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Nearly Degenerate Neutrinos and Broken Flavour Symmetry *

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Abstract

Theories with non-Abelian flavour symmetries lead at zeroth order to neutrino degeneracy and massless charged fermions. The flavour symmetry is spontaneously broken sequentially to give hierarchies $\Delta m^2_{atm} \gg \Delta m^2_{\odot}$ and $m_\tau \gg m_\mu \gg m_e$, and a misalignment of the vacuum between neutrino and charged sectors gives large θ_{atm}. Explicit models are given which determine $\theta_{atm} = 45°$. A similar construction gives vacuum misalignment for the lighter generations and gives a vanishing $\beta\beta_{0\nu}$ rate, so that there is no laboratory constraint on the amount of neutrino hot dark matter in the universe, and the solar mixing angle is also maximal, $\theta_{\odot} = 45°$.

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1 Introduction

Measurements of both solar and atmospheric neutrino fluxes provide evidence for neutrino oscillations. With three neutrinos, this implies that there is negligible neutrino hot dark matter in the universe unless the three neutrinos are approximately degenerate in mass. In this letter we construct theories with approximately degenerate neutrinos, consistent with the atmospheric and solar data, in which the lepton masses and mixings are governed by spontaneously broken flavour symmetries.

The Super-Kamiokande collaboration has measured the magnitude and angular distribution of the ν_μ flux originating from cosmic ray induced atmospheric showers [1]. They interpret the data in terms of large angle ($\theta > 32°$) neutrino oscillations, with ν_μ disappearing to ν_τ or a singlet neutrino with Δm^2_{atm} close to $10^{-3}\mathrm{eV}^2$. Five independent solar neutrino experiments, using three detection methods, have measured solar neutrino fluxes which differ significantly from expectations. The data is consistent with ν_e disappearance neutrino oscillations, occuring either inside the sun, with $\Delta m^2_\odot$ of order $10^{-5}\mathrm{eV}^2$, or between the sun and the earth, with $\Delta m^2_\odot$ of order $10^{-10}\mathrm{eV}^2$. The combination of data on atmospheric and solar neutrino fluxes therefore implies a hierarchy of neutrino mass splittings: $\Delta m^2_{atm} \gg \Delta m^2_\odot$ [1].

In this letter we consider theories with three neutrinos. Ignoring the small contribution to the neutrino mass matrix which gives $\Delta m^2_\odot$, there are three possible forms for the neutrino mass eigenvalues:

$$\text{“Hierarchical”} \qquad \overline{m}_\nu = m_{atm} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} \tag{1}$$

$$\text{“Pseudo-Dirac”} \qquad \overline{m}_\nu = m_{atm} \begin{pmatrix} 1 & & \\ & 1 & \\ & & \alpha \end{pmatrix} \tag{2}$$

$$\text{“Degenerate”} \qquad \overline{m}_\nu = m_{atm} \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} + M \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \tag{3}$$

where m_{atm} is approximately 0.03 eV, the scale of the atmospheric oscillations. The real parameter α is either of order unity (but not very close to unity) or zero, while the mass scale M is much larger than m_{atm}. We have chosen to order the eigenvalues so that $\Delta m^2_{atm} = \Delta m^2_{32}$, while $\Delta m^2_\odot = \Delta m^2_{21}$ vanishes until perturbations much less than m_{atm} are added. An important implication of the Super-Kamiokande atmospheric data is that the mixing $\theta_{\mu\tau}$ is large. It is remarkable that this large mixing occurs between states with a hierarchy of Δm^2, and this places considerable constraints on model building.

What lies behind this pattern of neutrino masses and mixings? An attractive possibility is that a broken flavour symmetry leads to the leading order masses of (1), (2) or

[1] A problem in one of the solar neutrino experiments or in the Standard Solar Model could, however, allow comparable mass differences

(3), to a large θ_{atm}, and to acceptable values for $\theta_\odot$ and $\Delta m^2_\odot$. It is simple to construct flavour symmetries which lead to (1) or (2) with large (although not necessarily maximal) θ_{atm} [2]. For example, the hierarchical case results from integrating out a heavy Majorana right-handed neutrino which has comparable complings to ν_μ and ν_τ, and the pseudo-Dirac case when the heavy state is Dirac, with one component coupling to the $\nu_{\mu,\tau}$ combination and the other to ν_e.[2] However, in both hierarchical and pseudo-Dirac cases, the neutrino masses have upper bounds of $(\Delta m^2_{atm})^{\frac{1}{2}}$. In these schemes the sum of the neutrino masses is also bounded, $\Sigma_i m_{\nu i} \leq 0.1$ eV, implying that neutrino hot dark matter has too low an abundance to be relevant for any cosmological or astrophysical observation [4].

By contrast, it is more difficult to construct theories with flavour symmetries for the degenerate case [5], where $\Sigma_i m_{\nu i} = 3M$, which are therefore unconstrained by any oscillation data. While non-Abelian symmetries can clearly obtain the degeneracy of (3) at zeroth order, the difficulty is in obtaining the desired lepton mass hierarchies and mixing angles, which requires flavour symmetry breaking vevs pointing in very different directions in group space. We propose a solution to this vacuum misalignment problem, and use it to construct a variety of models, some of which predict $\theta_{atm} = 45°$. We also construct a model with bimaximal mixing [6] having $\theta_{atm} = 45°$ and $\theta_{12} = 45°$ [7].

2 Texture Analysis

What are the possible textures for the degenerate case in the flavour basis? These textures will provide the starting point for constructing theories with flavour symmetries. In passing from flavour basis to mass basis, the relative transformations of e_L and ν_L gives the leptonic mixing matrix V [8]. Defining V by the charged current in the mass basis, $\overline{e}V\nu$, we choose to parameterize V in the form

$$V = R(\theta_{23})R(\theta_{13})R(\theta_{12}) \tag{4}$$

where $R(\theta_{ij})$ represents a rotation in the ij plane by angle θ_{ij}, and diagonal phase matrices are left implicit. The angle θ_{23} is necessarily large as it is θ_{atm}. In contrast, the Super-Kamiokande data constrains $\theta_{13} \leq 20°$ [9], and if $\Delta m^2_{atm} > 2 \times 10^{-3}\text{eV}^2$, then the CHOOZ data requires $\theta_{13} \leq 13°$ [10]. For small angle MSW oscillations in the sun [11], $\theta_{12} \approx 0.05$, while other descriptions of the solar fluxes require large values for θ_{12} [12].

Which textures give such a V together with the degenerate mass eigenvalues of eqn. (3)? In searching for textures, we require that in the flavour basis any two non-zero entries are either independent or equal up to a phase, as could follow simply from flavour symmetries. This allows just three possible textures for m_ν at leading order

[2]The conventional paradigm for models with flavour symmetries is the hierarchical case with hierarchically small mixing angles, typically given by $\theta_{ij} \approx (m_i/m_j)^{\frac{1}{2}}$. If the neutrino mass hierarchy is moderate, and if the charged and neutral contributions to θ_{atm} add, this conventional approach is not excluded by the data [3].

$$\text{“}A'' \quad m_\nu = M\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + m_{atm}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} \tag{5}$$

$$\text{“}B'' \quad m_\nu = M\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix} + m_{atm}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix} \tag{6}$$

$$\text{“}C'' \quad m_\nu = M\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + m_{atm}\begin{pmatrix}0&0&0\\0&1&-1\\0&-1&1\end{pmatrix} \tag{7}$$

Alternatives for the perturbations proportonal to m_{atm} are possible. Each of these textures will have to be coupled to corresponding suitable textures for the charged lepton mass matrix m_E, defined by $\overline{e_L} m_E e_R$. For example, in cases (A) and (B), the big θ_{23} rotation angle will have to come from the diagonalization of m_E.

To what degree are the three textures A,B and C the same physics written in different bases, and to what extent can they describe different physics? Any theory with degenerate neutrinos can be written in a texture A form, a texture B form or a texture C form, by using an appropriate choice of basis. However, for certain cases, the physics may be more transparent in one basis than in another, as illustrated later.

3 A Misalignment Mechanism

The near degeneracy of the three neutrinos requires a non-Abelian flavour symmetry, which we take to be $SO(3)$, with the three lepton doublets, l, transforming as a triplet. This is for simplicity – many discrete groups, such as a twisted product of two Z_2s would also give zeroth order neutrino degeneracy. We expect the $SO(3)$ theories discussed below to have analogues with discrete non-Abelian symmetries [3].

We work in a supersymmetric theory and introduce a set of "flavon" chiral superfields which spontaneously break SO(3). For now we just assign the desired vevs to these fields; later we construct potentials which force these orientations. Also, for simplicity we assume one set of flavon fields, χ, couple to operators which give neutrino masses, and another set, ϕ, to operators for charged lepton masses. We label fields according to the direction of the vev, e.g. $\phi_3 = (0, 0, v)$. For example, texture A, with

$$m_E = \begin{pmatrix}0&0&0\\0&\delta_2&D_2\\0&\delta_3&D_3\end{pmatrix} \equiv m_{II}, \tag{8}$$

results from the superpotential

$$W = (l\cdot l)hh + (l\cdot\chi_3)^2 hh + (l\cdot\phi_3)\tau h + (l\cdot\phi_2)\tau h + (l\cdot\phi_3)\xi_\mu \mu h + (l\cdot\phi_2)\xi_\mu \mu h \tag{9}$$

[3] $SO(3)$ has been invoked recently in connection with quasi-degenerate neutrinos also in Refs [13].

where the coefficient of each operator is understood to be an order unity coupling multiplied by the appropriate inverse power of the large flavour mass scale M_f. The lepton doublet l and the ϕ, χ flavons are all $SO(3)$ triplets, while the right-handed charged leptons (e, μ, τ) and the Higgs doublets, h, are $SO(3)$ singlets. The electron mass is neglected. The form of eqn. (9) may be guaranteed by additional Abelian flavour symmetries; in the limit where these symmetries are exact, the only charged lepton to acquire a mass is the τ. These symmetries are broken by vevs of flavons $\xi_{e,\mu}$, which are $SO(3)$ and standard model singlet fields. The hierarchy of charged fermion masses is then generated by $\langle \xi_{e,\mu} \rangle / M_f$. The ratios $\langle \phi_{2,3} \rangle / M_f$ and $\langle \chi \rangle / M_f$ generate small dimensionless $SO(3)$ symmetry breaking parameters. The first term of (9) generates an $SO(3)$ invariant mass for the neutrinos corresponding to the first term in (5). The second term gives the second term of (5) with $m_{atm}/M = \langle \chi_3 \rangle^2 / M_f^2$. The remaining terms generate the charged lepton mass matrices. Note that the charged fermion masses vanish in the $SO(3)$ symmetric limit — this is the way we reconcile the near degeneracy of the neutrino spectrum with the hierarchical charged lepton sector. This is viable because, although the leading neutrino masses are $SO(3)$ invariant, they are second order $SU(2)$ violating and are suppressed relative to the electroweak scale $\langle h \rangle$ by $\langle h \rangle / M_f$, where M_f may be very large, of order the unification or Planck scale. On the other hand the charged lepton masses, although arising only via $SO(3)$ breaking, are only first order in $SU(2)$ breaking. Hence their suppression relative to $\langle h \rangle$ is of order $\langle \phi_i \rangle / M_f$. Since ϕ_i are $SU(2)$ singlets, they may have vevs much larger than h: the charged leptons can indeed be much heavier than the neutrinos.

In this example we see that the origin of large θ_{atm} is due to the misalignment of the ϕ vev directions relative to that of the χ vev. This is generic. In theories with flavour symmetries, large θ_{atm} can only arise because of a misalignment of flavons in charged and neutral sectors. To obtain $\theta_{atm} = 45°$, as preferred by the atmospheric data, requires however a very precise misalignment, which can occur as follows. In a basis where the χ vev is in the direction $(0, 0, 1)$, there should be a single ϕ field coupling to τ which has a vev in the direction $(0, 1, 1)$, where an independent phase for each entry is understood. As we shall now discuss, in theories based on $SO(3)$, such an alignment occurs very easily, and hence should be viewed as a typical expectation, and certainly not as a fine tuning.

Consider any 2 dimensional subspace within the l triplet, and label the resulting 2-component vector of $SO(2)$ as $\ell = (\ell_1, \ell_2)$. At zeroth order in SO(2) breaking only the neutrinos of ℓ acquire a mass, and they are degenerate from $\ell \cdot \ell h h$. Introduce a flavon doublet $\chi = (\chi_1, \chi_2)$ which acquires a vev to break $SO(2)$. If this field were real, then one could do an $SO(2)$ rotation to set $\langle \chi_2 \rangle = 0$. However, in supersymmetric theories χ is complex and a general vev has the form $\langle \chi_i \rangle = a_i + i b_i$. Only one of these four real parameters can be set to zero using $SO(2)$ rotations. Hence the scalar potential can determine a variety of interesting alignments. There are two alignments which are easily produced and are very useful in constructing theories:

$$\text{“SO(2)” Alignment:} \qquad W = X(\chi^2 - M^2); \qquad m_\chi^2 > 0; \qquad \langle \chi \rangle = M(0, 1). \quad (10)$$

The parameter M, which could result from the vev of some $SO(2)$ singlet, can be taken

real and positive by a phase choice for the fields. Writing $\langle\chi_i\rangle = a_i + ib_i$, with a_i and b_i real, an $SO(2)$ rotation can always be done to set $a_1 = 0$. The driver field X forces $\langle\chi^2\rangle = M^2$, giving $b_2 = 0$ and $a_2^2 = b_1^2 + M^2$ with b_1 undetermined. The potential term which aligns the directon of the χ vev is the positive soft mass squared $m_{\chi^2}\chi^*\chi$, which sets $b_1 = 0$.

The second example is:

$$\text{"U(1)" Alignment:} \qquad W = X\varphi^2; \qquad m_\varphi^2 < 0; \qquad \langle\varphi\rangle = V(1,i) \text{ or } V(1,-i). \tag{11}$$

It is now the negative soft mass squared which forces a magnitude $\sqrt{2}|V|$ for the vev. Using $SO(2)$ freedom to set $a_2 = 0$, $|F_X|^2$ provides the aligning potential and requires $\langle\varphi\rangle^2 = a_1^2 + 2ia_1b_1 - b_1^2 - b_2^2 = 0$, implying $b_1 = 0$ and $b_2 = \pm ia_1$. The $U(1)$ alignment leaves a discrete 2-fold degeneracy. In fact, the vevs $V(1,\pm i)$ do not require any particular choice of $SO(2)$ basis: performing $SO(2)$ transformation by angle θ on them just changes the phase of V by $\pm\theta$. The phases in $\langle\varphi\rangle$ are unimportant in determining the values of the neutrino mixing angles, so that the relative orientation of the vevs of (10) and (11) corresponds to 45° mixing.

The vev of the $SO(2)$ alignment, (10), picks out the original $SO(2)$ basis; however, the vev of the $U(1)$ alignment, (11), picks out a new basis (φ_+, φ_-), where $\varphi_\pm = (\varphi_1 \pm i\varphi_2)/\sqrt{2}$. If $\langle(\varphi_1,\varphi_2)\rangle \propto (1,i)$, then $\langle(\varphi_-,\varphi_+)\rangle \propto (1,0)$. An important feature of the $U(1)$ basis is that the $SO(2)$ invariant $\varphi_1^2 + \varphi_2^2$ has the form $2\varphi_+\varphi_-$. In the SO(3) theory, we usually think of $(l\cdot l)hh$ as giving the unit matrix for neutrino masses as in texture A. However, if we use the $U(1)$ basis for the 12 subspace, this operator actually gives the leading term in texture B, whereas if we use the $U(1)$ basis in the 23 subspace we get the leading term in texture C.

4 The Neutrinoless Double Beta Decay Constraint

Searches for neutrinoless double beta decay, $\beta\beta_{0\nu}$, place a limit $m_{\nu ee} < 0.5$ eV [14]. Consider texture A with $m_E = m_{II}$, so that the electron is dominantly in l_1. The $\beta\beta_{0\nu}$ limit implies $\Sigma_i m_{\nu i} < 1.5$ eV, and therefore places a constraint on the amount of neutrino hot dark matter in the universe

$$\Omega_\nu(l_1 \simeq e) < 0.05\left(\frac{0.5}{h}\right)^2. \tag{12}$$

While values of Ω_ν which satisfy this constraint can be of cosmological interest, it is also important to know whether this bound can be violated.

The bound is not greatly altered if texture A is taken with

$$m_E = \begin{pmatrix} 0 & \delta_1 & D_1 \\ 0 & \delta_2 & D_2 \\ 0 & \delta_3 & D_3 \end{pmatrix} \equiv m_{III}, \tag{13}$$

for generic values of D_1, D_2 and D_3. However, there is a unique situation where the $\beta\beta_{0\nu}$ bound on the amount of neutrino hot dark matter is evaded. It is convenient to discuss this special case in the basis in which it appears as texture B with $m_E = m_{II}$. To the order which we work, the electron mass eigenstate is then in the doublet $l_- = (l_1 - il_2)/\sqrt{2}$, where we label the basis by $(-,+,3)$ and, since there is no neutrino mass term l_-l_-hh, the rate for neutrinoless double beta decay vanishes. This important result is not transparent when the theory is described by texture A. In this case $m_E = m_{III}(\delta_1 = i\delta_2, D_1 = iD_2)$ and the electron is in a linear combination of l_1 and l_2. There are contributions to $\beta\beta_{0\nu}$ from both l_1l_1hh and l_2l_2hh operators, and these contributions cancel.

As an illustration of the utility of the $U(1)$ vev alignment, this theory with vanishing $\beta\beta_{0\nu}$ rate is described by the superpotential

$$W = (l \cdot l)hh + (l \cdot \chi_3)^2 hh + (l \cdot \phi_3)\tau h + (l \cdot \phi_-)\tau h + (l \cdot \phi_3)\xi_\mu \mu h + (l \cdot \phi_-)\xi_\mu \mu h. \quad (14)$$

Comparing with the theory for texture A with $m_E = m_{II}$, described by (9), the only change is the replacement of a vev in the 2 direction with one in the $-$ direction.

In theories of this sort, it is likely that a higher order contribution to $\beta\beta_{0\nu}$ will result when perturbations are added for m_e and $\Delta m^2_\odot$. For example, if the electron mass results from mixing with the second generation by an angle $\theta \simeq (m_e/m_\mu)^{\frac{1}{2}}$, then $\beta\beta_{0\nu}$ is reintroduced. However, the resulting limit on Ω_ν is weaker than (12) by about an order of magnitude, corresponding to this mixing angle. Large values of Ω_ν in such theories could be probed by further searches for neutrinoless double beta decay.

5 Models For Large θ_{atm}

Along the lines described above, we first construct a model with large, but undetermined, θ_{atm}, which explicitly gives both the Yukawa couplings and the orientation of the flavon vevs. Introduce two SO(3) triplet flavons, carrying discrete symmetry charges so that one, χ, gives only neutrino masses, while the other, ϕ, gives only charged lepton masses:

$$W = (l \cdot l)hh + (l \cdot \chi)^2 hh + (l \cdot \phi)\tau h. \quad (15)$$

Suppose that both flavons are forced to acquire vevs using the "$SO(2)$" alignment of (10):

$$W = X(\chi^2 - M^2) + Y(\phi^2 - M'^2) + Z(\chi \cdot \phi - M''^2); \qquad m^2_\chi > 0, \qquad m^2_\phi > 0 \quad (16)$$

where we have also added a Z driving term to fix the relative orientation of $\langle\chi\rangle$ and $\langle\phi\rangle$. As before we may take M, M' and M'' real by a choice of the phases of the fields. Minimizing the potential from $|F_X|^2$ the $SO(3)$ freedom allows the choice: $\langle\chi\rangle = M(0,0,1)$. The minimization of $|F_Y|^2$ is not identical, because now there is only a residual $SO(2)$ freedom, which allows only the general form $\langle\phi_i\rangle = a_i + ib_i$, with $a_1 = 0$. Setting $F_Y = 0$ and minimizing $\phi^*\phi$ gives $\langle\phi\rangle = M'(0, \sin\theta, \cos\theta)$, with θ undetermined. The Z driver fixes

$\cos\theta = MM'/M''^2$ which is of order unity if M, M' and M'' are comparable. If all other flavons couple to the leptons through higher dimension operators, $\theta_{atm} = \theta$.

Perhaps a more interesting case is to generate maximal mixing. To achieve this, change $\langle\phi\rangle$ to the "$U(1)$" alignment of (11)

$$W = X(\chi^2 - M^2) + Y\phi^2 + Z(\chi\cdot\phi - M''^2); \qquad m_\chi^2 > 0, \qquad m_\phi^2 > 0. \tag{17}$$

As before, $SO(3)$ freedom allows $\langle\chi\rangle = M(0,0,1)$ and $\langle\phi_i\rangle = a_i + ib_i$, with $a_1 = 0$. Setting $F_Z = 0$ aligns $b_3 = 0$ and $a_3 = M''^2/M \equiv V$, while $F_Y = 0$ forces $b_1^2 + b_2^2 = V^2 + a_2^2$ and $a_2 b_2 = 0$. With $m_\phi^2 > 0$, the remaining degeneracy is completely lifted by the soft mass squared term, giving $a_2 = 0$ and $(b_1, b_2) = V(\sin\theta, \cos\theta)$. Since $a_1 = a_2 = 0$, the $SO(2)$ freedom has not been used up, and we can choose an $SO(2)$ basis in which $\theta = 0$:

$$\langle\chi\rangle = M\begin{pmatrix}0\\0\\1\end{pmatrix} \qquad\qquad \langle\phi\rangle = V\begin{pmatrix}0\\i\\1\end{pmatrix}. \tag{18}$$

As expected, these vevs show that χ has an "$SO(2)$" alignment, while ϕ has a "$U(1)$" alignment. The alignment term ensures that ϕ and χ vevs are not orthogonal. The lepton masses from (15) now give $\theta_{atm} = 45°$, up to corrections of relative order m_μ/m_τ. In the $(1,-,+)$ basis, this theory has the leading terms of texture C with

$$m_E = \begin{pmatrix}0 & 0 & 0\\ 0 & \delta_2 & 0\\ 0 & 0 & D_3\end{pmatrix} \equiv m_I \tag{19}$$

6 Models With Large Ω_ν and Large θ_{atm}

The key to avoiding the $\beta\beta_{0\nu}$ constraint (12) [6, 7] is to have a $U(1)$ vev alignment in the 12 space so that the electron is in l_-. In this basis the $SO(3)$ invariant neutrino mass term is $2l_+l_- + l_3l_3$, as shown in texture B, and gives a vanishing $\beta\beta_{0\nu}$ rate. Thus we seek to modify the model of eqn (15), which generates large θ_{atm}, to align the electron along l_-. The interactions of (15) are insufficient to identify the electron. We must add perturbations for the muon mass, which will identify the electron as the massless state. Hence we extend (15) to

$$W_1 = (l\cdot l)hh + (l\cdot\chi)^2 hh + (l\cdot\phi_\tau)\tau h + (l\cdot\phi_\mu)\xi_\mu\mu h, \tag{20}$$

and seek potentials where $\langle\phi_{\tau,\mu}\rangle$ have zero components in the + direction.

To obtain a χ vev in the 3 direction, and a $U(1)$ alignment in the 12 space, we use (17), with $M'' = 0$ to enforce the orthogonality of ϕ with χ

$$W_2 = X(\chi^2 - M^2) + Y\phi_\mu^2 + Z\chi\cdot\phi_\mu; \qquad m_\chi^2 > 0, \qquad m_{\phi_\mu}^2 < 0, \tag{21}$$

which gives

$$\langle\chi\rangle = M\begin{pmatrix}0\\0\\1\end{pmatrix} \qquad \langle\phi_\mu\rangle = V\begin{pmatrix}1\\i\\0\end{pmatrix}. \tag{22}$$

Large θ_{atm} requires $\langle\phi_\tau\rangle$ to have large components in both $-$ and 3 directions, and results from the addition

$$W_3 = Z'\,\phi_\mu\cdot\phi_\tau; \qquad m^2_{\phi_\tau} < 0. \tag{23}$$

In the (1,2,3) basis

$$\langle\phi_\tau\rangle = V'\begin{pmatrix}1\\i\\\sqrt{2}x\end{pmatrix}. \tag{24}$$

This theory, described by $W_1 + W_2 + W_3$, determines the vev orientations so that Ω_ν is unconstrained by $\beta\beta_{0\nu}$ decay. The value of θ_{atm} is generically of order unity, but is not determined.

Additional potential terms can determine x and hence θ_{atm}. For example, maximal mixing can be obtained in a theory with three extra triplets, $\phi_{1,2,3}$. Discrete symmetries are introduced so that none of these fields couples to matter: the matter interactions remain those of (20). The field ϕ_1 is driven to have an $SO(2)$ alignment, and also the product $\phi_1\cdot\chi$ is driven to zero. The $SO(2)$ freedom, not specified until now, then allows the form $\langle\phi_1\rangle = V_1(1,0,0)$. The other two triplets ϕ_2 and ϕ_3 are driven just like ϕ_μ and ϕ_τ respectively: $\phi_2^2, \phi_2\cdot\chi$ and $\phi_2\cdot\phi_3$ are all forced to vanish. However, the vevs are not identical to those of $\phi_{\mu,\tau}$, because $\phi_2\cdot\phi_\mu$ is forced to be non-zero, so that the discrete $\pm$ choice of the $U(1)$ alignment is opposite for $\phi_{2,3}$ compared with $\phi_{\mu,\tau}$:

$$\langle\phi_2\rangle = V_2\begin{pmatrix}1\\-i\\0\end{pmatrix} \qquad \langle\phi_3\rangle = V_3\begin{pmatrix}1\\-i\\\sqrt{2}y\end{pmatrix}. \tag{25}$$

Maximal mixing follows from two further constraints: forcing $\phi_3\cdot\phi_\tau$ to zero imposes $xy = -1$, while forcing $\epsilon\phi_1\phi_3\phi_\tau$ to zero (ϵ is the tensor totally antisymmetric in $SO(3)$ indices) sets $y = -x$. Hence $(x,y) = (\pm1,\mp1)$, giving $\theta_{atm} = 45°$. The complete theory is described by $W_1 + W_2 + W_3 + W_4$, where

$$\begin{aligned} W_4 \;=\; & X_1(\phi_1^2 - M_1^2) + X_2\,\phi_1\cdot\chi + X_3\phi_2^2 + X_4\,\phi_2\cdot\chi + X_5\,\phi_2\cdot\phi_3 \\ & + X_6\,(\phi_2\cdot\phi_\mu - M_2^2) + X_7\,\phi_3\cdot\phi_\tau + X_8\,\epsilon\phi_1\phi_3\phi_\tau. \end{aligned} \tag{26}$$

There are other options for constructing theories with interesting vacuum alignments. For example, doublets may be used as well as triplets, and if $SO(3)$ is gauged, the aligning potential may arise from D terms as well as F terms.

7 Conclusions

In this letter we made a counter-intuitive proposal for a theory of lepton masses; in the limit of exact flavour symmetry, the three neutrinos are massive and degenerate, while the three charged leptons are massless. Such zeroth-order masses result when the three lepton doublets form a real irreducible representation of some non-Abelian flavour group — for example, a triplet of $SO(3)$. A sequential breaking of the flavour group then produces both a hierarchy of charged lepton masses and a hierarchy of neutrino Δm^2. The Majorana neutrino masses are small because, as always, they are second order in weak symmetry breaking.

We showed that the $SO(3)$ symmetry breaking may follow a different path in the charged and neutral sectors, leading to a vacuum misalignment with interesting consequences. There can be large leptonic mixing angles, with 45° arising from the simplest misalignment potentials. Such mixing can explain the atmospheric neutrino data while allowing large amounts of neutrino hot dark matter. The latter is consistent with the bounds on the $\beta\beta_{0\nu}$ process because the symmetry suppresses the Majorana mass matrix element $m_{\nu ee}$. Such a model can give bimaximal mixing [6, 7] with the large mixing angles very close to 45°.

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References

[1] Super-Kamiokande Collaboration, Y. Fukuda et al. Phys.Lett.**B433** (1998) 9;**B436** (1998) 33; Kamiokande Collaboration, S. Hatakeyama et al., Phys.Rev.Lett.**81** (1998) 2016.

[2] N. Irges, S. Lavignac, P. Ramond, Phys. Rev. **D58** (1998) 035003, hep-ph/9802334; J. Sato and T. Yanagida, hep-ph/9809307; H. Fritzsch and Z. Xing, Phys. Lett. B327(1996)265; hep-ph/9807234; M. Jezabek and Y. Sumino, hep-ph/9807310; M. Tanimoto, hep-ph/9807283; G. Altarelli and F. Feruglio, Phys. Lett. **B439**, 112 (1998),hep-ph/9807353,9809596; B. Allanach, (1998), hep-ph/9806294; S. Davidson and S. F. King, hep-ph/9808296; C. Jarlskog, M. Matsuda, S. Skadhauge, and M. Tanimoto, hep-ph/9812282(1998); R. Barbieri, L. Hall, D. Smith, A. Strumia and N. Weiner, to appear on JHEP, hep-ph/9807235; R.Barbieri, L. Hall and A. Strumia, hep-ph/9808333.

[3] G. K. Leontaris, S. Lola and G. Ross, *Nucl. Phys.* **B454** (1995) 25, hep-ph/9505402; K.S. Babu, J.C. Pati and F. Wilczek, hep-ph/9812538.

[4] Eric Gawiser and Joseph Silk, Science 280 (1998), astro-ph/9806197.

[5] B. Bamert and C. P. Burgess, Phys. Lett. **329B** (1994) 289; D. Caldwell and R. N. Mohapatra, Phys. Rev. **D50** (1994) 3477; A. Ioannissyan and J. W. F. Valle, Phys. Lett. **332B** (1994) 93; D.G. Lee and R. N. Mohapatra, Phys. Lett. **329B** (1994) 463; A. S. Joshipura, Z. Phys. **C64** (1994) 31; A. Ghosal, Phys. Lett. **B398**, 315 (1997); A. K. Ray and S. Sarkar, Phys. Rev. **D58**, 055010 (1998); U.Sarkar, hep-ph/9808277.

[6] M. Fukugita, M. Tanimoto and T. Yanagida, Phys. Rev. **D57** (1998) 4429; V. Barger, P. Pakvasa, T. J. Weiler, and K. Whisnant, hep-ph/9806387; Y.Nomura and T. Yanagida, hep-ph/9807325; S.M. Bilenky and C. Giunti, hep-ph/9802201; R. Mohapatra and S. Nussinov, Phys. Lett. **B346**, 75 (1995), hep-ph/9808301, 9809415; C. Giunti, hep-ph/9810272; A. Baltz, A. S. Goldhaber and M. Goldhaber, hep-ph/9806540; S.K. Kang and C.S. Kim, hep-ph/9811379.

[7] F. Vissani, hep-ph/9708483; H.Georgi and S.L.Glashow, hep-ph/9808293.

[8] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[9] V. Barger, T.J. Weiler and K. Whisnant, hep-ph/9807319; G.L.Fogli, E. Lisi, A. Marrone and G. Scioscia, hep-ph/9808205.

[10] M. Apollonio et. al (CHOOZ collaboration), hep-ex/9711002.

[11] L. Wolfenstein, Phys. Rev. **D 17**, 2369 (1978); S. P. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1985); Nuovo Cimento **9 C**, 17 (1986).

[12] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys.Rev.Lett.**81** (1998) 1158.GALLEX Collaboration, P. Anselmann et al.,Phys. Lett.**327B** (1994) 377; **388B** (1996) 384;SAGE Collaboration, J. N. Abdurashitov et al., Phys.Rev.Lett.**77** (1996) 4708;Homestake Collaboration, R. Davis et al., Nucl. Phys. **B38** (Proc. Suppl.) (1995) 47.

[13] C. Carone and M. Sher, Phys. Lett. **B420** (1998) 83; E. Ma, hep-ph/9812344; C. Wetterich, hep-ph/9812426; Y.-L. Wu, hep-ph/9810491.

[14] L. Bodis et al, *Phys. Lett.* **B407**, (1997) 219; for a recent review see H. V. Klapdor-Kleingrothaus, hep-ex/9802007.

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