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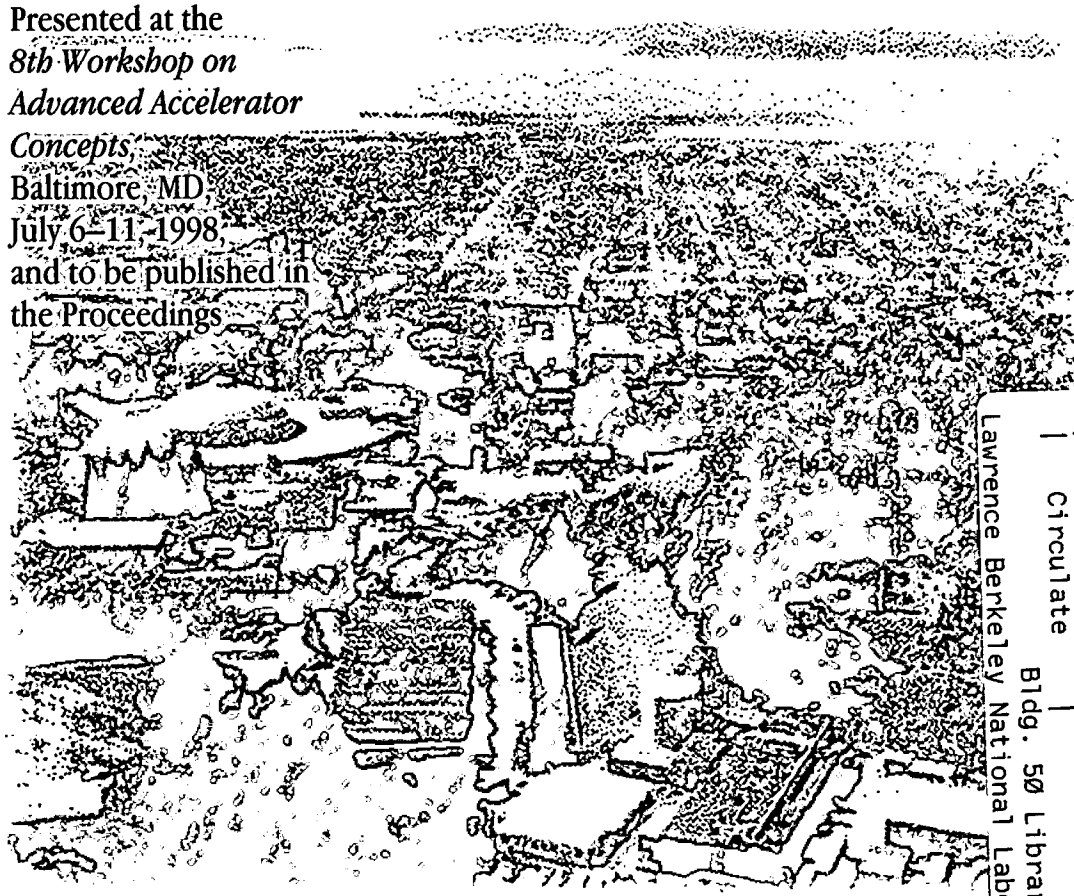

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Laser Acceleration in Vacuum, Gases, and Plasmas with Capillary Waveguide

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Abstract. I propose a new method for laser acceleration of relativistic electrons using the leaky modes of a hollow dielectric waveguide. The hollow core of the waveguide can be either in vacuum or filled with uniform gases or plasmas. In case of vacuum and gases, TM_{01} mode is used for direct acceleration. In case of plasmas, EH_{11} mode is used to drive longitudinal plasma wave for acceleration. Structure damage due to high power laser can be avoided by choosing a core radius sufficiently larger than laser wavelength. Effect of nonuniform plasma density on waveguide performance is also analyzed.

MODE PROPERTIES

The capillary waveguide considered here is made of a hollow core with an index of refraction ν_1 and radius R, embedded in a dielectric medium with an index of refraction ν_2. We are interested only in oversized waveguide satisfying the condition $\lambda_1/R \ll 1$, where $\lambda_1 = \lambda/\nu_1$ and λ is the wavelength in vacuum. As a result, EM wave in the core is dominantly transverse. Choosing appropriate dielectric medium such that $\sqrt{\nu^2 - 1} \gg \lambda_1/R$, where $\nu = \nu_2/\nu_1$, the eigenmodes of the waveguide can be solved following the same procedure by Marcatili and Schmeltzer [1].

Expressing the eigenmodes in the following form

$$\left\{ \begin{array}{c} \mathcal{E}(r,\phi,z,t) \\ \mathcal{H}(r,\phi,z,t) \end{array} \right\} = \left\{ \begin{array}{c} \mathbf{E}_{lm}(r,\phi) \\ \mathbf{H}_{lm}(r,\phi) \end{array} \right\} e^{i(\beta_{lm} z - \omega t) - \alpha_{lm} z} , \tag{1}$$

the eigenvalues are given by

$$\beta_{lm} = k_1(1 - 1/2\gamma_g^2) , \quad \alpha_{lm} = \chi/\gamma_g^2 R , \tag{2}$$

where $k_1 = \nu_1 k$, $k = 2\pi/\lambda$, $\gamma_g = 2\pi R/U_{lm}\lambda_1 \gg 1$, and U_{lm} is the mth root of the equation, $J_{l-1}(U_{lm}) = 0$.

There are three types of modes, corresponding to

$$\chi = \begin{cases} \frac{1}{\sqrt{\nu^2 - 1}} & : \quad TE_{0m} \ (l = 0) \\ \frac{\nu^2}{\sqrt{\nu^2 - 1}} & : \quad TM_{0m} \ (l = 0) \\ \frac{\nu^2 + 1}{2\sqrt{\nu^2 - 1}} & : \quad EH_{lm} \ (l \neq 0) \ . \end{cases} \tag{3}$$

For laser acceleration, we are interested primarily in two low-order modes: TM_{01} mode for acceleration in vacuum and gases, and EH_{11} mode for acceleration in plasmas. Consequently, we consider three cases: $\delta\nu_1 = 0$ when the core is in vacuum, $\delta\nu_1 > 0$ and $\delta\nu_1 < 0$ when the core is filled with uniform gases and plasmas, respectively, where $\delta\nu_1 = \nu_1 - 1$ and $|\delta\nu_1| \ll 1$. It is noted that EH_{11} mode is often designated as HE_{11} mode elsewhere in the literature.

The electric field within the core $r \leq R$ are given by

$$TM_{01} \quad : \quad \begin{cases} E_z = E_a J_0(k_{r1} r) \\ E_r = -i(\Gamma/k_{r1}) E_a J_1(k_{r1} r) \ , \end{cases} \tag{4}$$

$$EH_{11} \quad : \quad \begin{cases} E_y = E_0 J_0(k_{r1} r) \\ E_z = -i(k_{r1}/\Gamma) E_0 J_1(k_{r1} r) \sin\phi \ , \end{cases} \tag{5}$$

where E_a is the peak acceleration field for TM_{01} mode, E_0 is the peak transverse field for EH_{11} mode, $\Gamma = \beta_{lm} + i\alpha_{lm}$, and

$$k_{r1} = (U_{lm} - i\chi/\gamma_g)/R \ . \tag{6}$$

To leading order, $\Gamma/k_{r1} = \gamma_g$, since by definition, $k_1^2 = \Gamma^2 + k_{r1}^2$. Given electric field, magnetic field of a mode can be determined by

$$\begin{aligned} \mathbf{H}_t &= \hat{\mathbf{z}} \times (\Gamma \mathbf{E}_t + i\nabla_t E_z)/kZ_0 \\ H_z &= (i/\Gamma)\nabla_t \cdot \mathbf{H}_t \ , \end{aligned} \tag{7}$$

where subscript t denotes transverse component of a vector or operator, $\hat{\mathbf{z}}$ is a unit vector in z-direction, and Z_0 is the vacuum impedance.

As seen from Eq.(4) and Eq.(5), the transverse field dominates over the longitudinal one by a large factor, γ_g. For TM_{01} mode, E_r is peaked at $r = r_p$ with a maximum value

$$E_r^{max} = \gamma_g E_a J_1^{max}(U_{01} r_p/R) \ , \tag{8}$$

where $r_p/R = 0.481$ and $J_1^{max}(U_{01} r_p/R) = 0.582$. For EH_{11} mode, E_y is peaked on-axis.

A crucial factor of concern for using a waveguide for laser acceleration is structure damage due to high power laser. To evaluate surface field, E_s, at dielectric boundary we expand the Bessel function in the transverse field given by Eq.(4) and Eq.(5), using the expression for k_{r1}, Eq.(6), to obtain

$$\begin{array}{lllll} E_s/E_a & \equiv & |E_r(r=R)|/E_a & = & \chi|J_0(U_{01})| & : & TM_{01} \\ E_s/E_0 & \equiv & |E_y(r=R)|/E_0 & = & \chi|J_1(U_{11})|/\gamma_g & : & EH_{11}\ . \end{array} \tag{9}$$

It is important to note that for TM_{01} mode surface field is of the same order as the peak acceleration field, and for both modes surface field is much smaller than the peak transverse field.

Within the dielectric medium $r \geq R$, the maximum field intensities occur at the boundary $r = R$, and all fields have the radial dependence $\exp(ik_{r2}r)/\sqrt{r}$, where to leading order $k_{r2} = k_1\sqrt{\nu^2 - 1}$. A non-vanishing imaginary part of ν due to even slightly lossy dielectric medium could give rise to a rapid exponential decay of fields in radial direction. Thus the power carried in each mode is distributed dominantly within the core and can be expressed by

$$P(z) \;=\; P_0\, e^{-z/L_{attn}}\ , \tag{10}$$

where $L_{attn} = \gamma_g^2 R/2\chi$ is power attenuation length due to refractive loss, and

$$P_0 = \int_0^R\!\!\int_0^{2\pi} |E_t|^2/2Z_0\; r dr d\phi = \left\{ \begin{array}{lll} \pi R^2\gamma_g^2 E_a^2 J_0(U_{01})^2/2Z_0 & : & TM_{01} \\ \pi R^2 E_0^2 J_1(U_{11})^2/2Z_0 & : & EH_{11}\ . \end{array} \right. \tag{11}$$

It is noted that EH_{11} mode is linearly polarized, whereas TM_{01} mode is radially polarized. However, when necessary, a linearly polarized mode can be formed by a proper mixing of TM_{01} with EH_{21} mode [2]. The electric fields for the mixed mode $TM_{01} + EH_{21}$ are given by

$$TM_{01} + EH_{21} \quad : \quad \left\{ \begin{array}{l} E_z = E_a[J_0(k_{r1}r) + J_2(k_{r1}r)\cos 2\phi] \\ E_y = -2i(\Gamma/k_{r1})E_a J_1(k_{r1}r)\sin\phi\ . \end{array} \right. \tag{12}$$

To preserve the acceleration gradient on-axis, E_a, same as the TM_{01} mode, the mixed mode requires a factor of two more laser power. For the three modes we have $U_{11} = 2.405$, $U_{01} = U_{21} = 3.832$, also, $J_0(U_{01}) = -0.403$ and $J_1(U_{11}) = 0.519$.

Coupling between the waveguide modes and free space Gauss-Laguerre modes (also known as TEM modes [3]) can be very efficient. When focused at the waveguide input cross section, power coupling from a radially polarized TEM_{01} mode to the TM_{01} mode reaches a maximum of 97% at $w_0/R = 0.56$, this is true also for coupling from a lineally polarized TEM_{01} mode to the mixed $TM_{01} + EH_{21}$ mode; and coupling from a TEM_{00} mode to EH_{11} mode is 98% at $w_0/R = 0.64$, where w_0 is the Gaussian beam waist. Despite the fact that the modes are leaky due to refractive loss, optical guiding is quite effective as the losses for low-order modes can be made very small by choosing R sufficiently large relative to λ, as seen from Eq.(2).

ACCELERATION IN VACUUM

According to Eq.(2), phase velocity, v_p, of the TM_{01} mode is larger than the speed of light, c

$$v_p = \frac{\omega}{\beta_{01}} = \frac{c}{1 - 1/2\gamma_g^2} . \tag{13}$$

We define an acceleration phase slippage length over which a relativistic electron with energy $W_0 = \gamma mc^2$, while being accelerated, slips a full π phase with respect to the fast acceleration wave

$$L_a = \frac{\lambda}{1/\gamma_g^2 + 1/\gamma^2} . \tag{14}$$

Over this distance, energy gain of the electron on-axis is

$$\Delta W_a = eE_a \int_0^{L_a} \sin(\pi z/L_a) dz = eE_a L_a T_a , \tag{15}$$

where $T_a = 2/\pi$ is a reduction factor due to a π phase slippage during acceleration. Here we have neglected the small attenuation of the acceleration field due to waveguide loss over the distance L_a. In parallel, we may also define a deceleration phase slippage length, L_d, over which the electron slips another π phase while losing energy amounted to $\Delta W_d = eE_a L_d T_d$, where T_d can be different from T_a if $L_d/L_a \neq 1$. The average acceleration gradient during a period of 2π phase slippage is then given by

$$G = \frac{\Delta W_a - \Delta W_d}{L_a + L_d} = G_a \frac{1 - (L_d/L_a)(T_d/T_a)}{1 + L_d/L_a} , \tag{16}$$

where $G_a = \Delta W_a/L_a = eE_a T_a$. To have net acceleration, the ratio L_d/L_a should be made small. This can be done by introducing a static magnetic field during the half period of deceleration. The effect of the magnetic field is to reduce the longitudinal velocity of the electron such that it slips faster, thus taking shorter distance, L_d, in the field of deceleration.

For simplicity, we assume the magnetic field is sinusoidal as in a wiggler, $B_y = B_0 \cos(2\pi z/\lambda_w)$, with a period λ_w. Then L_d is defined by

$$\left[\frac{1}{\gamma_g^2} + \frac{1}{\gamma^2} + \frac{a_w^2}{\gamma^2}\right] \frac{\pi L_d}{\lambda} - \frac{a_w^2 \lambda_w}{2\gamma^2 \lambda} \sin\left[\frac{4\pi L_d}{\lambda_w}\right] = \pi , \tag{17}$$

where $a_w = eB_0\lambda_w/2\pi\sqrt{2}mc$. If we set $\lambda_w = L_d$ then

$$L_d = \frac{\lambda}{1/\gamma_g^2 + 1/\gamma^2 + a_w^2/\gamma^2} , \tag{18}$$

and a_w is now determined by

$$a_w = \sqrt[3]{Q_1 + \sqrt{Q_1^2 + Q_2^3}} + \sqrt[3]{Q_1 - \sqrt{Q_1^2 + Q_2^3}} \ , \tag{19}$$

where $Q_1 = eB_0\lambda\gamma^2/4\pi\sqrt{2}mc$ and $Q_2 = [1 + (\gamma/\gamma_g)^2]/3$. Due to longitudinal oscillation in electron orbit, T_d is different from T_a and given by

$$T_d = \frac{1}{\pi}\int_0^\pi \sin\,[\theta - \kappa \sin\,(4\theta)]d\theta \ , \tag{20}$$

where $\kappa = (1 - L_d/L_a)/4$. The value of T_d varies in the range $\{1.84 \leftrightarrow 2\}/\pi$ for L_d/L_a in the range $\{0 \leftrightarrow 1\}$.

We have assumed the electron is decelerated by the on-axis value of E_z, but as the electron is deflected off-axis, it will see a weaker longitudinal field and a stronger transverse EM fields. The maximum orbital offset in the x-direction due to the wiggler field is $\Delta X_{max} = \sqrt{2}a_w\lambda_w/\pi\gamma$.

Because of the magnetic deflection, the electron will radiate and lose energy. The radiative energy loss per wiggler period is

$$\Delta W_r \ = \ \frac{8\pi^2 mc^2}{3}\left(\frac{r_e}{\lambda_w}\right) a_w^2\gamma^2 \ , \tag{21}$$

where r_e is the classical radius of electron. The maximum possible energy that can be accelerated with this method can be determined by the condition: $\Delta W_a > \Delta W_d + \Delta W_r$.

Transverse force on a relativistic electron due to an EM wave does not vanish to order of $1/\gamma^2$ in a waveguide mode or when the index of refraction differs from unity. To see this, we note the magnetic field of the TM_{01} mode can be expressed as $H_\varphi = (1 + 1/2\gamma_g^2 + \delta\nu_1)E_r/Z_0$. Correspondingly, the transverse force on a relativistic electron is

$$F_r = -e(1/2\gamma^2 - 1/2\gamma_g^2 - \delta\nu_1)E_r \ , \tag{22}$$

which can be either focusing or defocusing depending on acceleration phase, ϕ_a, which varies constantly due to slippage. The beta function for an electron near the axis in vacuum is found to be

$$\beta_t = \gamma_g\lambda\sqrt{(\gamma mc^2/\pi e\lambda E_a \sin\phi_a)/[1 - (\gamma_g/\gamma)^2]} \ . \tag{23}$$

To avoid strong nonlinear transverse EM force when the electron is deflected off-axis by the wiggler field, B_y, during deceleration, the mixed mode $TM_{01} + EH_{11}$ may be used since it has zero transverse force along the x-axis, as seen from Eq.(12).

An example is given in Table 1 for acceleration in vacuum with the TM_{01} mode. Here in calculating β_t we set $\sin\phi_a$=1, and I_s is the laser intensity on the waveguide surface.

Table 1. Example for Laser Acceleration in Vacuum.

λ [μm]	1	γ_g	410	E_a [GV/m]	3.7
R/λ	250	L_a [cm]	16	E_s [GV/m]	3.0
ν_2	1.5	L_d [cm]	6.2	I_s [TW/cm^2]	1.2
W_0 [GeV]	1	L_{attn} [m]	10	G [GV/m]	1.1
P_0 [TW]	100	β_t [cm]	12	ΔW_a [GeV]	0.38
B_0 [T]	1.5	$\Delta X_{max}/R$	0.35	ΔW_d [GeV]	0.14
a_w	6.2	T_d	0.61	ΔW_r [eV]	88

ACCELERATION IN GASES

The phase velocity of the TM_{01} mode in uniform gases is given by

$$v_p = \frac{\omega}{\beta_{01}} = \frac{c}{1 - 1/2\gamma_g^2 + \delta\nu_1}, \tag{24}$$

which corresponds to an acceleration length or phase slippage length

$$L_{slip} = \frac{\lambda}{|1/\gamma_g^2 + 1/\gamma^2 - 2\delta\nu_1|}. \tag{25}$$

The phase matching condition is obtained by making the denominator zero

$$\delta\nu_1 = 1/2\gamma_g^2 + 1/2\gamma^2. \tag{26}$$

This condition suggests an alternative way to maintain phase matching as γ increases during acceleration: instead of varying $\delta\nu_1$ by adjusting gas pressure along the waveguide, one may change γ_g by tapering waveguide radius. For highly relativistic electron satisfying the condition $\gamma \gg \gamma_g$, a steady state phase matching condition, $\delta\nu_1 = 1/2\gamma_g^2$, is approached. An example is given in Table 2 for laser acceleration in gases in the highly relativistic limit. The beta function in gases is smaller than that in vacuum, Eq.(23), by a factor of $\sqrt{2}$. The maximum acceleration gradient is limited by various processes occurring in gases in the field of high power laser, such as nonlinear self-focusing and gas breakdown. Here we assume the limit is set by $E_r^{max} \leq 10$GV/m.

Table 2. Example for Laser Acceleration in Gases.

λ [μm]	10	P_0 [GW]	50	E_a [GV/m]	0.21
R/λ	50	γ_g	82	E_s [GV/m]	0.17
ν_2	1.5	$\delta\nu_1$ [10^{-5}]	7.4	L_{attn} [m]	0.84

ACCELERATION IN PLASMAS

Wave equation for laser field propagation in weakly relativistic plasma under cold fluid condition is governed by [4]

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{E}_l = \frac{\omega_p^2}{c^2}\left[1 + \frac{\delta n}{n_0} - \frac{a^2}{2}\right]\mathbf{E}_l \ , \tag{27}$$

where $\omega_p = \sqrt{e^2 n_0/\epsilon_0 m}$ is the electron plasma frequency, n_0 the ambient plasma density, and ϵ_0 the dielectric constant in vacuum. The plasma density modulation, $\delta n/n_0$, driven by the ponderomotive potential of the laser field, $a^2 = <|e\mathbf{E}_l/mc\omega|^2>$, will generate a wakefield, $\mathbf{E}_w = -\nabla\Phi$, where the wake potential, Φ, is determined by [5,6]

$$\left[\frac{\partial^2}{\partial t^2} + \omega_p^2\right]\Phi = \omega_p^2 \frac{mc^2}{e}\frac{a^2}{2} \ . \tag{28}$$

To close the loop, $\delta n/n_0$ is related to the wake potential by Poisson's equation

$$\nabla^2\Phi = (e/\epsilon_0)\delta n \ . \tag{29}$$

Under the condition $a^2 \ll 1$, we will have $\delta n/n_0 \ll 1$, as will be shown later. As a result, the second and third term on the right hand side of Eq.(27) can be dropped and the wave equation is then decoupled from the plasma equations, Eq.(28) and Eq.(29). The only effect of the plasma on laser propagation is through an index of refraction $\nu_1 = 1 - \omega_p^2/2\omega^2$.

We now consider laser wakefield acceleration [7] in a capillary waveguide filled with a uniform plasma. A laser pulse propagating through the waveguide will excite a wakefield with phase velocity equals the group velocity of the laser pulse. For EH_{11} mode, the group velocity is given by

$$v_g = \frac{d\omega}{d\beta_{11}} = \frac{c}{1 + 1/2\gamma_g^2 + 1/2\gamma_p^2} \ , \tag{30}$$

where $\gamma_p = \omega/\omega_p \gg 1$. Correspondingly, the slippage length by definition is

$$L_{slip} = \frac{\lambda_p}{|1/\gamma_g^2 + 1/\gamma_p^2 - 1/\gamma^2|} \ . \tag{31}$$

To solve the plasma equations, we take the approach in parallel to the 1D linear analysis of laser wakefield by Gorbunov and Kirsanov [5], except here the solution we provide is in full 3D. Introducing a variable $\zeta = z - v_g t$, Eq.(28) can be solved as

$$\Phi = -(k_p mc^2/e)\int_\zeta^\infty d\zeta' \sin[k_p(\zeta - \zeta')]\frac{a^2}{2} \ , \tag{32}$$

where $k_p = \omega_p/v_g$. For a Gaussian pulse of EH_{11} mode, we have from Eq.(5)

$$a^2(\rho,\zeta) = \frac{a_0^2}{2} J_0^2(U_{11}\rho) e^{-\zeta^2/2\sigma_z^2 - z/L_{attn}} \ , \tag{33}$$

where $\rho = r/R$, the wake potential behind the laser pulse is

$$\Phi = -\Phi_0 J_0^2(U_{11}\rho) e^{-z/L_{attn}} \sin(k_p z - \omega_p t) \ , \tag{34}$$

where

$$\Phi_0 = (\sqrt{2\pi} mc^2/4e)\ a_0^2\ k_p\sigma_z\ e^{-(k_p\sigma_z)^2/2} \ . \tag{35}$$

The longitudinal wakefield is then given by

$$E_{wz} = E_a J_0^2(U_{11}\rho) e^{-z/L_{attn}} \cos(k_p z - \omega_p t) \ , \tag{36}$$

and the transverse wakefield by

$$E_{wr} = -2(\gamma_p/\gamma_g) E_a J_0(U_{11}\rho) J_1(U_{11}\rho) e^{-z/L_{attn}} \sin(k_p z - \omega_p t) \ , \tag{37}$$

where the peak acceleration field, $E_a = \Phi_0 k_p$, is maximized if the laser pulse length is chosen according to the condition, $k_p\sigma_z = 1$. From here on, we will use this optimal condition wherever relevant.

As wakefield is excited in the plasma channel, energy in the driver pulse, Eq.(33), will be depleted. To characterize this process, a pump depletion length, L_{pump}, can be defined by the condition, $W_l = W_w$, where W_l is the initial energy of the laser pulse

$$W_l = \sqrt{\pi/8} J_1^2(U_{11}) \epsilon_0 \lambda_p R^2 E_0^2 \ , \tag{38}$$

and W_w is the energy in the wakefield the laser pulse left behind as it propagates a distance L_{pump}

$$W_w = (\pi mc/4e)^2 [\epsilon_0/\mathrm{Exp}(1)] a_0^4 \omega_p^2 R^2 L_{pump} [I_z + (\gamma_p/\gamma_g)^2 I_r] \ . \tag{39}$$

The two terms above on the right hand side correspond to energy in the longitudinal and transverse wakefield, respectively, where $I_z = \int_0^1 d\rho\rho J_0^4(U_{11}\rho) = 7.62\times10^{-2}$ and $I_r = \frac{1}{4}\int_0^1 d\rho\rho J_0^2(U_{11}\rho) J_1^2(U_{11}\rho) = 6.35\times10^{-3}$. We then obtain

$$L_{pump} = [4\sqrt{2\pi} J_1^2(U_{11})\mathrm{Exp}(1)/\pi^2] \frac{\lambda_p}{a_0^2(I_z/\gamma_p^2 + I_r/\gamma_g^2)} \ . \tag{40}$$

In addition, we may define a characteristic pulse dispersion length, L_{disp}, over this propagation distance the driver pulse will double its length, σ_z. Given group velocity dispersion by Eq.(30), we have

$$L_{disp} = \frac{\sqrt{3}\,\gamma_p^2\,\lambda}{\pi(1/\gamma_g^2 + 1/\gamma_p^2\,)} \ . \tag{41}$$

There is yet another characteristic length, the beta function, due to the transverse wakefield. For electron near the axis the beta function is given by

$$\beta_t = (2/U_{11})[\mathrm{Exp}(1)/2\pi]^{1/4}\sqrt{\frac{\gamma}{\sin\phi_a}}\frac{R}{a_0} \ . \tag{42}$$

Finally, the plasma density modulation is

$$\frac{\delta n}{n_0} = \sqrt{\pi/8\mathrm{Exp}(1)}a_0^2\{1 + 2(\gamma_p/\gamma_g)^2[1 - J_1^2(U_{11}\rho)/J_0^2(U_{11}\rho)]\}$$
$$J_0^2(U_{11}\rho)e^{-z/L_{attn}}\sin(k_p z - \omega_p t) \ . \tag{43}$$

Indeed, we have $\delta n/n_0 \ll 1$, if $a_0^2 \ll 1$.

An example is given in Table 3 for laser acceleration in plasmas. Here the energy gain per stage is defined by, $\Delta W_a = eE_a L_{slip} T_a$, the energy gain per slippage length. In calculating β_t we set $\sin\phi_a = 1$.

Table 3. Example for Laser Acceleration in Plasmas.

λ [μm]	1	n_0 [10^{17}/cm^3]	1.1	E_a [GV/m]	0.94
R/λ	150	$(\delta n/n_0)_{\max}$	0.034	E_s [GV/m]	1.7
ν_2	1.5	γ_p	100	I_s [TW/cm^2]	0.39
W_0 [GeV]	1	γ_g	392	L_{slip} [m]	0.94
P_0 [TW]	20	β_t [cm]	1.6	L_{attn} [m]	7.9
W_l [J]	2.7	σ_z [μm]	16	L_{disp} [m]	52
a_0	0.28	ΔW_a [GeV]	0.56	L_{pump} [m]	126

We have analyzed the capillary waveguide when the core is filled with a plasma of uniform density, n_0. In reality, this can only be an approximation. To evaluate the effect of nonuniformity of the plasma, let's consider a special case when the uniform background is modified by a parabolic profile given by

$$n \ = \ n_0 + \Delta n[1 - (r/R)^2] \ . \tag{44}$$

Such a profile has a defocusing effect on the mode if $\Delta n > 0$, thus making the waveguide less effective. However, the attenuation length for the EH_{11} mode is reduced by a factor of two at most, if the following criteria is satisfied

$$\frac{\Delta n}{n_0} \ \leq \ 4.6\,\frac{\gamma_p^2}{\gamma_g^2} \ . \tag{45}$$

For the example in Table 3, this corresponds to $\Delta n/n_0 \leq 30\%$. Thus we may infer from here that the guiding provided by the capillary waveguide can also be rather stable, against either systematic or random variations in plasma density. The derivation of the criteria, Eq.(45), will be published elsewhere.

CONCLUSIONS

I have introduced the concepts and techniques that are crucial for advancing the current development of laser acceleration into a new, more realistic stage. Of what being accomplished here, the most notable is the significant increase in acceleration distance by using the leaky modes of an oversized capillary waveguide. In vacuum, a new mechanism for energy transfer from laser to electron is proposed, and with which both limits on acceleration distance set by diffraction and phase slippage are overcome. In gases, a tuning technique is provided to maintain the phase matching over a longer acceleration distance. In plasmas, the approach taken here is radically different in concept from the current mainstream development. First of all, the prevailing notion on optical guiding in plasma is based on an analogy to optical fiber in which the index of refraction is maximal on axis, opposite to what proposed here. Secondly, it is commonly believed that no guiding structure could sustain the laser power required for acceleration without being, at least partially, turned into plasma. Therefore the only method considered so far for optical guiding is to tailor the plasma itself in transverse density profile one way or the other, by either relativistic self-focusing [6], charge displacement [8], or capillary discharge [9]. By relieving the duty of optical guiding from the plasma, the medium for acceleration, to an external waveguide, our approach opens up a new avenue to more effective, stable and practical optical guiding and acceleration. In addition, the acceleration structure proposed here has the following advantages: dielectric damage [10] by high power laser is shown not to be a problem; the large waveguide cross section is favorable for achieving better electron beam quality; the efficient coupling between waveguide modes and free space modes eases mode handling such as mode injection, transport and recycling, thus leading to a better overall system efficiency; last but not least, all aforementioned desirable features are achieved without sacrificing a virtue of practical importance: the simplicity. This work was supported by the U.S. Department of Energy under contract No.DE-AC03-76SF00098.

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