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L.G. Moretto, L. Beaulieu, L. Phair, and G.J. Wozniak

Nuclear Science Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

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Source Sizes in Nuclear Multifragmentation and the Statistical Exploration of Fragmentation Phase Space

L.G. Moretto, L. Beaulieu, L. Phair, and G.J. Wozniak
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720
(January 13, 1999)

The multiplicity distributions for individual fragment Z values in nuclear multifragmentation are binomial. The extracted maximum value of the multiplicity, m_Z, is found to depend on Z according to $m_Z = Z_0/Z$, where Z_0 is the source size. This is shown to be a strong indication of statistical coverage of fragmentation phase space. The inferred source sizes coincide with those extracted from the analysis of fixed multiplicity charge distributions.

The characterization of multifragmenting sources is seen as an essential requisite for the description of nuclear multifragmentation reactions [1,2]. Although the clustering of events in velocity space gives a visual impression of these sources, a more specific evidence, directly related to the mechanism of multifragmentation itself, is needed.

In previous work [3], it was empirically shown that the observed charge distributions resulting from nuclear multifragmentation obey the following invariant form:

$$P_n(Z) \propto \exp\left(-\frac{B(Z)}{\sqrt{E_t}} - cnZ\right) \tag{1}$$

where n is the total intermediate mass fragment (IMF:$3 \leq Z \leq 20$) multiplicity of the event; E_t is the total transverse energy ($E_t = \sum_i E_i \sin^2\theta_i$, where E_i is the kinetic energy of charged particle i in an event, and θ_i is its polar angle); and $B(Z)$ is the "barrier" distribution.

From thermodynamic considerations and percolation simulations, it was shown that c in Eq. (1) vanishes when the gas of IMFs is in equilibrium with a liquid (residue source, or percolating cluster), and assumes a value $\propto 1/Z_0$ (Z_0 being the source size) when the source is wholly vaporized [3–5]. In other words, a vanishing parameter c indicates "coexistence" between liquid and gas (univariance), while a non-zero value indicates the presence of a single gas phase (bivariance).

Experimentally, the parameter c undergoes an evolution with (transverse) energy from approximately zero to a non-zero positive constant [4]. Thus the source size evolves from near infinity (an "infinite" reservoir of fragments) to the actual size of the source. This interpretation, although tentative, gives insight both into the prevailing equilibrium conditions and also into the source size.

A new empirical feature observed in many reactions has led us to a complementary method of independently determining the source size.

It has been shown [5] that the total fragment multiplicity distribution P_n at any given E_t is *empirically* given by a binomial distribution [5]

$$P_n = \frac{m!}{n!(m-n)!} p^n (1-p)^{m-n}. \tag{2}$$

This implies that fragments are emitted nearly independently of each other, so that the probability P_n of observing n fragments can be written by combining a single one-fragment emission probability p according to Eq. (2). The parameter m (the total number of throws) represents the maximum possible number of fragments, which is immediately related to the source size.

The simplest statistical model of multifragmentation has exactly the structure of Eq. (2). Let us assume that the source is made up of m fragments. The "outside" fragments have energy ϵ_2, and those "inside" have energy ϵ_1. A generic partition of n fragments outside and $m-n$ fragments inside has the probability:

$$P_n = \frac{m!}{n!(m-n)!} \frac{e^{-(n\epsilon_2+(m-n)\epsilon_1)/T}}{(e^{-\epsilon_1/T}+e^{-\epsilon_2/T})^m} \tag{3}$$

which leads to Eq. (2) when

$$p = \frac{e^{-\epsilon_2/T}}{e^{-\epsilon_1/T}+e^{-\epsilon_2/T}}. \tag{4}$$

Thus, a simple way to obtain the size of the source is to multiply m by the fragment size.

Unfortunately, when all charges which constitute an IMF are considered simultaneously (IMF: $Z_{th} \leq Z \leq 20$, with Z_{th} usually equal to 3), one should multiply m by a "suitably" averaged Z. In fact, a dependence of m on the lower threshold Z_{th} was already found such that $m(Z_{th}) \times Z_{th} \approx constant$ [5].

The natural next step is to restrict the fragment definition to a single atomic number Z.

A straightforward generalization of Eq. (3) to the production of fragments with charges 1, 2, ...Z_0 is given by the multinomial distribution

$$P = \frac{Z_0!}{n_1!n_2!...n_{Z_0}!} p_1^{n_1} p_2^{n_2} ... p_{Z_0}^{n_{Z_0}} \tag{5}$$

with

$$Z_0 = \sum_Z Z n_Z. \tag{6}$$

Here there is no single quantity m as in Eq. (3), since the constraint is now on the total charge rather than on the total number of fragments. However, this implies immediately a simple scaling that must be obeyed if the fragmentation phase space is to be completely explored.

Let us indicate with $W(Z_0, T, ...)$ the statistical weight associated with a particle number Z_0, temperature T, etc. If we now secure at least n_Z fragments of charge Z, the associated weight or probability of the residue is given by (assuming $T \approx$ constant):

$$P_{n_Z,Z} \propto W(Z_0 - n_Z Z, T, ...). \tag{7}$$

But, since the charge constraint is to be applied "minimally," what counts is only the product Zn_Z, rather than the individual Z values. Consequently, the weight of the residue does not change if we substitute $n_Z Z$ with $n_{Z'} Z'$ provided that

$$n_Z Z = n_{Z'} Z'. \tag{8}$$

In other words,

$$P_{n_Z,Z} \propto P_{n_{Z'},Z'} \tag{9}$$

if $n_Z Z = n_{Z'} Z'$.

This gives immediately the scaling laws

$$n_Z / n_{Z'} = Z'/Z \tag{10}$$

or for the extreme value of $n_Z = m_Z$,

$$m_Z = Z_0 / Z \tag{11}$$

This feature follows directly from the uniform exploration of the fragmentation phase space, and it acquires a significance on par with the Arrhenius plot which demonstrates the canonical population of the "energy" phase space.

When the restriction to *individual* Z values is made experimentally [6], the multiplicity distributions become nearly Poissonian, namely $mp << m$. This introduces interesting simplifications in the analysis and interpretation of the data, but at the cost of a loss of scale. In the Poisson limit the average multiplicity $\langle n \rangle = mp$ is the only accessible parameter, and the decomposition into m and p becomes impossible. The recovery of scale for an individual Z is highly desirable in view of the possibility that the number of throws m_Z (for a single species Z) might obey the simple scaling of Eq. (11).

Thus, we have attempted binomial fits of the multiplicity distributions for *individual* Z values in an effort to extract m_Z. Fortunately, a number of reactions (^{36}Ar+^{197}Au at 35 to 110 AMeV [7], ^{129}Xe+^{27}Al,^{51}V,^{nat}Cu, ^{89}Y, at 50 AMeV [8], and ^{129}Xe+^{197}Au at 50 to 60 AMeV [9]) has been studied with good Z resolution and high statistics. We first consider the asymmetric, intermediate-energy reactions

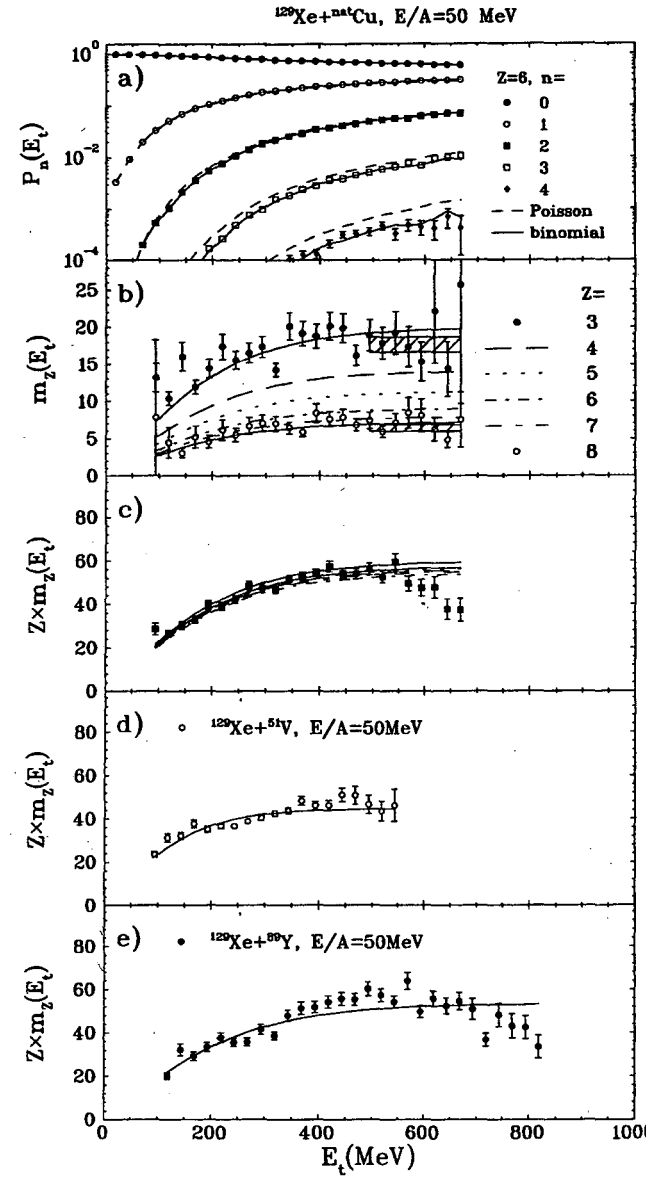

FIG. 1. Panel a): the n-fold probability distribution is plotted as a function of transverse energy for carbon fragments emitted from the reaction ^{129}Xe+^{nat}Cu at 50 AMeV. The dashed and solid curves are Poisson and binomial fits, respectively, to the excitation functions. Panel b): the extracted binomial parameter m_Z (the number of "throws") is plotted as a function of transverse energy for lithium (solid circles) and oxygen (open circles) emission. The solid lines are hyperbolic tangent fits to the indicated data (see Eq. (12)). The other lines are fits to data not shown. The two hatched regions represent weighted averages of the top 5 % most central collisions of the m_Z values for lithium and oxygen. Panel c): The fits from panel b) are scaled by the atomic number Z of the emitted particle. The square symbols represent an "average" source size calculated with Eq. (13). Panel d): The symbols represent an "average" source size calculated with Eq. (13) for ^{129}Xe+^{51}V. The line is a hyperbolic tangent fit to the data (see Eq. (12). Panel e): the same as d) but for ^{129}Xe+^{89}Y.

in reverse kinematics exemplified by ^{129}Xe+^{nat}Cu at 50 AMeV, for which we can expect a single dominant fragment source.

Examples of both binomial and Poisson fits to the carbon yield from this reaction are shown in panel a) of Fig. 1. An improvement of the fit by using the binomial expression is observed for large fold numbers. A similar improvement is observed for each Z in all reactions listed in this letter.

The E_t dependence of the parameters m_Z from the binomial fits to the multiplicity distributions associated with each fragment atomic number leads to several observations.

For each Z value, m_Z increases to a near constant value with increasing E_t. We approximate this behavior with the form

$$m_Z = m_Z^0 \tanh f E_t. \tag{12}$$

The parameter m_Z^0 represents the saturation value of m_Z for large E_t and f controls the rise of m_Z with increasing E_t. The solid lines in panel b) of Fig. 1 are fits to m_Z values extracted for lithium and oxygen emission from the reaction ^{129}Xe+^{nat}Cu at 50 AMeV. The other discontinuous lines are fits to data not shown (Z=4-7).

Furthermore, at all E_t values there is an overall decrease of m_Z with increasing fragment Z value in agreement with the expected scaling $Zm_Z = Z_0$. The remarkable precision of this dependence is exemplified in panel c) of Fig. 1 and in Fig. 2. By applying the expected scaling (Zm_Z), all of the fits to the ^{129}Xe+^{nat}Cu data collapse together, resulting in the approximate source "size" as a function of E_t. A weighted average ($\langle Z_0 \rangle$) of the data over different exit channels, constructed according to

$$\langle Z_0 \rangle (E_t) = \sum_Z Zm_Z(E_t) a_Z, \tag{13}$$

is shown by the symbols in panels c), d), and e) of Fig. 1. The weight a_Z is the standard weight (proportional to the inverse square of the individual errors). A similar behavior is observed in the two additional asymmetric reactions ^{129}Xe+^{51}V and ^{129}Xe+^{89}Y (see Fig. 1d,e).

The E_t dependence of the source size is tantalizing. The source size increases quickly to a saturation value. The fact that E_t is related to impact parameter as well as to the total excitation energy may explain the observed features. In the highly asymmetric reverse kinematic reactions one quickly achieves a sufficient overlap to produce a dominant Xe-like source as one moves from peripheral to central collisions.

As a special case of the $1/Z$ scaling, the "saturation" m_Z values from central collisions of ^{129}Xe+^{nat}Cu (the top 5% of the E_t scale), shown by the hatched regions in panel c) of Fig. 1 for lithium and oxygen, are shown in Fig. 2. The solid lines show a fit with the expected $1/Z$ dependence. The same data in the form Zm_Z vs. Z are

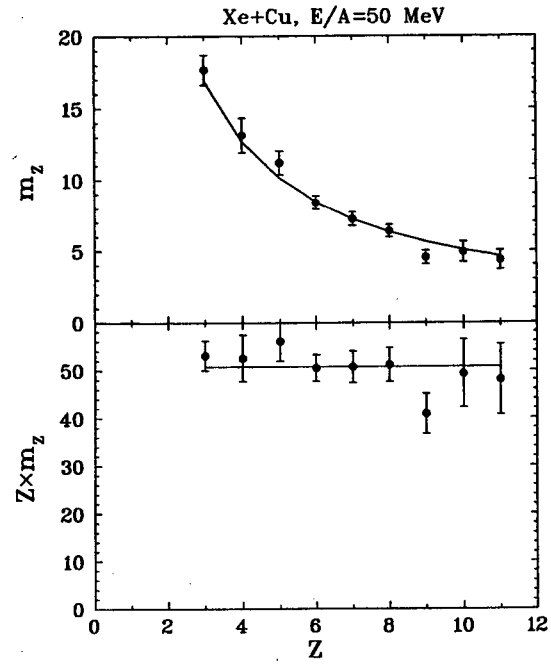

FIG. 2. Top panel: The m_Z values extracted for the 5% most central collisions are plotted as a function of Z for the reaction ^{129}Xe+^{nat}Cu at 50 AMeV. The solid line is a fit of the form given by Eq. (11). Bottom panel: the scaled values $Z \times m_Z$ are plotted as a function of Z.

shown in the bottom panel of Fig. 2, where the accuracy of the $1/Z$ dependence is manifested by the constancy of Zm_Z vs. Z. This striking $1/Z$ dependence of the parameter m_Z is observed for all asymmetric systems measured (top panel of Fig. 3).

These overall results for asymmetric reactions suggest the dominance of a single source, *strongly support the hypothesis of uniform (statistical) exploration of the fragmentation phase space*, and lead to the interpretation of $Z_0 = Zm_Z$ as the source "size."

In the less asymmetric reactions ^{129}Xe+^{197}Au at 50 and 60 AMeV for which at least two sources are plausible, we shall refer directly to $Z_0 = Zm_Z$ as the source size, although we shall see that now it depends on Z value as well as on E_t. The weak decrease of the source size with increasing Z, already observable in ^{129}Xe+^{89}Y (Fig. 3), becomes more visible in the case of the ^{197}Au target. At low Z values the source size Z_0 is ≈ 70 and it decreases monotonically with increasing Z to Z_0 approximately 40-50. The Z dependence seems to suggest that for the ^{89}Y target and, most of all, for the ^{197}Au target there may be a distribution of sizes, the higher Z fragment being emitted preferentially by the smaller source(s).

The reactions ^{36}Ar+^{197}Au at 35, 50, 80, 110 AMeV give a picture intermediate between the ^{129}Xe+^{197}Au and the ^{129}Xe induced reverse kinematics reactions. They also give information of the source size dependence on bombarding energy. The source size at low fragment Z increases from $Z_0 \approx 30$ to $Z_0 \approx 60$ as the bombarding

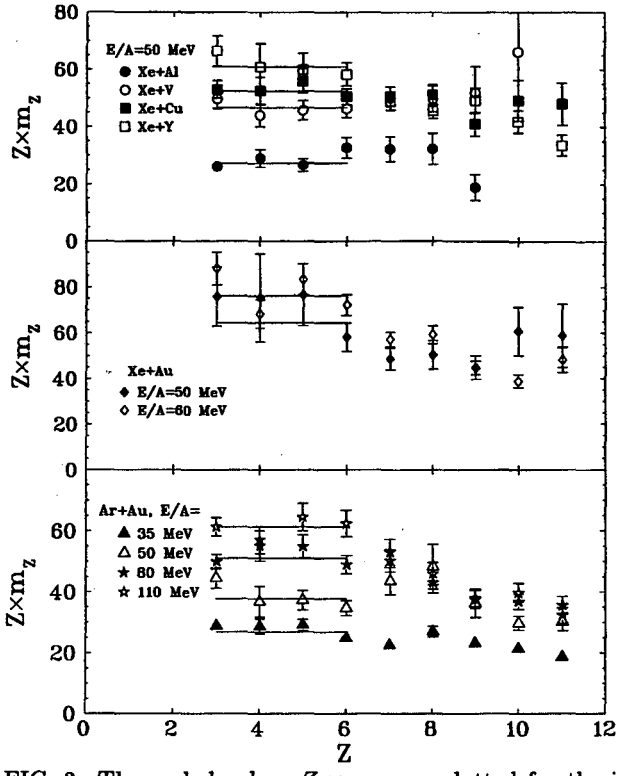

FIG. 3. The scaled values $Z \times m_Z$ are plotted for the indicated reactions of the 5% most central collisions. The solid lines represent weighed averages from $Z = 3$ to 6.

energy increases from 35 to 110 AMeV, while at higher fragment Z the source size increases from $Z_0 \approx 20$ to $Z_0 \approx 40$.

Let us now return to the source size determined from the parameter c as in refs. [3,4]. The parameter c is observed to increase from ≈ 0 and to saturate with increasing E_t. If the parameter c is interpreted as $1/Z_0$, the source size calculated from c is large (near infinity) at low E_t, reflecting the fact that the source residue acts as an "infinite" reservoir of charge. At high E_t, however, when the source residue disappears, the extracted value of Z_0 should stabilize around the actual size of the source. With the exception of the reactions ^{129}Xe+^{27}Al,^{51}V, Cu, ^{89}Y, this limit is attained. Thus, it is possible to plot the value of Z_0 determined from m_Z against that obtained from the c parameter (both for the top 5% most central collisions in E_t). Such a plot is shown in Fig. 4. The result is striking. Not only are the two quantities well correlated, but they also agree in absolute value within the precision of the measurements! The strong corroboration that the two approaches offer to each other gives confidence that we have gained direct access to the source size. This source (sources) is specifically the entity that generates the fragments through "chemical equilibrium". It does not contain the pre-equilibrium part which is often incorporated in other source reconstruction methods.

In conclusion, we have shown that:

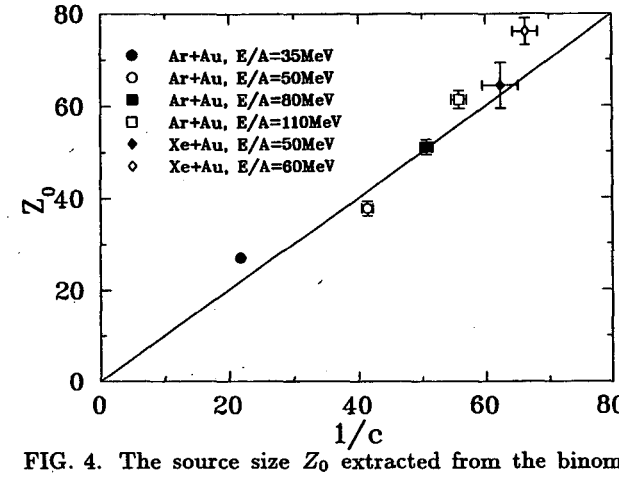

FIG. 4. The source size Z_0 extracted from the binomial analysis as a function of the source size $(1/c)$ determined as per ref. [3] from central collision of the indicated reactions.

a) The binomial (nearly Poissonian) multiplicity distributions for individual fragment atomic numbers permit the extraction of the parameter m_Z, the number of throws.

b) m_Z for the reactions where a single source is clearly dominant has the form $m_Z = Z_0/Z$.

c) The $1/Z$ dependence is a dramatic proof that the fragmentation phase space is statistically explored.

d) Source size or sizes can be extracted. They should reflect the regions where chemical (as opposed to physical) equilibrium is achieved.

e) These source sizes agree with the sizes obtained from the analysis of multiplicity selected charge distributions in the E_t range where a single gas phase, or thermodynamic bivariance, prevails.

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Ernest Orlando Lawrence Berkeley National Laboratory
One Cyclotron Road | Berkeley, California 94720

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