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Gaugino Masses in Modular Invariant Supergravity*

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Abstract

We calculate gaugino masses in string-derived models with hidden-sector gaugino condensation. The linear multiplet formulation for the dilaton superfield is used to implement perturbative modular invariance. The contribution arising from quantum effects in the observable sector includes the term recently found in generic supergravity models. A much larger contribution is present if matter fields with Standard Model gauge couplings also couple to the Green-Schwarz counter term. We comment on the relation of our Kähler $U(1)$ superspace formalism to other calculations.

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It was recently pointed out [1, 2] that the super-Weyl anomaly of standard $N = 1$ supergravity generates a gaugino mass proportional to the beta-function coefficient, which may solve the problem of small gaugino masses found in certain classes of models.

In this paper we consider a class of string-derived models [3]–[5] in which gaugino condensation occurs in a hidden sector with modular invariant couplings. That is, the field theoretic quantum anomaly that breaks invariance under the modular transformation (T-duality)

$$T \to \frac{aT - ib}{icT + d}, \quad ab - cd = 1, \quad a, b, c, d \in Z, \tag{1}$$

is explicitly canceled by a universal Green-Schwarz (GS) counter term together with model-dependent string threshold corrections. Gaugino masses in these models were found in [5] to be suppressed with respect to the gravitino mass–although not as severely as in some gauge-mediated models [6]. However, in [5] a contribution was omitted that generates, among others, the term found in [1, 2]. In this paper we correct this omission. The additional correction is obtained by an analysis of the superspace expression for the loop correction, as well as by an explicit calculation using component fields and Pauli-Villars regularization. In these models the anomaly associated with the Kähler transformation (1) is explicitly canceled. Because Kähler and super-Weyl transformations are intimately connected in the Kähler $U(1)$ superspace formalism [7] that we use, one might expect the mass term found in [1, 2] to be absent in this class of models. However, this term has its origin in the running of the couplings from the string scale to the condensation scale, and is therefore independent of the string scale physics. In addition, we find a contribution that depends on the unknown couplings of matter fields in the GS term – a situation similar to the case for scalar masses discussed in [5].

In the linear supermultiplet formulation the dilaton ℓ is the lowest component of a vector superfield L that satisfies the modified linearity condition

$$-\left(\bar{\mathcal{D}}^2 - 8R\right) L = W^\alpha W_\alpha, \quad -\left(\mathcal{D}^2 - 8\bar{R}\right) L = \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}, \quad L\Big| = \ell, \tag{2}$$

where the superfield R is related to elements of the supervielbein, W^α is a Yang-Mills superfield strength, and the summation over gauge indices is suppressed. The Bianchi identity

$$\left(\mathcal{D}^2 - 24\bar{R}\right) W^\alpha W_\alpha - \left(\bar{\mathcal{D}}^2 - 24R\right) \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha} = \text{total derivative} \tag{3}$$

follows immediately from (2). To describe gaugino condensation [8, 9], a vector multiplet V is introduced whose components include those of a linear multiplet L as well as chiral and anti-chiral superfields $U, \bar{U}$ that are the (anti-)chiral projections of V:

$$-\left(\bar{\mathcal{D}}^2 - 8R\right) V = U, \quad -\left(\mathcal{D}^2 - 8\bar{R}\right) V = \bar{U}, \tag{4}$$

and are interpreted as condensate superfields for a strongly coupled (confined) hidden Yang-Mills sector: $U \simeq (W^\alpha W_\alpha)_h$. With this construction the superfield U has the correct Kähler $U(1)$ weight as well as the correct constraint, that is, the counterpart of the Bianchi identity (3). This construction was generalized in [4] to the case of several gaugino condensates, and it was found that the results are dominated by the condensate of the gauge group $\mathcal{G}_a = \mathcal{G}_+$ with the largest β-function coefficient $b_a = b_+$, where

$$b_a = \frac{1}{8\pi^2}\left(C_a - \frac{1}{3}C_a^M\right), \quad C_a^M = \sum_A C_a^A, \tag{5}$$

with C_a and C_a^A quadratic Casimir operators in the adjoint and matter representations, respectively. For this reason we include only a single condensate here. When both the condensate and the weakly coupled, unconfined Yang-Mills sectors are included, the linearity condition takes the form

$$\begin{aligned} -\left(\bar{\mathcal{D}}^2 - 8R\right) V &= U + \sum_a (W^\alpha W_\alpha)_a, \\ -\left(\mathcal{D}^2 - 8\bar{R}\right) V &= \bar{U} + \sum_a (\overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha})_a, \end{aligned} \tag{6}$$

We consider a class of orbifold compactifications with three untwisted moduli chiral superfields T^I and matter chiral superfields Φ^A. The Kälher potential is

$$K = k(V) + \sum_I g^I + \sum_A e^{q_I^A}|\Phi^A|^2 + O(\Phi^4), \quad g^I = -\ln(T^I + \bar{T}^I), \tag{7}$$

where the parameters q_I^A are the modular weights of Φ^A, and the relevant part of the Lagrangian is

$$\mathcal{L}_{eff} = \mathcal{L}_1 + \sum_a \mathcal{L}_a, \quad \mathcal{L}_1 = \mathcal{L}_{KE} + \mathcal{L}_{GS} + \mathcal{L}_{th} + \mathcal{L}_{VY} + \mathcal{L}_{pot}, \tag{8}$$

where

$$\mathcal{L}_{KE} = \int \mathrm{d}^4\theta\, E\,[-2 + f(V)], \quad k(V) = \ln V + g(V), \tag{9}$$

contains the kinetic energy terms for the dilaton, chiral and gravity superfields, as well as the tree-level Yang-Mills terms. The functions $f(V), g(V)$ parameterize non-perturbative string effects. They satisfy the conditions

$$Vg'(V) = f - Vf'(V), \quad g(0) = f(0) = 0, \tag{10}$$

which ensure that the Einstein term has canonical form [3], and that they vanish in weak coupling limit: $g^2/2 = \langle \ell \rangle = \langle V| \rangle \to 0$. The term

$$\begin{aligned} \mathcal{L}_{VY} &= \frac{1}{8}\int \mathrm{d}^4\theta\, \frac{E}{R} U \left[b'_+ \ln(e^{-K/2}U) + \sum_\alpha b^\alpha \ln \Pi^\alpha \right] + \text{h.c.}, \\ b'_a &= \frac{1}{8\pi^2}\left(C_a - C_a^M\right), \quad \sum_\alpha b^\alpha = \frac{1}{12\pi^2} C_+^M, \end{aligned} \tag{11}$$

is the generalization to supergravity [10, 11] of the Veneziano-Yankielowicz superpotential term [12], including [13] gauge invariant composite matter fields Π^α, and

$$\mathcal{L}_{pot} = \frac{1}{2}\int \mathrm{d}^4\theta\, \frac{E}{R} e^{K/2} W(\Pi^\alpha, T^I) + \text{h.c.} \tag{12}$$

is a superpotential for the matter condensates. In (11) C_+^M refers to the confined matter superfields Φ^A of the strongly coupled sector.

The operators $\mathcal{L}_a$ are the quantum corrections from light field loops to the unconfined Yang-Mills couplings:

$$\mathcal{L}_a = \frac{1}{64\pi^2}\int d^4\theta \frac{E}{R} \left(W_\alpha P_\chi \left[f_a(\Box_\chi) - B_a\right] W^\alpha\right)_a + \text{h.c.}, \tag{13}$$

where $\Box_\chi^{-1}$ is the chiral superfield propagator [14]:

$$\langle \Box_\chi \rangle = \left\langle \Box + \frac{1}{2} R\mathcal{D}^2 + O(R\bar{R}) \right\rangle, \tag{14}$$

in our notation,[1] and P_χ is the chiral projection operator: $P_\chi W^\alpha = W^\alpha$, that reduces in the flat space limit to $(16\Box)^{-1}\bar{\mathcal{D}}^2\mathcal{D}^2$. The function [15]

$$\begin{aligned} B_a &= \sum_I \left(C - b_a^I\right) g^I + \left(C^a - C_M^a\right) k(V) + 2\sum_A C_A^a \ln\left(1 + p_A V\right), \\ b_a^I &= C - C_a + \sum_A \left(1 - 2q_I^A\right) C_a^A, \quad C = C_{E_8} = 30, \end{aligned} \tag{15}$$

determines the renormalized coupling constant [5, 10, 16] $g_a(\mu_s)$ at the string scale μ_s:

$$\begin{aligned} g_a^{-2}(\mu_s) &= \left\langle \frac{1+f}{2\ell} - b_a' k(\ell) + \sum_A \frac{C_a^A}{8\pi^2} \ln(1 + \ell p_A) \right. \\ &\quad \left. - \sum_I \frac{b_a^I}{16\pi^2} \ln\left[|\eta(it^I)|^2 (t^I + \bar{t}^I)\right] \right\rangle, \quad \mu_s = \left\langle e^{\frac{1}{2}(k-1)} \right\rangle, \end{aligned} \tag{16}$$

and the functions

$$\frac{1}{8\pi^2} f_a(\mu^2) = g_a^2(\mu^2) - g_a^2(\mu_s^2) \tag{17}$$

govern the running of the gauge couplings from the string scale to the normalization scale $\mu^2 = - < \Box >$. $\mathcal{L}_a$ and $\mathcal{L}_{VY}$ are anomalous under (1). This anomaly is canceled by two counter terms: the GS term [17]

$$\begin{aligned} \mathcal{L}_{GS} &= \int \mathrm{d}^4\theta \, EVV_{GS}, \quad b = \frac{C}{8\pi^2} = b_{E_8}, \\ V_{GS} &= b\sum_I g^I + \sum_A p_A e^{\sum_I q_I^A g^I} |\Phi^A|^2 + O(\Phi^4), \end{aligned} \tag{18}$$

[1]We set the background space-time curvature scalar r to zero throughout this paper. A term proportional to $r\lambda\lambda$ would result in a contribution to the gaugino mass through a Weyl rescaling, but we find that such terms are suppressed by powers of μ^{-2} or m^{-2} where m is the Pauli-Villars mass introduced below.

and the term induced by string loop corrections [18]

$$\mathcal{L}_{th} = -\frac{1}{64\pi^2}\sum_I \int \mathrm{d}^4\theta \frac{E}{R} \ln \eta^2(iT^I) \left(b_+^I U + \sum_a b_a^I (W^\alpha W_\alpha)_a \right) + \text{h.c.}. \tag{19}$$

The parameters b_a^I vanish for orbifold compactifications with no $N = 2$ supersymmetry sector [19]. For $a = +$, the q_I^A are modular weights of the confined matter superfields. Note that we have not introduced kinetic terms for the condensate superfield; that is, we are treating the condensate as static. A dynamical condensate has been studied [20] in the case of an E_8 gauge group, and it was found that the bound state masses are above the condensation scale; when these states are integrated out the theory reduces to the static case considered here.

To evaluate the gaugino masses, we set all matter fields to zero in the vacuum: $< \phi^A >=< \Phi^A| >= 0$. First recall that "D-terms" like $\mathcal{L}_{GS}$ and $\mathcal{L}_{KE}$ can be cast in the form of "F-terms" by integration by parts:

$$\mathcal{L} = \int \mathrm{d}^4\theta E\phi = -\frac{1}{16}\int \mathrm{d}^4\theta \frac{E}{R}\left(\bar{\mathcal{D}}^2 - 8R\right)\phi + \text{h.c.} \tag{20}$$

These and the remaining "F-terms" can be evaluated using the standard construction [7]

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\int \mathrm{d}^4\theta \frac{E}{R}\mathbf{r} = \sqrt{g}\left(s - r\bar{M}\right) + \text{h.c.} + O(\psi_\mu), \\ r &= \mathbf{r}\Big|, \quad s = -\frac{1}{4}\mathcal{D}^2\mathbf{r}\Big|, \end{aligned} \tag{21}$$

with

$$\begin{aligned} W^\alpha W_\alpha\Big| &= -\lambda\lambda, \quad u = U\Big|, \quad M = -6R\Big| = (\bar{M})^*, \quad \ell = V\Big|, \\ t^I &= T^I\Big|, \quad F^I = -\frac{1}{4}\mathcal{D}^2 T^I\Big|, \quad F_u = -\frac{1}{4}\mathcal{D}^2 U\Big|, \\ -\frac{1}{4}\mathcal{D}^2 W^\alpha W_\alpha\Big| &= -\frac{1}{2}\left(F^2 + iF\cdot\tilde{F}\right) - \bar{M}\lambda\lambda + 2\bar{\lambda} i \not{\mathcal{D}}\lambda + \cdots. \end{aligned} \tag{22}$$

For example,

$$\mathcal{L}_{GS} = -\frac{1}{16}\int \mathrm{d}^4\theta \frac{E}{R}\left(\bar{\mathcal{D}}^2 - 8R\right) V V_{GS} + \text{h.c.}$$

$$
\begin{aligned}
&= \frac{1}{16}\int \mathrm{d}^4\theta \frac{E}{R}\left(\sum_a (W^\alpha W_\alpha)_a V_{GS} + V\bar{\mathcal{D}}^2 V_{GS}\right) + \text{h.c.} + \cdots \\
&= \frac{b}{8}\sum_I \left(\bar{\lambda} i \not{\mathcal{D}} \lambda g^I - \lambda\lambda F^I g'^I\right) + \text{h.c.} + \cdots . \qquad (23)
\end{aligned}
$$

The evaluation of the component form of $\mathcal{L}_1$ is rather involved and has been carried out explicitly in [3, 4] neglecting the unconfined Yang-Mills fields. To include the latter terms we need only make the substitutions

$$
u \;\to\; u - \sum_a (\lambda\lambda)_a, \quad F_u \to F_u - \frac{1}{4}\sum_a \mathcal{D}^2 (W^\alpha W_\alpha)_a \Big| \qquad (24)
$$

in those results. Using the vacuum values found in [4]:

$$
\left\langle F^I \right\rangle = 0, \quad \left\langle \bar{M} \right\rangle = \frac{3}{4}\left\langle \left(b'_+ \bar{u} - 4\overline{W} e^{K/2}\right)\right\rangle = \frac{3}{4} b_+ \left\langle \bar{u} \right\rangle = -3 m_{\tilde{G}}, \qquad (25)
$$

the contribution of $\mathcal{L}_1$ to observable sector gaugino masses found in [5] is

$$
\mathcal{L}_1 \ni \frac{1}{2g_s^2}\bar{\lambda} i \not{\mathcal{D}} \lambda + \frac{1}{16\ell^2}\left(1 + \ell g'\right)\left(1 + b_+ \ell\right) \bar{u}\lambda\lambda + \text{h.c.}, \qquad (26)
$$

where $m_{\tilde{G}}$ is the gravitino mass, and

$$
g_s = \left\langle \sqrt{\frac{2\ell}{1+f}} \right\rangle \qquad (27)
$$

is the tree-level field theory coupling constant. The requirement that the vacuum energy vanishes gives the condition [4]

$$
\left\langle \left(1 + \ell g'\right)\left(1 + b_+ \ell\right)^2 \right\rangle = 3\left\langle \ell^2 \right\rangle b_+^2, \qquad (28)
$$

so that, taking into account gauge coupling renormalization, one gets a contribution to the gaugino mass [5]

$$
m_a^{(1)}(\mu_c) = -\left\langle \frac{3 g_a^2(\mu_c) b_+^2 \bar{u}}{8(1 + b_+ \ell)} \right\rangle = \frac{3 g_a^2(\mu_c) b_+}{2(1 + b_+ \ell)} m_{\tilde{G}}, \qquad (29)
$$

where $\mu_c = |u|^{\frac{1}{3}}$ is the condensation scale in reduced Planck units. A gravitino mass in the TeV range requires $b_+ \simeq 1/30$, so this contribution to the gaugino masses

is quite small, although it is possible that two-loop renormalization effects between the condensation scale and the weak scale can bring masses of this order within experimental bounds [21].

To evaluate (13) we drop [2] terms of order $< R\bar{R}/\Box > \doteq -m_{\tilde{G}}^2/4\mu^2$ to obtain

$$\left\langle [\mathcal{D}^2, f(\Box_\chi)] \right\rangle \Phi = \left\langle 8\bar{R}\left[f'(\Box_\chi)\Box + R\left\{ f'(\Box_\chi) + f''(\Box_\chi)\Box \right\} \mathcal{D}^2 \right] \right\rangle \Phi. \tag{30}$$

where Φ is chiral; only the first term on the right hand side contributes to gaugino masses; this is the contribution found in [1, 2]. To lowest order in perturbation theory,

$$\begin{aligned} f_a(\Box) &= \left(3C_a - C_a^M\right) \ln\left(\Box/\mu_s^2\right), \\ \left\langle [\mathcal{D}^2, f_a(\Box)] \right\rangle \lambda_a &= 8\left\langle \bar{R} \right\rangle \left(3C_a - C_a^M\right) \lambda_a. \end{aligned} \tag{31}$$

In addition we have:

$$\left\langle \mathcal{D}^2 B_a | \right\rangle = -\left\langle 4\sum_I F^I \frac{\partial}{\partial T^I} B_a \Big| + \left(\bar{U} - 8V\bar{R}\right) \frac{\partial}{\partial V} B_a \Big| \right\rangle. \tag{32}$$

Using the vacuum values (25), we obtain

$$\begin{aligned} -\frac{1}{2} m_a^{(2)}(\mathcal{L}_a) &= \frac{1}{128\pi^2} \left\langle \mathcal{D}^2 \left[f_a(\Box_\chi) - B_a \right] \right\rangle \\ &= -\frac{\bar{u}}{128\pi^2} \Big\{ (3C_a - C_M) b_+ - (1 + b_+\ell) \times \\ &\quad \left[\ell^{-1}(\ell g' + 1)(C_a - C_M) + 2\sum_A C_a^A \frac{p_A}{(1 + \ell p_A)} \right] \Big\}, \end{aligned} \tag{33}$$

giving for the full contribution at the condensation scale μ_c

$$\begin{aligned} \mathcal{L}_1 + \sum_a \mathcal{L}_a \ni \sum_a \left[\frac{1}{2g_a^2(\mu_c)} \left(\bar{\lambda} i \not{\mathcal{D}} \lambda\right)_a + \frac{M_a}{16} \bar{u}(\lambda\lambda)_a + \text{h.c.} \right], \\ M_a = (1 + b_+\ell) \left[\ell^{-2} (1 + \ell g')(1 + b_a'\ell) + \sum_A \frac{C_a^A p_A}{4\pi^2(1 + \ell p_A)} \right] - 3b_a b_+. \end{aligned} \tag{34}$$

We now obtain for the gaugino masses

$$m_a(\mu_c) = \frac{g_a^2(\mu_c)}{2} \left[\frac{3b_+(1 + b_a'\ell)}{1 + b_+\ell} - 3b_a + \sum_A \frac{C_a^A p_A (1 + b_+\ell)}{4\pi^2 b_+ (1 + \ell p_A)} \right] m_{\tilde{G}}. \tag{35}$$

Next we explicitly calculate the gaugino masses using a Pauli-Villars (PV) regularization that has been formulated [22] for supergravity Lagrangians with the dilaton in a chiral multiplet. For present purposes, we need only consider the regulation of loops containing gauge-charged fields. Because the results below depend only on the Kähler potential for the PV fields and their couplings to the GS term, it is straightforward to transcribe the analysis to the case where the dilaton is described by a linear supermultiplet.

To regulate loop corrections to the Yang-Mills self-energy, one needs gauge-charged PV chiral supermultiplets: Z^A with signature η_A, that transform under gauge transformations according to representations R_A; Y_A with the same signature that transform according to the conjugate representation $\bar{R}_A$; and Φ^a_α, with signature η^a_α, that transforms according to the adjoint representation of the gauge group. In order to cancel the logarithmic divergences in the Yang-Mills self-energy, the Casimirs $\mathrm{Tr}_{R_A}\left(T^aT^b\right) = \delta_{ab}C^a_{R_A}$, and the signatures of the PV fields must satisfy

$$\sum_A \eta_A C^a_{R_A} = -C^M_a, \quad \sum_\alpha \eta^a_\alpha = 3. \tag{36}$$

The Kähler potential for these fields takes the form (setting light gauge-charged fields to zero in the background)

$$\begin{aligned} K_{PV} &= \sum_A \left[g^Z_A(V)e^{\sum_I \alpha^I_A g^I}|Z^A|^2 + g^Y_A(V)e^{\sum_I \beta^I_A g^I}|Y_A|^2\right] \\ &\quad + \sum_{a,\alpha} g_\alpha(V)e^{\alpha_\alpha \sum_I g^I}|\Phi^a_\alpha|^2 \equiv \sum_n g_n(V)G_n(T)|\Phi^n|^2, \end{aligned} \tag{37}$$

The V-dependence of K_{PV} requires an additional term $\mathcal{L}_{PV} = \int d^4\theta E f_{PV}(V)$ in the Lagrangian, where f_{PV} is related to K_{PV} by the differential equation in (10) that relates f to g:

$$f_{PV}(V) = \sum_n f_n(V)G_n(T)|\Phi^n|^2, \quad Vg'_n = f_n - Vf'_n, \quad \Phi^n = Z^A, Y_A, \Phi^a_\alpha. \tag{38}$$

The component Lagrangian can be obtained following the methods outlined in [3]. As shown in [15], supersymmetry of the modular anomaly from field theory quantum

corrections imposes constraints on the parameters in (37):

$$
\begin{aligned}
1 &= \sum_{\alpha} \eta^a_\alpha \alpha_\alpha, \quad \sum_A \eta_A \left(\alpha^I_A + \beta^I_A\right) C^a_{R_A} = -\sum_A C^A_a q^A_I, \\
0 &= \sum_A \eta_A \left(\ln h^Z_A + \ln h^Y_A\right) C^a_{R_A}, \quad \sum_\alpha \eta^a_\alpha \ln h_\alpha = k, \\
h_n(V) &= f_n(V) + g_n(V).
\end{aligned} \tag{39}
$$

in agreement with the requirements for full one-loop regularization [22]. Among the fields Z^A there is a subset $\tilde{Z}^A_\alpha$ with the same modular weights and the same gauge couplings as the light fields Φ^A. If the parameters p_A in the Green-Schwarz term are nonvanishing, we also require

$$
\begin{aligned}
V^{PV}_{GS} &= \sum_{A,\alpha} p_A e^{\sum_I q^A_I g^I} |\tilde{Z}^A_\alpha|^2, \quad g^{\tilde{Z}}_A = h^{\tilde{Z}}_A = 1, \\
\alpha^{\tilde{Z}^I}_A &= q^A_I, \quad \sum_\alpha \eta^{\tilde{Z}^A}_\alpha = -1.
\end{aligned} \tag{40}
$$

The potential for the PV scalar fields $\phi^n = \phi^a, z^A, y_A$ is

$$
\begin{aligned}
V &= \frac{1}{16\ell^2} \left(\ell g'(\ell) + 1\right) \left|u(1 + b_+\ell) - 4\ell W^{PV} e^{K/2}\right|^2 \\
&\quad - \frac{3}{16} \left|b_+ u - 4W^{PV} e^{K/2}\right|^2 + \hat{K}_{n\bar{m}} F^n \bar{F}^{\bar{m}},
\end{aligned} \tag{41}
$$

where

$$
\begin{aligned}
-\bar{F}^{\bar{m}} &= \hat{K}^{n\bar{m}} \left\{ \frac{u}{4} G_n \bar{\phi}^n \left[(p_n - b_+) h_n + h'_n (1 + \ell b_+)\right] + e^{K/2} W^{PV}_n \right\} + O(\phi^2_m), \\
\hat{K}_{n\bar{m}} &= \left(\hat{K}^{n\bar{m}}\right)^{-1} = G_n h_n \left[1 + p_n \ell\right] \delta_{n\bar{m}}, \quad p_{n \neq \tilde{Z}} = 0.
\end{aligned} \tag{42}
$$

The superpotential W^{PV} contains quadratic terms in the PV fields that give their masses. These give rise to "B-terms" in the potential (41) which take the form

$$
\begin{aligned}
V_B &= \frac{1}{2} \sum_{m,n} \mu_{mn} a_m \phi^m \phi^n + \text{h.c.}, \quad \mu_{mn} = e^{K/2} W^{PV}_{mn}, \\
a_n &= e^{K/2} \frac{\bar{u}}{4} \left\{ b_+ \ell + (1 + b_+ \ell) \left[\frac{2p_n}{1 + p_n \ell} + 2\frac{h'_n}{h_n} - \frac{1}{\ell} \left(\ell g' + 1\right) \right] \right\}.
\end{aligned} \tag{43}
$$

The first term in this expression is independent of the PV Kähler potential – *i.e.*, of the effective cut-offs – and of the details of the supersymmetry breaking mechanism. It is precisely the contribution found in [1, 2]. Here it arises from the presence of the Kähler potential in the condensate Lagrangian (11) as dictated by local supersymmetry. As noted previously [23], the structure of this term embeds the evolution of the gauge coupling constant from the string scale to the condensation scale. The PV Lagrangian also contains the terms (in four-component spinor notation)

$$\begin{aligned}\mathcal{L}_{PV} \ni & \sum_n \hat{K}_{n\bar{n}} \left[-\partial_a \bar{\phi}^{\bar{n}} \partial^a \phi^n + i\bar{\chi}_L^{\bar{n}} \not{\mathcal{D}} \chi_L^n + \sqrt{2}i \left(\bar{\lambda}_R^a (\bar{\phi}^{\bar{n}} T_a \chi_L^n) + \text{h.c.}\right)\right] \cdot \\ & - \sum_{n,m} \left[\mu_{mn}^2 |\phi^n|^2 + \frac{1}{2}\mu_{mn} \left(\bar{\chi}_R^n \chi_L^m + \text{h.c.}\right)\right], \end{aligned} \tag{44}$$

In terms of the normalized fields $\Phi_r^n = (\hat{K}^{\frac{1}{2}})_{n\bar{n}} \Phi^n$, the interaction terms are

$$\begin{aligned}\mathcal{L}_{Yuk} &= \sqrt{2}i \sum_n \bar{\lambda}_R^a (\bar{\phi}_r^{\bar{n}} T_a \chi_{rL}^n) + \text{h.c.}, \quad V = \frac{1}{2} \sum_{m,n} m_m a_m \phi_r^m \phi_r^n + \text{h.c.}, \\ m_n^2 &= \hat{K}^{n\bar{n}} \hat{K}^{m\bar{m}} \mu_{mn}^2, \end{aligned} \tag{45}$$

where m_n is the mass of Φ_r^n. The Feynman amplitude $F_a = -i < \lambda_R^a | \mathcal{L}_{eff} | \lambda_L^a >$ for $\lambda_L \to \chi_L^n + \bar{\phi}^{\bar{n}} \to \phi^m + \bar{\chi}_R^{\bar{m}} \to \lambda_R$ gives a contribution

$$\begin{aligned}\mathcal{L}_{eff} &\ni -\frac{1}{2} m_a \bar{\lambda}_R^a \lambda_L^a + \text{h.c.} = \frac{i}{2} F_a \bar{\lambda}_R^a \lambda_L^a + \text{h.c.}, \\ -\frac{1}{2} m_a^{(2)} &= -i \sum_n \eta_n C_n^a \int \frac{d^4p}{(2\pi)^4} \frac{a_n m_n^2}{(p^2 - m_n)^3} = -\sum_n \eta_n \frac{C_n^a a_n}{32\pi^2}, \end{aligned} \tag{46}$$

Using the constraints (36), (39) and (40), this reduces to the result found in (33).

Aside from the renormalization of the coupling constants, there are three contributions from $\mathcal{L}_a$ in (35). The first, proportional to b'_a, gives a negligible correction to (29). The second term modifies the result (29) of [5] by a factor (neglecting $b_+\ell \sim .03$)

$$\eta_a \simeq (1 - b_a/b_+) \simeq [0.6, 1.1, 1.8] \quad \text{for} \quad \mathcal{G}_a = [SU(3), SU(2), U(1)], \tag{47}$$

if we assume just the MSSM contribution to the β-functions. The third term depends on the unknown parameters p_A. It was found in [5] that the squark, slepton and Higgs

masses m_s also depend on these parameters. If the matter fields decouple from the GS term, one has

$$p_A = 0, \quad m_s = m_{\tilde{G}}, \tag{48}$$

and the full correction to the gaugino masses is given by (47). If the GS term is proportional to Kähler potential we get

$$p_A = b, \quad m_s \approx 10 m_{\tilde{G}}. \tag{49}$$

Analyses of dynamical symmetry breaking in the MSSM favor smaller masses for at least the stop and Higgs particles. Another possibility is that the GS term depends only on the metric of the compact 6-manifold, in which case it couples only to untwisted fields:

$$p_A^{untw} = b, \quad m_s^{untw} \approx 10 m_{\tilde{G}}, \quad p_A^{tw} = 0, \quad m_s^{tw} = 0, \tag{50}$$

resulting in a mass hierarchy among generations as has been proposed by some authors [24]. For a single Standard Model generation, $\sum_A C_a^A = 2$, so in this scenario, with n untwisted generations, the last term in (35) dominates, and one gets the following gaugino masses at the condensation scale:

$$m_a(\mu_c) \approx -\frac{n\alpha_a(\mu_c) b(1+\ell b_+)}{\pi b_+(1+\ell b)} m_{\tilde{G}} \approx -\frac{10 n \alpha_a(\mu_c)}{\pi} m_{\tilde{G}}. \tag{51}$$

Finally, we address the generality of the result (34) in the broader context of the class of string-derived models that we are considering. It would be modified if modular invariance is broken by string nonperturbative effects, such as a moduli-dependence of the functions $g(V), f(V)$ as was found for a particular orbifold [25]. Modular invariance of the effective Lagrangian for the condensate ensures that the moduli are stabilized at one of the two self-dual points in the fundamental domain: $t^I = 1,\ e^{i\pi/6}$. Together with the condition that the vacuum energy vanishes, this assures that their F-components vanish in the vacuum: $< F^I >= 0$. The potential for the moduli and the dilaton [4] is

$$16\ell^2 V = \left(1 + \ell \frac{dg}{d\ell}\right) |(1 + b_+ \ell)\, u_+|^2$$

$$-3\ell^2 |b_+ u_+|^2 + 4\ell^2 (1 + b\ell) \sum_I \left| \frac{F^I}{\mathrm{Re}t^I} \right|^2 , \tag{52}$$

so the condition (28), responsible for the suppression of gaugino masses if $p_A = 0$, holds only to corrections of order $(< F^I >)^2$. If $< F^I > \neq 0$, there are additional corrections to gaugino masses from:

$$\mathcal{L}_{th} + \mathcal{L}_{GS} + \sum_a \mathcal{L}_a \ni - \sum_{a,I} \frac{b_a^I}{32\pi^2} F^I \left[\frac{\eta'(t^I)}{\eta(t^I)} - \frac{1}{4\mathrm{Re}t^I} \right] . \tag{53}$$

Since the term in brackets vanishes at the self-dual points, both corrections are of order $(< F^I >)^2$, and will be small if the moduli are stabilized near the self-dual points. Moreover, the second contribution is absent in the Z_3, Z_7 orbifolds that appear promising for model building [26]. The condition (28) provides several phenomenologically desirable features [5] of our model, namely moduli masses much larger than the gravitino mass:

$$m_\ell \approx 50 m_{t^I} \approx 10^3 m_{\tilde{G}}, \tag{54}$$

and a suppression of the axion decay constant by a factor of about 50 with respect to earlier estimates [27]. Moreover, the result that $< F^I >= 0$ avoids a potential source of unwanted flavor-changing neutral currents. A detailed analysis of the phenomenology of this class of models will be given elsewhere [21].

In concluding, we wish to emphasize that the gaugino mass contribution equal to $m_0 = \beta(g^2) m_{\tilde{G}}/2g^2$ is a model independent result in our Kähler $U(1)$ superspace formalism, in agreement with the assertion made in [2]. In the formalism of [1], the auxiliary field of the supergravity multiplet differs from the field M used here by a Weyl rotation that depends on the Kähler potential. As a result, the analogous term that they find is not model independent; for example they get no contribution in no-scale models [28]. In our formalism, m_0 in this case is exactly canceled by the contribution from B_a in (13). Consider for example the simplest no-scale model with Kähler potential

$$K = -3 \ln(T + \bar{T} - \sum_A |\Phi^A|^2), \quad < \phi^A >= 0, \quad < W > \neq 0, \tag{55}$$

e.g. $W = \Phi^3 +$ constant. In this model (with no cancellation of the modular anomaly)

$$B_a = \left(C_a - \frac{1}{3}C_a^M\right) K, \quad \left\langle \mathcal{D}^2 B_a | \right\rangle = -\left\langle 4F^T \frac{\partial}{\partial T} B_a \right\rangle \Big|, \tag{56}$$

the vacuum values satisfy

$$F^T K_T = -3e^{K/2}W, \quad R| = \frac{1}{2}e^{K/2}W, \tag{57}$$

and we obtain

$$\left\langle \mathcal{D}^2 \left[f_a(\Box_\chi) - B_a \right] \right\rangle = 0, \tag{58}$$

in agreement with [28]. We find the same cancellation in a PV calculation for this model.

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