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Seong Kon Lee, Hee Joon Kim, Yoonho Song,
Frank H. Morrison, and Ki Ha Lee

Earth Sciences Division

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Seong Kon Lee,[1] Hee Joon Kim,[2] Yoonho Song,[3]
Frank H. Morrison,[4] and Ki Ha Lee[5]

[1]Seoul National University
[2]Pukyong National University
[3]Korean Institute of Geology, Mining and Materials
[4]University of California, Berkeley, California 94720
[5]Lawrence Berkeley National Laboratory

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A new formulation of magnetic field integral equation for 3-D EM modeling

Seong Kon Lee[1]; Hee Joon Kim [2]; Yoonho Song[3], Frank H. Morrison[4], and Ki Ha Lee[5]*
[1]Seoul National University, Seoul 151-742, Korea; [2] Pukyong National University, Pusan 608-737, Korea
[3]Korean Institute of Geology and Mining, Taejon 305-350, Korea; [4] University of California, Berkeley, CA 94720, USA; [5] Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

Summary

An efficient method to solve three-dimensional (3-D) electromagnetic (EM) scattering problems is in great demand for practical applications of EM methods. We have derived a new formulation of a magnetic field integral equation (MFIE) and investigated its theoretical aspects. There are several advantages of using an MFIE in obtaining solutions of 3-D EM problems since magnetic fields vary more smoothly than electric fields in an anomalous body if a non-magnetic medium is considered. The MFIE, derived from the electric field integral equation (EFIE) in a more familiar, and physically more meaningful manner, consists of terms expressed by an equivalent electric current density and magnetic moment source, which are generated by the presence of a conductivity anomaly.

It is also shown that the extended Born approximation to the MFIE results in an identity scattering tensor, which indicates that the Born approximation is more effective using the MFIE than the EFIE. It turns out that the Born and nonlinear Born (Habashy et al., 1993) approximations are the same for the MFIE. This analysis implies that an MFIE solution may be effective for practical 3-D EM modeling.

Introduction

Any rigorous numerical modeling method of three-dimensional (3-D) electromagnetic (EM) problems is time-consuming and takes a large amount of computer memory. As a result, 3-D EM data analysis is very difficult unless the algorithm is implemented on a massively parallel computing machine or a multiple processor environment (Alumbaugh and Newman, 1995). Hence, alternative attempts have been made to seek approximate but good solutions, in most cases, based on the Born approximations. Alumbaugh and Morrison (1993) developed a 2-D iterative Born inversion for imaging the subsurface conductivity distribution. The Born approximation technique, however, tends to be inaccurate for high conductivity contrast and at higher frequencies. To overcome these difficulties, Habashy et al. (1993) published the extended Born approximation method in which the electric fields inside inhomogeneities are approximated by the product of a 'scattering tensor' and the incident field. 2-D and 2.5-D numerical algorithms have been published based on this approximation (Torres-Verdin and Habashy, 1994; 1995) and the method has been successfully extended to 3-D problems (Tseng et al., 1996). Zhdanov and Fang (1996) presented a similar method called 'quasi-linear approximation' in which the electric fields are evaluated by a linear transformation of the primary field and apply this method to inversion. All these methods mentioned above utilize the electric field in the formulation of the problem. Since the normal component of the electric field is discontinuous across the conductivity boundary, the effectiveness of the extended Born approximation, for example, may be limited.

Xie et al. (1995) derived a magnetic field integro-differential equation solution and Xie et al. (1996) applied Born approximations to combined solutions of finite element and integral equation schemes in forward and inverse problems. Lee et al. (1998) presented a preliminary analysis using the MFIE. They showed that the extended Born approximation to MFIE is the Born approximation itself and obtained a convergent iterative Born approximations to MFIE. In this paper, we further modify the MFIE and show that the MFIE has a physical representation in terms of how the magnetic field and 'anomalous' currents are related through the conductivity anomaly in a inhomogeneous medium.

Magnetic Field Integral Equation

Using Maxwell's equation and the dyadic Green's function, one can find the solution of the EM scattering problem in the form of the electric field integral equation (EFIE) (Hohmann, 1975; Tseng et al., 1996)

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_b(\mathbf{r}) - \hat{z}\int_V \underline{\underline{\mathbf{G}}}_{EJ}(\mathbf{r},\mathbf{r}')\cdot\Delta\hat{y}(\mathbf{r}')\mathbf{E}(\mathbf{r}')d\mathbf{r}' \qquad (1)$$

where

- $\mathbf{r}$ is the position where the total electric field is to be measured,
- $\mathbf{r}'$ is a position vector of any point in the anomalous body,
- $\mathbf{E}(\mathbf{r})$ and $\mathbf{E}_b(\mathbf{r})$ are the total and incident electric fields at $\mathbf{r}$, respectively,
- $\underline{\underline{\mathbf{G}}}_{EJ}(\mathbf{r},\mathbf{r}')$ is the electric dyadic Green's function which relates the electric field at $\mathbf{r}$ due to an electric current source at $\mathbf{r}'$,
- $\hat{z}(=i\omega\mu_0)$ is the impedivity, and
- $\Delta\hat{y}=\hat{y}(\mathbf{r}')-\hat{y}_b$ is the admittivity (or conductivity in quasi-static case) difference between the anomalous body and the background medium, and $\hat{y}(\mathbf{r}')=\sigma(\mathbf{r}')+i\omega\varepsilon(\mathbf{r}')$ is

the admittivity at $\mathbf{r}'$ and $\hat{y}_b$ is the admittivity of the background medium, respectively.

Taking curl of both sides of equation (1), Xie et al. (1995) obtained the magnetic field integro-differential equation,

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r}) - \hat{z}\int_V \phi(\mathbf{r}')\,\underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}')\cdot\nabla'\times\mathbf{H}(\mathbf{r}')\,d\mathbf{r}' \tag{2}$$

where

$$\underline{\underline{\mathbf{G}}}_{HJ} = -\nabla\times\underline{\underline{\mathbf{G}}}_{EJ}(\mathbf{r},\mathbf{r}')/\hat{z}, \tag{3}$$

$$\mathbf{S}(\mathbf{r}) = \hat{z}\int_V \phi(\mathbf{r}')\,\underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}_E(\mathbf{r}')\,d\mathbf{r}', \tag{4}$$

with the impressed electric source current density $\mathbf{J}_E$, and the conductivity function defined as

$$\phi(\mathbf{r}') = \frac{\Delta\hat{y}(\mathbf{r}')}{\hat{y}(\mathbf{r}')}. \tag{5}$$

Here we assume that the boundary of the anomalous body is within the volume, so that the conductivity function at the boundary of the volume is zero. For a current dipole located at $\mathbf{r}_s$ with dipole moment $Id\mathbf{l}$, then equation (4) reduces to

$$\mathbf{S}(\mathbf{r}) = \hat{z}\phi(\mathbf{r}_s)\,\underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}_s)\cdot Id\mathbf{l}, \tag{6}$$

which exists only when the conductivity at the source region differs from that of background medium.

Using the identity (Van Bladel, 1984)

$$\begin{aligned}&\nabla'\times\mathbf{H}(\mathbf{r}')\cdot\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}\\ &=\nabla'\cdot\left\{\mathbf{H}(\mathbf{r}')\times\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}\\ &+\mathbf{H}(\mathbf{r}')\cdot\nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}\end{aligned} \tag{7}$$

equation (2) is modified as

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r})\\ &-\hat{z}\int_V \nabla'\cdot\left\{\mathbf{H}(\mathbf{r}')\times\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}'\\ &-\hat{z}\int_V \mathbf{H}(\mathbf{r}')\cdot\nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}',\end{aligned} \tag{8}$$

where superscript T denotes transpose of a dyadic. Applying the divergence theorem to the first integral in the right-hand side of equation (8), and noting that the conductivity function is zero at the boundary, we find

$$\begin{aligned}&\int_V \nabla'\cdot\left\{\mathbf{H}(\mathbf{r}')\times\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}'\\ &=\int_S \mathbf{n}'\cdot\left\{\mathbf{H}(\mathbf{r}')\times\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}' = 0.\end{aligned} \tag{9}$$

Finally, we arrive at the magnetic field integral equation (MFIE) (Lee et al., 1998)

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r}) - \hat{z}\int_V \mathbf{H}(\mathbf{r}')\cdot\nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}'. \tag{10}$$

Now, we will further modify equation (10) to a familiar form that we can work easily with. We first break the integral into two parts, and use the identity (Van Bladel, 1984)

$$\begin{aligned}&\mathbf{H}(\mathbf{r}')\cdot\nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\right\}\\ &=\{\mathbf{H}(\mathbf{r}')\times\nabla'\phi(\mathbf{r}')\}\cdot\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\\ &+\{\mathbf{H}(\mathbf{r}')\phi(\mathbf{r}')\}\cdot\nabla'\times\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}'),\end{aligned} \tag{11}$$

then equation (10) can be rewritten as

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r})\\ &-\hat{z}\int_V \{\mathbf{H}(\mathbf{r}')\times\nabla'\phi(\mathbf{r}')\}\cdot\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')d\mathbf{r}'\\ &-\hat{z}\int_V \{\mathbf{H}(\mathbf{r}')\phi(\mathbf{r}')\}\cdot\nabla'\times\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')\,d\mathbf{r}'.\end{aligned} \tag{12}$$

Using the identity of a transpose of a dyadic

$$\mathbf{a}\cdot\underline{\underline{\mathbf{G}}}^T = \underline{\underline{\mathbf{G}}}\cdot\mathbf{a} \tag{13}$$

the first integral on the right-hand side of equation (12) is rewritten as

$$\begin{aligned}&-\hat{z}\int_V \{\mathbf{H}(\mathbf{r}')\times\nabla'\phi(\mathbf{r}')\}\cdot\underline{\underline{\mathbf{G}}}_{HJ}^T(\mathbf{r},\mathbf{r}')d\mathbf{r}'\\ &=-\hat{z}\int_V \underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}')\cdot\{\mathbf{H}(\mathbf{r}')\times\nabla'\phi(\mathbf{r}')\}d\mathbf{r}'\end{aligned} \tag{14}$$

To modify the second integral of equation (12), we first invoke the general reciprocity theorem

$$\underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}') = -\frac{1}{\hat{z}}\underline{\underline{\mathbf{G}}}^{T}_{EM}(\mathbf{r}',\mathbf{r}), \tag{15}$$

which is valid if the following representation

$$\nabla \times \mathbf{E} = -\hat{z}(\mathbf{H}+\mathbf{M}) \tag{16}$$

is used to include the magnetic moment $\mathbf{M}$ in the derivation of the dyadic Green's function. Then the dyadic part of the second integral in equation (12) is modified to

$$\nabla' \times \underline{\underline{\mathbf{G}}}^{T}_{HJ}(\mathbf{r}',\mathbf{r}) = -\frac{1}{\hat{z}}\nabla' \times \underline{\underline{\mathbf{G}}}_{EM}(\mathbf{r}',\mathbf{r}). \tag{17}$$

Now, since the right-hand side may be substituted by the relationship

$$\nabla' \times \underline{\underline{\mathbf{G}}}_{EM}(\mathbf{r}',\mathbf{r}) = -\hat{z}\underline{\underline{\mathbf{G}}}_{HM}(\mathbf{r}',\mathbf{r}), \tag{18}$$

equation (17) can be further modified to

$$\nabla' \times \underline{\underline{\mathbf{G}}}^{T}_{HJ}(\mathbf{r},\mathbf{r}') = \underline{\underline{\mathbf{G}}}_{HM}(\mathbf{r}',\mathbf{r}). \tag{19}$$

Again, invoking the reciprocity theorem

$$\underline{\underline{\mathbf{G}}}_{HM}(\mathbf{r}',\mathbf{r}) = \underline{\underline{\mathbf{G}}}^{T}_{HM}(\mathbf{r},\mathbf{r}'), \tag{20}$$

one arrives at

$$\nabla' \times \underline{\underline{\mathbf{G}}}^{T}_{HJ}(\mathbf{r},\mathbf{r}') = \underline{\underline{\mathbf{G}}}^{T}_{HM}(\mathbf{r},\mathbf{r}'). \tag{21}$$

Using equations (21) and (13), in that order, we can finally rewrite the second integral of equation (12) as

$$\begin{aligned} &-\hat{z}\int_V \{\mathbf{H}(\mathbf{r}')\phi(\mathbf{r}')\} \cdot \nabla' \times \underline{\underline{\mathbf{G}}}^{T}_{HJ}(\mathbf{r},\mathbf{r}')\, d\mathbf{r}' \\ &= -\hat{z}\int_V \underline{\underline{\mathbf{G}}}_{HM}(\mathbf{r},\mathbf{r}') \cdot \{\mathbf{H}(\mathbf{r}')\phi(\mathbf{r}')\}\, d\mathbf{r}'. \end{aligned} \tag{22}$$

Finally, MFIE (12) takes a familiar form

$$\begin{aligned} \mathbf{H}(\mathbf{r}) &= \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r}) \\ &-\hat{z}\int_V \underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}') \cdot \{\mathbf{H}(\mathbf{r}') \times \nabla' \phi(\mathbf{r}')\}\, d\mathbf{r}' \\ &-\hat{z}\int_V \underline{\underline{\mathbf{G}}}_{HM}(\mathbf{r},\mathbf{r}') \cdot \{\mathbf{H}(\mathbf{r}')\phi(\mathbf{r}')\}\, d\mathbf{r}'. \end{aligned} \tag{23}$$

If we identify an 'excess' electric current source defined by

$$\mathbf{H}(\mathbf{r}') \times \nabla' \phi(\mathbf{r}') = \mathbf{J}_{H\phi}(\mathbf{r}'), \tag{24}$$

and an 'excess' magnetic source

$$\mathbf{H}(\mathbf{r}')\phi(\mathbf{r}') = \mathbf{M}_{H\phi}(\mathbf{r}'), \tag{25}$$

then the MFIE can be simplified to

$$\begin{aligned} \mathbf{H}(\mathbf{r}) &= \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r}) \\ &-\hat{z}\int_V \underline{\underline{\mathbf{G}}}_{HJ}(\mathbf{r},\mathbf{r}') \cdot \mathbf{J}_{H\phi}(\mathbf{r}')\, d\mathbf{r}' \\ &-\hat{z}\int_V \underline{\underline{\mathbf{G}}}_{HM}(\mathbf{r},\mathbf{r}') \cdot \mathbf{M}_{H\phi}(\mathbf{r}')\, d\mathbf{r}'. \end{aligned} \tag{26}$$

From the equations (24) and (25), we notice that the 'excess' electric current source is normal to the gradient of conductivity function, with its amplitude equal to the gradient of the conductivity function times the magnetic field perpendicular to it. On a boundary between elements of different conductivity, for example, the current is the tangential magnetic field times the conductivity anomaly normalized by the conductivity itself, and bounded to the surface. On the other hand, the 'excess' magnetic moment is the magnetic field itself times the conductivity function. It is a volume distribution, and exists whenever there is a conductivity anomaly.

Extended Born approximation to MFIE

Habashy et al. (1993) suggested an alternate way to linearize the EFIE by rearranging equation (1) as

$$\mathbf{E}(\mathbf{r}) \approx \underline{\underline{\Gamma}}(\mathbf{r}) \cdot \mathbf{E}_b(\mathbf{r}), \tag{27}$$

where $\underline{\underline{\Gamma}}(\mathbf{r})$ is called the depolarization tensor or scattering tensor defined as

$$\underline{\underline{\Gamma}}(\mathbf{r}) = \left[\underline{\underline{\mathbf{I}}} + \hat{z}\int_V \underline{\underline{\mathbf{G}}}_{EJ}(\mathbf{r},\mathbf{r}')\Delta\hat{y}(\mathbf{r}')\, d\mathbf{r}'\right]^{-1}, \tag{28}$$

and $\underline{\underline{\mathbf{I}}}$ is the 3×3 identity dyadic.

The above non-linear approximation of EFIE is valid only if the electric field changes smoothly within the inhomogeneity. The electric field, however, can vary

significantly within the inhomogeneity when there is resistivity contrast in the inhomogeneity or electric field normal to this contrast boundary exists. Furthermore, a conventional Born approximation of electric field breaks down when the inhomogeneity is large compared to the wavelength or at higher frequencies. In contrast, the magnetic field within the inhomogeneity is much smoother than the electric field if the medium is non-magnetic.

Applying a non-linear approximation to equation (10), we have

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= \mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r}) \\ &\quad -\hat{z}\int_V \{\mathbf{H}(\mathbf{r}') - \mathbf{H}(\mathbf{r})\}\cdot\nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}^T_{HJ}(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}' \\ &\quad -\hat{z}\,\mathbf{H}(\mathbf{r})\cdot\int_V \nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}^T_{HJ}(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}'. \qquad (29)\end{aligned}$$

With the following identity and radiation condition,

$$\begin{aligned}&\int_V \nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}^T_{HJ}(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}' \\ &= \int_S \mathbf{n}'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}^T_{HJ}(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}' = 0, \qquad (30)\end{aligned}$$

equation (29) can be rewritten as

$$\begin{aligned}\mathbf{H}(\mathbf{r}) &= \underline{\underline{\mathbf{I}}}\cdot[\mathbf{H}_b(\mathbf{r}) + \mathbf{S}(\mathbf{r}) \\ &\quad -\hat{z}\int_V \{\mathbf{H}(\mathbf{r}') - \mathbf{H}(\mathbf{r})\}\cdot\nabla'\times\left\{\phi(\mathbf{r}')\underline{\underline{\mathbf{G}}}^T_{HJ}(\mathbf{r},\mathbf{r}')\right\}d\mathbf{r}'] \qquad (31)\end{aligned}$$

If we ignore the last term in the brace as we do in the extended Born approximation of EFIE, the depolarization tensor in the magnetic formulation becomes the identity tensor. In other words, the non-linear approximation is equivalent to Born approximation in the MFIE. Lee et al. (1998) applied iterative Born approximations to MFIE and obtained convergent solution.

Conclusions

We have derived a useful representation for the MFIE in which two distinctive 'fictitious' scattering currents are clearly identified and responsible for generating the secondary field. It shows that the secondary field is caused by excess electric and magnetic currents generated by conductivity anomaly and the magnetic field itself. The Born approximation to MFIE was shown to be equivalent to the extended Born approximation, and preliminary results (Lee et al., 1998) show that application of iterative Born approximation to MFIE can be effective for EM modeling.

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ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720

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