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Fermion masses and symmetry breaking of a U(2) flavour symmetry*

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Abstract

We show how a specific sequential breaking pattern of a U(2) flavour symmetry occurs automatically in a broad framework. The relative orientation in U(2) space of the spurion fields that breaks the U(2) symmetry is uniquely fixed, thus determining the form of the fermion mass matrices in a predictive way.

1 Introduction and main results

In previous papers [1], some of us have pointed out that a U(2) symmetry might be relevant to understand several features of flavour physics. We have in mind both a qualitative and partly quantitative explanation of the pattern of fermion masses and mixings as well as a possible understanding of the Flavour Changing Neutral Current

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problem in supersymmetric extensions of the Standard Model [2, 1]. Along these lines we want to discuss in this paper the problem of the relative orientation, in U(2) space, of the fields that break the U(2) symmetry, thus determining in a unique way the form of the fermion mass matrices. A refined discussion of the predictions of the U(2) symmetry is given in Ref. [3].

The flavour U(2) group acts on the lighter 2 generations ψ_a, $a = 1, 2$ as a doublet and on the third generation ψ_3, like on the Higgs fields, H, as trivial singlets. In the limit of unbroken U(2), only the third generation of fermions can acquire a mass, whereas the first two generations of scalar superpartners are exactly degenerate. While the first property is not a bad approximation of the fermion spectrum, the second one is what one needs to keep under control FCNC and CP-violating phenomena generated by superparticle exchanges. Furthermore, a two step breaking pattern of U(2) accommodates the double hierarchy $m_3 \gg m_2 \gg m_1$ among different generations in the fermion spectrum. Although it is natural to view U(2) as a subgroup of U(3), the maximal flavour group in the case of full intra-family gauge unification, U(3) will be anyhow strongly broken to U(2) by the large top Yukawa coupling.

Since the Higgs bosons are flavour singlets, the Yukawa interactions transform under U(2) as: $(\psi_3\psi_3)$, $(\psi_3\psi_a)$, $(\psi_a\psi_b)$. Hence the only relevant U(2) representations for the fermion mass matrices are 1, ϕ^a, S^{ab} and A^{ab}, where S and A are symmetric and antisymmetric tensors, and the upper indices denote a U(1) charge opposite to that of ψ_a. We view ϕ^a, S^{ab} and A^{ab} as "effective flavon" fields—in general they are polynomials of the fundamental flavon fields of the theory. The Yukawa potential has the form

$$\mathcal{L}_Y = H\left(\psi_3\psi_3 + \psi_3\frac{\phi^a}{M}\psi_a + \psi_a\frac{S^{ab}}{M}\psi_b + \psi_a\frac{A^{ab}}{M}\psi_b\right) , \tag{1}$$

where M is a mass scale weighting all non-renormalizable interactions and intra-family (vertical) indices and dimensionless couplings are omitted.

The most general form for the vacuum expectation values (vevs) of the flavon fields is, in a suitable basis,

$$\langle A\rangle = \begin{pmatrix} 0 & v \\ -v & 0 \end{pmatrix} \quad \langle\phi\rangle = \begin{pmatrix} 0 \\ V \end{pmatrix} \quad \langle S\rangle = \begin{pmatrix} s_2 & s_1 \\ s_1 & s_0 \end{pmatrix} \tag{2}$$

with V, v real and positive and s_i in general complex. Let us consider first the possible breaking patterns due to the flavon fields A and ϕ only, leaving S apart for the moment. With $\langle A\rangle$ and $\langle\phi\rangle$ there are only two ways of breaking U(2) depending on which one of the two scales V or v is larger:

$$i)\ \text{if } V > v \quad U(2) \xrightarrow[\langle\phi\rangle]{V} U(1) \xrightarrow[\langle A\rangle]{v} \{e\} , \tag{3}$$

$$ii)\ \text{if } V < v \quad U(2) \xrightarrow[\langle A\rangle]{v} SU(2) \xrightarrow[\langle\phi\rangle]{V} \{e\}, \tag{4}$$

where U(1) corresponds, in the chosen basis, to the subgroup of phase rotations of the first generation and e is the unity of U(2). Since *ii)* would give approximately equal

masses for the first two generations of fermions, the phenomenology selects case *i)* and indicates that $V \gg v$.

Let us now consider the flavon S. To preserve this breaking pattern requires $s_0 \equiv \langle S^{22} \rangle$, which breaks U(2) to the same U(1) as $\langle \phi \rangle$, at the scale V or less and s_1 and s_2, which break the remaining U(1), at the scale v or less. Otherwise, the hierarchy between the first two generations associated with the ratio v/V could be spoiled. Furthermore, in order to preserve the phenomenologically successful relations [4, 5]

$$|V_{us}| = \left| \sqrt{\frac{m_d}{m_s}} + e^{i\Phi} \sqrt{\frac{m_u}{m_c}} \right|, \tag{5a}$$

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\frac{m_u}{m_c}}, \tag{5b}$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}} \tag{5c}$$

among the CKM matrix elements, the masses of light quarks and the CP-violating phase Φ, stronger constraints on s_1, s_2 must be fulfilled [6, 1]:

$$|s_1| \ll v, \quad |s_2| \ll \frac{v^2}{V}. \tag{6}$$

Should relations (6) be considered an "ad hoc" hypothesis or is it possible to justify them?

This paper answers this question for theories having fundamental flavons ϕ, S, A, their U(2) conjugates $\bar{\phi}, \bar{S}, \bar{A}$ and possibly U(2) singlets X_i. In these theories (6) is a prediction, following from the most general softly broken supersymmetric potential (including possible non-renormalizable terms) which yields the breaking pattern (3) with $V \gg v$. This result adds confidence to the U(2) scheme and strengthens its predictions. More precisely, for generic values of the parameters in the potential of the flavon fields, we show in Sect.s 2 and 3 that the minimum is non degenerate and that, at the minimum, in an appropriate U(2)-basis,

$$\frac{\phi}{M} = \mathcal{O}\begin{pmatrix} \epsilon\epsilon' \\ \epsilon \end{pmatrix} \quad \frac{S}{M} = \mathcal{O}\begin{pmatrix} \epsilon'^2 & \epsilon\epsilon' \\ \epsilon\epsilon' & \epsilon \end{pmatrix} \quad \epsilon \equiv \frac{V}{M} \quad \epsilon' \equiv \frac{v}{M}, \tag{7}$$

with $\bar{\phi}$, $\bar{S}$ having similar magnitude and orientation as ϕ and S respectively.

By inserting (7) in (1), we find the Yukawa matrices in flavour space for the quarks and charged leptons with the texture

$$\lambda = \begin{pmatrix} \ll \epsilon'^2/\epsilon & \epsilon' & \ll \epsilon' \\ -\epsilon' & \epsilon & \mathcal{O}(\epsilon) \\ \ll \epsilon' & \mathcal{O}(\epsilon) & \mathcal{O}(1) \end{pmatrix}, \tag{8}$$

up to irrelevant phase factors. The parameters ϵ, ϵ' will depend in general upon the fermion charge. As shown in [6, 1] this texture for quarks leads to the relations (5a-c) as well as to the qualitative relation $|V_{cb}| \sim m_s/m_b$. The phenomenological consequences of (8) are carefully analysed in Ref. [3].

2 Minimizing the potential in the supersymmetric limit

Since we work in a supersymmetric framework, the potential V consists of a supersymmetry conserving and a supersymmetry breaking piece

$$V\left(\phi, S, A, \bar{\phi}, \bar{S}, \bar{A}; X_i\right) = V^{\text{susy}} + V^{\text{breaking}} \tag{9}$$

both invariant under a global U(2) symmetry. V^{susy} is determined by a superpotential W and is characterized by a scale M, e.g. the GUT scale, much bigger than the scale m, which controls the size of V^{breaking} in the usual way and is of the order of the electroweak scale.

Let us consider first the supersymmetric limit. Neglecting non renormalizable terms, the most general W, after a rescaling of the fields, is

$$W^{(r)} = \phi\bar{S}\phi + \bar{\phi}S\bar{\phi} + X_\phi\phi\bar{\phi} + X_S\bar{S}S + X_A\bar{A}A. \tag{10}$$

X_ϕ, X_S and X_A are linear combinations of one or more singlet fields and of possible mass terms. The part of the superpotential only dependent on the singlet fields X's does not affect any of our considerations and it is therefore not explicitly shown. Couplings between A, $\bar{A}$ and the other fields are forbidden by the antisymmetry of A, $\bar{A}$ in the $U(2)$ indices.

If $X_S \neq 0$, there is a supersymmetric minimum where

$$\begin{cases} S^{ab} = -\dfrac{\phi^a\phi^b}{X_S} \\ \bar{S}_{ab} = -\dfrac{\bar{\phi}_a\bar{\phi}_b}{X_S} \end{cases} \tag{11}$$

and

$$\phi\bar{\phi} = \frac{X_S X_\phi}{2}. \tag{12}$$

To show that this solution is preferable to $\phi = \bar{\phi} = 0$ supersymmetry breaking must be considered, as done in Section 3.

The minimum equations for A, $\bar{A}$

$$X_A A^{ab} = 0 \quad X_A\bar{A}_{ab} = 0 \tag{13}$$

decouple, at renormalizable level. For $X_A \neq 0$, they give $A = \bar{A} = 0$. Unlike the case for S, $\bar{S}$, ϕ, $\bar{\phi}$, the introduction of non-renormalizable interactions cannot be treated pertubatively.

Let us therefore consider first, in the case of a general superpotential, the minimum equations for S, $\bar{S}$. For field vevs small relative to M, standard inversion theorems guarantee that the minimum equations can be solved for S, $\bar{S}$ functions of ϕ, $\bar{\phi}$ as for eq. (11). ¿From general U(2) covariance,

$$S^{ab} = \Sigma_1\phi^a\phi^b + \Sigma_2\left[(A\bar{\phi})^a\phi^b + \phi^a(A\bar{\phi})^b\right] + \Sigma_3(A\bar{\phi})^a(A\bar{\phi})^b \equiv \hat{S}^{ab} \tag{14 a}$$

$$\bar{S}_{ab} = \bar{\Sigma}_1\bar{\phi}_a\bar{\phi}_b + \bar{\Sigma}_2\left[(\bar{A}\phi)_a\bar{\phi}_b + \bar{\phi}_a(\bar{A}\phi)_b\right] + \bar{\Sigma}_3(\bar{A}\phi)_a(\bar{A}\phi)_b \equiv \hat{\bar{S}}_{ab}, \tag{14 b}$$

with Σ_i, $\bar{\Sigma}_i$ functions of the invariants $\phi\bar{\phi}$, $A\bar{A}$. More explicitly, to leading order in $1/M$,

$$S^{ab} = \sigma_1 \frac{\phi^a \phi^b}{X_S} + \frac{\sigma_2}{M X_S}\left[(A\bar{\phi})^a \phi^b + \phi^a (A\bar{\phi})^b\right] + \frac{\sigma_3}{M X_S^2}(A\bar{\phi})^a (A\bar{\phi})^b \tag{15 a}$$

$$\bar{S}_{ab} = \bar{\sigma}_1 \frac{\bar{\phi}_a \bar{\phi}_b}{X_S} + \frac{\bar{\sigma}_2}{M X_S}\left[(\bar{A}\phi)_a \bar{\phi}_b + \bar{\phi}_a (\bar{A}\phi)_b\right] + \frac{\bar{\sigma}_3}{M X_S^2}(\bar{A}\phi)_a (\bar{A}\phi)_b, \tag{15 b}$$

where $\sigma_1 = \bar{\sigma}_1 = -1$ and $\sigma_{2,3}$, $\bar{\sigma}_{2,3}$ are polynomial in $\phi\bar{\phi}/X_S^2$, $A\bar{A}/X_S^2$, X_A/X_S. It is possible to give examples of explicit non-renormalizable potential that generate non vanishing $\sigma_{2,3}$, $\bar{\sigma}_{2,3}$.

To solve the minimum equations $\partial W/\partial\phi = \partial W/\partial\bar{\phi} = 0$, it is useful to define

$$\hat{W}(\phi\bar{\phi}, A\bar{A}) \equiv W\left(\hat{S}(\phi,\bar{\phi},A,\bar{A}), \hat{\bar{S}}(\phi,\bar{\phi},A,\bar{A}), \phi, \bar{\phi}, A, \bar{A}\right). \tag{16}$$

Since $\hat{S}$, $\hat{\bar{S}}$ solve $\partial W/\partial S = \partial W/\partial \bar{S} = 0$, it is immediate that

$$0 = \frac{\partial W}{\partial \bar{\phi}_a} = \frac{\partial \hat{W}}{\partial \phi\bar{\phi}}\phi^a \quad 0 = \frac{\partial W}{\partial \phi^a} = \frac{\partial \hat{W}}{\partial \phi\bar{\phi}}\bar{\phi}_a \tag{17}$$

which, again disregarding the possibility $\phi = \bar{\phi} = 0$, are equivalent to the unique equation

$$\frac{\partial \hat{W}}{\partial \phi\bar{\phi}} = 0\,. \tag{18}$$

This allows to compute $\phi\bar{\phi}$ in terms of $A\bar{A}$ and of the singlet fields. As before, to leading order in $1/M$, from eq. (18),

$$\phi\bar{\phi} = \frac{X_S}{2}\left(X_\phi + \frac{\sigma_\phi(A\bar{A}/X_S^2)}{M}\right), \tag{19}$$

with σ_ϕ polynomial in its variable.

Analogously, $\partial\hat{W}/\partial A\bar{A} = 0$ is the unique equation to be solved in $A\bar{A}$. Examples of non renormalizable interactions that fix $A\bar{A}$ at a nonvanishing vev are easy to construct. We assume $|A\bar{A}| \ll |\phi\bar{\phi}|$.

These considerations make clear that the supersymmetric minimum is highly degenerate even for a non renormalizable potential. Other than the degeneracy related to U(2) invariance, the surface of minima is flat in directions corresponding to the relative orientation of ϕ and $\bar{\phi}$ and to the rescalings

$$\phi \to x\phi, \quad \bar{\phi} \to \bar{\phi}/x, \quad A \to yA, \quad \bar{A} \to \bar{A}/y \tag{20}$$

with x, y real. This degeneracy is removed by the introduction of the supersymmetry breaking potential, as we show in the next Section. Furthermore, if the parameters in

Since $z = z^{(0)} + z^{(1)}$ is a minimum, by differentiating (23) with respect to $z^{(1)}$ one obtains

$$\frac{\partial^2 V_0}{\partial z_i \partial z_j^\dagger}\bigg| z^{(1)\dagger}_j + m\,\frac{\partial f}{\partial z_i}\bigg| = 0 \tag{24}$$

which, substituted into (23), leads to

$$V = V_2 + m\frac{\partial f^\dagger}{\partial z_i^\dagger} z_i^{(1)\dagger} = V_2 + m\frac{\partial f}{\partial z_i} z_i^{(1)} \equiv V_{\text{eff}}, \tag{25}$$

where c has been omitted. V_{eff}, with $z^{(1)}$ given by (24) in terms of $z^{(0)}$ and proportional to the holomorphic supersymmetry breaking terms, can be viewed as a simplified "effective" potential to be minimized in $z_i^{(0)}$, $z_i^{(0)\dagger}$ on the surface $\mathcal{S}$, thus removing the degeneracy of the supersymmetric minimum.

Notice that the contribution due to the holomorphic supersymmetry breaking terms vanishes if they are universal, namely if $f \propto W$. In this case we have in fact $\partial f/\partial z \propto \partial W/\partial z = 0$ on $\mathcal{S}$.

Let us therefore consider the general problem of the minimization on $\mathcal{S}$ of a U(2)-symmetric potential V_{eff}, function of the flavon fields and their hermitian conjugates. On $\mathcal{S}$ we can use

$$V_{\text{eff}}\left(S = \hat{S}, \bar{S} = \hat{\bar{S}}, \phi, \bar{\phi}, \text{h.c.}\right) \equiv \hat{V}_{\text{eff}}\left(\phi\phi^\dagger, \bar{\phi}\bar{\phi}^\dagger, AA^\dagger, \bar{A}\bar{A}^\dagger, \bar{\phi}A\phi^\dagger, \phi\bar{A}\bar{\phi}^\dagger, \bar{\phi}^\dagger A^\dagger\phi, \phi^\dagger\bar{A}^\dagger\bar{\phi}\right), \tag{26}$$

where the variables of $\hat{V}_{\text{eff}}$ are all the possible U(2)-invariants one can build with the flavons and their hermitian conjugates besides $\phi\bar{\phi}$, $\phi^\dagger\bar{\phi}^\dagger$, $A\bar{A}$, $A^\dagger\bar{A}^\dagger$ that are constant on $\mathcal{S}$. $\hat{V}_{\text{eff}}$ has to be minimized under the constraint $\phi\bar{\phi} = (\phi\bar{\phi})^{(0)}$, $\phi^\dagger\bar{\phi}^\dagger = (\phi^\dagger\bar{\phi}^\dagger)^{(0)}$, $A\bar{A} = (A\bar{A})^{(0)}$, $A^\dagger\bar{A}^\dagger = (A^\dagger\bar{A}^\dagger)^{(0)}$. Introducing the Lagrange multipliers

$$\hat{V} \equiv \hat{V}_{\text{eff}} - \lambda_\phi(\phi\bar{\phi} - (\phi\bar{\phi})^{(0)}) - \lambda_\phi^\dagger(\phi^\dagger\bar{\phi}^\dagger - (\phi^\dagger\bar{\phi}^\dagger)^{(0)}) - \lambda_A(A\bar{A} - (A\bar{A})^{(0)}) - \lambda_A^\dagger(A^\dagger\bar{A}^\dagger - (A^\dagger\bar{A}^\dagger)^{(0)}), \tag{27}$$

and projecting the minimum equation for ϕ along two orthogonal directions one gets

$$0 = (\bar{\phi}^\dagger)^a \frac{\partial\hat{V}}{\partial\phi^a} = \frac{\partial\hat{V}_{\text{eff}}}{\partial\phi\phi^\dagger}\phi^\dagger\bar{\phi}^\dagger - \lambda_\phi\bar{\phi}\bar{\phi}^\dagger \tag{28 a}$$

$$0 = \phi_a^\dagger A^{ab}\frac{\partial\hat{V}}{\partial\phi^b} = \frac{\partial\hat{V}_{\text{eff}}}{\partial\phi\phi^\dagger}\bar{\phi}A\phi^\dagger - \frac{1}{2}\left(\frac{\partial\hat{V}_{\text{eff}}}{\partial\phi\bar{A}\bar{\phi}^\dagger}\bar{\phi}\bar{\phi}^\dagger A\bar{A} - \frac{\partial\hat{V}_{\text{eff}}}{\partial\bar{\phi}^\dagger A^\dagger\phi}\bar{\phi}\bar{\phi}^\dagger AA^\dagger\right) \tag{28 b}$$

$$0 = \frac{\partial\hat{V}}{\partial A^{ab}}A^{ab} = \frac{\partial\hat{V}_{\text{eff}}}{\partial\bar{\phi}A\phi^\dagger}\bar{\phi}A\phi^\dagger + \frac{\partial\hat{V}_{\text{eff}}}{\partial AA^\dagger}AA^\dagger. \tag{28 c}$$

the potential related to fields and bar-fields have similar order of magnitude, we show that

$$\bar{\phi}_1\phi_2^\dagger - \bar{\phi}_2\phi_1^\dagger = \mathcal{O}\left(\frac{V^2 v}{M}\right), \quad \phi\phi^\dagger \sim \bar{\phi}\bar{\phi}^\dagger, \quad AA^\dagger \sim \bar{A}\bar{A}^\dagger \tag{21}$$

which, together with $X_S \sim X_\phi$, proves the results stated in Sect. 1. This is made manifest by choosing a basis where $\phi_1 = 0$, since, from (21), $\bar{\phi}_2/M \simeq \phi_2/M \simeq \epsilon$, $\bar{\phi}_1/M \simeq \epsilon\epsilon'$ and S^{ab}, by inserting these ϕ, $\bar{\phi}$ vevs in (15 a), has the same form as in (7). The proof of (21) is lengthy. We establish it by discussing a general method to minimize the potential in presence of supersymmetry breaking.

3 Minimizing the full potential

Denoting by z_i the collection of all fields, the potential V consists of three pieces, $V = V_0 + V_1 + V_2$, with the general structure

$$V_0 = \frac{\partial W^\dagger}{\partial z_i^\dagger}\frac{\partial W}{\partial z_i} \tag{22 a}$$

$$V_1 = mf(z) + mf^\dagger(z) \tag{22 b}$$

$$V_2 = \sum_i m_i^2 |z_i|^2 , \tag{22 c}$$

and $f(z)$ holomorphic in z. Let us write the position of the minimum of the full potential as $z_i = z_i^{(0)} + z_i^{(1)}$ where $z^{(0)}$ is on the surface $\mathcal{S}$ of minima of V_0 , as defined by (20), and $z^{(1)}$ is a correction, due to the supersymmetry breaking terms, orthogonal to it. Note that, due to the structure of the supersymmetric minimum, each holomorphic U(2)-invariant function of the flavon fields ϕ, S, A, $\bar{\phi}$, $\bar{S}$, $\bar{A}$ or of their hermitian conjugates, separately, is constant on $\mathcal{S}$. Therefore to resolve the degeneracy of $z^{(0)}$ an expansion of V to first order in m or $z^{(1)}$ is not sufficient. Expanding $V(z^{(0)} + z^{(1)})$ up to second order around $z^{(0)}$ gives

$$V(z^{(0)} + z^{(1)}) = \\ c + \left.\frac{\partial^2 V_0}{\partial z_i \partial z_j^\dagger}\right|_{z^{(0)}, z^{(0)\dagger}} z_i^{(1)} z_j^{(1)\dagger} + m \left.\frac{\partial f}{\partial z_i}\right|_{z^{(0)}} z_i^{(1)} + m \left.\frac{\partial f^\dagger}{\partial z_i^\dagger}\right|_{z^{(0)\dagger}} z_i^{(1)\dagger} + V_2|_{z^{(0)}, z^{(0)\dagger}} , \tag{23}$$

where $c = V_0(z^{(0)}, z^{(0)\dagger}) + mf(z^{(0)}) + mf^\dagger(z^{(0)\dagger}) = mf(z^{(0)}) + mf^\dagger(z^{(0)\dagger})$ is actually independent of $z^{(0)}, z^{(0)\dagger}$ and all other terms are of second order in the supersymmetry breaking scale m. While the constant c is independent of the position on the surface $\mathcal{S}$, it does distinguish this surface from the alternative solution in which $\phi = \bar{\phi} = 0$. In order to select the desired solution, c must be negative at the minimum. It is easy to convince oneself that this is the case in a large region of the parameter space.

Similarly one gets other 9 barred and hermitian conjugate equations. The lagrange multipliers can be eliminated from this total of 12 equations leaving only the following 4 independent ("real") equations:

$$\frac{\partial \hat{V}_{\text{eff}}}{\partial \phi\phi^\dagger}\phi\phi^\dagger = \frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{\phi}\bar{\phi}^\dagger}\bar{\phi}\bar{\phi}^\dagger \tag{29 a}$$

$$\frac{\partial \hat{V}_{\text{eff}}}{\partial AA^\dagger}AA^\dagger + \frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{\phi}A\phi^\dagger}\bar{\phi}A\phi^\dagger = \frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{A}\bar{A}^\dagger}\bar{A}\bar{A}^\dagger + \frac{\partial \hat{V}_{\text{eff}}}{\partial \phi\bar{A}\bar{\phi}^\dagger}\phi\bar{A}\bar{\phi}^\dagger \tag{29 b}$$

$$\frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{\phi}\bar{\phi}^\dagger}\frac{\bar{\phi}A\phi^\dagger}{\phi\phi^\dagger} = \frac{1}{2}\left(\frac{\partial \hat{V}_{\text{eff}}}{\partial \phi\bar{A}\bar{\phi}^\dagger}A\bar{A} - \frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{\phi}^\dagger A^\dagger\phi}AA^\dagger\right) \tag{29 c}$$

$$\frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{\phi}\bar{\phi}^\dagger}\frac{\bar{\phi}^\dagger A^\dagger\phi}{\phi\phi^\dagger} = \frac{1}{2}\left(\frac{\partial \hat{V}_{\text{eff}}}{\partial \phi^\dagger\bar{A}^\dagger\bar{\phi}}A^\dagger\bar{A}^\dagger - \frac{\partial \hat{V}_{\text{eff}}}{\partial \bar{\phi}A\phi^\dagger}A^\dagger\bar{A}^\dagger\right). \tag{29 d}$$

The number of degree of freedom is 4 (real) too, because all the U(2)-invariants can be expressed on $\mathcal{S}$ in terms of $\phi\phi^\dagger$, $AA^\dagger$, $\bar{\phi}A\phi^\dagger$, $\bar{\phi}^\dagger A^\dagger\phi$ through

$$\bar{A}\bar{A}^\dagger = \frac{(A\bar{A})^{(0)}(A^\dagger\bar{A}^\dagger)^{(0)}}{AA^\dagger}, \quad \bar{\phi}\bar{\phi}^\dagger = \frac{(\phi\bar{\phi})^{(0)}(\phi^\dagger\bar{\phi}^\dagger)^{(0)}}{\phi\phi^\dagger} + 2\frac{(\phi^\dagger\bar{A}^\dagger\bar{\phi})(\bar{\phi}A\phi^\dagger)}{\phi\phi^\dagger AA^\dagger},$$
$$\phi\bar{A}\bar{\phi}^\dagger = \bar{\phi}^\dagger A^\dagger\phi\frac{(A\bar{A})^{(0)}}{AA^\dagger}, \quad \phi^\dagger\bar{A}^\dagger\bar{\phi} = \bar{\phi}A\phi^\dagger\frac{(A^\dagger\bar{A}^\dagger)^{(0)}}{AA^\dagger}.$$

Notice that the same 4 equations (29) could be recovered by using the previous relations to parametrize the surface $\mathcal{S}$ and to express $\hat{V}_{\text{eff}}$ in terms of $\phi\phi^\dagger$, $AA^\dagger$, $\bar{\phi}A\phi^\dagger$, $\bar{\phi}^\dagger A^\dagger\phi$ and by differentiating $\hat{V}_{\text{eff}}$ with respect to them.

Let us consider now as an example of application of the previous formalism the simple renormalizable case in which the holomorphic supersymmetry breaking terms are proportional to the superpotential. In this case V_{eff} is of the form

$$V_{\text{eff}} = V_2 = m_S^2 S^\dagger S + m_{\bar{S}}^2\bar{S}^\dagger\bar{S} + m_\phi^2\phi\phi^\dagger + m_{\bar{\phi}}^2\bar{\phi}\bar{\phi}^\dagger + m_A^2 A^\dagger A + m_{\bar{A}}^2\bar{A}^\dagger\bar{A} \tag{30}$$

and therefore

$$\hat{V}_{\text{eff}} = m_S^2\frac{(\phi\phi^\dagger)^2}{X_S^\dagger X_S} + m_{\bar{S}}^2\frac{(\bar{\phi}\bar{\phi}^\dagger)^2}{X_S^\dagger X_S} + m_\phi^2\phi\phi^\dagger + m_{\bar{\phi}}^2\bar{\phi}\bar{\phi}^\dagger + m_A^2 A^\dagger A + m_{\bar{A}}^2\bar{A}^\dagger\bar{A}. \tag{31}$$

The equations (29) simplify to

$$2m_S^2\frac{(\phi\phi^\dagger)^2}{X_S^\dagger X_S} + m_\phi^2\phi\phi^\dagger = 2m_{\bar{S}}^2\frac{(\bar{\phi}\bar{\phi}^\dagger)^2}{X_S^\dagger X_S} + m_{\bar{\phi}}^2\bar{\phi}\bar{\phi}^\dagger \tag{32 a}$$

$$m_A^2 A^\dagger A = m_{\bar{A}}^2\bar{A}^\dagger\bar{A} \tag{32 b}$$

$$\bar{\phi}^\dagger A^\dagger\phi = 0 \tag{32 c}$$

$$\bar{\phi}A\phi^\dagger = 0. \tag{32 d}$$

Since $\bar{\phi}A\phi^\dagger = 0$, $\bar{\phi}$ and $\phi^\dagger$ are aligned and $\phi\phi^\dagger = xX_S^\dagger X_S/2$, $\bar{\phi}\bar{\phi}^\dagger = \bar{x}X_S^\dagger X_S/2$ with $x\bar{x} = 1$ and x determined by eq. (32 a) provided that a positive solution exists. Therefore $\phi\phi^\dagger$ and $\bar{\phi}\bar{\phi}^\dagger$ are of the same order of magnitude if $m_S^2 \sim m_{\bar{S}}^2$ and $m_\phi^2 \sim m_{\bar{\phi}}^2$. Analogously for $AA^\dagger$ and $\bar{A}\bar{A}^\dagger$.

The more general renormalizable case in which the A-terms are generic can be solved more easily by using a symbolic manipulation program. However, more than the explicit form of the equations the important outcomes are that: *i)* in a large region of the parameter space, eqs. (29a, b) have a solution, the degeneracy is then removed and for similar values of parameters and "barred" parameters $|\phi\phi^\dagger| \sim |\bar{\phi}\bar{\phi}^\dagger| \sim V^2$, $|AA^\dagger| \sim |\bar{A}\bar{A}^\dagger| \sim v^2$; *ii)* V_{eff} does not depend on $\bar{\phi}A\phi^\dagger$, as it can be easily seen. Point *i)* assures that, in a neighborhood of the $1/M = 0$ case, the general non-renormalizable case can be solved pertubatively from the renormalizable one, thereby resolving the degeneracy. Point *ii)*, together with eqs. (29c, d), assures that $\bar{\phi}A\phi^\dagger = 0$ and therefore that ϕ and $\bar{\phi}$ are aligned in the renormalizable case. Moreover, from perturbative expansion one gets $|\bar{\phi}A\phi^\dagger| \sim v^2V^2/M$ and therefore

$$|\bar{\phi}_1| \sim \frac{vV}{M}$$

in the basis in which $\phi^1 = 0$. This completes the proof of (21).

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