LBNL-42812
Preprint

ERNEST ORLANDO LAWRENCE
BERKELEY NATIONAL LABORATORY

Extra Dimensions and the Muon Anomalous Magnetic Moment

M.L. Graesser

Physics Division

September 1999

Submitted to
Physical Review D

REFERENCE COPY
Does Not
Circulate
Bldg. 50 Library - Ref.
Lawrence Berkeley National Laboratory

LBNL-42812
Copy 1

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LBNL-42812

Extra Dimensions and the Muon Anomalous Magnetic Moment

M.L. Graesser

Department of Physics
University of California, Berkeley

and

Physics Division
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

September 1999

This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098. The author was also supported by the National Energy Research Scientific Computing Division.

September 1, 1999

LBNL-42812
UCB-PTH-99/03

Extra Dimensions and the Muon Anomalous Magnetic Moment [1]

M. L. Graesser [2]
Theoretical Physics Group, Lawrence Berkeley National Laboratory
and
Department of Physics, University of California
Berkeley, California 94720

Abstract

It has been proposed recently that the scale of strong gravity can be very close to the weak scale. Dimensions of sizes anywhere from ~mm to ~TeV^{-1} can be populated by bulk gravitons, vector bosons and fermions. In this paper the one-loop correction of these bulk particles to the muon magnetic moment (MMM) are investigated. In all the scenarios considered here it is found that the natural value for the MMM is $O(10^{-8}-10^{-9})$. One main result is that the contribution of each Kaluza–Klein graviton to the MMM is remarkably finite. The bulk graviton loop implies a limit of ~ 400 GeV on the scale of strong gravity. This could be pushed up to $\sim 1-2$ TeV, even in the case of six extra dimensions, if the BNL E821 experiment reaches an expected sensitivity of $\sim 10^{-9}$. Limits on a bulk $B-L$ gauge boson are interesting, but still allow for forces 10^6-10^7 times stronger than gravity at mm^{-1} distances. The correction of a bulk right-handed neutrino to the MMM in one recent proposal for generating small Dirac neutrino masses is considered in the context of a two Higgs doublet model, and is found to be close to 10^{-9}. The contributions of all these bulk particles to the MMM are (roughly) independent of both the total number of extra dimensions and the dimension of the subspace occupied by the bulk states. Finally, limits on the size of "small" compact dimensions gotten from the MMM and atomic parity violation are determined and compared.

[1]This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098. The author was also supported by NSERC.
[2]email: graesser@thwk8.lbl.gov

1 Introduction.

In the past year there has been a remarkable proposal by Arkani-Hamed, Dimopoulos and Dvali (ADD) for solving the hierarchy problem [1, 2, 3]. In their vision the scale of strong quantum gravity is lowered to the weak scale. A weak-scale valued Planck scale M_* and Newton's constant are reconciled by allowing gravity to propagate in n extra *large* compact dimensions. An application of Gauss' Law quantifies the relationship between $\bar{M}_{PL}^2 = (8\pi G_N)^{-1}$, M_*, and the volume of the compact space $V_n = R^n$, to be

$$2\bar{M}_{PL}^2 = M_*^{n+2} R^n. \tag{1}$$

In the case of 2 extra dimensions and the phenomenologically preferred value $M_* \sim$TeV, $R \sim$mm and future short-distance experiments may observe a transition in Newton's Law from a r^{-2} to r^{-4} force law. In the case of 6 extra dimensions and $M_* \sim$TeV, $R^{-1} \sim$MeV, which is large compared to $M_* \sim$TeV. The non-observation of Kaluza-Klein (KK) towers for the Standard Model (SM) particles, for example, requires that the SM particles not propagate in the large extra dimensions. This requires that they be fixed to a 3–dimensional wall, with a thickness smaller than $\sim O((700 \text{ GeV})^{-1})$. This may be accomplished using purely field-theoretic techniques [1], or by using the D-branes of string theory [2]. This remarkable proposal allows for many new interesting phenomena to be discovered at future experiments. These include dramatic model-independent signals at high-energy colliders from bulk graviton production [4, 5], signatures of the spin-2 structure of the KK gravitons [6], distortions in the Drell-Yan energy distribution [6], and enhancements in the f^+f^-, WW, ZZ, $\gamma\gamma$ production cross-sections at linear colliders [6, 7], as well as other interesting collider signatures [8] and implications for gravitational processes at high energies [9]. Future colliders may also discover Kaluza-Klein and string excitations of the Standard Model particles [10, 11, 2]. The existence of either *large* or TeV^{-1} sized dimensions also opens new territory for model-building. In an important early paper [10] by Antoniadis, the author advocates the existence of TeV^{-1} sized dimensions to break supersymmetry at low-energies. Other prospects include new ideas for obtaining gauge coupling unification [12, 13], suppressing proton decay, and flavor physics[14, 15]. Existing phenomena already strongly constrain M_*. These include astrophysical processes (such as SN1987A in the case $n = 2$) [3], bulk graviton production [4, 5], and WW, ZZ production at LEP2 [7].

In this paper the bulk contributions at the one-loop level to the anomalous magnetic moment of the muon (MMM) are computed. These include contributions from bulk gravitons, bulk $B-L$ gauge bosons, and, in a two Higgs doublet model, those from bulk right-handed (RH) neutrinos. The correction to the MMM is found to be $\sim m_\mu^2/M_*^2$, and is therefore in a first approximation *independent of n*, or, in the case of bulk fermions or vector bosons, independent of the *dimension of the subspace* inhabited by the bulk states.

Now it may seem strange to compute a one-loop correction to the MMM when a correction from a TeV-scale suppressed operator may be present at tree-level. The important point is that

the tree-level operator contains an unknown coefficient, whereas the one-loop corrections are present, calculable and essentially model-independent. They thus represent a lower value to the full bulk correction to the MMM.

In this paper it is found that the "expected" one-loop correction to the MMM is typically $O(10^{-8} - 10^{-9})$ for values of M_* from ~ 400 GeV to $\sim O(\text{TeV})$. For comparison, this is the same magnitude as the correction expected in low-energy supersymmetry at large $\tan\beta$ [16]. The bulk correction to the MMM should be compared to its extremely well-measured value of [17]

$$a_\mu^{exp} = \frac{g-2}{2} = (116592.30 \pm 0.8)) \times 10^{-8}. \tag{2}$$

The BNL experiment (E821) hopes to lower the error to 40×10^{-11} [18], a factor of ~ 20 below the existing sensitivity. The prediction in the SM is [19, 20]

$$a_\mu^{SM} = 116591.739(154) \times 10^{-8}, \tag{3}$$

and is in very good agreement with the current measurement. The theoretical error is dominated by the hadronic contributions. The error in the lowest order hadronic contribution is ~ 60 ppm of a_μ, but there is hope that this error can be reduced to ~ 0.5 ppm [18, 21]. Another large source of theoretical error is the hadronic "light-on-light" scattering correction to a_μ. This error is estimated to be ~ 0.15 ppm [20]. These translate into an expected theoretical error of $\sim 5 \times 10^{-10}$.

A main result of this paper is the computation of the one-loop correction from bulk gravitons and radions to the MMM. The corrections from a single KK graviton and KK radion are found to each be miraculously finite. The full one-loop bulk contribution is gotten by summing over all the KK states, and results in a correction to the MMM which is essentially model-independent and independent of n, depending only on the scale of strong gravity. Requiring that the correction is smaller than the present experimental error implies, in the case $n = 6$, for instance, that the lower limit on M_* is competitive with limits gotten from other physical processes. It is also found that as a general rule the a_μ constraint provides a comparable limit to M_* for *larger* numbers of extra dimensions. This is in contrast to astrophysical or terrestrial constraints gotten from the direct production of gravitons, where the effect of a physical process with characteristic energy scale E typically decouples as $\sim (E/M_*)^n$. As such, the a_μ constraint provides complementary information.

Bulk RH neutrinos were proposed in Ref. [15] to generate small Dirac neutrino masses, and can naturally account for the required mass splitting needed to explain the atmospheric neutrino anomaly. It is found that the correction to the MMM from the tower of RH neutrinos in a two Higgs doublet model is close to the future experimental sensitivity.

It is also found that while the constraint on the $B - L$ gauge coupling is quite impressive, a $B - L$ vector boson can still mediate isotope-dependent forces $10^6 - 10^7$ times stronger than gravity at sub-mm distance scales.

Finally, KK excitations of the SM gauge bosons are also probed by the MMM and atomic parity violation measurements. It is found that the stronger limits on the size of extra "small" dimensions are gotten from the atomic parity violation experiments. The limit is ~ 1 TeV (~ 3.6 TeV) for KK excitations into 1 (2) extra dimensions.

The outline of the paper is as follows. Section 1 discusses some machinery necessary to perform the one-loop graviton correction to the MMM. Section 2 discusses the results and limits gotten from the one-loop graviton computation. Section 3 discusses the bulk $B-L$ and KK photon contributions to the MMM, and concludes with a discussion of the KK Z_0 correction to atomic parity violation measurements. Section 4 discusses the correction to the MMM in a recent proposal for generating small Dirac masses within the context of a two Higgs-doublet model.

2 Preliminaries and Some Machinery.

The effective field theory approach described by Sundrum in Ref. [22] allows one to determine the couplings of the higher dimensional gravitons to some matter stuck on a wall. The interactions are constrained by local 4-dimensional Poincare invariance (which interestingly enough, arise from the coordinate reparameterisation invariance of the wall embedding) and the full $(4+n)-$dimensional general coordinate invariance, as well as any local gauge symmetries on the wall. At lowest order in $1/M_*$, all the higher dimensional gravitons interact only through the induced metric on the wall. Since the interaction of the induced metric with on-the-wall matter is fixed by 4−D general coordinate invariance, the lowest order couplings of the KK gravitons to the on-the-wall matter are fixed by the ordinary graviton coupling to matter, and are obviously universal. This is important, since the interactions of matter to the KK gravitons are therefore determined by replacing $h_{\mu\nu} \to \sum_n h^{(n)}_{\mu\nu}$ in the ordinary gravity-matter interactions. With $g = \eta + \kappa h$, where $\kappa = \sqrt{32\pi G_N}$, $h_{\mu\nu}(x) = \sum_n h^{(n)}_{\mu\nu}(x)$, the gravitons couple to the conserved symmetric stress-energy tensor with normalisation

$$-\kappa h^{\mu\nu} T_{\mu\nu}/2. \tag{4}$$

There is one subtlety though. The 4−D decomposition of the $4+n$ dimensional graviton results in a KK tower of spin-2 gravitons, $n-1$ vector bosons, and $n(n-1)/2$ spin-0 bosons. The vector bosons correspond to fluctuations in $g_{i\mu}$. In the linearised theory they couple only to $T^{i\mu}$, which vanishes if the wall has no momentum in the extra dimensions. The zero mode of the vector bosons acquires a mass $\sim M_*^2/M_{PL}$ from the spontaneous breaking of translation invariance in the extra dimensions [23]. The zero mode of one of the spin-0 bosons corresponds to fluctuations in the size of the compact dimensions (the "radion") and couples to T^μ_μ. It acquires a mass $\sim 10^{-3}$eV to MeV from the dynamics responsible for stabilising the size of the extra dimensions [23]. The remaining spin-0 fields do not couple to T^μ_ν [4, 24].

The end result is that only the tower of KK gravitons and radions couple to the matter on the wall. Since T^μ_μ is proportional to the fermion masses, it is important to include the radion and its tower of KK states in computing the correction to $g-2$. A careful computation [4, 24] determines the normalisation of the coupling of the radion to T^μ_μ. The correct prescription is then to replace

$$h_{\mu\nu}(x) \to \sum_n \left(h^{(n)}_{\mu\nu}(x) - \sqrt{\omega}\eta_{\mu\nu}\phi^{(n)}(x)\right). \tag{5}$$

For a canonical normalisation of ϕ, $\omega = (n-1)/3(n+2)$. The couplings of the KK tower of gravitons and radions to matter is then gotten from the ordinary graviton coupling to matter by using the substitution in Eqn. 5.

Next I present some of the machinery which is needed to perform the one-loop bulk graviton correction to the MMM. The reader is referred to References [25] for a far more careful treatment of this subject, and for early quantum gravity computations. The recent References [4, 24] present a clear discussion of the proper procedure for determining the bulk graviton couplings to matter.

The Lagrangian coupling gravity to on-the-wall photons and fermions is

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left(-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2\xi}(g^{\mu\nu}D_\mu A_\nu)^2 + \bar{\psi}i\gamma^a e^\mu_a D_\mu\psi - m\bar{\psi}\psi\right), \tag{6}$$

with $D_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\rho_{\mu\nu}A_\rho$, Γ is the Christoffel symbol, and ξ is the $U(1)$ gauge fixing parameter. The e^μ_a fields are the vielbeins satisfying $g^{\mu\nu} = e^\mu_a\eta^{ab}e^\nu_b$ and $\eta^{ab} = e^{\mu a}g_{\mu\nu}e^{\nu b}$. The fermionic covariant derivative is $D_\mu = \partial_\mu - ieA_\mu - \frac{i}{2}\omega^{ab}_\mu\sigma_{ab}$, with $\sigma_{ab} = \frac{i}{4}[\gamma_a, \gamma_b]$. Finally, the spin connection is

$$\omega^{ab}_\mu = \frac{1}{2}\left(e^{\nu a}(\partial_\mu e^b_\nu - (\mu \leftrightarrow \nu)) - (a \leftrightarrow b)\right) - \frac{1}{2}e^{\nu a}e^{\rho b}e_{\mu d}\left(\partial_\nu e^d_\rho - (\nu \leftrightarrow \rho)\right). \tag{7}$$

Inserting $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, $e_{\mu a} = \eta_{\mu a} + \kappa h_{\mu a}/2$, and expanding Eqn. 6 to $O(h)$ gives

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}(h=0) + \frac{1}{2}\kappa h^{\mu\nu}\left(\eta^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} - \frac{1}{4}\eta_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}\right) \\
&+ \frac{1}{2\xi}\kappa(\partial\cdot A)\left(-\frac{1}{2}h(\partial\cdot A) + h^{\mu\nu}(\partial_\mu A_\nu + \mu \leftrightarrow \nu) + A^\alpha\eta^{\mu\nu}\left(h_{\alpha\mu,\nu} + h_{\alpha\nu,\mu} - h_{\mu\nu,\alpha}\right)\right) \\
&+ \frac{\kappa}{2}\left((h\eta^{\mu\nu} - h^{\mu\nu})\bar{\psi}i\gamma_\mu D_\nu\psi - mh\bar{\psi}\psi + \frac{1}{2}(\partial_\mu h - \partial^\nu h_{\mu\nu})\bar{\psi}i\gamma^\mu\psi)\right).
\end{aligned} \tag{8}$$

The minimal coupling of gravity to the matter stress-energy tensor, Eqn. 4, is gotten by integrating Eqn. 8 by parts to remove the derivatives on h.

The propagator for a massive spin-2 particle, with mass m_N, is

$$D(k^2, m_N^2)_{\mu\nu,\rho\sigma} = \frac{1}{2}\frac{iP_{\mu\nu,\rho\sigma}}{k^2 - m_N^2}. \tag{9}$$

The tensor structure in the numerator is

$$\begin{aligned}
P_{\mu\nu,\rho\sigma} &= \left(\eta_{\mu\rho} - \frac{k_\mu k_\rho}{m_N^2}\right)\left(\eta_{\nu\sigma} - \frac{k_\nu k_\sigma}{m_N^2}\right) + \left(\eta_{\mu\sigma} - \frac{k_\mu k_\sigma}{m_N^2}\right)\left(\eta_{\nu\rho} - \frac{k_\nu k_\rho}{m_N^2}\right) \\
&- \frac{2}{D-1}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_N^2}\right)\left(\eta_{\rho\sigma} - \frac{k_\rho k_\sigma}{m_N^2}\right)
\end{aligned} \tag{10}$$

and is uniquely fixed by several requirements [25]. The first is that the graviton is the excitation of the metric, so P is symmetric in $\mu \leftrightarrow \nu$ and $(\mu\nu) \leftrightarrow (\rho\sigma)$. Next, unitarity implies that on-shell P is positive-definite and that the spin-1 and spin-0 components are projected out. Finally, the unitarity requirement $P(k^2 = m_N^2, m_N^2)_{\mu\nu,\rho\sigma} = \sum_S \epsilon^S_{\mu\nu}\epsilon^S_{\rho\sigma}$, where $\epsilon^S_{\mu\nu}$ is one of 5 polarisation tensors, fixes the normalisation of P.

It will be useful to compare the ordinary massless graviton one-loop correction to a_μ [26] to the massive spin-2 one-loop correction in the limit $m_N \to 0$. The massless spin-2 propagator is required for the former computation. In the harmonic gauge it is

$$D(k^2)_{\mu\nu,\rho\sigma} = \frac{1}{2}\frac{iP_{\mu\nu,\rho\sigma}}{k^2}, \quad P(k^2)_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}. \tag{11}$$

It will be important to note that the coefficient of $\eta_{\mu\nu}\eta_{\rho\sigma}$ in the numerator of the graviton propagator differs between the massless and massive cases.

Each Kaluza-Klein (KK) mode n of a bulk graviton contributes to a_μ. It is conveinent to express the contribution of a Feynman diagram i, with internal KK mode n, to a_μ as

$$\Delta a_\mu^{(n,i)} = \Delta^{(n)}(g-2)_i/2 = G_N m_F^2 \mathcal{A}_i^{(n)}/2\pi. \tag{12}$$

The fermion mass is m_F. The total correction to a_μ is then gotten by summing over all the KK modes and Feynman diagrams. The final result is

$$\frac{G_N}{2\pi} m_F^2 \sum_n \mathcal{A}_F^{(n)}(m_F^2, m_N^2), \quad \mathcal{A}_F^{(n)} = \mathcal{A}_1^{(n)} + \cdots + \mathcal{A}_K^{(n)}, \text{for } K \text{ diagrams.} \tag{13}$$

This formula can be trivially modified to include bulk fermion or vector boson corrections to the MMM. Since the coupling of matter to the KK graviton, vector boson or fermion modes is universal [3], the only dependence of $a_i^{(n)}$ on the KK quantum number n enters through the mass of the KK mode, $m_N = n/R$. Replacing the sum in Eqn. 13 with an integral gives the final result

$$\Delta a_\mu = \frac{G_N R^n}{2\pi} m_F^2 \frac{\pi^{n/2}}{\Gamma[n/2]} \int dm^2 (m^2)^{(n/2-1)} \mathcal{A}_F(m_F^2, m^2) \ . \tag{14}$$

This is a good approximation since the mass splitting is tiny: $\sim 10^{-3}$ eV for $n = 2$, ~ 5 MeV for $n = 6$.

In evaluating the one-loop contributions to the MMM, the dominant correction will occur from those states in the loop with masses close to the cutoff. This is a direct consequence of the large multiplicity of states at large KK masses. The total correction to the MMM is then well-approximated by substituting into Eqn. 14 the large m_N limit of $\mathcal{A}_F$, and cutting off the (power-divergent) integral over m_N^2 at KK masses $m_N = M_*$. For the cases considered in this paper, in the large m_N limit $\mathcal{A}_F \to c/(m_N^2)^W$ with $W = 0$ or 1. Then using the relation $(4\pi G_N)^{-1} = M_*^{n+2} R^n$, a good approximation to Eqn. 14 is, for $W = 0$ say,

$$\Delta a_\mu = \frac{\lambda}{8\pi^2} c \frac{2\pi^{n/2}}{n\Gamma[n/2]} \frac{m_F^2}{M_*^2}. \tag{15}$$

[3] I assume for simplicity that the SM fermions do not have KK excitations.

The unknown coefficient λ is a gross parameterisation of our ignorance about what is regulating the integral, and can only be computed in a finite theory of quantum gravity, e.g. string theory. The expectation is that $\lambda \sim O(1)$ since, for example, in string theory the amplitude $\mathcal{A}_F$ becomes exponentially suppressed at KK masses larger than the string scale. So whereas λ is an unknown $O(1)$ coefficient, the limits on M_* gotten from the bulk graviton loop scale as $\lambda^{1/2}$ and are therefore not so sensitive to its value. Finally, it is consistent to keep the factors of $\pi^{n/2}$, etc. in Eqn. 15, since this factor counts the degeneracy of states, which can only increase in the full theory.

3 Bulk Gravity.

a. Spin-2 Graviton

Using the Lagrangian in Eqn. 8 the Feynman rules for the graviton and fermion, photon interactions can be gotten[4]. Using this, and the massive graviton propagator, Eqn. 9, it is straightforward although tedious to compute the one-loop correction to a_μ. The Feynman diagrams relevant to the calculation are shown in Fig. 1. Those terms in the numerator of the graviton propagator containing two or more k_αs do not contribute to a_μ at this order in G_N because of the gravitational Ward identity. I have also verified this by an explicit computation. With this in mind, naive power counting implies that the contribution of each Feynman diagram and for a given KK mode n to a_μ is logarithmically divergent in the ultraviolet. The explicit calculation presented below reveals though that these divergences cancel in the sum, so that the total contribution of each KK mode to a_μ is remarkably finite! A similar miraculous cancellation was observed in the ordinary (massless) graviton one-loop correction to a_μ [26].

In the notation of Eqn. 12 ($\mathcal{A}_i^{(n)} = a_i^{(n)}$), the partial amplitudes $a_i^{(n)}$ are ($D = 4 - \epsilon$)

$$a_1^{(n)} = a_2^{(n)} = -\frac{11}{3}\frac{1}{\epsilon} + \frac{1}{3} + \frac{11}{6}\ln m_F^2 - \frac{1}{2}\int dx \frac{R(x, m_F^2, m_N^2)}{L(x, m_F^2, m_N^2)} A(x), \tag{16}$$

$$a_3^{(n)} = a_4^{(n)} = \frac{4}{\epsilon} + 4 - 2\ln m_F^2 - \int dx \left(R(x, m_F^2, m_N^2) B(x) + 8\frac{m_F^2}{L(x, m_F^2, m_N^2)} C(x) \right), \tag{17}$$

$$a_5^{(n)} = -\frac{2}{3}\frac{1}{\epsilon} + \frac{5}{8} + \frac{1}{3}\ln m_F^2 - \int dx \left(R(x, m_F^2, m_N^2) D(x) + \frac{m_F^2}{L(x, m_F^2, m_N^2)} E(x) \right) \tag{18}$$

where $L(x, m_F^2, m_N^2) = x^2 m_F^2 + (1-x) m_N^2$ contains the only dependence of $a_i^{(n)}$ on the KK quantum number. The other functions are $R(x, m_F^2, m_N^2) = (2x m_F^2 - m_N^2)/L(x, m_F^2, m_N^2)$, $A(x) = 4x - x^3/3$, $B(x) = 14x^2/3 - 20x/3$, $C(x) = x^2 - x^3/2$, $D(x) = 3x^2/2 - 5x^3/3 + x^4/2$, and $E(x) = 16x^2/3 - 20x^3/3 + 3x^4 - x^5/2$. As promised above, the sum over all the 5 diagrams is finite: the coefficient of $1/\epsilon$ is $-22/3 + 8 - 2/3 = 0$! The final result is

$$a_F^{(n)} = a_1^{(n)} + \cdots + a_5^{(n)} = \frac{223}{24} - \int dx \frac{2x m_F^2 - m_N^2}{L(x, m_F^2, m_N^2)} H(x) - \int dx \frac{m_F^2}{L(x, m_F^2, m_N^2)} P(x), \tag{19}$$

[4]See references [4, 24] for an explicit presentation of the Feynman rules.

where $H(x) = x(1-x)(-28/3 + 3x/2 - x^2/2)$ and $P(x) = -x^5/2 + 3x^4 - 44x^3/3 + 64x^2/3$. An interesting limit is $m_N^2 \gg m_F^2$. In this case the last term in Eqn. 19 can be neglected, and $a_F^{(n)} = 223/24 - 14/3 + 1/2 - 1/8 = 5$. Note that the heavy KK states exhibit *non − decoupling*. A short comment about this will be made later.

The one-loop correction from the ordinary massless graviton was computed years ago in Ref. [26]. A comparison of the massive spin-2 correction, Eqns. 16, 17, 18, in the limit $m_N^2 \to 0$ to their results provides a non-trivial check on the calculation. Naively the contribution of each KK bulk graviton to a_μ in this limit should equal the ordinary massless graviton. This is because the coupling of matter to all the graviton KK modes is universal, including the zero mode. This expectation is incorrect though, since the tensor structure of the massive and massless graviton propagators are different. Essentially there are more helicity states flowing around the massive graviton loop than there are in the massless graviton loop. Specifically, the coefficient of the $\eta_{\mu\nu}\eta_{\rho\sigma}$ term in the numerator of the graviton propagator is $-1/(D-1)$ and $-1/(D-2)$, respectively, and they are not the same. Also note that the $\eta_{\mu\nu}\eta_{\rho\sigma}$ contribution is proportional to the radion correction, discussed below. The massless graviton correction to a_μ is then gotten by adding and subtracting this "$\eta - \eta$" contribution evaluated at $m_N^2 = 0$, weighted by either $-1/(D-1)$ or $-1/(D-2)$. That is, the ordinary graviton correction to $(g-2)/2$ should be

$$\begin{aligned} b_i(\text{massless graviton}) &= a_i^{(n)}(m_N^2 \to 0) + \frac{1}{D-1}\tilde{a}_i^{(n)} - \frac{1}{D-2}\tilde{a}_i^{(n)} \\ &= a_i^{(n)}(m_N^2 = 0) - \frac{1}{D-1}\frac{1}{D-2}\tilde{a}_i^{(n)}, \end{aligned} \tag{20}$$

where $\tilde{a}_i^n$ is the contribution to $(g-2)/2$, evaluated at $m_N^2 = 0$, of the $\eta_{\mu\nu}\eta_{\rho\sigma}$ term in the massive graviton propagator. That is, $\tilde{a}_i^{(n)} = \omega^{-1} r_i^{(n)}(m_N^2 = 0)$ where $r_i^{(n)}$ is the contribution of a KK radion, discussed below.

In this limit Eqns. 16, 17 and 18 reduce to $a_1^{(n)} = a_2^{(n)} = -11/3\epsilon - 32/9 + 11/6 \ln m_F^2$, $a_3^{(n)} = a_4^{(n)} = 4/\epsilon + 20/3 - 2\ln m_F^2$, and $a_5^{(n)} = -2/3\epsilon - 26/9 + 1/3 \ln m_F^2$. Now an explicit computation gives $\tilde{a}_1^{(n)} = \tilde{a}_2^{(n)} = 0$, $\tilde{a}_3^{(n)} = \tilde{a}_4^{(n)} = -2$, $\tilde{a}_5^{(n)} = 3$. The correction from each diagram is evidently finite! Using Eqn. 21 and the $m_N^2 = 0$ limit of Eqns. 16, 17 and 18 presented above, one gets $b_1 = b_2 = -11/3\epsilon - 32/9$, $b_3 = b_4 = 4/\epsilon + 20/3 + 2/6 = 4/\epsilon + 7$ and $b_5 = -2/3\epsilon - 26/9 - 1/2 = -2/3\epsilon - 61/18$. These results are in agreement with the computation given in Ref. [26].

b. Radion

The contribution of the tower of KK radions to a_μ is gotten by computing the Feynman diagrams in Fig. 1, using the Lagrangian in Eqn. 8 and the expansion of the induced metric, Eqn. 5. It is remarkable that, before summing over the KK states, the contribution of each Feynman diagram to a_μ is finite! In the notation of Eqn. 12 ($\mathcal{A}_i^{(n)} = r_i^{(n)}$), the result is

$$\begin{aligned} r_1^{(n)} = r_2^{(n)} &= 0, \\ \omega^{-1} r_3^{(n)} = \omega^{-1} r_4^{(n)} &= -4 - 2\int dx \left(R(x, m_F^2, m_N^2) F(x) + \frac{m_F^2}{L(x, m_F^2, m_N^2)} G(x) \right), \end{aligned}$$

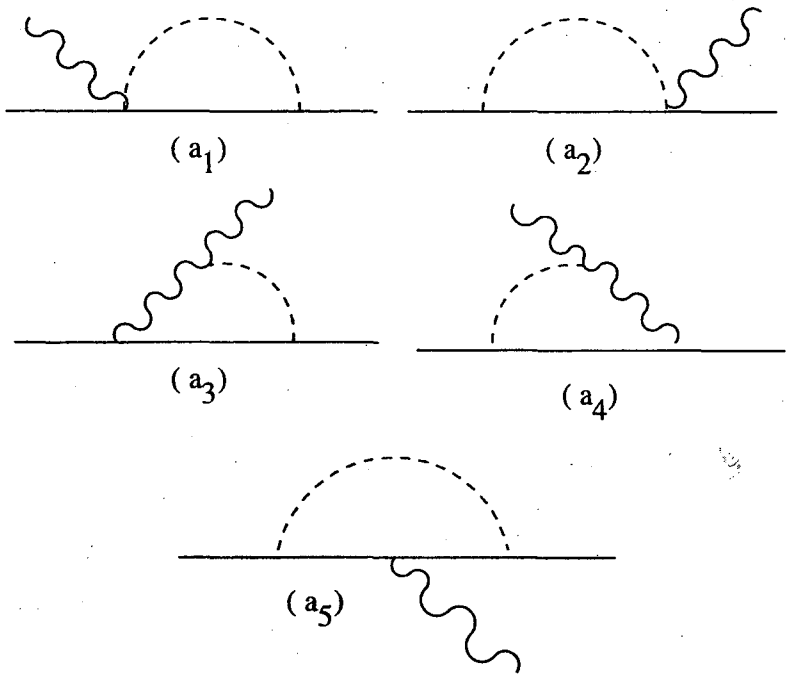

Figure 1: Bulk gravity contribution at the one-loop level to the anomalous magnetic moment. Dashed lines denote either bulk spin-2 graviton or bulk spin-0 radion. Wavy and solid lines denote on-the-wall photons and fermions, respectively.

$$\omega^{-1} r_5^{(n)} = -1 - 3 \int dx \left(R(x, m_F^2, m_N^2) K(x) + \frac{2}{3} \frac{m_F^2}{L(x, m_F^2, m_N^2)} G(x) \right), \quad (21)$$

where $\sqrt{\omega}$ is the coupling of the radion to the trace of the stress-energy momentum tensor, $\omega = (n-1)/3(n+2)$. The functions are: $F(x) = x(1-x)(3x/2+1)$, $G(x) = x^2(3x^2/2 - 2x - 2)$, and $K(x) = x^2(1-x)$. In the limit $m_N^2 \to 0$, $\omega^{-1} r_i^{(n)} \to \tilde{a}_i^{(n)}$. In the limit $m_N^2 \gg m_F^2$, $\omega^{-1} r_3^{(n)} = \omega^{-1} r_4^{(n)} \to -2$, and $\omega^{-1} r_5^{(n)} \to 0$. So in this limit $r_F^{(n)} = -4\omega$.

c. Applications

To constrain M_* it is useful to consider the cases $m_N \ll m_F$ and $m_N \gg m_F$. In the former case $a_1^n + \cdots + a_5^n$ reduces to $10/3$, and $r_1^n + \cdots + r_5^n$ reduces to $-\omega$. In the $m_N \gg m_F$ case $a_1^n + \cdots + a_5^n \to 5$, and $r_1^n + \cdots + r_5^n \to -4\omega$. In view of the final sum over KK states, and the fact that the sum in either limit is $O(1)$, the (by far) dominant contribution to a_μ occurs for those KK modes with large n, since the degeneracy of states is much greater. In this case the integral in Eqn. 14 over the KK number phase space is trivial and power divergent. Applying the formula of Section 2, Eqn. 15, and $\omega = (n-1)/3(n+2)$, the total correction to a_μ is

$$\begin{aligned} \Delta a_\mu &= \frac{\lambda}{8\pi^2} \left(5 - \frac{4}{3} \left(\frac{n-1}{n+2} \right) \right) \frac{2\pi^{n/2}}{n\Gamma[n/2]} \frac{m_F^2}{M_*^2} \\ &= \frac{7\lambda}{12\pi} \left(\frac{m_\mu}{M_*} \right)^2 , \; n = 2 , \\ &= \frac{25\pi\lambda}{288} \left(\frac{m_\mu}{M_*} \right)^2 , \; n = 6. \end{aligned} \quad (22)$$

The first contribution is from the tower of spin-2 KK states, and the second is the contribution from the tower of KK radion states. Note that this result scales as M_*^{-2}, and is *independent of n*. The presence of the factor $2\pi^{n/2}/n\Gamma[n/2]$ from the phase space integration substantially enhances the correction. For instance, for $n \sim 6$ this factor is $\sim \pi^3/6$ and compensates the loop suppression. Requiring that Δa_μ is less than the current 2σ limit of $2 \times 0.8 \times 10^{-8}$ gives the following 2σ limits :

$$\text{(current limit) } n = 2: \quad M_* > 340 \text{ GeV } \lambda^{1/2} \ , \ n = 4, 6: \quad M_* > 410 \text{ GeV } \lambda^{1/2}. \tag{23}$$

Note that since the limit on $M_* \sim \lambda^{1/2}$, it is not so strongly sensitive to the value of λ. The limit for $n = 2$ is considerably weaker than existing accelerator limits. For instance, the good agreement between existing LEP2 data and the predicted SM cross-section for γ+missing energy final states constrains the bulk graviton production rates [5, 4], and implies $M_* > 1200$ GeV for $n = 2$. The a_μ constraint for $n = 6$ is competitive with the limit from bulk graviton production which for $n = 6$ is $M_* > 520$ GeV [4, 5].

The future prospects are more promising though. The BNL E821 experiment plans to decrease the experimental error in a_μ^{exp} by a factor of ~ 20, i.e., down to $\Delta a_\mu^{exp} = 4 \times 10^{-10}$. This should be compared to the KK graviton and radion contribution to the MMM. Inserting the relevant numbers into Eqn. 22 gives for $n = 2$ (upper) and $n = 6$ (lower):

$$\Delta a_\mu = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \times 10^{-9} \times \lambda \times \left(\frac{\text{TeV}}{M_*}\right)^2. \tag{24}$$

So an $O(10^{-9})$ correction to the MMM is not unreasonable.

Finally, the final formulas for the correction to MMM, and the non-decoupling of the KK states, can be understood by computing the one-loop diagram in the full theory. Recall that the correction to the MMM requires a helicity flip. This pulls out a factor of $p_\alpha \gamma^a p_a$, lowering the degree of divergence by two. Then naive power counting gives

$$\Delta a_\mu \sim \frac{m_F^2}{M_*^{n+2}} \int dk k^{n+3} k^{-2} k^{-1} k k^{-2} \sim m_F^2 \frac{\Lambda^n}{M_*^{n+2}} \sim \frac{m_F^2}{M_*^2}, \tag{25}$$

for $\Lambda \sim M_*$. The other factors appearing in the integrand are also easily understood. The first factor k^{n+3} is the $4+n$ dimensional phase space factor. The factors of k^{-2} and k^{-1} are from the the graviton and muon propagators. Finally, the third factor counts the momentum factors at the graviton vertices; for example, in Fig.1(a) the graviton–fermion–fermion interaction contains $\sim k$. This naive explanation then reproduces Eqn. 22. The KK states exhibit non-decoupling since their coupling to matter is stronger at higher energies.

4 Bulk Vector Bosons

a. Bulk Vector Bosons

In this Section I compute one-loop correction of a bulk X gauge boson to the MMM. A $U(1)_X$ gauge field in the bulk that couples to leptons contributes to a_μ by the one-loop diagram in Fig. 2a. It could be a gauged $B-L$, or the higher KK modes of the photon, for example. As with the bulk gravitons, the bulk vector boson interactions are constrained by the measured value of a_μ. In this case the limits are more model-dependent. Each KK mode couples universally to leptons with an unknown 4-dimensional gauge coupling g_X. This bulk vector boson also does not have to live in the full n-dimensional bulk. It could instead live in a $p-$dimensional subspace with volume V_p. It will be shown though, that those regions of parameters resulting in $O(10^{-8}-10^{-9})$ corrections to the MMM will also be probed by future short-distance force experiments.

While the limit from a_μ implies that the gauge coupling must be microscopic (see Eqn. 36 below), it is also quite natural for the 4$-$dimensional coupling to be small [3]. In the full $4+p$ dimensional gauge theory the gauge coupling between a vector boson and fermions is $g_X^{(p)}/M_*^{p/2}$. Confining the fermions to a 3$-$dimensional wall, by either a topological defect or just using a $p-$dimensional delta function, and performing the usual KK decomposition of the vector boson results in a universal coupling g_X of each KK mode to fermions. It also gives

$$\alpha_X = \alpha_X^{(p)}/(M_*^p V_p), \tag{26}$$

the relation between the fine structure constants in the 4-D effective theory and the fundamental $p-$dimensional theory. As a result, it is very natural for the vector boson to be very weakly coupled. For example, in the case $p=n$, $\alpha_X = \alpha_X^{(n)} M_*^2/\bar{M}_{PL}^2$, independent of n. In this case $\alpha_X \sim 2\times 10^{-30}\times \alpha_X^{(n)}\times (M_*/3\text{ TeV})^2$.

This also makes it very natural for the X boson to be extremely light. Note that even if a $B-L$ gauge boson is lighter than the proton (as will be the case here), the proton remains stable since the $B-L$ gauge boson does not carry $B-L$ charge. These facts were pointed out in Ref. [3], where it was argued that a $B-L$ vector boson could mediate forces at the sub-mm distance scale 10^7-10^8 times stronger than gravity. To avoid conflicts with these short-distance force experiments, the mass of the zero mode of a new $U(1)_X$ coupled to baryons must be larger than $\sim 10^{-4}$eV. This requires that the gauge symmetry is spontaneously broken in the bulk by the vev v of some scalar field ϕ. The mass term for the gauge field in the full theory, written using 4-D canonically normalised fields, is

$$\int d^{4+p}x \frac{1}{2}\frac{g_{4+p}^2}{M_*^p}\frac{\phi^2}{V_p}\frac{A_{X\mu}}{\sqrt{V_p}}\frac{A_X^\mu}{\sqrt{V_p}}. \tag{27}$$

Inserting the vev, the usual KK expansion for A_X, integrating over the $p-$volume, and using Eqn. 26 gives

$$m_X = g_X v \tag{28}$$

for the mass of the zero mode. If instead the scalar field ϕ is confined to a distant 3−D wall, then in Eqn. 27 I should replace $\phi^2/V_p \to \phi^2$ and insert the approximation [15] $\phi(x,y) \sim \delta^{(p)}(y)\phi(x)$. This results in the same expression for the mass, Eqn. 28. Consistency of this field theoretic approach requires that the vev must satisfy $v < M_*$. This implies an upper bound of $g_X M_*$ to the mass of the zero mode. When this is combined with the constraint on g_X from a_μ, an interesting *upper bound* to the mass of the zero mode is gotten. The masses of the other KK states are

$$m_N^2 = g_X^2 v^2 + \frac{n^2}{R^2}. \tag{29}$$

An evaluation of Fig. 2a gives the contribution of one KK state to a_μ. The result is finite and

$$\Delta a_\mu^{(n)} = \frac{\alpha_X}{\pi} \int dx \frac{m_F^2}{L(x, m^2, m_N^2)} x^2(1-x). \tag{30}$$

The normalisation of the $U(1)_X$ charge is chosen such that the X charge of a lepton is 1. The function $L(x, m^2, m_N^2)$ is defined in the previous section. In the limit $m_N \to 0$ Eqn. 30 reduces to the famous result from Schwinger [27].

I now want to consider performing the sum over all the KK states. The contribution from those states $m_N \lesssim m_F$ is obviously finite. In the opposite limit $m_N \gg m_F$, $\Delta a_\mu^{(n)} \to (\alpha_X/3\pi) m_F^2/m_N^2$. In this case the correction decouples as m_N^{-2}. Since the degeneracy of the KK states increases as $(m_N^2)^{p/2-1}$, however, the resultant sum is power divergent as $(m_N^2)^{p/2-1}$ for $p > 2$, logarithmically divergent for $p = 2$, and finite for $p = 1$. As in the case of the bulk graviton loop, a cutoff $\Lambda = M_*$ is used to regulate these divergences, and an overall factor of $\lambda \sim O(1)$ is inserted to parameterise the result of an actual one-loop computation in a more complete theory of quantum gravity. The contribution of those modes $m_N \gg m_F$ to a_μ is then, using a trivial modification of Eqn. 15,

$$\Delta a_\mu = \lambda \frac{\alpha_X}{3\pi} m_F^2 R^2 \left(\frac{\pi^2}{6}\right), \; p = 1 \tag{31}$$

$$\Delta a_\mu = 2\lambda \frac{\alpha_X}{3\pi} \left(\frac{2}{p-2} \frac{\pi^{p/2}}{\Gamma[p/2]}\right) \left(\frac{\bar{M}_{PL} m_F}{M_*^2}\right)^2 \left(\frac{M_*^p V_p}{M_*^n V_n}\right), \; p > 2 \tag{32}$$

$$= 2\lambda \frac{\alpha_X^{(p)}}{3\pi} \left(\frac{2}{p-2} \frac{\pi^{p/2}}{\Gamma[p/2]}\right) \left(\frac{m_F}{M_*}\right)^2, \; p > 2 \tag{33}$$

$$\Delta a_\mu = \lambda \frac{\alpha_X}{3} m_F^2 R^2 \ln M_*^2/m_F^2, \; p = 2. \tag{34}$$

To arrive at Eqn. 31, $p = 1$, I have neglected the mass of the zero mode. Also note that in this case the result is finite. For $p > 1$ the symmetry breaking contribution is irrelevant since the resulting sum is divergent. To get from Eqn. 32 to Eqn. 33, I have used the relation $2\bar{M}_{PL}^2 = M_*^{n+2} V_n$ and the relation Eqn. 26 between the 4−dimensional and $(4+p)$−dimensional fine structure constants.

These expressions for the bulk vector boson contribution to a_μ can also be understood by considering computing the one–loop diagram in the full theory. Recall that the correction to the

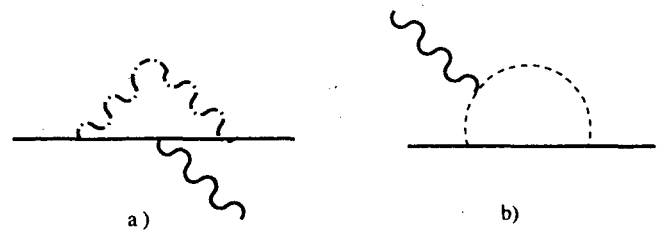

Figure 2: One-loop correction to the anomalous magnetic moment from: a) abelian bulk vector bosons (dashed wavy line), b) charged Higgs (dotted line) and bulk RH neutrinos. Wavy and solid lines denote on-the-wall photons and fermions, respectively.

MMM requires a helicity flip in the loop diagram. This pulls out a factor of $p_\alpha p^a \gamma_a$, lowering the degree of divergence by two. Then naive power counting gives for $p > 2$

$$\Delta a_\mu \sim m_F^2 \frac{\alpha_X^{(p)}}{M_*^p} \int dk k^{p+3} k^{-2} k^{-2} k^{-2} \sim m_F^2 \frac{\alpha_X^{(p)}}{M_*^p} \Lambda^{p-2}. \tag{35}$$

The first two factors of k^{-2} in the integrand are for the KK vector boson and fermion propagators. Substituting $\Lambda \sim M_*$ further simplifies this result to $\Delta a_\mu \sim \alpha_X^{(p)} m_F^2 / M_*^2$, which is Eqn. 33. Note that this result is independent of the dimension of the subspace. This implies that the correction to MMM is independent of V_p, and depends only on M_* and $\alpha_X^{(p)}$, the fine structure constant in the full theory. The dependence on V_p only enters once the correction to the MMM is expressed in terms of the 4–dimensional fine structure constant.

b. Applications

Requiring that $\Delta a_\mu < 2 \times 0.8 \times 10^{-8}$ gives the following 2σ limits for the interesting cases $p = n = 2$, and $p = n = 6$:

$$p = n = 2: \ \alpha_X < 3.3 \times 10^{-32} \times \frac{1}{\lambda} \left(\frac{M_*}{\text{TeV}}\right)^4 , \ p = n = 6: \ \alpha_X < 2.4 \times 10^{-31} \times \frac{1}{\lambda} \left(\frac{M_*}{\text{TeV}}\right)^4 . \tag{36}$$

The limit in the case $n = 2$ is ~ 10 below the "natural" value of $\sim 2 \times 10^{-31}$ for $M_* = 1$ TeV. I emphasize that these tiny limits apply only to the situation where the vector boson occupies the whole bulk. The limits on α_X are considerably weaker if the vector boson lives in a subspace. That is, for other choices of $p < n$, the limit on α_X is weakened by the amount $M_*^n V_n / M_*^p V_p$.

The limits in Eqn. 36 translate into the following 2σ upper bounds on the mass of the zero mode, using $m_X < g_X M_*$:

$$p = n = 2: \quad m_X < 1.7 \times 10^{-2} \times \frac{1}{\sqrt{\lambda}} \left(\frac{M_*}{3 \text{ TeV}}\right)^3 \text{ eV} , \tag{37}$$

$$p = n = 6: \quad m_X < 4.7 \times 10^{-2} \times \frac{1}{\sqrt{\lambda}} \left(\frac{M_*}{3 \text{ TeV}}\right)^3 \text{ eV}. \tag{38}$$

In the case that $X = B - L$, the gauge boson can mediate an isotope dependent force at sub-mm distance scales. Even though the gauge coupling is microscopic, the force can still be

huge compared to the gravitational force at those distance scales. The limits in Eqn. 36 quantify an upper limit to this signal. From the upper bound on the gauge boson mass, Eqn. 38, it is clear that there is still an allowed region where $m_X \ll \mathrm{mm}^{-1}$. Then the ratio of the forces is, for $p = n = 2$ to $p = n = 6$,

$$F_X : F_{\mathrm{grav}} = \alpha_X : G_N m_{\mathrm{nucl}}^2 \lesssim 10^{6-7} \left(\frac{M_*}{\mathrm{TeV}}\right)^4 . \tag{39}$$

The conclusion is that even though the $(g-2)$ constraint is strong in the case $p = n$, the $B - L$ gauge boson can still mediate forces $10^6 - 10^7$ times stronger than gravity at sub-mm distances.

Next I apply these results to the scenario of [12], where KK excitations of the SM gauge particles and Higgs bosons (and their superpartners) result in a power law evolution of the gauge couplings. This allows for a unification scale as low as few decades above the TeV scale. In this picture the SM gauge bosons, for example, have KK excitations in one or two extra dimensions. These will all contribute to the MMM. In what follows however, I only concentrate on the contribution from the KK photons for the reason that their contribution (and hence also those from KK Z's and W's) is rather small (see Eqn. 40 below). It is then seen that the constraint from a_μ provides a rather weak upper limit to R, the size of the subspace that the gauge bosons occupy. Using Eqns. 31, 34, $\alpha_{em}^{-1} = 137$, $M_* \sim 10$ TeV, and the 2σ experimental error on the muon $(g-2)/2$ gives

$$p = 1 : \; R^{-1} > 30 \text{ GeV} \; , \; p = 2 : \; R^{-1} > 190 \text{ GeV}. \tag{40}$$

The limit for $p = 1$ is finite, whereas the case $p = 2$ is logarithmically sensitive to the cutoff. Since these two constraints satisfy $R^{-1} \gg m_\mu$, the use of Eqns. 31 and 34 is consistent. The rather weak bound for $p = 1$ is due to the α_{EM} suppression, and the constant density of KK states.

Finally, KK excitations of the Z_0 gauge boson result in a tree–level correction to the theoretical predictions for atomic parity violation parameters. The measurement of atomic parity violation therefore constrain the size of some extra "small" dimensions. These experiments measure a quantity Q_W which is the coherent sum of the Z boson vector couplings to the nucleus. The measured value for Cesium is [17]

$$Q_W^{exp} = -72.41 \pm 0.25 \pm 0.80, \tag{41}$$

which agrees very well with the theoretical prediction [17]

$$Q_W^{SM} = -73.12 \pm 0.06 \pm 0.01. \tag{42}$$

The exchange of a tower of KK Z_0 bosons trivially modifies (at tree-level) the SM prediction to[5]

$$Q_W^{SM} \to Q_W^{th} = Q_W^{SM} + Q_W^{KK} = Q_W^{SM} \left(1 + m_Z^2 R^2 \sum_n \frac{1}{n^2}\right). \tag{43}$$

[5] I assume that the SM fermions do not have any KK excitations below M_*.

The m_Z contribution to the KK mass n/R has been neglected. Note that the KK correction to Q_W pushes the theoretical prediction farther away from the measured value. To limit R, I require that $Q_W^{exp} - Q_W^{th} < 2\sigma$, or $-Q_W^{KK} < 0.89$. Using the measured value given above, and performing the sum over KK states results in the following 2σ limits

$$p = 1: \; R^{-1} > 1100 \text{ GeV} \; ; \; p = 2: \; R^{-1} > 3.6 \text{ TeV}\sqrt{\frac{\ln M_* R}{\ln 20}}. \tag{44}$$

So it is quite clear that the stronger limits on TeV−sized extra dimensions are gotten from atmoic parity violation rather than the MMM. Compare the results in Eqn. 44 to Eqn. 40.

5 Bulk Right-Handed Neutrinos

The authors of Ref. [15] proposed several mechanisms for generating $\sim O(\text{eV})$ sized neutrino masses. In one of their models which is the attention of this section, small Dirac neutrino masses are generated by introducing bulk RH neutrinos. I now briefly explain their idea. The interaction

$$\int d^4x \; kLH \sum_n v_c^{(n)} \tag{45}$$

gives a Dirac mass $kv/\sqrt{2}$ to the LH and RH zero mode neutrino. The higher KK states also mix with the LH neutrino, but what will eventually concern us for the MMM are the extremely heavy KK states, which have very tiny mass mixing with the LH neutrinos. In their model this interaction arises from a higher dimension operator in the full theory. More specifically, the operator

$$\int d^4x d^p y \frac{k_0}{M_*^{p/2}} L(x,y) H(x,y) \frac{v^c(x,y)}{\sqrt{V_p}}, \tag{46}$$

where I have allowed the fields to propagate in p extra dimensions, and k_0 is a dimensionless constant. Inserting the approximation $\psi_{SM}(x,y) \sim \sqrt{\delta^{(p)}(y)}\psi_{SM}(x)$ for the SM zero modes [15], and the usual KK mode expansion for v^c gives the relation between k and k_0:

$$k = k_0 \frac{1}{\sqrt{M_*^p V_p}} = k_0 \left(\frac{M_*}{\bar{M}_{PL}}\right)^{p/n}, \tag{47}$$

where to obtain the second relation I used $(4\pi G_N)^{-1} = M_*^{n+2} R^n$ and assumed $V_p = R^p$ for simplicity. Consequently, the coupling k can be very tiny, and neutrino masses of the correct order of magnitude required to explain the atmospheric and solar neutrino anomalies can be obtained.

To obtain a correction to the MMM at the one-loop level, I assume:

i) the muon neutrino obtains a Dirac mass by mixing with a sterile neutrino, as in Eqn. 45;

ii) the Higgs sector is extended to include an extra Higgs doublet which also acquires a vev.

The second assumption is realised in low-energy supersymmetric extensions to the SM, for example. The point of this assumption is to obtain a trilinear coupling between the physical charged Higgs, the muon, and the bulk RH neutrinos. Consequently, these interactions contribute to the MMM at one-loop, as shown in Fig. 2b.

An evaluation of Fig. 2b gives the following correction to the MMM:

$$\Delta a_\mu^{(n)} = \frac{2k^2}{16\pi^2}\int dx \frac{m_F^2}{\tilde{L}(x, m_F^2, m_{\phi+}^2, m_N^2)} x^2(1-x), \tag{48}$$

where I have denoted by k the trilinear coupling of the physical charged Higgs, charged LH fermion, and bulk RH neutrinos. It is related to a linear combination of the trilinear couplings in the gauge basis. The function $\tilde{L}$ is $\tilde{L}(x, m_F^2, m_{\phi+}^2, m_N^2) = -x(1-x)m_F^2 + xm_{\phi+}^2 + (1-x)m_N^2$, where $m_{\phi+}$ is the mass of the charged Higgs. In the limit $m_N^2 \gg m_F^2, m_{\phi+}^2$,

$$\Delta a_\mu^{(n)} = \frac{2}{3}\frac{k^2}{16\pi^2}\frac{m_F^2}{m_N^2}. \tag{49}$$

As in the case of the bulk gravitons and bulk vector bosons, the dominant correction to the MMM occurs from the superheavy KK RH neutrinos. In fact, the sum over KK states is identical (up to numerical factors) to the KK sum for the vector bosons. Integrating Eqn. 49 for $p > 2$, and using trivial modifications to Eqns 14 and 15, gives

$$\Delta a_\mu = \frac{2}{3}k^2\frac{\lambda}{8\pi^2}\frac{2}{p-2}\frac{\pi^{p/2}}{\Gamma[p/2]}\frac{m_F^2}{M_*^2}\left(\frac{\bar{M}_{PL}}{M_*}\right)^{2p/n}. \tag{50}$$

Note that since $k = k_0(M_*/M_{PL})^{p/n}$, the strong dependence on $M_*/\bar{M}_{PL}$ cancels, leaving

$$\Delta a_\mu = \frac{2}{3}k_0^2\frac{\lambda}{8\pi^2}\frac{2}{p-2}\frac{\pi^{p/2}}{\Gamma[p/2]}\frac{m_F^2}{M_*^2}. \tag{51}$$

Choosing $p = 5$, for example, gives

$$\Delta a_\mu \sim 8k_0^2\lambda \times 10^{-10}\left(\frac{\text{TeV}}{M_*}\right)^2 \sim k_0^2\lambda \times 10^{-9}\left(\frac{\text{TeV}}{M_*}\right)^2. \tag{52}$$

So since both k_0 and $\lambda \sim O(1)$, it is possible to get $\Delta a_\mu \sim 10^{-9}$. This may be within the reach of the BNL E821 experiment.

6 Acknowledgements

The author would like to thank N. Arkani-Hamed and M. Suzuki for discussions and encouraging me to complete this project. The author also thanks T. Moroi for useful communications about the BNL E821 experiment. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U. S. Department of Energy under Contract DE-AC03-76SF00098. The author also thanks the Natural Sciences and Engineering Research Council of Canada (NSERC) for their support.

References

[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys.Lett. **B429** (1998) 263.

[2] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys.Lett.**B436** (1998) 257; G. Shiu and S. Tye, Phys.Rev. **D58** (1998) 106007.

[3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. **D59** (1999) 086004.

[4] G. Giudice, R. Rattazzi and J. Wells, Nucl. Phys. **B544** (1999) 3.

[5] E. Mirabelli, M. Perelstein and M. Peskin, Phys. Rev. Lett.**82** (1999) 2236.

[6] J. L. Hewett, Phys. Rev. Lett. **82** (1999) 4765.

[7] K. Agashe and N. Deshpande, Phys. Lett. **B456** (1999) 60.

[8] P. Mathews, S. Raychaudhuri, K. Sridhar, Phys. Lett. **B450** (1999) 343; P. Mathews, S. Raychaudhuri, K. Sridhar, Phys. Lett. **B455** (1999) 115.

[9] S. Nussinov and R. Shrock, Phys. Rev. **D59** (1999) 105002.

[10] I. Antoniadis, Phys. Lett. **B246** (1990) 377.

[11] I. Antoniadis and K. Benakli, Phys. Lett. **B326** (1994) 69; I. Antoniadis, K. Benakli and M. Quirós, Phys. Lett. **B331** (1994) 313.

[12] K. Dienes, E. Dudas, T. Gherghetta, Phys. Lett. **B436** (1998) 55; K. Dienes, E. Dudas, T. Gherghetta, Nucl. Phys. **B537** (1999) 47.

[13] Z. Kakushadze, Nucl.Phys. **B548** (1999) 205.

[14] N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353; Z. Berezhiani and G. Dvali, Phys. Lett. **B40** (1999) 24.

[15] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, hep-ph/9811448.

[16] T. Moroi, Phys.Rev. **D53** (1996) 6565, Erratum-ibid. **D56** (1997) 4424.

[17] Caso *et al.*, Particle Data Group: Review of Particle Physics, Euro. Phys. J. **C3** (1998) 1.

[18] By E821 BNL Collaboration (B. L. Roberts *et al.*), Published in the Proceedings of the XXVIII International Conference on High Energy Physics, Warsaw, Poland, 1996, River Edge, NJ World Scientific, 1997, p. 1035.

[19] T. Kinoshita, Phys. Rev. Lett. **75** (1995) 4728 ; A. Czarnecki, B. Krause and W. J. Marciano, Phys. Rev. Lett. **76** (1996) 3267.

[20] M. Hayakawa, T. Kinoshita and A. I. Sanda, Phys. Rev. Lett. **75** (1995) 790.

[21] Private communication with Takeo Moroi.

[22] R. Sundrum, Phys. Rev. **D59** (1999) 085009; R. Sundrum, Phys. Rev. **D59** (1999) 085010.

[23] N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124.

[24] T. Han, J. Lykken and R-J Zhang, Phys. Rev. **D59** (1999) 105006.

[25] M. J. G. Veltman, "Quantum Theory of Gravitation" in *Methods in Field Theory*, North-Holland, Amsterdam (1976); M. T. Grisaru, P. van Nieuwenhiuzen and C. C. Wu, Phys. Rev. bf D 12 (1975) 1813; A. van Proeyen, Nucl. Phys. **B174** (1980) 189; P. van Nieuwenhiuzen, Phy. Rep **68**, 121 (1981).

[26] F.A. Berends and R. Gastmans, Phys. Lett. **B55**, 311 (1975).

[27] J. Schwinger, Phys. Rev. **73** (1948) 416; J. Schwinger, Phys. Rev **75** (1949) 898.

Ernest Orlando Lawrence Berkeley National Laboratory
One Cyclotron Road | Berkeley, California 94720

Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098