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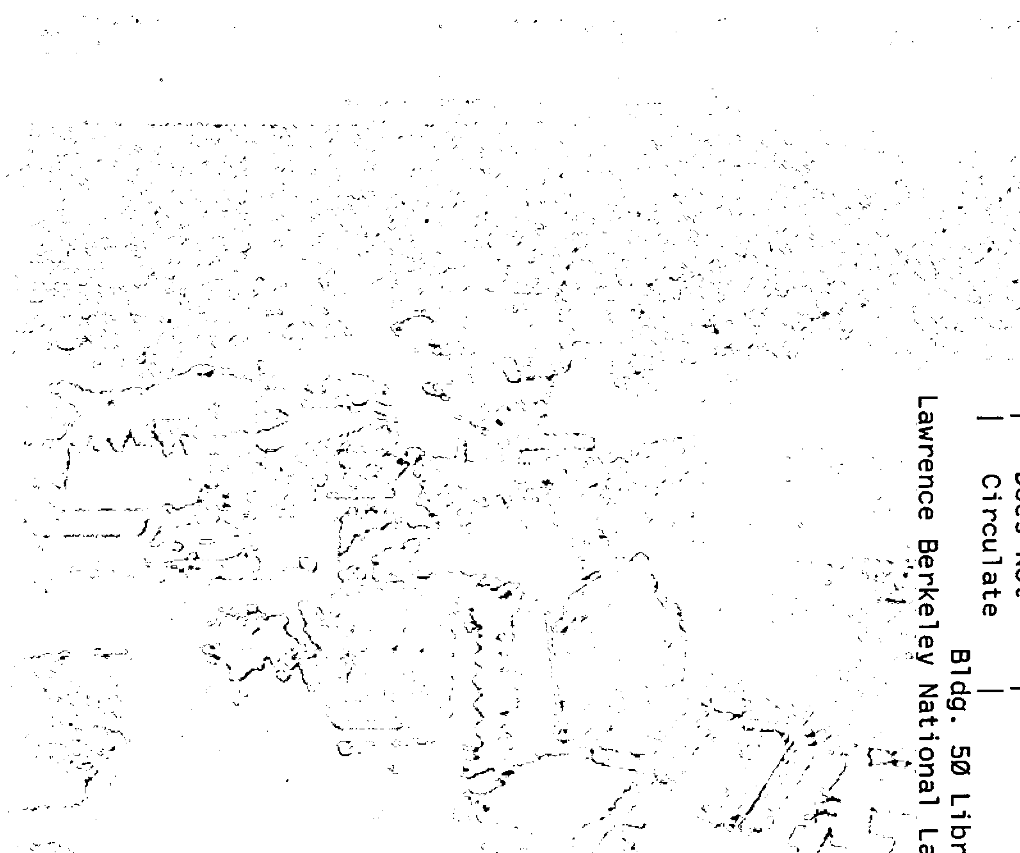

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Textures for Atmospheric and Solar Neutrino Oscillations *

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Abstract

In theories with three light neutrinos, a complete list of five zeroth-order textures for lepton mass matrices is found, which naturally yield: $m_{e,\mu} = 0, m_\tau \neq 0, \Delta m^2_\odot = 0$, together with $\theta_{e\tau} = 0, \theta_{\mu\tau} \approx 1$. These textures provide suitable starting points for constructing theories which account for both atmospheric and solar neutrino fluxes. Using flavor symmetries, two schemes for such lepton masses are found, and example constructions exhibited.

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1 The Super-Kamiokande Collaboration has provided strong evidence that atmospheric ν_μ are depleted as they traverse the Earth [1, 2]. For example, the up/down event ratio shows a 6σ statistical significance, and could only be explained by some systematic effect an order of magnitude larger than those already considered. With two flavor $\nu_\mu \to \nu_\tau$ oscillations, the mixing angle is large, $\sin^2 2\theta > 0.82$, and Δm^2_{ATM} is within a factor of three of 1.5×10^{-3} eV^2 at the 90% C.L. [2].

In this letter we study lepton masses in theories with three light neutrinos. With the assumptions given below, we find a complete list of five zeroth-order textures, which can account both for this atmospheric data and for the presence of solar neutrino oscillations at some frequency $\Delta m^2_\odot \ll \Delta m^2_{ATM}$. Requiring the textures to follow from symmetries, we are able to find only two zeroth-order schemes for the light lepton mass matrices which give both the observed atmospheric and solar neutrino fluxes.

2 Both charged and neutral lepton mass matrices involve small dimensionless parameters: $m_e/m_\tau \ll m_\mu/m_\tau \ll 1$ and $\Delta m^2_\odot/\Delta m^2_{ATM} \ll 1$. In the flavor basis, the lepton mass matrices, $\bar{\ell}_L m_E \ell_R$ and $\nu_L^T m_{LL} \nu_L$, can be written as a perturbation series in a set of small parameters ϵ:

$$m_E = v\left(\lambda_E^{(0)} + \lambda_E^{(1)}(\epsilon) + ...\right) \tag{1}$$

$$m_{LL} = \frac{v^2}{M}\left(\lambda_\nu^{(0)} + \lambda_\nu^{(1)}(\epsilon) + ...\right) \tag{2}$$

where v is the electroweak symmetry breaking scale and M is some new large mass scale, so that the $\lambda^{(i)}$ are dimensionless contributions at ith order in perturbation theory. The matrices $\lambda_E^{(0)}$ and $\lambda_\nu^{(0)}$ have entries which are either zero or of order unity, and the possible forms for these matrices, which we call zeroth-order textures, are the subject of this letter.[1]

Diagonalization of m_E and m_{LL} gives both the lepton mass eigenvalues and the leptonic mixing matrix V, defined by the charged current interaction $\bar{\nu} V^T \gamma^\mu \ell\, W_\mu$. Ignoring phases, which are irrelevant for our purposes, we define the three Euler angles by $V = V_e^\dagger V_\nu = R_{23}(\theta_{23}) R_{13}(\theta_{13}) R_{12}(\theta_{12})$. The rotation matrices $V_{e,\nu}$, which diagonalize $m_{E,LL}$, are the same functions of the angles $\theta_{E,\nu ij}$ as V is of θ_{ij}. We require that the diagonalization of $\lambda_{E,\nu}^{(0)}$ leads to zeroth-order eigenvalues

$$m_{E1}^{(0)} = m_{E2}^{(0)} = 0 \tag{3}$$

$$m_{E3}^{(0)} \simeq m_\tau \tag{4}$$

$$\Delta m_\odot^{2(0)} \equiv m_{\nu 1}^2 - m_{\nu 2}^2 = 0. \tag{5}$$

[1] Our classification of textures does not assume any flavor symmetry. However, perturbation series of the type (1) and (2) could result from a flavor symmetry which defines the flavor basis, allows the terms $\lambda_{E,\nu}^{(0)}$, and leads to higher order corrections as a power series in the small flavor symmetry breaking parameters ϵ.

In most cases $\lambda_\nu^{(0)}$ will lead to a non-zero value for Δm^2_{ATM}; however, this is not a strict requirement.

To diagonalize $\lambda_E^{(0)}$, we first rotate the right-handed charged leptons to obtain the form:

$$\lambda_E^{(0)} = \begin{pmatrix} 0 & 0 & C \\ 0 & 0 & B \\ 0 & 0 & A \end{pmatrix}.$$

We diagonalize this form by choosing $\theta^{(0)}_{E12} = 0$, which defines a basis for the light states, and determines $\theta^{(0)}_{E13,23}$ in terms of A, B and C. We further require that the diagonalization of $\lambda^{(0)}_{E,\nu}$ gives zeroth-order leptonic mixing angles

$$\theta^{(0)}_{23} \approx 1 \tag{6}$$

and

$$\theta^{(0)}_{13} = 0. \tag{7}$$

The requirement $\theta^{(0)}_{13} = 0$ follows from recent fits to the Super-Kamiokande atmospheric data [3], which find θ_{13} less than approximately 20°, for a hierarchy of Δm^2 values such as we assume. Furthermore, for $\Delta m^2_{ATM} > 2 \cdot 10^{-3}$ eV^2, the CHOOZ experiment requires $\theta_{13} < 13°$. We therefore take the view that a non-zero value of θ_{13} can be at most of order ϵ. [2]

Many theories of flavor with hierarchical fermion masses yield small mixing angles — as in the well-known example of the Cabibbo angle $\theta_c \approx (m_d/m_s)^{1/2}$. Conversely, theories with large mixing angles typically do not have mass hierarchies. To avoid this typical situation, the textures for $\lambda^{(0)}_{E,\nu}$ become highly constrained. One must search either for a $\lambda^{(0)}_\nu$ which gives a large $\theta^{(0)}_{\nu 23}$, while maintaining the degeneracy $\Delta m^{2(0)}_\odot = 0$; or a $\lambda^{(0)}_E$ which gives a large $\theta^{(0)}_{E23}$, while maintaining the charged lepton mass hierarchy $m^{(0)}_{e,\mu} = 0$.[3] There are few solutions to this puzzle, and therefore few zeroth-order textures.

[2] Suppose a zeroth-order texture gives $\theta^{(0)}_{13} \approx 1$. Since perturbations may give $\theta_{E12} \approx 1$, it is conceivable that on computing V one finds a precise cancellation, so that $\theta_{13} = 0$. In general such cases are fine tuned, and we do not consider them further. For the textures which do give $\theta^{(0)}_{13} = 0$, it is important that the perturbations to λ_E do not induce $\theta_{E12} \approx 1$, since this will in general lead to $\theta_{13} \approx 1$.

[3] It has been noted [4] that the atmospheric data does not exclude a conventional interpretation, with small angles following from mass hierarchies, because the relevant mass hierarchies may not be large. With $\theta^{(0)}_{E,\nu 23} = 0$, at order ϵ one typically finds $|\theta_{\nu 23}| = (m_\odot/m_{ATM})^{1/2}$ and $|\theta_{E23}| = (m_\mu/m_\tau)^{1/2}$. For $\Delta m^2_\odot = 10^{-5}$ eV^2 and $\Delta m^2_{ATM} = 10^{-3}$ eV^2, and opposite signs for $\theta_{\nu 23}$ and θ_{E23}, one finds $\sin^2 2\theta_{23} = 0.82$, at the edge of the Super-Kamiokande 90% C.L. allowed region. It is very interesting that this case is not excluded, but it is not favored, and we study the more revolutionary case that at least one of $\theta^{(0)}_{\nu,E23}$ is non-zero. With a suitable see-saw structure $|\theta_{\nu 23}| = (m_\odot/m_{ATM})^{1/4}$ is also possible [5].

3 In searching for textures for $\lambda^{(0)}_{\nu,E}$, we allow two types of relations between otherwise independent, non-zero matrix elements.

- Any two entries of a matrix may be set equal. This can frequently result from a symmetry, but we do not require that a symmetry origin can be found.
- The determinant of each matrix, or any of its 2×2 sub-determinants, can be set to zero.

We do not allow other precise relations. The determinantal conditions arise naturally when heavy states are integrated out, as in the seesaw and Froggatt-Nielsen mechanisms. This may be due to zeroth order textures of the light-heavy couplings for any (non singular) mass matrix of the heavy states, or to a mass hierarchy of the heavy states themselves. Suppose that a heavy charged lepton couples to combinations $\Sigma_i a_i e_{Li}$ and $\Sigma_i b_i e_{Ri}$ of left- and right-handed charged lepton flavor eigenstates. On integrating out the heavy state, $\lambda^{(0)}_{Eij} \propto a_i b_j$ and has only a single non-zero eigenvalue. Integrating out each heavy state leads to a single eigenvalue in the light matrix. If there is a hierarchy amongst the contribution of various heavy states, then there will be a hierarchy of light eigenvalues. Only the dominant contributions are included in $\lambda^{(0)}_{\nu,E}$, which may therefore have zero determinants and sub-determinants.

4 There are three zeroth-order neutrino mass matrix textures which satisfy (5), (6) and (7) (with $\theta \to \theta_\nu$), which we label I, II and III:

$$\lambda_\nu^{(0)I} = \begin{pmatrix} 0 & B & A \\ B & 0 & 0 \\ A & 0 & 0 \end{pmatrix} \qquad \lambda_\nu^{(0)II} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{B^2}{A} & B \\ 0 & B & A \end{pmatrix} \qquad \lambda_\nu^{(0)III} = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & A \\ 0 & A & 0 \end{pmatrix}. \tag{8}$$

Texture I leads to a heavy pseudo-Dirac neutrino:

$$I: \quad \text{Pseudo-Dirac} \qquad m^{(0)}_{\nu 1} = m^{(0)}_{\nu 2} \gg m^{(0)}_{\nu 3} = 0 \tag{9}$$

while texture II gives the zeroth-order hierarchical eigenvalue pattern:

$$II: \quad \text{Hierarchical} \qquad m^{(0)}_{\nu 3} \gg m^{(0)}_{\nu 1} = m^{(0)}_{\nu 2} = 0. \tag{10}$$

The third texture leads to complete degeneracy at zeroth-order:

$$III: \quad \text{Degeneracy} \qquad m^{(0)}_{\nu 1} = m^{(0)}_{\nu 2} = m^{(0)}_{\nu 3} \neq 0. \tag{11}$$

Textures I and II can both be obtained by the seesaw mechanism. A simple model for texture II has a single heavy Majorana right-handed neutrino, N, with interactions $l_{2,3}NH + MNN$, which could be guaranteed, for example, by a Z_2 symmetry with $l_{2,3}$, N odd and l_1 even. A simple model for texture I has two heavy right-handed neutrinos which form the components of a Dirac state and have the interactions $l_1N_1H + l_{2,3}N_2H + MN_1N_2$. These interactions could result, for example, from a U(1) symmetry with $N_1, l_{2,3}$ having charge +1, and N_2, l_1 having charge −1. In both cases, the missing

right-handed neutrinos can be heavier, and/or have suitably suppressed couplings.

The search for charged lepton textures is very similar to that for neutrinos, except that the zeroth-order eigenvalues must be hierarchical, not pseudo-Dirac or degenerate. There is only one zeroth-order charged lepton mass matrix texture which satisfies (3), (4), (6) and (7) (with $\theta \to \theta_E$) and $\theta^{(0)}_{E12} = 0$, which we label IV

$$\lambda_E^{(0)IV} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & 0 & A \end{pmatrix} \tag{12}$$

which is the charged analogue of the hierarchical neutrino texture II;

we have rotated the right-handed charged leptons so that the entries of the first two columns vanish. Textures (I), (II) and (IV) have the special case $A = B$.

A special hierarchical texture,

$$\lambda_E^{(0)V} = \begin{pmatrix} A & A & A \\ A & A & A \\ A & A & A \end{pmatrix} \tag{13}$$

often considered in the literature [6], gives $\theta^{(0)}_{13} = 0$ only when $\theta^{(0)}_{E12}$ takes the non-zero value corresponding to the light eigenstates being $(1, 1, -2)$ and $(1, -1, 0)$, and is therefore an exception to our usual requirement that $\theta^{(0)}_{E12} = 0$. In turn, these two special directions will have to be picked out by the perturbation which gives m_μ. $\theta^{(0)}_{E12} = 0$. Note that texture V is not allowed for the neutrino mass matrix, since it gives $\theta^{(0)}_{\nu 13} \simeq 35°$, and clearly violates (7).

Texture II has been obtained by the seesaw mechanism by adding an extra singlet neutrino, or by adding R parity violating interactions in supersymmetric theories [7]. The possibility of complete degeneracy via texture III has also been noted [8]. A search for theories giving the observed atmospheric and solar neutrino

fluxes with three light neutrinos, but with more stringent assumptions than those adopted here, found both textures I and II [9]. It is remarkable that all theories, which explain both atmospheric and solar neutrino fluxes with three light neutrinos, must yield one of these zeroth-order textures. Any exception must involve more complicated relations between matrix elements — for example, with entries differing by a multiple or a sign [10].

Texture III is unique: it is the only case where the zeroth-order texture does not provide Δm^2_{ATM} (for neutrinos) or m_τ (for charged leptons). Apparently this texture does not guarantee a large value for θ_{23}: at zeroth-order the three neutrinos are all degenerate, so that degenerate perturbation theory may lead to a large value for $\theta_{23}^{(1)}$, perhaps cancelling $\theta_{23}^{(0)}$. This is not correct — small perturbations to a Dirac mass term yield a pseudo-Dirac neutrino with a mixing angle close to 45° — texture (III) yields the result $\theta_{23}^{(0)} = 45°$, and this receives only small corrections from higher order. This is to be compared with the Super-Kamiokande data: $\theta_{23} = 45° \pm 13°$ at 90% C.L.

5 In listing textures I–V, we have not discussed the form of the lepton mass matrix which is not explicitly displayed. A neutrino texture from (8) could be paired with a diagonal charged lepton mass matrix, or with texture IV.

It is straightforward to find symmetries which yield textures I and II; an example of $L_e - L_\mu - L_\tau$ was given in [9]. Any symmetry yielding these textures does not distinguish l_μ from l_τ, and hence requires both of these textures to be paired with the charged lepton mass matrix of texture IV. We find two zeroth-order schemes for lepton masses:

$$\text{Scheme A} \qquad \lambda_\nu^{(0)I} \oplus \lambda_E^{(0)IV} \tag{14a}$$

$$\text{Scheme B} \qquad \lambda_\nu^{(0)II} \oplus \lambda_E^{(0)IV}. \tag{14b}$$

For these schemes the special case $A = B$, in both charged and neutral matrices, gives $\theta_{23}^{(0)} = 0$, and is not allowed. Even if $A = B$ in just one of the matrices, neither scheme can claim to predict θ_{23} near 45°, as suggested by the Super-Kamiokande data.

We do not know how texture III can be obtained from symmetry arguments. The democratic texture V follows from the symmetry group $S_{3L} \times S_{3R}$ [6]. With ℓ_L (ℓ_R) transforming as $1_L \oplus 2_L$ ($1_R \oplus 2_R$) of $S_{3L}(S_{3R})$, $\lambda_E^{(0)} = A(1_L 1_R)$, which is the democratic form of (13). However, a scheme for both charged and neutral lepton masses does not result at zeroth order. The neutrino mass matrix involves two invariants: $\lambda_\nu^{(0)} = B(1_L 1_L) + C(2_L 2_L)$, and is diagonalized by the rotation matrix $V_\nu^{(0)} = V_e^{(0)}$, giving $V^{(0)} = I$. Even if B and C are fine-tuned, or a suitably extended symmetry is spontaneously broken in a special direction [11], so that $\lambda_\nu^{(0)} \propto I$, the lepton mixing angles are then all determined by the higher order perturbations.

The only zeroth-order schemes for lepton masses which we can justify from symmetries are A and B. In scheme A the solar neutrino oscillations are large angle , while in B the angle may be large or small, and is entirely determined by the perturbations. Neutrino hot dark matter and a $\beta\beta_{0\nu}$ signal are possible only in texture III, and hence do not occur in either scheme.

Models for the zeroth-order schemes A and B, in which the neutrino masses arise from the seesaw mechanism, are easy to construct, and can also be extended to SU(5) unification. Scheme B occurs in theories with a single heavy Majorana right-handed neutrino, N, and any flavor symmetry with $N, l_{2,3}, e_3, H_{u,d}$ transforming trivially, but

$l_1, e_{1,2}$ transforming non-trivially such that $l_1 e_{1,2} H_d$ is forbidden. An example is provided by a U(1) symmetry with $l_1, e_{1,2}$ positively charged. This can be extended to a theory with three right-handed neutrinos by introducing N_1 and N_2, with equal and opposite charges, such that the only allowed renormalizable operator is the mass term $M' N_1 N_2$.

Scheme A occurs in a theory which has two heavy right-handed neutrinos, which form the components of a Dirac state, and the interactions $l_1 N_1 H_u + l_{2,3} N_2 H_u + M N_1 N_2 + l_{2,3} e_3 H_d$. For example, this follows from a U(1) flavor symmetry with $N_1, l_{2,3}$ having charge $+1$, $N_2, l_1, e_{1,2}, H_d$ having charge -1, H_u and e_3 being neutral. This can be extended to a theory with three right-handed neutrinos by introducing the neutral state N_3.

In our example $U(1)$ theories, for both schemes A and B, in the trivial extensions to $SU(5)$ where each field is replaced by its parent $SU(5)$ multiplet ($e_3 \rightarrow T_3$ etc), the only additional invariant interaction is $T_3 T_3 H_u$, yielding the top quark Yukawa coupling at zeroth order.

6 We have studied textures for lepton mass matrices with three light neutrinos, which at zeroth-order give a large mixing angle for the oscillation $\nu_\mu \rightarrow \nu_\tau$, and a vanishing mixing angle for $\nu_\mu \rightarrow \nu_e$. We allow textures with zero determinants and sub-determinants, and we allow independent entries to be set equal. In the case of the neutrino mass matrix, requiring two degenerate neutrinos to allow $\Delta m^2_\odot \ll \Delta^2_{ATM}$, there are just three such zeroth-order textures; while in the charged case, requiring a zeroth-order mass for m_τ but not for m_μ or m_e, there are only two possibilities. For a flavor symmetry to yield both charged and neutral textures with these zeroth-order properties, only two schemes are possible, both of which can be obtained in simple seesaw models.

Many other theories of neutrino masses exist which can account for both atmospheric and solar neutrino fluxes. They involve more than three light neutrinos, textures with entries related in some special way, or theories where m_τ, θ_{23} and/or θ_{13} are determined only at higher order. However, the zeroth-order textures and schemes which we have found appear particularly simple.

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