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THE CONFINING $N = 1$ SUPERSYMMETRIC GAUGE THEORIES: A REVIEW[a]

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We give a classification and overview of the confining $N = 1$ supersymmetric gauge theories. For simplicity we consider only theories based on simple gauge groups and no tree-level superpotential. Classification of these theories can be done according to whether or not there is a superpotential generated for the confined degrees of freedom. The theories with the superpotential include s-confining theories and also theories where the gauge fields participate in the confining spectrum, while theories with no superpotential include theories with a quantum deformed moduli space and theories with an affine moduli space.

1 Introduction

In this talk, we give an overview of the confining $N = 1$ supersymmetric gauge theories. Before jumping into the details of the classification of such models, one has to answer the question of what we mean by a confining theory. The definition we will be using throughout this talk is the following: *we call a theory confining, if there is a low-energy description purely in terms of composite gauge singlets* (that is, the low-energy effective theory is a Wess-Zumino model for the gauge singlet fields, there are no massless gauge degrees of freedom in the IR theory). This broad definition of confinement does not automatically imply that there would be an area law for the Wilson loop, or a linear potential between external test charges. The reason is that in some cases (when there are massless dynamical fields in a faithful representation of the gauge group), the external charges can be screened, and instead of a linear potential there will be no potential at all. In this case there is no phase boundary between the

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Higgs and the confining phases, and there is no invariant distinction between these two phases. This is what will actually happen in most of the examples reviewed below. Keeping this broad definition of confinement in mind, we are ready to discuss the classification of these models.

For simplicity, we will consider only theories based on simple gauge groups and no tree-level superpotential. Then the confining theories can be classified into two broad categories, according to whether or not there is a superpotential generated for the composite fields. These two categories can be further refined: for the case of theories with a confining superpotential, one can distinguish between theories where the composites contain only chiral superfields (these are the s-confining theories), or theories where the gauge field W_α also participates in forming the composites. In the case of theories with no superpotential, one can distinguish between theories where there are classical constraints relating the composites and theories where there are no such constraints. These categories will be discussed in detail below. A final category which we will not discuss in detail is when the low-energy effective theory is empty, that is there is a mass gap, and no massless chiral superfields are present. This is the case for example for $N = 1$ pure Yang-Mills theories. However, we expect such theories to be very rare for the following reason. If there is an exact continuous global symmetry present in the theory, then it is either spontaneously broken or not. If it is spontaneously broken, we expect massless Goldstone-bosons, if it is not broken, then the 't Hooft anomaly matching conditions have to be satisfied, implying the presence of massless fermions. Thus we expect that only theories like pure $N = 1$ Yang-Mills, with no continuous global symmetries to exhibit such behavior. Finally, a warning: the four categories to be explained below contain all confining theories known up today. However, it is possible, that there might be a lot more confining theories around, which might not fit into the above classification scheme.

2 Theories with a Non-vanishing Confining Superpotential

2.1 The S-confining Theories

S-confining theories are defined as follows[1]:

- there is a non-vanishing superpotential for the confined degrees of freedom (non-singular at the origin)

- the composites involve only chiral superfields

- the description in terms of gauge invariant composites is valid everywhere on the moduli space.

The first example of an s-confining theory has been found by Seiberg[2]. We will use this example ($SU(N)$ theory with $F = N + 1$ flavors) to explain the most important properties of such theories. The field content and global symmetries of the theory, together with the confining spectrum is given below.

	$SU(N)$	$SU(N+1)$	$SU(N+1)$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	1	1	$\frac{1}{N+1}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	$\frac{1}{N+1}$
$M=(Q\bar{Q})$		$\square$	$\square$	0	$\frac{2}{N+1}$
$B=(Q^N)$		$\bar{\square}$	1	N	$\frac{N}{N+1}$
$\bar{B}=(\bar{Q}^N)$		1	$\bar{\square}$	$-N$	$\frac{N}{N+1}$

The confining superpotential is

$$\frac{1}{\Lambda^{2N-1}}(\det M - \bar{B}MB) \tag{1}$$

There is ample of evidence that this is indeed the correct low-energy description of the original $SU(N)$ theory [2]. First of all, the confined degrees of freedom M, B and $\bar{B}$ satisfy the 't Hooft anomaly matching conditions. Second, the classical limit is correctly reproduced by the superpotential, since the equations of motion result exactly in the classical constraints of the theory. Finally, integrating out flavors results in the correct descriptions of the theories with less flavors. Subsequently, several other s-confining theories have been found[3,4]. The natural question to ask is how to find all other s-confining theories. We will answer this question below.

The most severe constraint on s-confining theories comes from the requirement that there is a non-vanishing confining superpotential. Global symmetries fix this superpotential to be of the form [1]

$$W \propto \Lambda^3 \left[\prod_i \frac{\Phi_i^{\mu_i}}{\Lambda^{\mu_i}}\right]^{\frac{2}{\sum_i \mu_i - G}}, \tag{2}$$

where Φ_i are the underlying chiral superfields (not the composites), μ_i is the Dynkin index with respect to the gauge group of the i^{th} chiral superfield given by $\mathrm{Tr}\, T^i_A T^i_B = \mu_i \delta_{AB}$, where the T's are the generators of the gauge group in the i^{th} representation, and G is the Dynkin index of the adjoint. For example in the case of Seiberg's example $SU(N)$ with $N+1$ flavors $\Phi_i = Q_i, \bar{Q}_i$ (i is the flavor index $i = 1, \ldots, N+1$), $\mu_i = 1$, $G = 2N$, thus the superpotential has the form $Q^{N+1}\bar{Q}^{N+1}/\Lambda^{2N-1}$, which can be written in terms of the confined degrees of freedom either as $\det M$ or $\bar{B}MB$.

Examining the form of (2) one can observe, that the confining superpotential is singular at the origin unless the overall exponent is an integer, implying the index constraint

$$\sum_i \mu_i - G = 2 \text{ or } 1. \tag{3}$$

This is a very severe constraint on the matter content of a given theory. In fact, it restricts the candidates for s-confining theories to a finite set. This set of theories for the case of $SU(N)$ groups is given in Table 1. In order to find out which of those theories listed in Table 1 are actually s-confining, we note one more necessary condition the s-confining theories have to satisfy: an s-confining theory flows only to s-confining theories. The reason behind this is simple. An s-confining theory is described by a set of gauge invariant operators. Going along a flat direction in this language just means giving VEV's to some gauge invariant fields, thus the resulting theory also has to be describable in terms of a theory of gauge invariants. Using this condition one can go ahead and check the various flows of the candidate theories listed in Table 1. The theories where a flow results in a non-s-confining theory can be excluded. For the remaining examples one can explicitly find the confined spectrum and show that the consistency conditions are all satisfied. This way one can find all s-confining theories based on simple groups. The results for $SU(N)$ theories are listed in Table 1. Here we give just one more simple s-confining example, which is based on $SU(5)$ with three antisymmetric tensors and three antifundamentals. The detailed description of the remaining $SU(N)$ theories together with the theories based on other groups can be found in [1].

	$SU(5)$	$SU(3)$	$SU(3)$	$U(1)$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0
$\bar{Q}$	$\overline{\square}$	1	$\square$	-3	$\frac{2}{3}$
$A\bar{Q}^2$		$\square$	$\overline{\square}$	-5	$\frac{4}{3}$
$A^3\bar{Q}$		$\begin{smallmatrix}\square&\square\\\square&\end{smallmatrix}$	$\square$	0	$\frac{2}{3}$
A^5		$\square\square$	1	5	0

$$W_{dyn} = \frac{1}{\Lambda^9}\left[(A^5)(A^3\overline{Q})(A\overline{Q}^2) + (A^3\overline{Q})^3\right]$$

2.2 Composites Contain W_α

This category does not have its own name, since there is only one known example [5]. This example is based on an $SO(N)$ theory with $N-3$ vectors. Intriligator and Seiberg argued, that there is a branch on which the theory

Table 1: All SU theories satisfying $\sum_i \mu_i - G = 2$. This list is finite because the indices of higher index tensor representations grow very rapidly with the size of the gauge group. We give the gauge group in the first column, and the field content in the second column. In the third column, we indicate which theories are s-confining. For the theories which do not s-confine we give the flows to non s-confining theories or indicate that there is a Coulomb branch on the moduli space.

$SU(N)$	$(N+1)(\Box + \overline{\Box})$	s-confining
$SU(N)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + N\overline{\Box} + 4\Box$	s-confining
$SU(N)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} + 3(\Box + \overline{\Box})$	s-confining
$SU(N)$	Adj $+\Box + \overline{\Box}$	Coulomb branch
$SU(4)$	Adj $+\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	Coulomb branch
$SU(4)$	$3\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2(\Box + \overline{\Box})$	$SU(2)$: $8\Box$
$SU(4)$	$4\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \Box + \overline{\Box}$	$SU(2)$: $\Box\Box + 4\Box$
$SU(4)$	$5\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	Coulomb branch
$SU(5)$	$3(\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\Box})$	s-confining
$SU(5)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\Box + 4\overline{\Box}$	s-confining
$SU(5)$	$2(\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}})$	$Sp(4)$: $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2\Box$
$SU(5)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} + 2\overline{\Box} + \Box$	$SU(4)$: $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2(\Box + \overline{\Box})$
$SU(6)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 5\overline{\Box} + \Box$	s-confining
$SU(6)$	$2\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} + 2\overline{\Box}$	$SU(4)$: $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 2(\Box + \overline{\Box})$
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + 4(\Box + \overline{\Box})$	s-confining
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 3\overline{\Box} + \Box$	$SU(5)$: $2\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} + 2\overline{\Box} + \Box$
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	$Sp(6)$: $\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \Box$
$SU(6)$	$2\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \Box + \overline{\Box}$	$SU(5)$: $2(\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}})$
$SU(7)$	$2(\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 3\overline{\Box})$	s-confining
$SU(7)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + 4\overline{\Box} + 2\Box$	$SU(6)$: $\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + 3\overline{\Box} + \Box$
$SU(7)$	$\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}} + \Box$	$Sp(6)$: $\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix} + \begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \Box$

confines with the following spectrum:

	$SO(N)$	$SU(N-3)$	$U(1)_R$	Z_{2N-6}
Q	$\square$	$\square$	$\frac{-1}{N-3}$	1
$M=(Q^2)$		$\square\square$	$\frac{-2}{N-3}$	2
$b=(W_\alpha^2 Q^{N-4})$		$\bar{\square}$	$1+\frac{1}{N-3}$	$N-4$

The confining superpotential is given by

$$W = Mb^2.$$

There are lots of checks that this spectrum is indeed correct [5,6], including integrating out a flavor from the theory with $F = N-2$ and obtain this branch, continuous and discrete anomaly matching, and integrating out one more flavor. However, this is the only known example of this kind, and it would be very interesting to find more confining theories of this sort.

3 Theories with a Vanishing Confining Superpotential

There are two broad classes of known confining theories with vanishing superpotential. One class includes the famous theories with a quantum deformed moduli space, while the other class contains the theories with an "affine moduli space" of vacua. These can be distinguished by noting, that in the first case there are non-trivial classical constraints among the basic composite invariants, while in the second case there are none.

3.1 Theories with Constraints: Quantum Deformed Moduli Space

The first example of a theory with a quantum modified constraint has been discovered by Seiberg[2]. The example is SUSY QCD with the number of colors equal to the number of flavors, $SU(N)$ with $F = N$. The field content and global symmetries of the theory, together with the confining spectrum is given below.

	$SU(N)$	$SU(N)$	$SU(N)$	$U(1)_B$	$U(1)_R$
Q	$\square$	$\square$	1	1	0
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	0
$M=(Q\bar{Q})$		$\square$	$\square$	0	0
$B=(Q^N)$		1	1	N	0
$\bar{B}=(\bar{Q}^N)$		1	1	$-N$	0

The composites M, B and $\bar{B}$ satisfy the classical constraint $\det M - B\bar{B} = 0$. In the infrared, quantum effects modify this constraint to $\det M - B\bar{B} = \Lambda^{2N}$.

Table 2: The SU theories satisfying the index constraint $\sum_i \mu_i = G$. The first column gives the gauge group, the second column the field content and the third column gives the phase of the low-energy theory. QDMS stands for confining with a quantum deformed moduli space, and i and c distinguish between the cases where the constraint which is quantum modified is invariant or covariant under the global symmetries of the theory [9]. Note, that all theories satisfying $\sum_i \mu_i = G$ are either confining with a quantum modified constraint or in the Coulomb phase. The theories in the Coulomb phase have been discussed in Ref. [12].

$SU(N)$	$N(\Box + \overline{\Box})$	i-QDMS
$SU(N)$	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + (N-1)\overline{\Box} + 3\Box$	i-QDMS
$SU(N)$	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + \overline{\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array}} + 2(\Box + \overline{\Box})$	i-QDMS
$SU(N)$	Adj	Coulomb phase
$SU(4)$	$3\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + (\Box + \overline{\Box})$	c-QDMS
$SU(4)$	$4\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array}$	Coulomb phase
$SU(5)$	$2\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + \Box + 3\overline{\Box}$	i-QDMS
$SU(5)$	$2\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + \overline{\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array}} + \overline{\Box}$	c-QDMS
$SU(6)$	$2\begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + 4\overline{\Box}$	i-QDMS
$SU(6)$	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \\ \hline \end{array} + 3(\Box + \overline{\Box})$	i-QDMS
$SU(6)$	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{\|c\|}\hline \\ \hline \\ \hline \end{array} + 2\overline{\Box}$	c-QDMS
$SU(6)$	$2\begin{array}{\|c\|}\hline \\ \hline \\ \hline \\ \hline \end{array}$	Coulomb phase
$SU(7)$	$\begin{array}{\|c\|}\hline \\ \hline \\ \hline \\ \hline \end{array} + 4\overline{\Box} + 2\Box$	c-QDMS

Again there is a lot of evidence that this is indeed what happens. The 't Hooft anomaly matching conditions are not satisfied at the origin, but they are satisfied at any point on the quantum deformed moduli space (from which the origin is excluded). Integrating out one flavor reproduces the well-known Affleck-Dine-Seiberg superpotential [7], and higgsing the gauge group will also give a consistent result. Later more theories with a quantum modified constraint have been identified [3,4,8].

One can again try to find all theories that similarly have a quantum modified constraint. In these theories a classical constraint of the form $\sum(\Pi_i X_i) = 0$ (where X_i are gauge invariant operators) is modified quantum mechanically to $\sum(\Pi_i X_i) = \Lambda^p \Pi_j X_j$. Here, the X_j are some other combination of the gauge invariant operators, including the possibility that the quantum modification is just Λ^p. The power p must necessarily be positive to reproduce the correct classical limit. Such a modification of the classical constraint is only possible in theories where $\sum \mu_i - G = 0$. To show this, consider assigning R-charge zero to every chiral superfield. This R-symmetry is anomalous and the anomaly has to be compensated by assigning R-charge $\sum \mu_i - G$ to the scale of the gauge group raised to the power of its one loop β function coefficient $\Lambda^{(3G-\sum \mu_i)/2}$. Since the constraints have to respect this R-symmetry one immediately sees that Λ can only appear in a constraint if it has vanishing R-charge. Therefore, we conclude that only theories with $\sum \mu_i - G = 0$ may exhibit quantum deformed moduli spaces. We can find all theories satisfying $\sum \mu_i - G = 0$ by simply leaving out a flavor from the matter contents listed in Table 1. The resulting theories are displayed in Table 2. The theories based on SU groups of Table 1 have been examined in detail by Grinstein and Nolte [9], and those based on other groups by Grinstein and Nolte [10] and Cho [11]. Again, based on the flows one can exclude all theories from Table 1 which do not flow to a confining theory. In the remaining examples one can find the quantum modified constraint either by integrating out one flavor from an s-confining theory, or if the theory with one more flavor is not s-confining, then one has to consider the flows along various flat directions in order to find what the quantum modified constraint is. It has been found in [9,10] that there are two types of theories with a quantum modified constraint. One possibility is that the constraint is invariant under all global symmetries, then the quantum modified constraint has the form $\Pi_i X_i = \Lambda^p$, and the origin is excluded from the moduli space by the quantum modification. The other possibility is that the constraint carries a non-vanishing global charge, and thus the quantum modification must be field dependent, of the form $\Pi_i X_i = \Lambda^p X_1$, where X_1 is a single composite field. In this case, the origin of the moduli space is not excluded, and the 't Hooft anomaly matching conditions have to be satisfied after the field X_1 is eliminated from the spectrum. Below, we present an example where the quantum modified constraint is covariant under the global symmetries. The example is based on an $SU(4)$ gauge theory with matter in $3\begin{smallmatrix}\Box\\\Box\end{smallmatrix} + \Box + \overline{\Box}$. The theory with an additional flavor is not s-confining. The confining spectrum is given in the table below.

	$SU(4)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	0	1	0
Q	$\square$	1	1	-3	0
$\bar{Q}$	$\bar{\square}$	1	-1	-3	0
A^2		$\square\!\square$	0	2	0
$QA^2\bar{Q}$		$\bar{\square}$	0	-4	0
$Q\bar{Q}$		1	0	-6	0
A^3Q^2		1	2	-3	0
$A^3\bar{Q}^2$		1	-2	-3	0

The quantum modified constraint is

$$\frac{1}{6}(Q\bar{Q})^2(A^2)^3 + 4(A^2)(QA^2\bar{Q})^2 + 64(A^3Q^2)(A^3\bar{Q}^2)^2 = \Lambda^8(Q\bar{Q})$$

Note that one can eliminate the field $(Q\bar{Q})$ from the theory by solving the quantum modified constraint. The remaining fields match all anomalies of the ultraviolet theory.

Finally, we note that there are no known confining theories, where classical constraints among the basic invariants do exist, but none of them is quantum modified. However, there is no argument why theories like that could not exist.

3.2 No Constraints Among Invariants: Affine Moduli Space

The first and perhaps most famous example of this class of theories is the ISS model [13], which is an $SU(2)$ gauge theory with one chiral superfield Q in the spin 3/2 representation of the gauge group. Classically this theory has a single independent gauge invariant Q^4, which satisfies the 't Hooft anomaly matching conditions. Therefore it is widely believed that this theory confines without generating a confining superpotential. Theories which have at least a branch on which they behave analogously have been later found in Refs. [5,4,1]. The classification of such theories has been done by Dotti and Manohar. They obtain a list of all theories where there is no constraint among the fundamental composites (which they call theories with an affine moduli space), and on these theories they explicitly check whether the 't Hooft anomaly matching conditions are satisfied or not. The resulting theories are given in Table 3. The first six theories in Table 3 have a confining branch with no superpotential generated in addition to a branch with a dynamically generated superpotential. The seventh theory is the ISS model which as explained above presumably only has a confining phase with no superpotential generated. The phase of

Table 3: The theories which have no classical constraints among the basic invariants and satisfy 't Hooft anomaly matching from Ref.[14]. The first column gives the gauge group, the second column the matter content. S stands for the spinor of the given SO group.

$SU(2N)$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}+\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$
$SU(6)$	$\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\\\Box\end{smallmatrix}$
$Sp(2N), N \geq 2$	
$SO(N), N \geq 5$	$(N-4)\Box$
$SO(12)$	$2S$
$SO(14)$	S
$SU(2)$	$\Box\!\Box\!\Box$
$SU(8)$	$\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}$
$Sp(8)$	$\begin{smallmatrix}\Box\\\Box\\\Box\\\Box\end{smallmatrix}$
$SO(N), N \geq 5$	$\Box\!\Box$
$SO(16)$	S

the last four theories is not very well established. The fact that the 't Hooft anomaly matching conditions are satisfied would suggest that these theories are confining just like the ISS model. However, a more careful analysis of the different branches of these theories shows that it is unlikely that these theories confine at the origin, instead they are likely to be in an interacting non-Abelian Coulomb phase[15].

Finally, we note that Dotti and Manohar have also shown that the only theories with no classical invariants at all (which are believed to break supersymmetry dynamically) are the two well-known examples: $SU(5)$ with $10+\bar{5}$ and $SO(10)$ with a single spinor.

4 Conclusions

There have been a lot of new results recently concerning the low-energy dynamics of $N=1$ supersymmetric gauge theories. The simplest of these theories are the confining ones, where the low-energy effective theory is simply a theory of gauge singlets. The known confining theories can be classified according to whether or not there is a superpotential generated for the confined degrees of freedom. The theories which do have a confining superpotential include the s-confining theories and the theories where the composites involve the gauge field W_α. The class of theories where there is no superpotential for the confined degrees of freedom contains the theories with a quantum deformed moduli space and the theories with an affine quantum moduli space. Some of these categories (s-confining, quantum deformed moduli space, affine moduli space) have been exhaustively studied for the case of simple gauge group and no tree-level superpotential. Others are not well understood, and perhaps there might be completely new types of confining theories waiting to be discovered.

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