the empty background. Here, the second term in Eq. (B.1) is the induced term, and $\bar{\phi}$ is given by

$$\bar{\phi} \sim \max\left[|X|, (H^2/m_{3/2}^2) M_*\right]. \tag{B.2}$$

(In this section, we neglect $O(1)$ coefficients which do not change our argument.) Notice that $\bar{\phi}$ is the shifted minimum of the potential, and that it is time-dependent.[#14] With the

[#14] Other terms (higher order terms) are less significant for our argument, and they do not change the following discussion. For example, if the potential has a term of the form $H^2\phi^2$, solution to the equation of motion contains a term of $O(H^2/m_{3/2}^2)\phi_{\mathrm{osc}}$, with ϕ_{osc} being the solution to the equation of motion with $H = 0$. However, this is much smaller than the original amplitude ϕ_{osc} since $H \ll m_{3/2}$, and hence this effect is negligible.

above potential, equation of motion for ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + m_{3/2}^2(\phi - \bar{\phi}) = 0. \tag{B.3}$$

Solution to this equation can be written as

$$\phi = \phi_{\rm osc} + \delta\phi, \tag{B.4}$$

where $\phi_{\rm osc}$ is the oscillating solution with $\bar{\phi} = 0$, while $\delta\phi$ is a perturbation induced by the new terms. Notice that $\phi_{\rm osc}$ obeys the original equation of motion:

$$\ddot{\phi}_{\rm osc} + 3H\dot{\phi}_{\rm osc} + m_{3/2}^2\phi_{\rm osc} = 0, \tag{B.5}$$

and hence its averaged amplitude is proportional to $R^{-3/2}$. Thus, $\phi_{\rm osc}$ obeys the behavior discussed in Section 4.

If $\bar{\phi} \sim |X|$, the shift cannot be larger than the averaged amplitude of $\phi_{\rm osc}$ (see Eq. (4.11)). In this case, the original amplitude is always larger than the shift of the minimum, and hence the extra contribution is negligible. Thus, in the following discussion, we concentrate on the case where $\bar{\phi}$ is dominated by the Hubble-induced term.

In order to consider the Hubble-induced term, we approximate the potential of X as

$$V \sim \frac{\zeta_2}{(16\pi^2)^2} m_{3/2}^2 M_*^2 \left(\ln \frac{X^* X}{v^2}\right)^2, \tag{B.6}$$

with ζ_2 being a constant. This potential has a minimum at $|X| = v$, and increases logarithmically for large $|X|$. Therefore, this potential reproduces the important feature of the potential of X (especially for $|X| \sim v$). When the Universe is dominated by X, expansion rate is estimated as

$$H \sim \frac{\sqrt{\zeta_2}}{16\pi^2} L m_{3/2}, \tag{B.7}$$

with

$$L \equiv \ln \frac{X^* X}{v^2}. \tag{B.8}$$

Importantly, this expansion rate is much smaller than the gravitino mass. With the above expansion rate, we denote

$$\bar{\phi} = \frac{k_H H^2}{m_{3/2}^2} M_*, \tag{B.9}$$

where k_H is an unknown constant expected to be of $O(1)$. We consider the case where the energy density of the Universe is dominated by that of X, so H decreases as the amplitude of X approaches to v.

As shown in Eq. (B.4), solution to Eq. (B.3) is given by the sum of the oscillating solution $\phi_{\rm osc}$ which is from the non-perturbed equation of motion and $\delta\phi$ induced by extra terms with non-vanishing H. We have already understood the behavior of $\phi_{\rm osc}$, so now we consider $\delta\phi$.

The important point in deriving $\delta\phi$ is that the expansion rate H is much smaller than the gravitino mass $m_{3/2}$. Because of this, we can expand $\delta\phi$ by powers of $(H^2/m_{3/2}^2)$. First, let us consider a simple case where $\dot{H}$ is proportional to H^2:

$$\dot{H} = -c_H H^2, \tag{B.10}$$

with c_H being a constant of $O(1)$. For example, if X has a parabolic potential, $c_H = 3/2$. In this case, solution to Eq. (B.3) is obtained as

$$\phi = \phi_{\rm osc} + \left[\frac{k_H H^2}{m_{3/2}^2} - 6c_H(c_H - 1)\frac{k_H H^4}{m_{3/2}^4} + O\left(\frac{H^6}{m_{3/2}^6}\right)\right] M_*. \tag{B.11}$$

Notice that the term of $O(H^2/m_{3/2}^2)M_*$ is exactly equal to $\bar{\phi}$. In the case where the potential of X is logarithmic like Eq. (B.6), formula for $\dot{H}$ is slightly different:

$$\dot{H} \sim -\frac{1}{1+L}H^2. \tag{B.12}$$

In this case, ϕ is given as

$$\phi = \phi_{\rm osc} + \left[\frac{k_H H^2}{m_{3/2}^2} - \frac{36(5+2L-L^2)}{(1+L)^3}\frac{k_H H^4}{m_{3/2}^4} + O\left(\frac{H^6}{m_{3/2}^6}\right)\right] M_*, \tag{B.13}$$

and the $O(H^2/m_{3/2}^2)M_*$ term agrees with $\bar{\phi}$ again. In general, as far as $H \ll m_{3/2}$ and $\dot{H} \lesssim O(H^2)$, the leading correction is always equal to $\bar{\phi}$. This is because, in Eq. (B.3), first two terms become of $O(H^4/m_{3/2}^4)M_*$, and hence $\bar{\phi}$ has to be cancelled out by $O(H^2/m_{3/2}^2)M_*$ term in ϕ. As a result, the deviation from the shifted minimum is always of $O(H^4/m_{3/2}^4)M_*$.

Since $\bar{\phi}$ smoothly goes to 0 as the Universe expands, $\bar{\phi}$ term in ϕ is harmless. In other words, in the early stage, ϕ oscillates around the shifted minimum $\bar{\phi}$, but this minimum approaches to the true minimum as $H \to 0$. Therefore, we just have to consider the deviation from $\phi = \bar{\phi}$.

In our situation, potential of X changes its behavior at $|X| \sim v$; it is logarithmic for $|X| \gtrsim v$, and parabolic potential is relevant once X is trapped in the SUSY breaking minimum. Thus, the solution to Eq. (B.3) changes its behavior at $|X| \sim v$. For example, if we match two cases at $L = 1$ (though the matching point is quite uncertain), $\phi_{\rm osc}$ is shifted as

$$\phi_{\rm osc} \to \phi_{\rm osc} - \frac{45}{2}\frac{k_H H^4}{m_{3/2}^4}M_*. \tag{B.14}$$

Notice that this shift is not $O(H^2/m_{3/2}^2)M_*$, but $O(H^4/m_{3/2}^4)M_*$. In general, shift of this order may be possible, especially when the potential changes its behavior. However, shift cannot be $O(H^2/m_{3/2}^2)M_*$.

If this shift is larger than the averaged amplitude of $\phi_{\rm osc}$, our argument may break down. Therefore, we require

$$\phi_{\rm osc} \gtrsim \frac{H^4}{m_{3/2}^4} M_*. \tag{B.15}$$

When $|X| \gtrsim v$, $\phi_{\rm osc}$ is proportional to $|X|^{1/2}$ while H depends on $|X|$ only logarithmically. On the other hand, once X is trapped in the SUSY breaking minimum, $\phi_{\rm osc}$ and H are both proportional to $(|X| - v)$. Therefore, this constraint becomes most stringent when $|X| \sim v$. Since the expansion rate is much smaller than the gravitino mass (see Eq. (B.7)), above constraint is very weak. Numerically, this requires $v \gtrsim 100$ GeV, even for $\zeta_2 \sim 1$. Of course, if we adopt smaller value of ζ_2, lower bound on v becomes less severe. Therefore, in our case, this constraint is weak enough, and Hubble-induced term does not change our argument.

C Evolution of n_R

In this Appendix, we discuss how the R-number density evolves with time. In particular, since R-symmetry breaking terms exist in the potential, we need to know the evolution of n_R with them.

We consider a case where X oscillates with a large amplitude. In this case, frequency of the oscillation is roughly given by $\sim m_{\rm eff}$, where the effective mass $m_{\rm eff}$ is given by (see Appendix D)

$$m_{\rm eff}^2 = \frac{1}{2}\left(\frac{1}{X^*}\frac{\partial V}{\partial X} + \frac{1}{X}\frac{\partial V}{\partial X^*}\right). \tag{C.1}$$

Notice that $m_{\rm eff} \gg H$ when X is oscillating. Therefore, we consider the time scale $m_{\rm eff}^{-1} \ll \delta t \ll H^{-1}$, for which we neglect the change of $|X|$ and H. With the expansion of the Universe, the amplitude of X decreases as

$$|X| R^p = \text{const.}, \tag{C.2}$$

with p being a positive $O(1)$ constant. (For example, $p = 3/2$ for parabolic potential, and $p = 3$ for logarithmic one.) Therefore, we approximate the motion of X as

$$X \sim |X_0| e^{(i m_{\rm eff} - pH)t}. \tag{C.3}$$

With the R-symmetry breaking potential

$$V_{\not{R}} \sim -\frac{F_X^2}{M_*}(X^* + X) \times f(X^* X / M_*^2), \tag{C.4}$$

with $f(x) = k_0 + k_1 x + \cdots$, equation for the evolution of $n_R = i(X^* \dot{X} - \dot{X}^* X)$ is given by

$$\dot{n}_R + 3H n_R = 2F_X^2 {\rm Im}(X/M_*) \times f(|X|^2/M_*^2). \tag{C.5}$$

For the following discussion, it is more convenient to consider the R-number in a comoving volume. For this quantity, the above equation leads to

$$\frac{d(R^3 n_R)}{dt} = R^3 \times 2F_X^2 \mathrm{Im}(X/M_*) \times f(|X|^2/M_*^2). \tag{C.6}$$

We need to solve the above equation to obtain the resultant R-number. For this purpose, we first take the average of the right-hand side of this equation for the time scale $m_{\rm eff}^{-1} \ll \delta t \ll H^{-1}$. For this time scale, we approximate $|X|$ and H to be (almost) constant. On the other hand, the change of $\mathrm{Im}X$ is extremely non-adiabatic. In the flat background (i.e., if $H = 0$), average of $\mathrm{Im}X$ is supposed to vanish since X is in a periodic motion. However, in the actual situation, H is non-vanishing. By using Eq. (C.3), average of $\mathrm{Im}X$ is estimated as

$$\langle \mathrm{Im}X \rangle \sim |X| \frac{H}{m_{\rm eff}} \sim \frac{1}{m_{\rm eff}} \frac{d|X|}{dt}, \tag{C.7}$$

where we related the expansion rate H to $d|X|/dt$ by using Eq. (C.2). In Eq. (C.7) and hereafter, we neglect possible $O(1)$ coefficients since they do not change the following argument. Combining this equation with Eq. (C.6), we obtain

$$\frac{d(R^3 n_R)}{d|X|} \sim R^3 \times \frac{2F_X^2}{M_*} \frac{f(|X|^2/M_*^2)}{m_{\rm eff}}. \tag{C.8}$$

By using the fact that $m_{\rm eff}|X|^2 R^3$ is a constant of motion, we integrate the above equation from $X = X_i$ to $X = X_f$ (with $X_i > X_f$):

$$R^3 n_R \sim m_{\rm eff}|X|^2 R^3 \times \frac{2F_X^2}{M_*} \times \sum_n \frac{k_n}{M_*^{2n}} \left(\frac{|X_f|^{2n-1}}{m_{\rm eff}^2(X_f)} - \frac{|X_i|^{2n-1}}{m_{\rm eff}^2(X_i)} \right). \tag{C.9}$$

If the second term in the parenthesis in Eq. (C.9) wins the first term, integration in large $|X|$ region is more important, and major part of the R-number is generated when X starts to move. On the other hand, if the first term is dominant, we cannot neglect the R-number generation in the later stage.

If the potential of X is dominated by the logarithmic piece, $m_{\rm eff}$ is proportional to $|X|^{-1}$. In this case, R-number generation at large amplitude is more important for $n \geq 0$. As a result, even if there is a linear R-symmetry breaking term in the potential, R-number is generated when X starts to move, and R-symmetry is conserved with a good accuracy for a smaller value of $|X|$. On the other hand, for parabolic potential, $m_{\rm eff}$ is a constant. If the R-number violating potential is dominated by the linear term ($n = 0$), contribution at small $|X|$ becomes important. However, in our scenario, we assume that the linear term is suppressed enough when X starts to move, and that the R-number violating effect starts with cubic term ($n = 1$). In this case, R-number asymmetry is again generated when X starts to move.

In the actual situation, X starts to move with a quadratic potential, and at some stage, logarithmic piece takes over. With the assumption that the linear term is suppressed enough, R-number is generated when X starts to move, and afterwards, R-number in the comoving volume is conserved well.

D Scalar Field in the Expanding Universe

In this Appendix, we derive a convenient formula for the evolution of the scalar field φ in periodic motion. For simplicity, we consider the case where the amplitude of the scalar field is (almost) constant in a time scale of the periodic motion and also the potential for φ depends only on $|\varphi|$.

From the virial theorem, we obtain

$$2\langle \mathcal{K} \rangle = \left\langle \varphi \frac{\partial V}{\partial \varphi} + \varphi^* \frac{\partial V}{\partial \varphi^*} \right\rangle, \tag{D.1}$$

where $\mathcal{K} = \dot{\varphi}^* \dot{\varphi}$ is the kinetic energy of φ, and the bracket represents the time average.

The field equation for φ is given by

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V}{\partial \varphi^*} = 0. \tag{D.2}$$

Multiplying this equation by $\dot{\varphi}^*$, and using the definition of $\mathcal{K}$, we obtain

$$\dot{\mathcal{K}} + 6H\mathcal{K} + \left(\dot{\varphi}^* \frac{\partial V}{\partial \varphi^*} + \dot{\varphi} \frac{\partial V}{\partial \varphi} \right) = 0. \tag{D.3}$$

Now, we are at the position to consider the evolution of the scalar field. For this purpose, we define

$$S^2 = |\varphi|^2 \left(\varphi \frac{\partial V}{\partial \varphi} + \varphi^* \frac{\partial V}{\partial \varphi^*} \right), \tag{D.4}$$

and consider the evolution of $\langle S^2 \rangle$. By taking the derivative of $\langle S^2 \rangle$ with respect to time, we obtain

$$\begin{aligned} \frac{d\langle S^2 \rangle}{dt} &= \left\langle \frac{d|\varphi|^2}{dt} \left(\varphi \frac{\partial V}{\partial \varphi} + \varphi^* \frac{\partial V}{\partial \varphi^*} \right) + |\varphi|^2 \frac{d}{dt} \left(\varphi \frac{\partial V}{\partial \varphi} + \varphi^* \frac{\partial V}{\partial \varphi^*} \right) \right\rangle \\ &= \left\langle \frac{d|\varphi|^2}{dt} \left(\varphi \frac{\partial V}{\partial \varphi} + \varphi^* \frac{\partial V}{\partial \varphi^*} \right) + |\varphi|^2 \dot{\mathcal{K}} \right\rangle \\ &= -12H \left\langle |\varphi|^2 \mathcal{K} \right\rangle \\ &= -6H \langle S^2 \rangle, \end{aligned} \tag{D.5}$$

where we used the fact that the potential V is a function of $|\varphi|$. By solving the above equation, we obtain

$$\langle S^2 \rangle R^6 = \text{const.} \tag{D.6}$$

The scalar field evolves by following this relation.

For a more intuitive understanding, it is convenient to define the "effective mass" from the potential V:

$$m_{\rm eff}^2 = \frac{1}{2}\left(\frac{1}{\varphi^*}\frac{\partial V}{\partial \varphi} + \frac{1}{\varphi}\frac{\partial V}{\partial \varphi^*}\right). \tag{D.7}$$

With this effective mass, evolution of the scalar field is given by

$$m_{\rm eff}|\varphi|^2 R^3 = \text{const.} \tag{D.8}$$

For example, in the case of parabolic potential $V = m_\varphi^2|\varphi|^2$, $m_{\rm eff}$ does not depend on φ, and hence $|\varphi|$ scales as $R^{-3/2}$, while $|\varphi| \propto R^{-1}$ for quartic potential $V \propto |\varphi|^4$.

Notice that, in the flat space ($H = 0$),

$$\varphi = \varphi_0 e^{\pm i m_{\rm eff} t}, \tag{D.9}$$

satisfies the equation of motion of φ for any value of φ_0, if the potential of φ depends only on $|\varphi|$. (In Eq. (D.9), $m_{\rm eff}$ is evaluated at $\varphi = \varphi_0$.) Therefore, $m_{\rm eff}$ can be understood as a frequency of the periodic motion.

Since $m_{\rm eff}|\varphi|^2$ is proportional to the volume of the phase space for the periodic motion, a physical interpretation of Eq. (D.8) is that the phase space volume in a comoving volume is conserved as the Universe expands.

References

[1] M. Dine and A.E. Nelson, *Phys. Rev.* **D48**, 1277 (1993); M. Dine, A.E. Nelson and Y. Shirman, *Phys. Rev.* **D51**, 1362 (1995); M. Dine, A.E. Nelson, Y. Nir and Y. Shirman, *Phys. Rev.* **D53**, 2658 (1996).

[2] For review, see, for example, G.F. Giudice and R. Rattazzi, hep-ph/9801271.

[3] H. Pagels and J.R. Primack, *Phys. Rev. Lett.* **48** (1982) 223.

[4] T. Moroi, H. Murayama and M. Yamaguchi, *Phys. Lett.* **B303** (1993) 289.

[5] A. de Gouvêa, T. Moroi and H. Murayama, *Phys. Rev.* **D56** (1997) 1281.

[6] D.H. Lyth and E.D. Stewart, *Phys. Rev. Lett.* **75** (1995) 201; *Phys. Rev.* **D53** (1996) 1784.

[7] H. Murayama, *Phys. Rev. Lett.* **79** (1997) 18.

[8] S. Dimopoulos, G. Dvali, R. Rattazzi and G. Giudice, *Nucl. Phys.* **B510** (1998) 12.

[9] S. Dimopoulos, G. Dvali and R. Rattazzi, *Phys. Lett.* **B413** (1997) 336.

[10] K. Agashe, hep-ph/9804450.

[11] K.-I. Izawa, Y. Nomura, K. Tobe and T. Yanagida, *Phys. Rev.* **D56** (1997) 2886; Y. Nomura and K. Tobe, hep-ph/9708377 .

[12] E. Poppitz and S.P. Trivedi, *Phys. Rev.* **D55** (1997) 5508; N. Arkani-Hamed, H. Murayama and J. March-Russell, *Nucl. Phys.* **B509** (1998) 3; M.A. Luty, *Phys. Lett.* **B414** (1997) 71; Y. Shirman, *Phys. Lett.* **B417** (1998) 281; M.A. Luty and J. Terning, *Phys. Rev.* **D57** (1998) 6799; N. Arkani-Hamed, M.A. Luty and J. Terning, hep-ph/9712389.

[13] E. Witten, *Phys. Lett.* **B105** (1981) 267.

[14] K.-I. Izawa and T. Yanagida, *Prog. Theor. Phys.* **95** (1996) 829; K. Intriligator and S. Thomas, *Nucl. Phys.* **B473** (1996) 121.

[15] M. Dine, L. Randall and S. Thomas, *Phys. Rev. Lett.* **75** (1995) 398; *Nucl. Phys.* **B458** (1996) 291.

[16] I. Affleck and M. Dine, *Nucl. Phys.* **B249** (1985) 361.

[17] See, for example, G. Jungman, M. Kamionkowski and K. Griest, *Phys. Rep.* **267** (1996) 195.

[18] L.J. Hall, T. Moroi and H. Murayama, *Phys. Lett.* **B424** (1998) 305.

[19] S. Borgani, A. Masiero and M. Yamaguchi, *Phys. Lett.* **B386** (1996) 189.

[20] S. Dimopoulos, G.F. Giudice and A. Pomarol, *Phys. Lett.* **B389** (1996) 37; T. Han and R. Hempfling, *Phys. Lett.* **B415** (1997) 161.

[21] N. Arkani-Hamed, C.D. Carone, L.J. Hall and H. Murayama, *Phys. Rev.* **D54** (1996) 3072; I. Dasgupta, B.A. Dobrescu and L. Randall, *Nucl. Phys.* **B483** (1997) 95.

[22] G. Veneziano and S. Yankielowicz, *Phys. Lett.* **B113** (1982) 231.

[23] M.A. Luty, *Phys. Rev.* **D57** (1998) 1531; A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Phys. Lett.* **B412** (1997) 301.

[24] J. Bagger, E. Poppitz and L. Randall, *Nucl. Phys.* **B426** (1994) 3.

[25] A. Linde, *Phys. Lett.* **B129** (1983) 177.

[26] B.A. Campbell, M.K. Gaillard, H. Murayama and K.A. Olive, hep-ph/9805300.

[27] M. Kasuya and M. Kawasaki, *Phys. Rev.* **D56** (1997) 7597.

[28] M. Nagasawa and M. Kawasaki, *Phys. Rev.* **D50** (1994) 4821.

[29] See, for example, E. Holtmann, M. Kawasaki, K. Kohri and T. Moroi, hep-ph/9805405.

[30] S. Davidson, H. Murayama and K.A. Olive, *Phys. Lett.* **B328** (1994) 354.

[31] G.G. Raffelt, *Stars as Laboratories for Fundamental Physics: The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles* (University of Chicago Press, 1996).

[32] M.S. Turner, *Phys. Rev. Lett.* **59** (1987) 2489; M.T. Ressell, *Phys. Rev.* **D44** (1991) 3001; J.M. Overduin and P.S. Wesson, *Astrophys. J.* **414** (1993) 449.

[33] M.S. Turner, *Phys. Rev. Lett.* **60** (1988) 1797; A. Burrows, M.S. Turner and R.P. Brinkmann, *Phys. Rev.* **D39** (1989) 1020; A. Burrows, M.T. Ressell and M.S. Turner, *Phys. Rev.* **D42** (1990) 3297.

[34] J. Engel, D. Seckel and A.C. Hayes, *Phys. Rev. Lett.* **65** (1990) 960.

[35] M. Dine and A. Riotto, *Phys. Rev. Lett.* **79** (1997) 2632; S. Dimopoulos, G. Dvali and R. Rattazzi, *Phys. Lett.* **B410** (1997) 119.

[36] M. Kawasaki and T. Yanagida, *Phys. Lett.* **B399** (1997) 45; T. Asaka, J. Hashiba, M. Kawasaki and T. Yanagida, hep-ph/9802271.

[37] A.E. Nelson and N. Seiberg, *Nucl. Phys.* **B416** (1994) 46.

[38] L. Alvarez-Gaumé, M. Claudson and M. Wise, *Nucl. Phys.* **B207** (1982) 96.