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Michael S. Chanowitz

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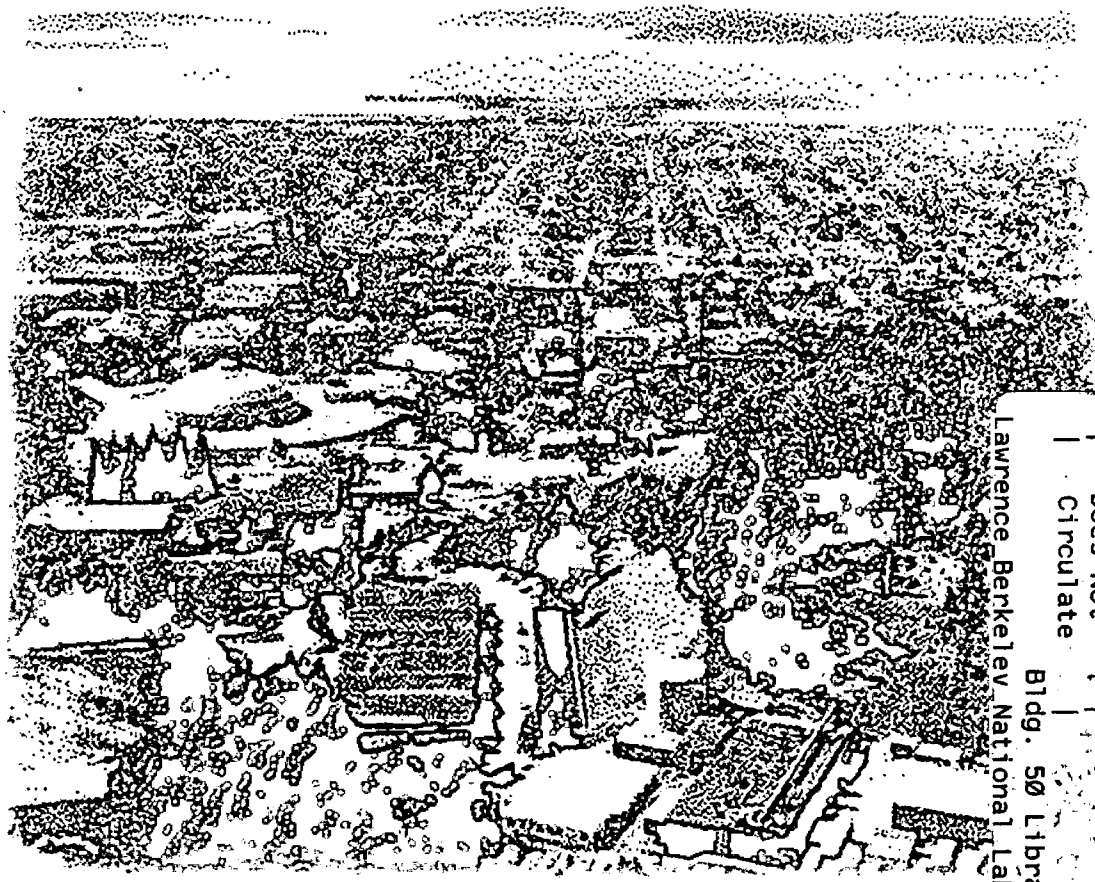

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Michael S. Chanowitz

Physics Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

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# Higgs boson mass constraints from precision data and direct searches[1]

Michael S. Chanowitz[2]

*Theoretical Physics Group*
*Lawrence Berkeley National Laboratory*
*University of California*
*Berkeley, California 94720*

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**Abstract**

Two of the nine measurements of $sin^2\theta_{eff}^{lepton}$, the effective weak interaction mixing angle, are found to be in significant conflict with the direct search limits for the Standard Model (SM) Higgs boson. Using a scale factor method, analogous to one used by the Particle Data Group, we assess the possible effect of these discrepancies on the SM fit of the Higgs boson mass. The scale factor fits increase the value of $sin^2\theta_{eff}^{lepton}$ by as much as two standard deviations. The central value of the Higgs boson mass increases as much as a factor of two, to $\simeq$ 200 GeV, and the 95% confidence level upper limit increases to as much as 750 GeV. The scale factor is based not simply on the discrepant measurements, as was the case in a previous analysis, but on an aggregate goodness-of-fit confidence level for the nine measurements and the limit. The method is generally applicable to fits in which one or more of a collection of measurements are in conflict with a physical boundary or limit. In the present context, the results suggest caution in drawing conclusions about the Higgs boson mass from the existing data.

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[2]Email: chanowitz@lbl.gov

## 1. Introduction

Beautiful measurements of $Z$ boson decay asymmetries at LEP and SLC[1] and of the top quark mass at Fermilab[2] appear to constrain the mass of the Standard Model (SM) Higgs boson at the level of a factor two or better. The combined fit of nine measurements of the effective leptonic weak interaction mixing angle yields $sin^2\theta_{eff}^{lepton} = 0.23148 \pm 0.00021$, which implies the SM Higgs boson mass $m_H = 86^{+84}_{-42}$ GeV and the upper limit $m_H < 260$ GeV at 95% confidence level (CL). In a previous letter[3] I observed that the most precise of the nine measurements, the left-right asymmetry $A_{LR}$, then implied $m_H = 16$ GeV and an *upper* limit $m_H < 77$ GeV at 95%CL, in contrast to the *lower* limit from direct searches, then given by $m_H > 77$ GeV, also at 95%CL. I analyzed the possible impact of this discrepancy on the SM fit of $m_H$ using a scale factor method inspired by a method the Particle Data Group[4] (PDG) has used to combine discrepant data. The conclusion was that both the central value and the upper limit on $m_H$ could be appreciably higher than in the conventional fit. Similar observations had been made previously, using different methods, by Gurtu[5] and Dittmaier, Schildknecht, and Weiglein.[6]

The work presented here differs significantly from reference [3] in which the discrepancy between the $A_{LR}$ measurement and the search limit was evaluated simply as the likelihood for a 95%CL upper limit at 77 GeV to be consistent with a 95%CL lower limit at the same mass, i.e., $2 \cdot 0.05 \cdot 0.95 \simeq 0.1$ or 10%. This may be a fair appraisal if we have an *a priori* reason to focus on the $A_{LR}$ measurement, such as for instance that it provides the most precise determination of $sin^2\theta_{eff}^{lepton}$, rather than choosing to consider it *because* we have noticed that it implies a value of $m_H$ below the SM search limit. In the latter case we need to consider the likelihood that *any* of the nine relevant measurements of $sin^2\theta_{eff}^{lepton}$ could fluctuate to produce a like discrepancy. It is fair to say that in this instance our attention is drawn to $A_{LR}$ by both its precision and the fact of its conflict with the SM search limits.

It may therefore be appropriate to approach the analysis from the perspective of the consistency of the complete ensemble of nine measurements with the SM search limit. That is the perspective of the analysis presented here, in which a suitable scale factor method is proposed. The method can be applied to a variety of different physical situations, for instance, the problem confronted by the

PDG of how to set an upper limit on the electron neutrino mass when several measurements (of the kinematic end-point in tritium decay) imply a tachyonic mass.[4] Here I will apply the method to the SM fit of $m_H$, using the Spring 1998 data, which differs appreciably from the Summer 1997 data used in the earlier analysis.

In the previous analysis the scale factor was introduced based on the goodness-of-fit CL between just the discrepant measurement and the limit. In the method presented here the scale factor is determined by the goodness-of-fit CL between the complete set of asymmetry measurements and the limit, therefore taking account of the likelihood that any measurement in the set might fluctuate into the low tail of the $sin^2\theta_{eff}^{lepton}$ distribution. The method is then truly analogous to the PDG method, which rescales the fit uncertainty by a scale factor determined by the goodness-of-fit CL of the chi-squared distribution of the complete data set.

It is important to keep in mind that the analysis presented here assumes the validity of the Standard Model (or the MSSM in the decoupling limit) and that in general, without a specific theoretical framework, the electroweak radiative corrections tell us nothing about the nature of electroweak symmetry breaking. In addition to quantum corrections from the Higgs sector, the value of $sin^2\theta_{eff}^{lepton}$ could be affected by quantum corrections from other sectors of new physics and/or from gauge boson mixing in theories with extended gauge sectors. The nature of electroweak symmetry breaking can only be definitively established by direct discovery and detailed study of the Higgs sector quanta at a high energy collider. Until then anything is possible: light Higgs scalars, dynamical symmetry breaking without Higgs scalars, or even that the Higgs mechanism is not realized in nature at all. Here we assume that no new physics contributes to $sin^2\theta_{eff}^{lepton}$ except the quantum corrections from the Higgs sector, and that any Higgs scalar decays as prescribed in the SM so that the Higgs boson search limits are applicable.

Section 2 is a brief review of the 1998 data and the SM fit of $m_H$. The uncertainties in the fit are examined for two different evaluations of $\alpha(m_Z)$.[7, 8] (The values quoted in this introductory section are based on reference [7].) Though the 1998 data set for $sin^2\theta_{eff}^{lepton}$ is more internally consistent than the 1997 data, its confidence level is still not robust and it continues to exhibit

discrepancies with the SM search limits. The central value of $m_H$ implied by $A_{LR}$ has increased to 25 GeV, but the direct search limit[9] has also increased, to $m_H > 89.3$ GeV at 95%CL, and the precision of the $A_{LR}$ measurement has improved. Putting all these changes together there is still a significant discrepancy, with $A_{LR}$ now implying $m_H < 89.3$ GeV at 93%CL.

A somewhat bigger discrepancy occurs in the less precise tau front-back asymmetry measurement, $A^{\tau}_{FB}$, which implies $m_H = 4$ GeV and $m_H < 89.3$ GeV at 95%CL. Although a single value of $sin^2\theta^{lepton}_{eff}$ is typically presented for the combined leptonic front-back asymmetry, $A^{l}_{FB}$, the measurements of $A^{e}_{FB}$, $A^{\mu}_{FB}$, and $A^{\tau}_{FB}$ are in fact quite distinct, each posing a unique set of experimental issues. As can be seen in table 2 below, $A^{\mu}_{FB}$, and $A^{\tau}_{FB}$ are individually at the same level of precision as all but the two most precise measurements, so it is most natural to consider them separately.

It is certainly the case that our attention is drawn to $A^{\tau}_{FB}$ by the low value of $m_H$ it implies, so in considering the conflict of $A_{LR}$ and $A^{\tau}_{FB}$ with the search limit we must assess the goodness-of-fit of the measurements with the search limit from the perspective of the complete set of nine measurements. The scale factors computed in this way then appropriately weight the increased likelihood of outlying measurements when $A^{l}_{FB}$ is disaggregated, with the number of $sin^2\theta^{lepton}_{eff}$ measurements increased from seven to nine.

Section 3 begins with a review of the PDG scale factor method for combining discrepant data and then presents a method to extend it to the case of measurements in conflict with a limit. The central observation of the PDG is that low CL data sets occur more often than expected by chance, and that historically many discrepancies are found to result from underestimated systematic errors. This should not be a surprise, since the estimation of systematic error is perhaps the most challenging task faced by experimenters in the analysis and presentation of their data. The PDG scaled error is meant to provide a more cautious interpretation of low CL data sets, with minimal impact on moderately discrepant data. After reviewing the motivation and formulation of the PDG scale factor, $S^*$, an analogous scale factor is constructed for situations in which the discrepancy is between a collection of measurements and a limit. Section 3 concludes with a brief discussion of the complementary relationship of the scale factor method with a recent analysis by Cousins and Feldman[10] of confidence

intervals near a physical boundary. Their construction is used to determine the upper limits on $m_H$ from the scaled fits.

Section 4 presents the application of the scale factor method to the fit of $m_H$ from the nine measurements of $sin^2\theta_{eff}^{lepton}$. The result is a continuum of fits which differ in how the scaling is shared between the two low measurements, $A_{LR}$ and $A_{FB}^{\tau}$. At one extreme, it suffices to scale the uncertainty of $A_{FB}^{\tau}$ by a factor 3 while leaving $A_{LR}$ unmodified; in this case the effect on the fit is small. At the other extreme, when the rescaling is dominantly applied to $A_{LR}$, the fitted central value of $m_H$ increases by a factor two relative to the conventional fit, while the 95%CL upper limit (in the Cousins-Feldman construction) increases by nearly a factor three relative to the conventional 95%CL limit. These extremes and a sample of intermediate cases are presented in Section 4.

The analysis in sections 2 – 4 assumes a perfect search limit, $m_H > 89.3$ GeV with 100% CL. In section 5 I show that the results obtained in this approximation apply to the actual, less than perfect experimental limits. The conclusion relies on the sharply increased confidence level obtained by the search experiments for values of $m_H^{\rm LIMIT}$ slightly below 89 GeV.

A brief summary and discussion are given in Section 6.

## 2. The electroweak data and the SM Higgs boson mass

Our strategy is to focus on the most direct determination of $m_H$, using the measurement of $sin^2\theta_{eff}^{lepton}$, augmented by the direct measurement of the top quark mass (by CDF and D0) together with the value of $\alpha(m_Z)$. The effective mixing angle, $sin^2\theta_{eff}^{lepton}$, has the greatest sensitivity to $m_H$ with the least collateral dependence on various other quantities such as the strong coupling constant $\alpha_S(m_Z)$ or the fraction of hadronic $Z$ decays to $b$ quarks, $R_b$. From the nine measurements of $sin^2\theta_{eff}^{lepton}$, which combine to yield $sin^2\theta_{eff}^{lepton} = 0.23148 \pm 0.00021$, and the conservative determination of $\alpha(m_Z) = (128.896 \pm 0.090)^{-1}$ by Eidelmann and Jegerlehner[7] I obtain using the state of the art radiative corrections of Degrassi et al.[11] $m_H = 86^{+84}_{-42}$ GeV, compared with the EWWG[1] global fit value $m_H = 66^{+74}_{-39}$ GeV (which also uses reference [7] for $\alpha(m_Z)$). Gaussian statistics are assumed for the $sin^2\theta_{eff}^{lepton}$ measurements, from which it follows in the SM fit that the logarithm of the Higgs boson mass, ln $m_H$, is Gaussian distributed.

The difference between the global fit and the fit based just on the $sin^2\theta_{eff}^{lepton}$ data is not great and is due primarily to the fact that the global fit uses the top quark mass, $m_t = 171.1 \pm 5.1$ GeV, determined from the combination of direct and indirect measurements, while in the fit restricted to the $sin^2\theta_{eff}^{lepton}$ data I have used the directly measured Fermilab value[4], $m_t = 173.8 \pm 5.1$ GeV. The smaller value of $m_t$ from the indirect determination is due principally to the remnant of the $R_b$ anomaly — since the current value of $R_b$ is 1.6 standard deviations above the SM fit value, the global fit prefers smaller values of $m_t$ in order to minimize the discrepancy. Because $m_t$ and $m_H$ are correlated in the fit, a higher value of $R_b$ thus leads indirectly to a lower value of $m_H$ in the global fit. Since in this paper I am assuming the validity of the Standard Model, the strategy followed seeks to minimize the extent of such indirect effects, which during the height of the $R_b$ anomaly (when $R_b$ was believed to be three standard deviations above the SM value) led to a serious distortion of the global fit of $m_H$.[6]

The uncertainty in the SM determination of $m_H$ is analyzed in table 1. The principal sources of uncertainty are the uncertainties in the measurements of $sin^2\theta_{eff}^{lepton}$ and $m_t$, and the evaluation of the fine structure constant at $m_Z$. I use $sin^2\theta_{eff}^{lepton} = 0.23148 \pm 0.00021$ from the conventional least square fit of the nine measurements and $m_t = 173.8 \pm 5.1$ GeV from the current PDG fit of the Fermilab top quark mass measurements. For $\alpha(m_Z)$ I use two values, $(128.896 \pm 0.090)^{-1}$ and $(128.933 \pm 0.021)^{-1}$. The former is the conservative evaluation by Eidelmann and Jegerlehner[7], while the latter, from Davier and Höcker[8], is one of several[12] recent, more optimistic evaluations, which rely on perturbative QCD down to lower energy scales. These typically have a smaller estimated error and a smaller central value, the latter implying a larger value of $m_H$. In this paper I will present results using both references [7] and [8]. Table 1 also displays much smaller contributions from the QCD coupling constant, $\alpha_S(m_Z) = 0.120 \pm 0.003$, and from uncomputed higher order corrections. For the latter I rely on the estimate of Degrassi et al.[11], whose compact representation of their calculations of the radiative corrections are used throughout this paper.[3]

[3]Weiglein and coworkers[13] have recently estimated a somewhat larger theoretical error for the results of reference [11]. However in any case the theoretical error is overwhelmed by the three dominant uncertainties in table 1.

Combined in quadrature the net uncertainty in $\ln(m_H)$ is $\pm 0.67$ or $\pm 0.52$ for the two evaluations of $\alpha(m_Z)$, corresponding respectively to a factor 2 or 1.7 uncertainty in $m_H$.

The measurements of $sin^2\theta_{eff}^{lepton}$ have been characterized by three discrepancies, which persevere, though at a diminished level, in the Spring 1998 data. In the Summer 1997 data the two most precise measurements, $A_{LR}$ and $A_{FB}^b$, differed by $3.1\sigma$ (CL = 0.002), and $A_{LR}$ differed from the LEP average by $2.9\sigma$ (CL = 0.005). In the Spring 1998 data $sin^2\theta_{eff}^{lepton}$ from $A_{LR}$ has increased by $0.7\sigma$ while $sin^2\theta_{eff}^{lepton}$ from $A_{FB}^b$ has decreased by $0.6\sigma$, so that the corresponding discrepancies are $2.3\sigma$ (CL = 0.02) and $2.4\sigma$ (CL = 0.015). The chi-squared for the nine measurements has improved from $\chi^2/dof = 14.5/8$ (CL = 0.07) to a more acceptable $\chi^2/dof = 10.7/8$ (CL = 0.2). The nine measurements are shown in table 2 along with their "pulls", defined as the number of standard deviations that each measurement differs from the least-squares fit value $0.23148 \pm 0.00021$. As another estimator of the consistency of the nine measurements I have used a Monte Carlo to compute the confidence level to replicate the observed distribution of the *absolute values* of the pulls, obtaining a probability of 0.07.[4]

Tables 3 and 4 (corresponding to $\alpha(m_Z)$ from references [7] and [8] respectively) shows the Higgs boson mass predictions of each of the nine $sin^2\theta_{eff}^{lepton}$ measurements listed in order of precision. For each measurement the tables display the central value for $m_H$, the symmetric (in $\ln(m_H)$) 90% confidence interval, and the implied probability that $m_H$ lies *below* 89.3 GeV, which is the current 95%CL lower limit from the LEP direct searches.[9]. To compute the confidence intervals in $\ln(m_H)$ and the implied probabilities for $m_H < 89.3$ GeV we must of course include the parametric errors shown in table 1, for instance, by treating $\ln(m_H)$ as a Gaussian statistical variable for each measurement, combining in quadrature the uncertainty arising from the particular measurement of $sin^2\theta_{eff}^{lepton}$ with the other parametric errors shown in table 1. Equivalently, as a matter of convenience, one may express the parametric errors as effective errors in $sin^2\theta_{eff}^{lepton}$ (e.g., for fixed, known $m_H$) and combine them in quadrature

[4]That is, 0.07 is the probability that the absolute value of the largest pull is $\geq 1.61$, the second $\geq 1.57, \ldots$, and the ninth $\geq 0.01$.

with the experimental $\delta(sin^2\theta_{eff}^{lepton})$.[5]

The question we wish to consider is whether/how the discrepancies of $A_{LR}$ and $A_{FB}^{\tau}$ with the SM Higgs boson search limits should affect the SM fit of the Higgs boson mass. The first part of the question is how big in fact is the discrepancy? The answer depends on precisely how we frame the question. If, without considering the particular central value obtained, we had an *a priori* reason to focus on a particular measurement, say on $A_{LR}$ because it is the most precise and therefore most important single measurement in the fit, then the discrepancy could be read off from table 3 or 4 (though also including the effect of the less than perfect 95% confidence level of the search limit) and the analysis might then proceed as in reference [3]. However it is fair to say that in the present context our attention is drawn to $A_{LR}$ and $A_{FB}^{\tau}$ by the fact of their conflict with the search limits. In that case the appropriately framed question is how likely is it that any two of the nine measurements could fluctuate to provide discrepancies with the search limits equal or greater than the observed discrepancies? We obtain an upper limit on that probability by assuming that the true value of $m_H$ is precisely at the value of the direct search lower limit, $m_H = 89.3$ GeV.

Let $p_\tau$ and $p_{LR}$ be the probabilities implied by the measurements of $A_{LR}$ and $A_{FB}^{\tau}$ that $m_H$ lies below 89.3 GeV. Then the upper limit on the probability that any two of nine measurements, $a$ and $b$, could fluctuate into the low tail of the $sin^2\theta_{eff}^{lepton}$ distribution such that $p_a \geq p_\tau$ and $p_b \geq p_{LR}$ is given by[6]

$$P_9(p_\tau, p_{LR}) = 1 - p_\tau^9 - 9(1 - p_\tau)p_{LR}^8. \tag{1}$$

Equation (1) is the goodness-of-fit CL between the nine measurements and the direct search limit in the Standard Model, assuming the search limits to be perfect. Taking $p_\tau$ and $p_{LR}$ from tables 3 and 4 we find $P_9(p_\tau, p_{LR}) = 0.12$ and 0.18 respectively. Though we assume here that the search limit has 100%CL,

[5] The theoretical uncertainties of the very large and very small values of $m_H$ in tables 3 and 4 are somewhat bigger than indicated in table 1. The largest values, $\gg 1$ TeV, have no precise meaning in any case. For the very small values, such as $m_H = 4$ MeV from $A_{FB}^{\tau}$, we are really only concerned with the implied probability $P(m_H < 89.3 \text{ GeV})$ which only depends on the relationship between $m_H$ and $sin^2\theta_{eff}^{lepton}$ at $m_H = m_H^{\text{LIMIT}}$ where table 1 does apply.

[6] That is, $P_9(p_\tau, p_{LR})$ is the complement of the probability that all nine measurements have $p_i < p_\tau$ or that one among them has $p_i > p_\tau$ while the other eight have $p_i < p_{LR}$.

it is shown in section 5 that essentially the same results are obtained when the actual confidence levels of the searches are taken into account.

These confidence levels, 0.12 and 0.18, might be characterized as marginal, not big enough to be considered "robust" nor small enough to force us to choose between the Standard Model and the experiments. They are in the gray area to which the Particle Data Group scaling factor $S^*$ would apply if similar CL's were obtained from the $\chi^2$ distribution of a collection of measurements, as discussed in the next section.

## 3. Scale factors for discrepant data

Having quantified the extent of the discrepancy between the search limit and the measurements of $sin^2\theta_{eff}^{lepton}$ in the SM, we now consider the more difficult aspect of the question: whether/how these discrepancies should affect the SM fit of the Higgs boson mass. There is no single "right" answer. A maximum likelihood fit including both the precision data and the direct search data would replicate the conventional fit if the central value lies above the lower limit, $m_H^{\rm LIMIT}$, from the direct searches. That is a defensible interpretation, since if the true value of $m_H$ were near $m_H^{\rm LIMIT}$ we would expect values of $m_H$ obtained from measurements of $sin^2\theta_{eff}^{lepton}$ to lie both above and below $m_H^{\rm LIMIT}$. By underweighting downward fluctuations while leaving upward fluctuations at their full weight, we risk skewing the fit upward. Mindful of this risk, it is still instructive to explore the sensitivity of the fit to the weight ascribed to measurements that are in significant contradiction with the direct search limit.

Clearly the direct search limit is not irrelevant. If, for instance, the only information available were the direct search limit and the $A_{LR}$ measurement, we would conclude that the standard model is excluded at 90% CL. Theorists would have flooded the Los Alamos server with papers on the death of the standard model and the birth of new theories W,X,Y,Z... In the SM fit the $A_{LR}$ measurement causes $m_H$ to shift by a factor two, from 170 to 85 GeV, and the 95% upper limit to fall from 570 to 260 GeV. It is fully weighted in the conventional standard model fit despite a significant contradiction with the standard model.

If the discrepancy were even greater — say, for instance, a precision mea-

surement implying [7] $m_H = 11$ MeV with a 99.9% CL upper limit at 89 GeV — we would be faced with three alternatives: 1) omit the measurement from the SM fit, presuming a plausible reason exists to suspect a large systematic error, 2) disregard the search limits, presuming them to be systematically flawed in some way, or 3) to abandon the Standard Model. On the other hand, a measurement one half standard deviation below the lower limit, with a $\simeq 30\%$ probability to be consistent with the limit, would surely be retained at essentially full weight.

The difficult question is how to resolve the intermediate cases in which the discrepancy is significant but not so significant that we are forced to choose between the data and the SM. Assuming the validity of the search limits and of the SM we consider a method that interpolates between the extremes of cases 1) and 2) above and which allows us to explore the sensitivity of the fit to the weight assigned to the discrepant measurements.

The problem of how to combine inconsistent data has led to the break-up of many beautiful friendships. The mathematical theory of statistics provides no magic bullets and ultimately the discrepancies can only be resolved by future experiments. The PDG[4] has for many years scaled the uncertainty of discrepant data sets by a factor

$$S^* = \sqrt{\chi^2/(N-1)} \tag{2}$$

where $N$ is the number of measurements being combined. They scale the uncertainty of the combined fit by the factor $S^*$ if and only if $S^* > 1$. This is a conservative prescription, which amounts to requiring that the fit have a good confidence level, ranging from 32% for $N = 2$ to $\simeq 44\%$ for $N \simeq 10$. If the confidence level is already good, the scale factor has little effect; it only has a major effect on very discrepant data. The PDG argues (see [15]) that low confidence level fits occur historically at a rate significantly greater than expected by chance, that major discrepancies are often, with time, found to result from underestimated systematic effects, and that the scaled error provides a more cautious interpretation of the data.

As an illustration we apply $S^*$ to the determination of $m_H$ from the nine

[7] In fact, parity violation in atomic Cesium currently implies $m_H \sim 11$ MeV (*M*eV is not a typographical error) though only $1.2\sigma$ from 89 GeV.[14] Its weight in the combined fit would be negligible.

measurements of $sin^2\theta_{eff}^{lepton}$. The chi-squared for the nine measurements is 10.7 for 8 degrees of freedom, corresponding to CL = 0.20. Then $S^* = \sqrt{10.7/8} = 1.16$ and the conventional fit $sin^2\theta_{eff}^{lepton} = 0.23148 \pm 0.00021$ is modified to $0.23148 \pm 0.00024$. The effect on $m_H$ is negligible: the central value is unchanged, while the 95%CL upper limit increases from 255 to just 272 GeV (using [7] for $\alpha(m_Z)$). The effect on $m_H$ is suppressed by the fact that the experimental error from $sin^2\theta_{eff}^{lepton}$ is dominated by the parametric error from $m_t$ and $\alpha(m_Z)$ shown in table 1. Even for the more discrepant Summer 1997 data, with $\chi^2 = 14.6$ for 8 d.o.f. and CL = 0.07, the effect of the $S^*$ factor is moderate, with the 95%CL upper limit increasing from 310 to 370 GeV.

We wish to construct an analogous method for situations in which the discrepancy is between some of a collection of measurements and a limit or physical boundary. In analogy to the $\chi^2$ confidence level for $S^*$ our point of departure is the G-O-F CL (goodness-of-fit confidence level) between the measurements and the limit, for instance, equation (1) for the case at hand. The method is to rescale the errors of the measurements that conflict with the limit by factors that increase the G-O-F CL of the rescaled data to a robust minimum value. Following the PDG the minimum CL is chosen to equal the CL corresponding to $\chi^2 = N - 1$ for $N - 1$ degrees of freedom. Regarding the limit as an additional degree of freedom we have $N = 10$ for the nine measurements and the limit. The minimum CL is then 0.44, corresponding to $\chi^2 = 9$ with 9 d.o.f.

Since there are two discrepant measurements, there are in general two different scale factors, $S_\tau$ and $S_{LR}$. In the notation of equation (1) the G-O-F CL requirement is

$$P_9(p'_\tau, p'_{LR}) = 0.44. \tag{3}$$

where $p'_\tau$ and $p'_{LR}$ are the values of $p_\tau$ and $p_{LR}$ after rescaling,

$$\delta(sin^2\theta_{eff}^{lepton})_\tau \rightarrow S_\tau \cdot \delta(sin^2\theta_{eff}^{lepton})_\tau \tag{4}$$

and

$$\delta(sin^2\theta_{eff}^{lepton})_{LR} \rightarrow S_{LR} \cdot \delta(sin^2\theta_{eff}^{lepton})_{LR}. \tag{5}$$

Equation (3) imposes one constraint, leaving a one dimensional parameter space within the $(S_\tau, S_{LR})$ plane to consider.

Before turning to the electroweak data, we conclude this section with a general formulation of the method. Consider a collection of $N$ measurements of a physical quantity $x$,

$$\{x_i, \delta_i\} \qquad\qquad i = 1, \ldots, N \tag{6}$$

where the $x_i$ are the individual measured values and $\delta_i$ are the one standard deviation uncertainties. Suppose there is an exact lower limit or physical boundary (this assumption is relaxed in section 5 for the Higgs boson search limits),

$$x_{\text{TRUE}} > x_{\text{LIMIT}}, \tag{7}$$

and that $n \leq N$ of the measurements fall below the limit,

$$\begin{aligned} x_i &< x_{\text{LIMIT}} \qquad & i &= 1, \ldots, n \\ x_i &> x_{\text{LIMIT}} \qquad & i &= n+1, \ldots, N. \end{aligned} \tag{8}$$

Furthermore assume, in analogy to $p_\tau$ and $p_{LR}$ defined above, that the probability density function associated with each of the $n$ low measurements, $PDF_i(x - x_i, \delta_i)$ implies a probability $p_i$ that the measurement conflicts with the limit (7),

$$p_i = \int_{-\infty}^{x_{\text{LIMIT}}} PDF_i(x - x_i, \delta_i)\, dx. \tag{9}$$

By analogy with equation (1) we compute an upper bound on the G-O-F CL between the $N$ measurements and the limit. We order the $n$ low measurements such that $p_1 > p_2 > \ldots > p_n$. The upper bound is then obtained by assuming

$$x_{\text{TRUE}} = x_{\text{LIMIT}} \tag{10}$$

and computing the probability that *any* $n$ of the $N$ measurements, designated by ordered integer $n$-tuples $\{a_1, \ldots, a_n\}$ chosen from the integers $\{1, \ldots, N\}$, ordered such that $p_{a_1} > p_{a_2} > \ldots > p_{a_n}$, satisfy the condition

$$p_{a_i} \geq p_i \tag{11}$$

for all $i = 1, \ldots, n$.

The combined PDF for the $N$ independent measurements is

$$PDF_N(\{x - x_i, \delta_i\}) = \prod_{i=1}^{N} PDF_i(x - x_i, \delta_i). \tag{12}$$

Finally we can write the upper bound on the G-O-F CL between the $N$ measurements and the limit in the general form

$$P_N(p_1, \ldots p_n) = \sum_{\{a_1,\ldots,a_n\}} \int_D PDF_N(\{x_{a_i} - x_{\text{LIMIT}}, \delta_{a_i}\})\, dx_{a_1} \ldots dx_{a_n} \tag{13}$$

where the sum is over all ordered integer $n$-tuples $\{a_1, \ldots, a_n\}$ chosen from the integers $\{1, \ldots, N\}$ and the domain of integration $D$ is defined by the condition

$$\frac{x_{\text{LIMIT}} - x_{a_i}}{\delta_{a_i}} \geq \frac{x_{LIMIT} - x_i}{\delta_i} \tag{14}$$

for all $i = 1, \ldots, n$.

Equations (12-14), in all their obtuse generality, are just the straightforward generalization of the G-O-F CL $P_9(p_1, p_2)$ given explicitly in equation (1). The general statement of the method now closely follows that example. We require a minimum G-O-F CL

$$P_N(p_1, \ldots p_n) \geq P_{\text{MIN}} \tag{15}$$

where $P_{\text{MIN}}$ is the confidence level corresponding to the chi-squared distribution with $\chi^2 = N$ for $N$ degrees of freedom. If equation (15) is satisfied by the data we combine the data without further ado. If equation (15) is not obeyed we rescale the errors of the $n$ low measurements,

$$\delta_i \rightarrow \delta_i' = S_i \delta_i \qquad (16),$$

so that the $p_i$ defined in equation (9) are replaced by $p_i'$

$$p_i' = \int_{-\infty}^{x_{\text{LIMIT}}} PDF_i(x - x_i, \delta_i')\, dx. \tag{17}$$

such that the G-O-F CL for the scaled data satisfies the requirement,

$$P_N(p_1', \ldots p_n') = P_{\text{MIN}}. \tag{18}$$

The condition equation (18) is satisfied by an $n-1$ dimensional subspace of the space of $n$-tuples $(S_1, \ldots, S_n)$.

This section concludes with a brief discussion of the relationship of the scale factor method to the Cousins-Feldman definition of confidence intervals near a physical boundary.[10] They observe that the standard construction of

confidence intervals near a physical boundary is flawed, in that it leads to intervals that in some instances "under-cover" (i.e., correspond to less than the nominal probability) and which have discontinuities as a function of the central value that are artifacts of the construction. Particularly germane to the method presented here is their observation that near a boundary the conventional construction confuses two aspects of the fit that are or should be conceptually distinct: that is, the goodness-of-fit CL between the measurement and the limit is typically assessed based on the extent that the conventional confidence intervals obtained from the fit overlap the region allowed by the boundary or limit. In contrast, the usual procedure for combining data (away from a boundary) uses the minimum of the chi-squared distribution to asses goodness-of-fit, while the confidence intervals are obtained quite independently from the shape of the chi-squared distribution.

They propose confidence intervals which rectify these shortcomings, at the cost of relaxing the upper limits near the boundary. In particular, their confidence intervals only have support in the allowed region, leaving the assessment of goodness-of-fit as a separate issue. In this paper I use a goodness-of-fit estimator, $P_N(p_1, \ldots, p_n)$, which is quite distinct from the confidence intervals that are the output of the fit. Rather the goodness-of-fit estimator is computed at the outset and is then used to constrain the scale factors that determine the final fit and confidence intervals. The upper limits on $m_H$ obtained from the scaled fits are given with the Cousins-Feldman construction, though for comparison the conventionally defined limits are also provided.

## 4. Scaled standard model fits

In this section the scale factor method is applied to the SM Higgs boson mass fit. We indicate how the scaled fit is obtained and present the results. The results in this section are obtained under the assumption that the search limit is perfect, i.e., $m_H > 89.3$ GeV at 100%CL. In section 5 I show that essentially the same results follow from the actual data of the search experiments, as a result of the rapidly rising confidence level for exclusion limits below 89.3 GeV.

The results are shown in tables 5 and 6 and in figure 1. Consider for instance the results using the more conservative evaluation[7] of $\alpha(m_Z)$, shown in table 5 and in the solid curves in figure 1. Recall from section 2 that the goodness-of-fit

CL between the nine measurements and a perfect lower limit at 89.3 GeV is 12%. Table 5 displays a selection of scaled fits with G-O-F CL of 44%. At one extreme the $A_{LR}$ measurement is unscaled, $S_{LR} = 1$, while $S_\tau = 3.5$. The effect on the SM fit is negligible: the central value and 95%CL upper limits for $m_H$ increase by just $\simeq 15\%$. At the other extreme, if we attempt to leave $A^\tau_{FB}$ unscaled, $S_\tau = 1$, we find that even if $A_{LR}$ is removed from the fit, $S_{LR} \to \infty$, the G-O-F CL is 39%. At this extreme in order to reach 44% it is necessary to set $S_\tau = 1.06$ and $S_{LR} \to \infty$. The effect on the fit is maximal: the central value increases to $m_H = 175$ GeV and the 95%CL upper limit increases to 750 GeV.

The scaled fits are obtained numerically, as described below. Consider for instance the entry in table 5 with $S_{LR} = 1$. From table 3 we see that $p'_{LR} = p_{LR} = 0.932$. Equation (1), $P_9(p'_\tau, 0.932) = 0.44$, is solved numerically to obtain $p'_\tau = 0.684$. Assuming Gaussian statistics we then deduce from the Gaussian distribution that $sin^2\theta^{lepton}_{eff}$ from $A^\tau_{FB}$ lies $0.475\delta'^{\,\mathrm{TOTAL}}_\tau$ below $sin^2\theta^{lepton}_{eff} = 0.23151$, the latter being the value of $sin^2\theta^{lepton}_{eff}$ corresponding to $m_H^{\mathrm{LIMIT}} = 89.3$ GeV. Here 'TOTAL' in $\delta'^{\,\mathrm{TOTAL}}_\tau$ denotes the sum in quadrature of the rescaled experimental error $\delta'_\tau$ and the parametric error from the sources shown in table 1,

$$\delta'^{\,\mathrm{TOTAL}}_\tau = \sqrt{\delta'^2_\tau + \delta^2_P}. \tag{19}$$

Taking $sin^2\theta^{lepton}_{eff} = 0.22987$ from $A^\tau_{FB}$ we then obtain $\delta'^{\,\mathrm{TOTAL}}_\tau = (0.23151 - 0.22987)/0.475 = 0.00345$. Using reference [7] the effective parametric error, expressed as an equivalent uncertainty in $sin^2\theta^{lepton}_{eff}$ is 0.00028, so that[8] $\delta'_\tau = 0.00344$, from which we finally obtain $S_\tau = \delta'_\tau/\delta_\tau = 0.00344/000.98 = 3.51$.

The fits for the intermediate cases are obtained similarly, by fixing either $S_\tau$ or $S_{LR}$ and computing the other. Equivalently, one may choose a grid in $p'_\tau$ or $p'_{LR}$ and compute the other, from which all other quantities in the fit can be obtained. (The latter was the procedure actually followed to construct tables 5 and 6).

Except for a small "central plateau" it is clear from the tables and figure that the value of $m_H$ is dominated by $S_{LR}$, as expected from the importance of $A_{LR}$ in the fit. In table 5 the "central plateau" occurs between $S_{LR} = 1.75$

[8]The parametric error is negligible compared to $\delta'_\tau$ but is important relative to more precise measurements such as $\delta_{LR}$.

and $S_{LR} = 2.01$, for which the inverse effects of increasing $S_{LR}$, and decreasing $S_\tau$ cancel one another. At the extreme of table 5, with $S_{LR} \to \infty$, the value of $sin^2\theta_{eff}^{lepton}$ is greater than the conventional fit value by two standard deviations, while the central value of $m_H$ is increased by one standard deviation. The shift in $m_H$ is smaller than the shift in $sin^2\theta_{eff}^{lepton}$ because of the diluting effect of the parametric error in table 1.

Table 6 and the dashed lines in figure 1 are based on $\alpha(m_Z)$ from reference [8]. They display the same general features as the fits based on [7]. The central values for $m_H$ are larger while the 95% CL upper limits are smaller, because reference [8] finds larger $\alpha(m_Z)$ but with smaller claimed uncertainty, and the latter effect dominates the former in the determination of the upper limit. Because the central values are larger, the discrepancies with the search limits are somewhat reduced (cf. tables 3 and 4) and consequently the scale factors are smaller. In the extreme case it is possible to satisfy the G-O-F CL requirement of 44% for $S_\tau = 1$ and finite $S_{LR}$. The fit in that case, with $S_{LR} = 3.6$, yields $m_H = 207$ GeV and $m_H < 670$ GeV. at 95% CL.

**5. Including the search limit confidence levels**

In the previous sections we regarded the search limit, $m_H > 89.3$ GeV, as an absolute boundary, neglecting the fact that it carries a less than perfect 95% confidence level. In this section we will see that the finite confidence level has negligible effect on the scaled fits and that the results presented in section 4 apply to the actual experimental situation.

The conclusion follows from the rather steep dependence of the Higgs boson search limit confidence level as a function of $m_H^{\rm LIMIT}$. For instance, preliminary data[16] from the ALEPH experiment show that the confidence level for $m_H > m_H^{\rm LIMIT}$ is 95% at $m_H^{\rm LIMIT} = 88$ GeV, rising to 99% at 83 GeV and to 99.9% at 78 GeV. These values are conservative since they follow from just one of the four LEP experiments. Furthermore the conclusion reached below that the results of section 4 apply to the real experimental limits does not depend at all sensitively on the values quoted above for $m_H^{\rm LIMIT}$ at 99% and 99.9%, since the dependence on $m_H^{\rm LIMIT}$ is logarithmic.

To get an upper limit on the correction to the "perfect search limit" results of section 4 we consider fits using the evaluation of $\alpha(M_Z)$ claiming greater

precision[8], since those fits are most sensitive to the value of $m_H^{\rm LIMIT}$. Consider the goodness-of-fit CL for the unscaled data. We refine the notation, making explicit the dependence of the probabilities $p_i$ defined in equation (9) on $m_H^{\rm LIMIT}$, by writing $p_\tau(m_H^{\rm LIMIT})$ and $p_{LR}(m_H^{\rm LIMIT})$. Notice from equation (9) that these probabilities are defined for perfect search limits. The actual goodness-of-fit CL can be obtained by weighting the value for a perfect search limit at 89.3 GeV by its actual 95% CL, i.e., $0.95 \cdot 0.181$, and then integrating over the corresponding larger goodness-of-fit CL's for smaller values of $m_H^{\rm LIMIT}$, $P_9(p_\tau(m_H^{\rm LIMIT}), p_{LR})(m_H^{\rm LIMIT}))$, weighted by the probability measure given by the derivative of the experimental search limit confidence level with respect to $m_H^{\rm LIMIT}$.

In practice it suffices to obtain an upper limit by approximating the integral by a discrete sum over a few regions, representing the goodness-of-fit CL for each region by the maximum for the region, which occurs at the lower boundary of the region in $m_H^{\rm LIMIT}$. In the present instance just two regions will suffice, corresponding to the 99 and 99.9% limits quoted above. To an accuracy of $\pm 0.001$ the upper limit on the true goodness-of-fit CL is given by

$$\begin{aligned} P_9^{\rm COMBINED} &= 0.95 P_9\left(p_\tau(89.3\ \text{GeV}), p_{LR}(89.3\ \text{GeV})\right) \\ &+ 0.049 P_9\left(p_\tau(83\ \text{GeV}), p_{LR}(83\ \text{GeV})\right) \\ &+ 0.001 P_9\left(p_\tau(78\ \text{GeV}), p_{LR}(78\ \text{GeV})\right). \end{aligned} \tag{20}$$

The relevant values of $p_\tau$, $p_{LR}$ and $P_9$ are given in table 7. Substituting those values into equation (20) we find that the actual G-O-F CL is bounded above by 0.183 with an uncertainty $\pm 0.001$. This value differs hardly at all from the 0.181 CL that corresponds to a perfect search limit at 89.3 GeV.

Since the scaled data is less precise, the correction due to the actual confidence limits of the searches will be even smaller and is therefore also perfectly negligible for the scaled fits. (I have verified this by applying the above analysis to some of the scaled fits, including the most sensitive case, from table 6 with $S_{LR} = 1$.) In fact, the numerical error in calculating tables 5 and 6 is of order 0.01, much bigger than the 0.002 correction from the finite confidence level of the search limits. We conclude that the fits shown in tables 5 and 6 do in fact reflect the actual experimental confidence levels of the direct search limits.

## 6. Conclusion

Motivated by the observation that within the SM framework two of the nine measurements of $sin^2\theta_{eff}^{lepton}$ are individually in significant conflict with the SM Higgs boson direct search limit, we constructed a scale factor method based on an aggregate goodness-of-fit confidence level between the complete set of nine measurements and the limit. Like an analogous scale factor used for many years by the Particle Data Group, the scale factor proposed here is intended to account for the possibility of underestimated systematic effects. It is applicable to other physical situations in which some of a set of measurements are in conflict with a physical boundary or experimental limit. Applied to the SM Higgs boson mass, the scaled fits exhibit the dependence of the fit on the weight accorded to the two measurements that are in conflict with the search limits. The fits in which the weight of $A_{LR}$ is reduced allow a central value of $m_H$ as large as $\simeq 200$ GeV and a 95%CL upper limit as large as 750 GeV. Relative to the conventional least-square fit, the central value of $sin^2\theta_{eff}^{lepton}$, increases by as much as two standard deviations while $m_H$ increases by as much as one standard deviation.

There is a tendency to think that the value of $sin^2\theta_{eff}^{lepton}$ is only of interest as a prognosticator of the Higgs boson mass, so that it will be of only secondary interest after/if a Higgs boson is discovered. This view underestimates the importance of $sin^2\theta_{eff}^{lepton}$ as a fundamental probe of a variety of new physics, not simply restricted to the Higgs sector. By comparing the measured value of $sin^2\theta_{eff}^{lepton}$ with the value predicted by the directly measured mass of the Higgs boson, we would have a probe of other possible new physics, such as for instance extended gauge sectors or nonsinglet heavy quanta. It would therefore be regrettable if the brilliant program of precision studies of $Z$ particle properties were to conclude with some measure of uncertainty as to how definitively the value of $sin^2\theta_{eff}^{lepton}$ has been determined.

There are a variety of possible explanations for the anomalies that have affected the measurements of $sin^2\theta_{eff}^{lepton}$, both the internal inconsistencies, which have diminished but continue to exist as of this writing, and the inconsistencies with the search limits that are the subject of this paper. They may in fact simply be the result of bad luck, chance fluctuations. They may result from underestimated systematic errors among some of the measurements. Or they may represent real effects and be harbingers of new physics. Hopefully the situation will be clarified by further experimental work, beginning with new

data and/or analyses to be presented at the Summer 1998 conferences.

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## Tables

Table 1. Uncertainties in the evaluation of the natural logarithm of the SM Higgs boson mass, ln $m_H$, from $sin^2\theta_{eff}^{lepton}$. The two values for $\alpha(m_Z)$ and 'Total' correspond to references [7] (larger values) and [8] (smaller values).

| Parameter | $\Delta(\ln m_H)$ |
|---|---|
| $sin^2\theta_{eff}^{lepton}$ | 0.40 |
| $\alpha(m_Z)$ | 0.46 or 0.11 |
| $m_t$ | 0.32 |
| $\alpha_S(m_Z)$ | 0.02 |
| theory | 0.07 |
| Total | 0.67 or 0.52 |

Table 2. Individual measurements of $sin^2\theta_{eff}^{lepton}$ with $1\sigma$ experimental errors and their pulls with respect to the least-square fit value $sin^2\theta_{eff}^{lepton} = 0.23148 \pm 0.00021$, listed in the order of the absolute value of the pulls.

| Measurement | $sin^2\theta_{eff}^{lepton}$ | Pull |
|---|---|---|
| $A_{FB}^{\tau}$ | 0.22987 (98) | -1.61 |
| $A_{LR}$ | 0.23084 (35) | -1.57 |
| $A_{FB}^{b}$ | 0.23211 (39) | +1.42 |
| $A_{\tau}$ | 0.23241 (80) | +1.12 |
| $< Q_{FB} >$ | 0.23210 (100) | +0.60 |
| $A_e$ | 0.23193 (90) | +0.48 |
| $A_{FB}^{c}$ | 0.23160 (110) | +0.12 |
| $A_{FB}^{e}$ | 0.23164 (145) | +0.11 |
| $A_{FB}^{\mu}$ | 0.23147 (82) | +0.01 |

Table 3. SM Higgs boson mass prediction for the individual measurements, based on $\alpha(m_Z)$ from reference [7], listed in order of the precision of the measurements. The central value of $m_H$ is shown along with the symmetric (in ln $m_H$) 90% confidence interval $m_{95}^{>}, m_{95}^{<}$ and the implied probability that $m_H < 89.3$ GeV.

| Measurement | $m_H$ (GeV) | $m_{95}^{>}, m_{95}^{<}$ | $P(m_H < 89.3$ GeV) |
|---|---|---|---|
| $A_{LR}$ | 25 | 6, 100 | 0.93 |
| $A_{FB}^b$ | 280 | 62, 1300 | 0.11 |
| $A_\tau$ | 500 | 35, 7100 | 0.14 |
| $A_{FB}^\mu$ | 83 | 5, 1300 | 0.52 |
| $A_e$ | 200 | 10, 3800 | 0.33 |
| $A_{FB}^\tau$ | 4 | 0.2, 95 | 0.95 |
| $Q_{FB}$ | 280 | 11, 7200 | 0.29 |
| $A_{FB}^c$ | 110 | 4, 2800 | 0.47 |
| $A_{FB}^e$ | 110 | 1, 12000 | 0.47 |

Table 4. Same as table 3 but with $\alpha(m_Z)$ from reference [8].

| Measurement | $m_H$ (GeV) | $m_{95}^{>}, m_{95}^{<}$ | $P(m_H < 89.3$ GeV) |
|---|---|---|---|
| $A_{LR}$ | 33 | 10, 110 | 0.91 |
| $A_{FB}^b$ | 370 | 100, 1400 | 0.04 |
| $A_\tau$ | 660 | 50, 8600 | 0.10 |
| $A_{FB}^\mu$ | 110 | 8, 1500 | 0.45 |
| $A_e$ | 260 | 15, 4700 | 0.27 |
| $A_{FB}^\tau$ | 5 | 0.2, 120 | 0.93 |
| $Q_{FB}$ | 360 | 15, 8800 | 0.24 |
| $A_{FB}^c$ | 140 | 6, 3400 | 0.41 |
| $A_{FB}^e$ | 150 | 2, 15000 | 0.42 |

Table 5. Fits based on $\alpha(m_Z)$ from reference [7]. The first line is the conventional fit while the other lines display scaled fits that meet the 44% minimum goodness-of-fit confidence level for the measurements and search limit. For each fit, specified by the pair of scale factors $S_\tau, S_{LR}$, the table displays the fitted value of $sin^2\theta_{eff}^{lepton}$ with $1\sigma$ uncertainty, the central value of $m_H$, the conventional 95% CL upper limit, $m_{95}$, and the Cousins-Feldman[10] 95% CL upper limit, $m_{95}^{\mathrm{CF}}$.

| $S_{LR}$ | $S_\tau$ | $sin^2\theta_{eff}^{lepton}$ | $m_H$ | $m_{95}$ | $m_{95}^{\mathrm{CF}}$ |
|---|---|---|---|---|---|
| 1 | 1 | 0.23148 (21) | 85 | 260 | 320 |
| 1 | 3.51 | 0.23155 (22) | 97 | 300 | 370 |
| 1.11 | 2.27 | 0.23160 (22) | 105 | 320 | 400 |
| 1.26 | 1.87 | 0.23165 (23) | 117 | 370 | 460 |
| 1.42 | 1.74 | 0.23170 (24) | 127 | 410 | 510 |
| 1.59 | 1.71 | 0.23173 (24) | 137 | 440 | 550 |
| 1.78 | 1.68 | 0.23177 (25) | 146 | 480 | 600 |
| 2.01 | 1.28 | 0.23177 (25) | 147 | 480 | 600 |
| 2.50 | 1.16 | 0.23180 (26) | 154 | 510 | 640 |
| $\infty$ | 1.06 | 0.23186 (27) | 175 | 590 | 750 |

Table 6. As in table 5 but with $\alpha(m_Z)$ from reference [8].

| $S_{LR}$ | $S_\tau$ | $sin^2\theta_{eff}^{lepton}$ | $m_H$ | $m_{95}$ | $m_{95}^{\mathrm{CF}}$ |
|---|---|---|---|---|---|
| 1 | 1 | 0.23148 (21) | 112 | 260 | 310 |
| 1 | 1.84 | 0.23154 (21) | 124 | 295 | 350 |
| 1.07 | 1.71 | 0.23157 (22) | 131 | 310 | 370 |
| 1.18 | 1.60 | 0.23161 (23) | 143 | 350 | 420 |
| 1.31 | 1.57 | 0.23165 (23) | 155 | 385 | 460 |
| 1.45 | 1.54 | 0.23169 (24) | 167 | 420 | 500 |
| 1.62 | 1.18 | 0.23170 (24) | 169 | 430 | 520 |
| 1.75 | 1.12 | 0.23171(24) | 173 | 440 | 530 |
| 2.00 | 1.08 | 0.23174 (25) | 182 | 470 | 570 |
| 3.62 | 1.00 | 0.23181 (26) | 207 | 550 | 670 |

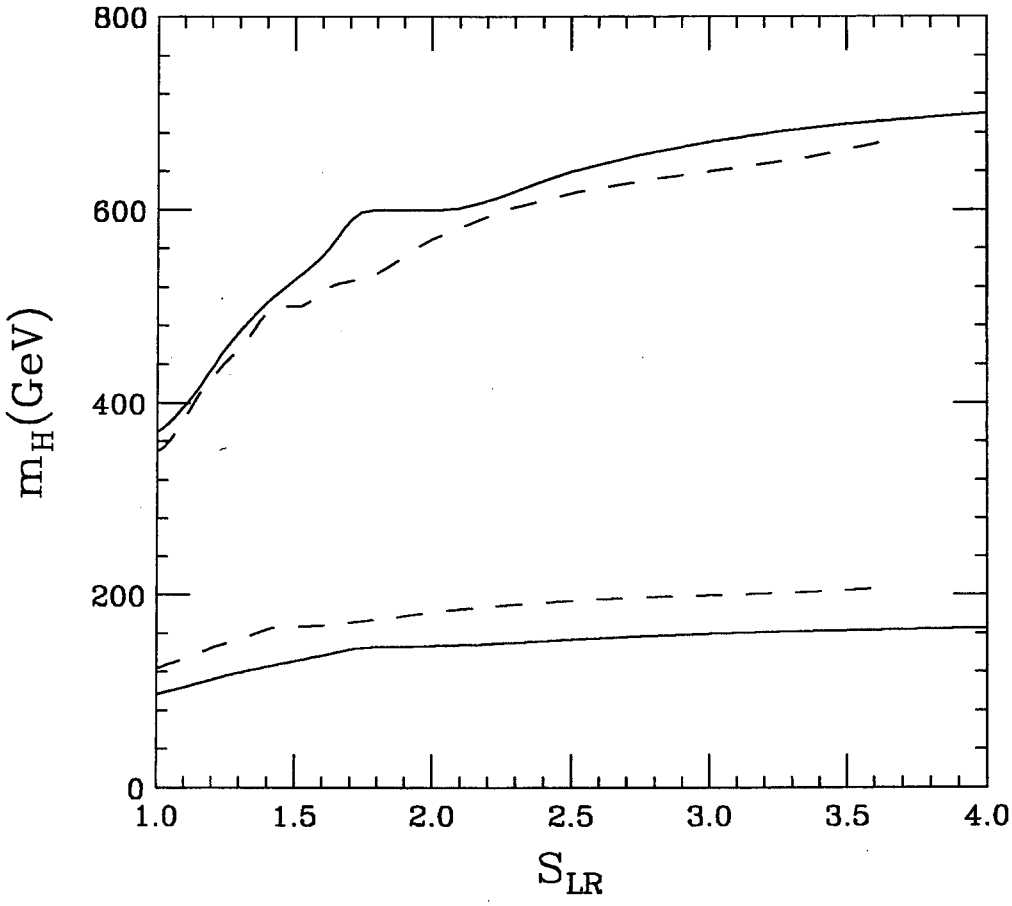
Table 7. The goodness-of-fit confidence level between the nine $sin^2\theta_{eff}^{lepton}$ measurements and the direct search limit for $m_H^{\mathrm{LIMIT}}$ corresponding to experimental confidence levels of 95%, 99%, and 99.9%. The G-O-F CL's, $P_9(p_\tau, p_{LR})$, are computed assuming perfect search limits at each $m_H^{\mathrm{LIMIT}}$, as discussed in the text. Reference [8] is used for $\alpha(m_Z)$.

| Search Limit CL | $m_H^{\mathrm{LIMIT}}$(GeV) | $p_\tau$ | $p_{LR}$ | $P_9(p_\tau, p_{LR})$ |
|---|---|---|---|---|
| 95% | 89.3 | 0.933 | 0.910 | 0.181 |
| 99% | 83 | 0.928 | 0.894 | 0.225 |
| 99.9% | 78 | 0.924 | 0.878 | 0.264 |

## Figure Caption

Figure 1. Scaled fits that meet the minimum goodness-of-fit criterion. The central value and 95%CL upper limit for the Higgs boson mass are plotted as a function of the scale factor for $sin^2\theta_{eff}^{lepton}$ from $A_{LR}$. Solid and dashed lines correspond to the evaluations of $\alpha(m_Z)$ from references [7] and [8] respectively.

Figure 1



Ernest Orlando Lawrence Berkeley National Laboratory
One Cyclotron Road | Berkeley, California 94720

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