LBNL-42512
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Physics Division

November 1998

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Physical Review D

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This work was supported by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and National Science Foundation Grant PHY-95-14797.

arXiv:hep-ph/9811299 v2 11 Nov 1998

UCB-PTH-98/54
LBNL-42512
hep-ph/9811299
November 9, 1998

U(2) and Maximal Mixing of ν_μ *

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Abstract

A $U(2)$ flavor symmetry can successfully describe the charged fermion masses and mixings, and supress SUSY FCNC processes, making it a viable candidate for a theory of flavor. We show that a direct application of this $U(2)$ flavor symmetry automatically predicts a mixing of 45° for $\nu_\mu \Rightarrow \nu_s$, where ν_s is a light, right-handed state. The introduction of an additional flavor symmetry acting on the right-handed neutrinos makes the model phenomenologically viable, explaining the solar neutrino deficit as well as the atmospheric neutrino anomaly, while giving a potential hot dark matter candidate and retaining the theory's predictivity in the quark sector.

*This work was supported in part by the U.S. Department of Energy under Contracts DE-AC03-76SF00098, in part by the National Science Foundation under grant PHY-95-14797.

1 Introduction

The pattern and origin of the quark and lepton masses and mixings remains a challenging question for particle physics. Although a detailed description of this pattern requires a theory of flavor with a certain level of complexity, the gross features may be described simply in terms of a flavor symmetry and its sequential breaking.

One simple flavor structure is motivated by four facts about flavor:

- The quarks and leptons fall into three generations, $\psi_{1,2,3}$, each of which may eventually have a unified description.
- The top quark is sufficiently heavy, that any flavor symmetry which acts on it non-trivially must be strongly broken.
- The masses of the two light generations imply a phenomenological description in terms of small dimensionless parameters, $\{\epsilon\}$.
- In supersymmetric theories, flavor-changing and CP violating phenomena suggest that the squarks and sleptons of the first two generations are highly degnerate.

It is attractive to infer that, at least at a phenomenological level, there is a non-Abelian flavor symmetry which divides the three generations according to

$$2 \oplus 1 : \qquad \psi_a \oplus \psi_3, \ a = 1, 2. \tag{1}$$

The four facts listed above follow immediately from such a structure, with $\{\epsilon\}$ identified as the small symmetry breaking parameters of the non-Abelian group. These control both the small values for quark masses and mixing angles, and also the small fractional non-degeneracies of the scalars of the first two generations.

The Super-Kamiokande collaboration has provided strong evidence for an anomaly in the flux of atmospheric neutrinos, which may be interpreted as large angle oscillations of ν_μ predominantly either to ν_τ or to ν_s, a singlet neutrino [1]. This observation provides a challenge to the non-Abelian $2 \oplus 1$ structure:

- ν_τ is expected to have a very different mass from that of $\nu_{e,\mu}$, and to only weakly mix with them.
- If the atmospheric oscillation is $\nu_\mu \to \nu_s$, what is the identity of this new singlet state, why is it light, and how could it fit into the $2 \oplus 1$ structure?

There are a variety of possible reactions to this challenge. One possibility is to drop the $2 \oplus 1$ idea; perhaps the CP and flavor violating problems of supersymmetry are solved by other means, or perhaps supersymmetry is not relevant to the weak scale. Another option is to retain the $2 \oplus 1$ structure for quarks, but not for leptons, where the flavor changing constraints are much weaker.

In this paper we study theories based on the flavor group $U(2)$, which immediately yields the structure (1), giving the $2 \oplus 1$ structure to both quarks and leptons [2]. The masses and mixings of the charged fermions and scalars resulting fron $U(2)$ have been studied in detail, and significant successes have been identified [3]. We add a right-handed neutrino to each generation, and find that the symmetry structure of the neutrino mass matrix automatically chooses ν_μ to be a pseudo-Dirac state coupled to one of the right-handed neutrinos, resulting in $\nu_\mu \to \nu_s$ with a mixing angle close to 45°.

2 U(2) Theories of Quark and Charged Lepton Masses.

The most general $U(2)$ effective Lagrangian for charged fermion masses, at leading order in the $U(2)$ breaking fields, is

$$\mathcal{L} = \psi_3\psi_3 h + \frac{1}{M}\left(\psi_3\phi^a\psi_a h + \psi_a(S^{ab} + A^{ab})\psi_b h\right) \tag{2}$$

where ϕ^a is a doublet, S^{ab} a symmetric triplet, A^{ab} an antisymmetric singlet of $U(2)$, and h are Higgs doublets. Coupling constants have been omitted, and M is a flavor physics mass scale. An entire generation is represented by ψ, so that each operator contains terms in up, down and charged lepton sectors, but unification is not assumed. For example, this theory follows from a renormalizable Froggatt-Nielsen model on integrating out a single heavy vector $U(2)$ doublet of mass M (see the second of [3]).

The hierarchical pattern of masses and mixings for charged fermions is generated by breaking $U(2)$ first to $U(1)$ with vevs $\phi^2, S^{22} \approx \epsilon M$, and then breaking $U(1)$ via the vev $A^{12} \approx \epsilon' M$. The symmetry breaking

$$U(2) \overset{\epsilon}{\Rightarrow} U(1) \overset{\epsilon'}{\Rightarrow} 1 \tag{3}$$

produces the Yukawa coupling textures

$$M_{LR} = v \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}. \tag{4}$$

3 General Effective Theory of Neutrino Masses.

Without right-handed neutrinos, the most general $U(2)$ effective Lagrangian for neutrino masses, linear in $U(2)$ breaking fields, is

$$\mathcal{L}^{\nu}_{eff} = \frac{1}{M} l_3 l_3 hh + \frac{1}{M^2}\left(l_3\phi^a l_a hh + l_a S^{ab} l_b hh\right). \tag{5}$$

where l_a, l_3 are lepton doublets. The term $l_a A^{ab} l_b hh$ vanishes by symmetry; hence the above vevs give the neutrino mass texture

$$M_{LL} = \frac{v^2}{M} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}. \tag{6}$$

so that the lightest neutrino is massless.[1] The mixing angle for $\nu_\mu \to \nu_\tau$ oscillations, $\theta_{\mu\tau}$, is of order ϵ — the same order as mixing of the quarks of the two heavier generations, V_{cb} — and is much too small to explain the atmospheric neutrino fluxes. However, in theories with flavor symmetries, the seesaw mechanism typically does not yield the most general neutrino mass matrix in the low energy effective theory. This apparent problem requires that we look more closely at the full theory, including the right-handed neutrinos.

4 The Seesaw Mechanism: A Single Light ν_R

Adding three right-handed neutrinos to the theory, $N_a + N_3$, the texture for the Majorana mass matrix is:

$$M_{RR} = M \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}. \tag{7}$$

with the 12 and 21 entries again vanishing by symmetry. In supersymmetric theories the zero eigenvalue is not lifted at higher order in the flavor symmetry breaking. This presents a problem for the 3×3 seesaw mechanism in $U(2)$ theories, since $M_{LL} = M_{LR} M_{RR}^{-1} M_{LR}^T$ and M_{RR} cannot be inverted.

One approach [4] is to allow further flavor symmetry breaking vevs, for example $\phi^1 \neq 0$, so that M_{RR} has no zero eigenvalues. Remarkably, taking $\phi^1/M \approx \epsilon'$, the seesaw gives $\theta_{\mu\tau} \approx 1$, as needed for the atmospheric neutrino anomaly. On the other hand, this pattern of neutrino masses cannot explain the solar neutrino fluxes, and the additional flavor breaking vevs remove two of the highly successful mass relation predictions of the quark sector.

In this paper we keep the minimal $U(2)$ symmetry breaking vevs and pursue the consequences of the light N_e state which results from (7). The singular nature of M_{RR}^{-1} is not a problem; it is an indication that N_e cannot be integrated out of the theory. However, N_τ and N_μ do acquire large masses, and when they are integrated out of the theory the low energy 4×4 neutrino mass matrix is:

$$M^{(4)} = \begin{pmatrix} & & & 0 \\ & M_{LL} & & \epsilon' v \\ & & & 0 \\ 0 & \epsilon' v & 0 & 0 \end{pmatrix} \tag{8}$$

[1] Including operators higher order in the $U(2)$ breaking fields, the lightest neutrino remains massless in a supersymmetric theory, but not in the non-supersymmetric case , where operators such as $l_a A^{ab} \phi_b^\dagger l_3 hh$ occur.

where M_{LL} is a 3×3 matrix in the (ν_a, ν_3) space, determined from seesawing out the two heavy right-handed states, and has one zero eigenvalue.

Because the $N_e - \nu_\mu$ mixing is weak scale, while all other couplings to ν_μ are suppressed, N_e and ν_μ are maximally mixed. Thus, we note that *a direct application of the $U(2)$ theory to the neutrino sector predicts a 45° mixing between ν_μ and ν_s!*

There is a significant phenomenological difficulty with this model. The mass of the $N_e - \nu_\mu$ pseudo-Dirac state is of order $\epsilon' v$. Using a value for ϵ' extracted from an analysis of the charged lepton sector, this is of order 1 GeV, well in excess of the 170 keV limit obtained from direct searches. One simple solution is to restrict the couplings of the right-handed neutrinos by an additional $U(1)_N$ approximate flavor symmetry. Each N field carries N charge $+1$, while the symmetry is broken by a field with charge -1, leading to a small dimensionless breaking parameter ϵ_N. The entries in the neutrino mass matrices receive further suppressions

$$M_{LR} \Rightarrow \epsilon_N M_{LR} \quad M_{RR} \Rightarrow \epsilon_N^2 M_{RR} \tag{9}$$

which, for the 4×4 light neutrino matrix, simply leads to the replacement $\epsilon' v \Rightarrow \epsilon_N \epsilon' v$ in the $N_e - \nu_\mu$ entry, giving

$$M^{(4)} = \begin{pmatrix} \frac{\epsilon'^2}{\epsilon}\frac{v^2}{M} & \epsilon'\frac{v^2}{M} & \epsilon'\frac{v^2}{M} & 0 \\ \epsilon'\frac{v^2}{M} & \epsilon\frac{v^2}{M} & \epsilon\frac{v^2}{M} & \epsilon_N\epsilon' v \\ \epsilon'\frac{v^2}{M} & \epsilon\frac{v^2}{M} & \frac{v^2}{M} & 0 \\ 0 & \epsilon_N\epsilon' v & 0 & 0 \end{pmatrix} \tag{10}$$

It is understood that all entries have unknown $O(1)$ coefficients.

Note that M_{LL} is unchanged. There is a simple reason for this. If we modify our right-handed couplings by the replacements $M_{LR} \to M_{LR}T$, $M_{RR} \to T^T M_{RR} T$, where T is any diagonal matrix, then

$$M_{LL} \Rightarrow M_{LR}T(T^T M_{RR} T)^{-1}(M_{LR}T)^T = M_{LL}. \tag{11}$$

It is interesting that the observed value of $\delta m^2_\odot$ can give the appearance that right-handed neutrinos receive GUT-scale masses, while their masses are in fact much lower.

If the $N_e - \nu_\mu$ entry dominates the mass of ν_μ, i.e. if $\epsilon_N \gg \frac{v}{M}$, this 4×4 matrix splits approximately into two 2×2 matrices, and maximal mixing is preserved. One 2×2 matrix describes the pseudo-Dirac state

$$\begin{pmatrix} \epsilon\frac{v^2}{M} & \epsilon_N\epsilon' v \\ \epsilon_N\epsilon' v & 0 \end{pmatrix} \tag{12}$$

while $\nu_e \Rightarrow \nu_\tau$ mixing is described by

$$\frac{v^2}{M}\begin{pmatrix} \frac{\epsilon'^2}{\epsilon} & \epsilon' \\ \epsilon' & 1 \end{pmatrix} \tag{13}$$

	m_{light}	m_{heavy}	δm^2	θ_{mix}
(1) Heavy states	$v\epsilon_N\epsilon' - \epsilon\frac{v^2}{2M}$	$v\epsilon_N\epsilon' + \epsilon\frac{v^2}{2M}$	$\frac{v^3}{M}\epsilon_N\epsilon\epsilon'$	45°
(2) Light states	$\frac{\epsilon'^2}{\epsilon}\frac{v^2}{M}$	$\frac{v^2}{M}$	$(\frac{v^2}{M})^2$	ϵ'

Table 1: General Theory: the masses, mixings, and splittings of the two sets of neutrinos.

The resulting masses and mixings are given in Table 1.

Since ϵ and ϵ' are determined by the charged fermion masses, in the neutrino sector there are two free parameters, ϵ_N and M, which describe five important observables: $\theta_\odot$, θ_{atm}, $\delta m^2_\odot$, δm^2_{atm} and m_ν, the mass of the pseudo-Dirac muon neutrino. However, the various predictions of the theory have varying levels of certainty. Because there are a large number of order one constants in the original formulation of the theory, we can end up with a prediction which has a coefficient of a product of some number of these quantities. To assess the level of certainty, we will include a quantity i, which we term the "stability index" of the prediction, which is simply the power of unknown order one coefficients appearing in the prediction.

Two of the three resulting predictions are the mixing angles

$$\sin\theta_\odot \approx \epsilon' \qquad [i=4]\,, \quad \theta_{atm} = 45^o \qquad [i=0]\,. \tag{14}$$

The postdiction of a maximal mixing angle for atmospheric oscillations is an important consequence of the $U(2)$ theory. The value of ϵ' extracted from the charged fermion sector is 0.004, within an order of magnitude of the central value $\theta_\odot = 0.037$ of the recent BP98 fit to the solar data, and within a factor of 4 of the minimal acceptable value of 0.016 [5]. Such a discrepancy is not a great concern, as we gain a comparable contribution from the charged lepton matrix. Furthermore, the prediction of $\theta_\odot$ involves the fourth power of unknown order one coefficients, thus $i = 4$, and is somewhat uncertain.

The relevant mass splitting for the $\nu_e \to \nu_\tau$ oscillations occuring in the sun is

$$\delta m^2_\odot \approx \left(\frac{v^2}{M}\right)^2 . \tag{15}$$

While this is not a prediction of the theory, it is intriguing, as has been noticed elsewhere in other contexts, that if M is taken close to the scale of coupling constant unification, $\delta m^2_\odot \approx 10^{-5}$ eV^2, in the right range for either small or large angle MSW oscillations.

The final free parameter ϵ_N is fixed by the observed mass splitting for atmospheric oscillations

$$\delta m^2_{atm} \approx \epsilon\epsilon'\epsilon_N\frac{v^3}{M} \approx \epsilon\epsilon'\epsilon_N v\sqrt{\delta m^2_\odot} \tag{16}$$

giving $\epsilon_N \approx 10^{-8}$ — the $U(1)_N$ symmetry is broken only very weakly.

The final prediction is for the mass of the heavy pseudo-Dirac $\nu_\mu N_e$ state:

$$m_\nu \approx \epsilon' \epsilon_N v \approx \frac{\delta m^2_{atm}}{\epsilon\sqrt{\delta m^2_\odot}} \approx 10^{0.4}\text{eV} - 10^2\text{eV}, \qquad [i=4] \tag{17}$$

where the given spread in mass is due to uncertainty in δm^2_{atm} and $\delta m^2_\odot$. While it is tempting to interpret this as a good candidate for hot dark matter, we will see later that KARMEN places stringent limits on the acceptable values of m_ν.

5 A Variant Theory

A variation on this breaking structure was explored in a particular model (see the second of [6]), and it is interesting to explore whether this same approach for neutrino masses can work within that model. In this variation, there is no S^{ab} field present, and the RR and LR masses are given by

$$M_{LR} = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \quad M_{RR} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \tag{18}$$

generating a light 4×4 mass matrix

$$M^{(4)} = \begin{pmatrix} \frac{\epsilon'^2}{\epsilon^2}\frac{v^2}{M} & \epsilon'\frac{v^2}{M} & \frac{\epsilon'}{\epsilon}\frac{v^2}{M} & 0 \\ \epsilon'\frac{v^2}{M} & 0 & \epsilon\frac{v^2}{M} & \epsilon_N\epsilon' v \\ \frac{\epsilon'}{\epsilon}\frac{v^2}{M} & \epsilon\frac{v^2}{M} & \frac{v^2}{M} & 0 \\ 0 & \epsilon_N\epsilon' v & 0 & 0 \end{pmatrix} \tag{19}$$

This matrix is problematic, because the 2×2 submatrix for the atmospheric neutrinos does not contain a splitting term. Of course, a splitting would be generated through interactions with the other left-handed states, we estimate

$$M^{(4)}_{\mu\mu} \approx \epsilon^2 \frac{1}{m_\nu}(\frac{v^2}{M})^2. \tag{20}$$

Consequently our atmospheric splitting is

$$\delta m^2_{atm} \approx \epsilon^2 (\frac{v^2}{M})^2. \tag{21}$$

Since we have $(\frac{v^2}{M})^2 = \delta m^2_\odot$, this would predict $\delta m^2_{atm} < \delta m^2_\odot$, which is unacceptable. One simple solution is to allow the appearance of the operators

$$(\frac{1}{M})^2 \phi^a \phi^b N_a N_b M_{GUT} \tag{22}$$

$$(\frac{1}{M})^2 \phi^a \phi^b N_a \nu_b H. \tag{23}$$

	m_{light}	m_{heavy}	δm^2	θ_{mix}
(1) Heavy states	$v\epsilon_N\epsilon' - \epsilon^2\frac{v^2}{2M}$	$v\epsilon_N\epsilon' + \epsilon^2\frac{v^2}{2M}$	$\frac{v^3}{M}\epsilon_N\epsilon^2\epsilon'$	45°
(2) Light states	$(\frac{\epsilon'}{\epsilon})^2\frac{v^2}{M}$	$\frac{v^2}{M}$	$(\frac{v^2}{M})^2$	$\frac{\epsilon'}{\epsilon}$

Table 2: Without S field: The masses, mixings, and splittings of the two sets of neutrinos.

The inclusion of one or both of these operators in our Lagrangian has the same effect on our final mass matrix, inducing $M^{(4)}_{\mu\mu} \approx \epsilon^2\frac{v^2}{M}$ and yielding the 2×2 submatrix

$$\begin{pmatrix} \epsilon^2\frac{v^2}{M} & \epsilon_N\epsilon' v \\ \epsilon_N\epsilon' v & 0 \end{pmatrix} \tag{24}$$

describing the pseudo-Dirac state, while $\nu_e \Rightarrow \nu_\tau$ mixing is now described by

$$\frac{v^2}{M}\begin{pmatrix} (\frac{\epsilon'}{\epsilon})^2 & \frac{\epsilon'}{\epsilon} \\ \frac{\epsilon'}{\epsilon} & 1 \end{pmatrix} \tag{25}$$

The resulting masses and mixings are given in table 2.

The mixing angles in this variation are predicted to be

$$\sin(\theta_\odot) \approx \frac{\epsilon'}{\epsilon} \qquad [i=5]\,, \quad \theta_{atm} = 45^o \qquad [i=0] \tag{26}$$

As the pseudo-Dirac muon neutrino is still present, the atmospheric angle is unchanged. However, the solar angle is changed somewhat. We should note that values for ϵ and ϵ' extracted for a fit of this model are different than for those of the previous model. Using values from fits in the charged fermion sector, we have $\epsilon \approx 0.03$ and $\epsilon' \approx 5\times 10^{-4}$ or $\epsilon' \approx 2.4\times 10^{-4}$ (depending on certain signs), yielding $\theta_\odot \approx O(1.5\times 10^{-2})$. Given the number of $O(1)$ parameters involved, this is again quite consistent with the BP98 small-angle MSW solution.

The solar splitting scale is unchanged, while the atmospheric splitting is further surpressed by a factor of ϵ.

$$\delta m^2_{atm} \approx \epsilon'\epsilon_N v\epsilon^2\sqrt{\delta m^2_\odot} \tag{27}$$

We fit this splitting again with the free parameter $\epsilon_N \approx 10^{-6} - 10^{-7}$. The resulting muon neutrino mass is then

$$m_\nu \approx \frac{\delta m^2_{atm}}{\epsilon^2\sqrt{\delta m^2_\odot}} \approx 10^{1.7}\text{eV} - 10^{3.5}\text{eV} \qquad [i=5] \tag{28}$$

Thus, while the explanations of the solar and atmospheric neutrinos remain, the neutrino becomes potentially dangerous in its cosmological implications. However, given the large stability index of this prediction, there are large uncertainties in the prediction for its mass.

6 KARMEN and LSND

The presence of an additional sterile state makes it possible that a signal would be seen in short baseline $\nu_\mu \to \nu_e$ oscillations, such as has been reported at LSND [7]. An estimate of the LSND mixing angle from the neutrino sector gives $\epsilon\epsilon'\delta m^2_\odot/\delta m^2_{atm}$, a very small result. Hence, this mixing originates from the charged lepton sector

$$\theta_{LSND} = \sqrt{\frac{m_e}{m_\mu}} \qquad [i = 0]\,. \tag{29}$$

The precise predictions for $1-2$ mixing angles in the charged sector is an essential feature of the $U(2)$ flavor symmetry. In the quark sector it is highly successful. In the lepton sector, $\theta_{LSND} = \sqrt{\frac{m_e}{m_\mu}}$ is only useful if the neutrino mixing is either predicted or small, as in this theory. Recently, the KARMEN experiment has placed limits on the allowed region for such oscillations, giving a limit $m_\nu \leq 0.6$ eV [8]. While the prediction for m_ν has a large stability index in both the general theory as well as the variant theory, because the initial range for m_ν is so high in the variant theory, it is disfavored by this bound.

The general theory is much safer, however. As we discuss in the appendix, the uncertainty due to order one coefficients would allow it to satisfy the KARMEN bound. Such a result would likely coincide with higher values of $\delta m^2_\odot$ and lower values of δm^2_{atm}.

7 Astrophysical and Cosmological Implications

There are three important cosmological implications of our theory.

1. We predict a small, but potentially significant amount of neutrino hot dark matter. The KARMEN bound limits us to a 0.6 eV neutrino, but because there are two massive states, it is still within the interesting region for HDM.
2. We predict abundances for light nuclei resulting from four light neutrino species. While newer data suggest D/H ratios lie in the low end of the range previously thought, and thus $N_\nu < 4$, this is still an open question.
3. There may be two further singlet neutrino states, dominantly N_μ and N_τ, at or below the weak scale. Successful nucleosynthesis requires that they decay before the era of nucleosynthesis. Because the mass eigenstates are slightly left-handed, the primary decay mode will be through the process shown in figure 1. This is similar to muon decay, which we use as a benchmark. For the lighter of the two states, we estimate its lifetime to be

$$\tau_{N_\mu} \approx \epsilon'^{-2} \left(\frac{\delta m^2_{atm}}{\delta m^2_\odot}\right)^2 \left(\frac{m_\mu}{m_{N_\mu}}\right)^5 \tau_\mu \tag{30}$$

The mass of this particle is

$$m_{N_\mu} \approx \frac{1}{\epsilon} \frac{(\delta m^2_{atm})^2}{(\delta m^2_\odot)^{3/2}} \qquad [i = 12] \tag{31}$$

for the general theory and

$$m_{N_\mu} \approx \frac{1}{\epsilon^2} \frac{(\delta m^2_{atm})^2}{(\delta m^2_\odot)^{3/2}} \qquad [i \approx 11 - 16] \tag{32}$$

in the variant theory. The stability index is approximate because it involves sums of order one coefficients of different powers. Furthermore, i will change depending on which of (23) are included.

The more dangerous case, the general theory, then has a mass $O(100MeV)$ and thus a lifetime $\tau_{N_\mu} \approx 10^3 s$, which is far too long to be acceptable. However, because the lifetime has a fifth power dependence on the mass, and because the prediction for the mass has index 12, deviations in the order one quantities could very easily push the lifetime down to an acceptable level. As we explore in the appendix, even conservatively we can only reasonably estimate the mass of this particle to be in the range (17MeV, 40GeV), which means that the lifetime could easily be $10^{-9}s$, without even beginning to push the limits of the order one quantities. The details are presented in the appendix.

8 Models

The theory described in this paper has a low energy effective Lagrangian of (2) for charged fermion masses, while the neutrino masses arise from the $U(2) \times U(1)_N$ effective Lagrangian

$$\begin{aligned} \mathcal{W} = \frac{\phi_N}{M} N_3 l_3 h + \frac{\phi_N}{M^2} \left(N_3 \phi^a l_a h + l_3 \phi^a N_a h + N_a (S^{ab} + A^{ab}) l_b h \right) \\ + \frac{\phi_N^2}{M^3} \left(N_3 N_3 M + N_3 N_a \phi^a M + N_a N_b S^{ab} M \right) \end{aligned} \tag{33}$$

where N_3 and N_a have $U(1)_N$ charges $+1$, while ϕ_N has $U(1)_N$ charged -1. The field ϕ_N gets a vev, breaking $U(1)_N$ and establishing an overall scale for these coefficients: $\frac{<\phi_N>}{M} = \epsilon_N$. This effective theory can result from a renormalizable model by integrating out heavy states, both singlet and doublet under $U(2)$, in the Froggatt-Nielsen mechanism.

This symmetry structure on the right-handed singlet sector is far from unique. Another possibility is for N_a to carry $U(1)_N$ charge, while N_3 is neutral under $U(1)_N$. This has no effect on any of our predictions, since the form of (10) for the light neutrino mass matrix

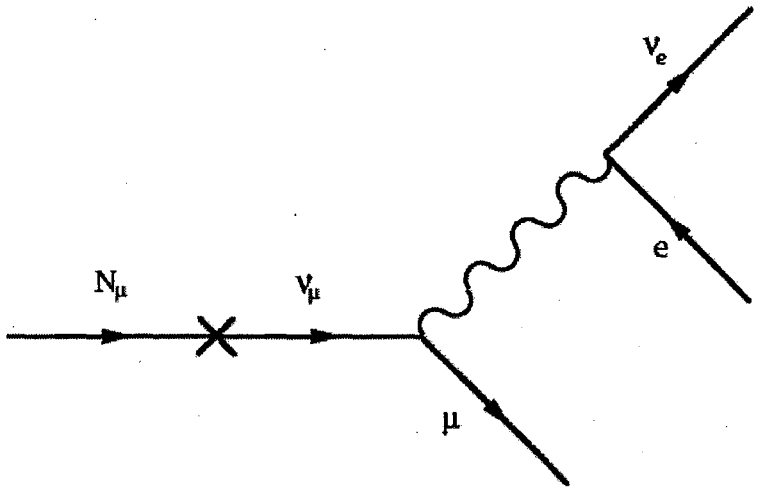

Figure 1: Principal decay mode for N_μ.

is unchanged. The only change is that N_3 has a mass of the order of the unification scale M rather than of order $\epsilon_N^2 M$.

Another possible symmetry structure for the theory is $U(2)_\psi \times U(2)_N$, where $U(2)_\psi$ acts as usual on all the matter with non-trivial $SU(3) \times SU(2) \times U(1)$ quantum numbers, while $U(2)_N$ acts only on the three right-handed neutrinos, with N_3 a singlet and N_a a doublet. The matrix M_{RR} now has the form of (7), and arises from the renormalizable interactions

$$W_{RR} = M N_3 N_3 + N_3 \phi^A N_A + N_A S^{AB} N_B \tag{34}$$

with vevs for S^{22} and ϕ^2 being of order ϵM and breaking $U(2)_N \to U(1)_N$. The interactions for M_{LR} are

$$W_{LR} = l_3 N_3 h + \frac{1}{M}\left(l_a \phi^a N_3 h + l_3 \phi^A N_A h + l_a R^{aA} N_A h\right) \tag{35}$$

where R^{aA} transforms as a (2,2). The vev for R^{22} is also of order ϵM, since this is the scale of breaking of $U(2)_\psi \times U(2)_N \to U(1)_\psi \times U(1)_N$. The breaking scale for $U(1)_\psi$ is $\epsilon' M$, so the vev of R^{12} takes this value. On the other hand, $U(1)_N$ is broken by R^{21}. We choose this scale to be smaller by a factor of ϵ_N, $< R^{21} > \approx \epsilon_N \epsilon' M$, giving

$$M_{LR} = v \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' \epsilon_N & \epsilon & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \tag{36}$$

Integrating out the heavy states N_2 and N_3, which now have masses of order the unification scale, this theory now reproduces (10) for the mass matrix of the four light neutrinos.

The common features of these models, which are inherent to our scheme, are:

- There is a $U(2)$ symmetry, which acts on the known matter as $\psi_3 \oplus \psi_a$, and is broken sequentially at scale ϵM and $\epsilon' M$.
- A $U(2)$ symmetry also acts on the three right-handed neutrinos with N_3 a singlet and $N_{1,2}$ a doublet. This $U(2)$, together with the symmetry of the Majorana mass, implies that N_1 does not have a Majorana mass and becomes a fourth light neutrino.
- There is an addition to the flavor group, beyond the $U(2)$ which acts on ψ. At least part of this additional flavor symmetry is broken at a scale very much less than M, leading to a small Dirac mass coupling of $\nu_\mu N_e$. Such a small symmetry breaking scale could be generated by the logarithmic evolution of a scalar m^2 term.

9 Conclusions

There are several theories with sterile neutrinos [9, 10, 11] some of which have 4×4 textures that split into two 2×2 matrices. Such theories provide a simple picture for atmospheric

oscillations via $\nu_\mu \to \nu_s$, and solar oscillations via $\nu_e \to \nu_\tau$, with $\delta m^2_\odot \approx \frac{v^2}{M} \approx 10^{-5}\text{eV}^2$ for $M \approx M_{unif}$. However, theories of this kind typically do not provide an understanding for several key points:

- Why is the Majorana mass of the singlet state ν_s small, allowing ν_s in the low energy theory?
- Why does ν_s mix with ν_μ rather than with ν_e or ν_τ?
- Why is the $\nu_s - \nu_\mu$ state pseudo-Dirac, leading to 45° mixing?
- How can this extended neutrino sector be combined with the pattern of charged quark and lepton masses in a complete theory of flavor?
- What determines the large number of free parameters in the neutrino sector?

In the theory presented here, all these questions are answered: the key tool is the $U(2)$ flavor symmetry, motivated several years ago by the charged fermion masses and the supersymmetric flavor problem. The simplest pattern of $U(2)$ symmetry breaking consistent with the charged fermion masses does not allow a Majorana mass for one of the three right-handed neutrinos. Furthermore, it is precisely this right-handed state that has a Dirac coupling to ν_μ but not to ν_e or ν_τ, guaranteeing that ν_μ is pseudo-Dirac with a 45° mixing angle.

Our theory provides a unified description of both charged fermion and neutrino masses, in terms of just three small symmetry breaking parameters and a set of order unity coefficients. Some predictions, such as $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}$ and $\theta_{atm} = 45^o$ are independent of the order unity coefficients and are precise. Other predictions, such as $|V_{cb}| \approx m_s/m_b$ and $\theta_\odot \approx \sqrt{m_e m_\mu/m_\tau^2}$ involve the order unity coefficients and are approximate. In the appendix we have introduced the "stability index" which attempts to quantify the uncertainty in such predictions according to the power of the unknown order unity coefficients appearing in the prediction.

There is one further free parameter of the theory—the overall mass scale M setting the normalization of the right handed Majorana mass matrix. If M is taken to be the scale of coupling constant unification $\delta m^2_\odot \approx 10^{-5} eV^2$.

The value of δm^2_{atm} is not predicted— this is the largest deficiency of the theory. It can be described by a very small flavor symmetry breaking parameter. Once this parameter is set by the observed value of δm^2_{atm}, it can be used to predict the approximate mass range of the pseudo-Dirac ν_μ to be in the range $10^{0.4} - 10^2$eV, with significant additional uncertainty due to order one coefficients. This, even with the KARMEN bound, allows for a neutrino of cosmological interest with $\sum_i m_{\nu_i} \approx 1$ eV. Such a neutrino could be seen at short baseline experiments, and may have already been seen by LSND. Searching for $\nu_\mu \to \nu_e$, with $\sin^2(2\theta) = 2 \times 10^{-2}$, below the current limit of δm^2 is an important experiment for the $U(2)$ theory, since it is this prediction which differentiates $U(2)$ from several other theories with a light singlet neutrino.

Experiment	Mode	Signal
Present solar ν exp.	$\nu_e \to \nu_\tau$	All data consistent with 2-flavor MSW
SNO	$\nu_e \to \nu_\tau$	Confirm SK measurement of B8. Measure $\frac{\phi_{NC}}{\phi_{CC}} \neq 1$
Borexino	$\nu_e \to \nu_\tau$	Consistent with small-angle 2-flavor MSW
KAMLAND	$\nu_e \to \nu_\tau$	No signal
LSND,KARMEN	$\nu_\mu \to \nu_e$	$\sin^2(2\theta) = 2 \times 10^{-2}$
K2K	$\nu_\mu \to N_e$	ν_μ disappearance. No e appearance
MINOS, ICARUS	$\nu_\mu \to N_e$	ν_μ disappearance. No τ appearance.
Atmospheric ν exp.	$\nu_\mu \to N_e$	Confirm 2 flavor $\nu_\mu \to \nu_s$ with 45° mixing.

Table 3: Experimental signals.

Predictions of the theory for experiments sensitive to neutrino oscillations are listed in table 3. We expect a small angle MSW solution to the solar neutrino anomaly, through a $\nu_e \Rightarrow \nu_\tau$ oscillation. The atmospheric neutrino anomaly is from $\nu_\mu \Rightarrow \nu_s$. This will be distinguishable from $\nu_\mu \Rightarrow \nu_\tau$ through a number of means: LBL experiments will see ν_μ disappearance, but no ν_e or ν_τ appearance. Improved statistics from Super-Kamiokande will be useful in distinguishing $\nu_\mu \Rightarrow \nu_\tau$ and $\nu_\mu \Rightarrow \nu_s$, for example via inclusive studies of multi-ring events [12].

A "Formalism" of the Stability Index

It is difficult to establish a formalism for the stability index, because it involves an inherently ill-defined quantity, namely, what constitutes an order one quantity. However, the potential instability of various predictions to variations in these order one parameters makes some attempt to quantify this necessary. Such a quantification should be relatively insensitive to what precisely constitutes an "order one quantity".

Therefore, we demand the following quantities of the index:

• An "order one" quantity should be defined as a quantity x with some probability distribution $P(x)$ to occur in an interval about 1. For reasons that will become clear later, it will be useful to consider instead the quantity $\overline{P}(y)$, where $x = 10^y$.

- This distribution should be "sensible", namely
  1. $P(x)$ should be an even function in $Log(x)$, that is, $\overline{P}(y)$ is even in y.
  2. $\overline{P}(y)$ should achieve its maximum value at 0.

3. $\overline{P}(y)$ should have a spread characterized by its variance,

$$v^2 = \int_{-\infty}^{\infty} y^2 \overline{P}(y)$$

the variance then quantifying what "order one" is numerically.

4. A product of two sensible distributions, correlated or uncorrelated, should be sensible.

- The index should have similar implications regardless of $\overline{P}(y)$, so long as it is sensible.
- The definition of $\overline{P}(y)$ should be the only necessary input.

We shall explore the motivation for these assumptions and will shortly see that the presented index nearly meets the requirements, and with minor modifications can meet them enitrely.

We assume that the expectation value of x, and of any products of x, is unity. It follows immediately that $\overline{P}(y)$ should be even in y. We do not have strong arguments in favor of this assumption, and if it were relaxed, the formalism could be suitably modified.

For instance, consider the seemingly sensible distribution

$$P(x) = \begin{cases} \frac{3}{8}, & \text{if } \frac{1}{3} \leq x \leq 3; \\ 0, & \text{otherwise.} \end{cases}$$

which has been normalized to give total probability 1. The expectation value of a product of n uncorrelated variables with such a distribution would be

$$< \overline{X} >= \int d^n x \prod_i P(x_i) x_i = (\frac{10}{3})^n. \tag{37}$$

Such a numerical pile-up of the central value of a product of order unity coefficients is excluded by our assumption.

What constitutes a "sensible" distribution is, of course, a judgement call. Examples of what we consider sensible distributions would be

- Flat distributions taking on the value $1/a$ from $-a/2$ to $a/2$
- Exponential distributions with standard deviation σ
- Linearly decreasing distributions of the form

$$\overline{P}(y) = \begin{cases} (\frac{1}{ab})(-\frac{b}{a}|y| + b), & \text{if } -a \leq y \leq a; \\ 0, & \text{otherwise.} \end{cases}$$

In fact, it can be shown that the last case it just the product of two uncorrelated quantities of the first type.

In all of these cases, the next moment (x^4) is irrelevant in quantifying the likelihood of the variable being within a particular region about zero. Requirement 3 is then simply a statement that a sensible distribution should simply have one quantity, its variance, to determine how confident we are that the variable is within that region. This will then allow us to be more confident in deducing the significance of the variance of some product.

This being stated, we can actually go about constructing some approximation of confidence intervals. The ability to describe the distribution of one variable by its variance is useful in allowing us to calculate the variances for higher products. We begin by writing the formal expression for the probability distribution of n uncorrelated variables $x_i = 10^{y_i}$ with probability distributions $\prod_i \overline{P}(y_i)$. We have

$$\overline{P}(z) = \int d^n y (\prod_i \overline{P}_i(y_i)) \delta(z - \sum_i y_i) \tag{38}$$

This expression is tedious to calculate for given $\overline{P}(y)$, particularly for large n. However, its variance is a relatively simply calculation.

$$v_z^2 = \int dz \overline{P}(z) z^2 = \int dz\, d^n y (\sum_i y_i)^2 (\prod_i \overline{P}_i(y_i)) \delta(z - \sum_i y_i) \tag{39}$$

Expanding the squared term we find terms

$$\int dz\, d^n y\, y_i\, y_j (\prod_i \overline{P}_i(y_i)) \delta(z - \sum_i y_i) = \begin{cases} 0, & \text{for } i \neq j; \\ v_i^2, & \text{if } i = j, \end{cases} \tag{40}$$

giving

$$v^2_{z\,uncorr} = \sum_i v_i^2 \tag{41}$$

For n correlated variables, a similar calculations yields

$$v^2_{z\,corr} = n^2 v_0^2 \tag{42}$$

where v_0^2 is the variance of the original variable.

Thus, a product of n correlated order one quantities is far more unstable than a product of n uncorrelated order one quantities. Simply counting the total number of order one coefficients is not sufficient. Thus we will refer to a product of the form

$$\prod_i x_i^{n_i} \tag{43}$$

as having index ($\sum_i n_i$) of type $(n_1, n_2, ..., n_m)$. If some of the n_i are repeated, we use the shorthand of writing n^j, if n is repeated j times. We assume all order one quantities have the same distribution. A product of type $(n_1, n_2, ..., n_m)$, has variance

$$v_{(n_i)}^2 = v_0^2 \sum_i n_i^2 \tag{44}$$

This works extremely well for products of order one coefficients. However, a sum of order one coefficients is not necessarily order one. In these cases, it is usually best to perform a Monte Carlo to determine the distribution.

B Sensible distributions

To characterize the probability of a general product to be within a certain region about 1, it is necessary to explore the particular forms of various distributions. We consider three reasonable distributions to be i) the flat distribution, ii) the Gaussian distribution, and iii) the linearly decreasing distribution.

A product of two equal width flat distributions yields a linear distribution, so we need only consider the flat and Gaussian cases. Gaussian distributions are well understood: products of variables with Gaussian $\overline{P}(y)$ functions are again Gaussian, allowing standard statistical techniques to be applied.

Products of flat distributions very quickly become characterized by Gaussian distributions. We have performed explicit Monte Carlos for $n = 1, 2, 3, 5, 7, 9$ uncorrelated variables. Even by $n = 2$ the Gaussian approximation is good, and for $n \geq 3$ it is very good. We thus believe it is reasonable to simply use Gaussian distributions, making a statistical interpretation of the variances simple.

For a standard, we propose using a distribution with variance $v = \sqrt{\frac{1}{12}}$, which corresponds to the variance of a flat distribution for $-\frac{1}{2} \leq y \leq \frac{1}{2}$. Changing the width of such a distribution from 1 to a would amount to multiplying this variance by a. Such generally mild sensitivity of the index to variations in the initial distribution is one its desirable qualities. We can then take "1-v" and "2-v" regions with $|y| \leq v$ and $|y| \leq 2v$, respectively. As should be clear, these should not be interpreted as the precise 67% and 95% 1 and 2-σ regions, because σ is not precisely defined. They are simply regions of medium and strong confidence, respectively.

As an example, consider a prediction with an unknown coefficient of order one quantities of the form $x_1^2 x_2 x_3$. We say this has index $2 + 1 + 1 = 4$ of type $(2, 1, 1)$, which we will write in shorthand as $(2, 1^2)$. Assuming the standard variance given above, this coefficient has variance $v = \sqrt{\frac{2^2+1^2+1^2}{12}} = \sqrt{\frac{1}{2}}$. Thus, we can have medium confidence that the prediction for x is known within a factor of $10^v = 5$, and strong confidence the the prediction is within a factor of $10^{2v} = 25$.

We can also see that this reduces to the expected prediction in the case of a variable of index 1 of type (1). It will have variance $v = \sqrt{\frac{1}{12}}$ which gives medium confidence that the prediction is known within a factor of 1.9, and strong confidence it is known within a factor of 3.8. This is a good consistency check that the index predicts what we would expect in the case of a single order one coefficient.

quantity	type (*=approx type)	Sign convention	Range of Med Confidence	Range of Strong Confidence
θ_{atm}	0	n/a	$\frac{\pi}{4}$ exact	exact
$\theta_{\odot}$	4*	+	$(0.002, 0.03)$	$(0.0006, 0.1)$
$\theta_{\odot}$	4*	-	$(0.005, 0.012)$	$(0.0001, 0.06)$
m_{ν}	$(2, 1^2)$	none	$(2, 50)$eV	$(0.4, 250)$eV
m_{N_L}	$(4, 3, 2^2, 1)$	none	(17MeV, 40GeV)	(300keV, 1.9TeV)
m_{N_H}	$(4, 2^4)$	none	(470MeV, 870GeV)	(11MeV, 37TeV)

Table 4: General Theory: uncertainties in predicitions. The regions listed here are simply for the uncertainty due to order one coefficients. Additional error due to uncertainty in input quantities, in particular in m_{ν}, can also be significant.

C Reassessing the uncertainties in the $U(2)$ neutrino model

In lieu of the preceding analysis, we address the index type of the predictions already presented, and thus assess strong and medium confidence regions of each prediciton. We list all uncertainties for the general theory in table 4.

In the general theory with the S-field, the atmospheric mixing angle is completely stable, while the solar angle is of approximate type $(1^4) = (1, 1, 1, 1)$. However, it involves a sum of order one coefficients, motivating the use of Monte Carlos. Since a sum is involved, the relative sign of the order one quantities becomes relevant, and we list those cases seperately. These Monte Carlos allow us to claim that we have medium confidence that $\theta_{\odot}$lies within $(0.002, 0.03)$ and strong confidence that it lies within $(0.0001, 0.1)$, giving large overlap of the BP98 region.

The mass of the pseudo-Dirac neutrino has stability index 5 of type $(2, 1^3)$, giving a medium confidence to know this within a factor of 5, and strong confidence within a factor of 25. Given the uncertainty in δm^2_{atm} and $\delta m^2_{\odot}$, which determine the prediction, m_{ν} could conceivably be as low as 0.1eV.

The masses of the right-handed states are not known so well. The mass prediction is, for the heavier state, of type $(4, 2^4)$, and, for the lighter state, of type $(4, 3, 2^2, 1)$. This would give medium confidence to know the masses at factors of 43 and 48, and strong confidence at factors of 1800 and 2300, respectively. The cosmological implications of these neutrinos are very uncertain, given that the lighter could be well over a TeV in mass.

Without the S field, certain uncertainties change. The precise nature of the changes depends on which splitting operators are included and what sign convention is taken. Because of the large number of permutations, we list only the basic results. The atmospheric angle is, as expected, completely certain. The solar angle becomes slightly more uncertain, but still overlaps BP98 well. The heaviest two righthanded masses typically become less certain by a factor of roughly 100, but the uncertainty is so large that the phenomenological predictions remain the same. The only dramatic difference in the variant theory is that ν_μ has a medium confidence region on its mass of (24eV, 4keV), and a strong confidence region of (2eV, 48keV). Including the uncertainties in the input quantities, the mass could be as low as $0.4eV$, which escapes the KARMEN bound, although narrowly.

Acknowledgements

This work was supported in part by the U.S. Department of Energy under Contracts DE-AC03-76SF00098, in part by the National Science Foundation under grant PHY-95-14797.

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Prepared for the U. S. Department of Energy under Contract No. DE-AC03-76SF00098