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Conformal Models of Thirring Type and the Affine Virasoro Construction [1]

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Abstract

We investigate a class of models in 1+1 dimensions with four fermion interaction term. At each order of the perturbation expansion, the models are ultraviolet finite and Lorentz non-invariant. We show that for certain privileged values of the coupling constants, Lorentz symmetry is restored, and indeed the model turns out to be conformally invariant. This phenomenon is both quantum mechanical and non-perturbative.

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1. Introduction

Conformal models have played an important role both in the statistical mechanics of lower dimensional systems and in the construction of viable string theories. In the absence of a systematic approach so far for the classification and the construction of conformal theories, various special models have been proposed and applied to string theory [1,2]. Among these are the conformal models based on the affine Virasoro construction [3,4], which are generalizations of the original Sugawara construction in terms of currents that satisfy an affine algebra. Apart from some isolated cases [5], this construction has so far not found widespread application in string theory. Among the reasons for this is the lack of a good understanding of the local field theory that is at the basis of this construction. Several formulations of the model based on various generalized sigma model type actions have been proposed [6,7,8], but more work is needed to make progress along this direction.

A different approach to the same problem is to start with various generalizations of the Thirring model and investigate possible non-trivial fixed points in the coupling constant space. This approach is motivated by the observation that the interaction in these models is of the form current$\times$current, which is very suggestive of the affine Virasoro construction. The original Thirring model has already been used in string compactification [9]. In their classical work, Dashen and Frishman [10] showed that a non-abelian generalization of the Thirring model symmetric under a Lie group is conformally invariant for certain quantized values of the coupling constant, and at these conformal points, the stress tensor admits a Sugawara construction. More recent work [11,12,13] suggests the possible existence of more general non-symmetric fixed points in the coupling constant space.

In this paper, we shall investigate a different generalization of the Thirring model. The models in question have some unusual properties. Since the coupling constants are dimensionless, one would expect the appearence of the usual renormalizable divergences in perturbation theory. Instead, it turns out that each order of perturbation expansion is ultraviolet finite. Another surprising feature is connected with Lorentz invariance. Superficially, the interaction term in these models seems to violate Lorentz invariance on the world sheet, and if true, this would disqualify them from being of use in string theory. We will however show that, if the coupling constants in the interaction term satisfy the so-called master equation [4], the corresponding

model is conformally invariant, and the stress tensor is given by the affine Virasoro construction. It then follows trivially that, since the conformal group includes Lorentz transformations, contrary to the appearances, the model is also Lorentz invariant. We would like to emphasize that Lorentz invariance is realized non-perturbatively only at the points in the coupling constant space that satisfy the master equation; individual terms in the perturbation expansion violate this symmetry. It is also of some interest to determine the transformation properties of the various fields that appear in the model under the Lorentz group, since in general fields could transform non-lineary in a complicated fashion. We have investigated the transformation properties of the basic fermion fields and the currents, which are bilinear composites of the fermions. It turns out that the fermions transform linearly, with, however an anomalous coefficient for the "spin" term. The currents also transform linearly, with a different coefficient for the spin term. Again, these simple transformation properties hold only at the conformal points in the space of coupling constants.

The paper is organized as follows. In section 2, we will introduce the model and argue for the lack of ultraviolet divergences in perturbation theory. In section 3, we will review free fields and the affine Virasoro construction. In section 4, we will show that if the coupling constants satisfy the master equation, the model is conformally invariant. The demonstration is based on the calculation of the operator form of the stress tensor in the standard interaction representation. In section 4, the Lorentz transformation properties of the fermions and the currents will be determined. Finally, the last section will summarize our conclusions.

2. The Model

The model is based on the following action:

$$I = I_0 + I', \tag{1}$$

where,

$$I_0 = \frac{i}{2} \int d^2\sigma \, \bar{\psi}^a \gamma^\mu \partial_\mu \psi^a, \tag{2}$$

and,

$$I' = \int d^2\sigma \left(g \, c_{ij} \, J^i_+ J^j_+ + g' \, c'_{ij} \, J^i_- J^j_- \right). \tag{3}$$

In these equations, ψ^a's are two component Majorana spinors in 1+1 dimensions. As is usual in string theory, the time coordinate is denoted by τ and

the space coordinate by σ. Again in keeping with the string usage, we will take σ to be compact and to range from 0 to 2π, although this is not important for most of the subsequent development. The coupling constants c_{ij} and c'_{ij} that appear in the interaction are real and symmetric in i and j. We have also introduced two redundant constant g and g' for later convenience. The currents $J^i_\pm$ of definite chirality are constructed from chiral fermions $\psi^a_\pm$:

$$J^i_\pm = \frac{1}{4}\psi^a_\pm \, \lambda^i_{ab} \, \psi^b_\pm, \tag{4}$$

where λ^i are matrices which act on the internal space labeled by a and b. They satisfy the commutation relations

$$[\lambda^i, \lambda^j] = i f^{ijk} \lambda^k,$$

and generate some Lie algebra. In what follows, we will take this algebra to be semi-simple with the metric given by identity, so that there will be no need to distinguish between upper and lower indices.

We shall now argue that the perturbation expansion for these models is ultraviolet finite. To start with, it is clear that the + and − chiralities never mix, and therefore they can be considered seperately in the perturbation expansion. As a simple example, consider the one loop contribution to the + chirality fermion-fermion scattering. If p is the total external momentum, suppressing all the dependence on the internal space indices and overall constants, one encounters a potentially divergent integral of the form

$$\begin{aligned} M &\cong \int \frac{d^2k}{(2\pi)^2} \frac{(k_0 - k_1)(p_0 - k_0 - p_1 + k_1)}{k^2(p-k)^2} \\ &= i \int_0^1 d\alpha \int_0^\infty k dk \int_0^{2\pi} d\theta \frac{\left(ike^{i\theta} + \alpha(p_0 - p_1)\right)\left((1-\alpha)(p_0 - p_1) - ike^{i\theta}\right)}{\left(k^2 + (\alpha^2 - \alpha)p^2\right)^2} \\ &= -\frac{i}{2}\frac{(p_0 - p_1)^2}{p^2}. \end{aligned} \tag{5}$$

The integral, which superficially appeared to be logarithmically divergent, is actually convergent. This is because the k dependent terms in the numerator on the second line of the equation, which would normally lead to a divergent integral, all vanish after the integration over the angle θ. Since the fermions in the Feynman graph all have positive chirality, the propagators always carry a factor $k_0 - k_1$, which after Euclidean rotation turns into $ik\, e^{i\theta}$. The important

point is that factors of k in the numerator always appear in the combination $k\,e^{i\theta}$ and, as a result, all of the θ dependent terms vanish upon integration. But since these are the only possible divergent terms, the integral must be finite. It is easy to see that this argument works also for higher order graphs, and it follows that all of them are finite. A similar argument, with a change of the sign of θ, shows that all of the graphs with negative chirality fermions are also finite. The only potentially divergent graphs are the ones that contain both positive and negative chirality fermions, but because of the form of the interaction (eq.(3)), there are no graphs of this type.

The price paid for the finiteness of the model is the loss of Lorentz invariance, at least in the perturbation expansion. Lorentz invariant interactions must conserve chirality, which is not the case in our model. One can check the breakdown of Lorentz invariance explicitly in the case fermion-fermion scattering process discussed above. Higher order graphs for this process will yield an answer proportional to a factor of the form

$$\left(\frac{(p_0 - p_1)^2}{p^2}\right)^n$$

where the integer n will depend on the order of perturbation theory. Since the above factor scales under Lorentz transformations, different orders in perturbation expansion will have different Lorentz transformation properties. This only means, however, that there is no Lorentz invariance for arbitrary values of the coupling constants. In section 4, we will show that, for special values of the coupling constants, Lorentz invariance is restored.

3. Free Field Constructions

In this section, we will review the free field limit of the model, with g set equal to zero, and introduce the affine Virasoro construction as a preparation for the next section. The free fermions of definite chirality, $\psi^a_{0,\pm}$, depend on the coordinates through the combinations $\sigma \mp \tau$, and they satisfy the following commutation relations:

$$[\psi^a_{0,\pm}(z), \psi^b_{0,\pm}(z')] = \delta_{a,b}\,\delta(z - z'). \tag{6}$$

In this equation, z stand for $\sigma - \tau$ for the + components and for $\tau + \sigma$ for the − components. Free currents $J^i_{0,\pm}$ are constructed from free fermions as in eq.(4), with a normal ordering prescription. They satisfy the commutation

relations

$$[J^i_{0,\pm}(z), J^j_{0,\pm}(z')] = i f_{ijk}\, \delta(z-z') - \frac{i\kappa}{2\pi}\, \delta_{i,j}\, \delta'(z-z'), \tag{7}$$

where z has the same meaning as before. The constant κ is the coefficient of the central term, and, given the representation matrices λ^i, it can easily be calculated.

Another quantity of interest is the stress tensor of the free field theory. Since there is no mass term, the stress tensor is traceless, and and the two independent components can be conveniently taken to be

$$T_{0,\pm} = \frac{1}{2}\left(T_0^{0,0} \pm T_0^{0,1}\right).$$

In terms of free fermions, they are given by the normal ordered expression

$$T_{0,\pm} = \pm\frac{i}{4}\left(: \partial_\sigma \psi^a_{0,\pm}\, \psi^a_{0,\pm} : - : \psi^a_{0,\pm}\, \partial_\sigma \psi^a_{0,\pm} :\right). \tag{8}$$

$T_{0,\pm}$ satisfy the conservation equations

$$(\partial_\tau + \partial_\sigma) T_{0,+} = (\partial_\tau - \partial_\sigma) T_{0,-} = 0, \tag{9}$$

and therefore they are functions of only the variables $\tau \mp \sigma$ respectively. We also note that their commutators generate the Virasoro algebra:

$$[T_{0,\pm}(z), T_{0,\pm}(z')] = \pm i\delta'(z-z')\left(T_{0,\pm}(z) + T_{0,\pm}(z')\right) + \frac{ic_0}{2\pi}(\partial_z^3 + \partial_z)\delta(z-z'), \tag{10}$$

where $z = \sigma \mp \tau$ as before. The numerical value of the coefficient c_0 of the central term will not be needed.

After this discussion of free fields, we will briefly review the affine Virasoro construction and the master equation. The affine Virasoro construction is an ansatz for the stress tensor in terms of free currents:

$$\begin{aligned} L_+(z) &= c_{ij} : J^i_{0,+}(z) J^j_{0,+}(z) :, \\ L_-(z) &= c'_{ij} : J^i_{0,-}(z) J^j_{0,-}(z) :, \end{aligned} \tag{11}$$

where the double dots imply normal ordering in order to have a well defined product of the currents. The basic idea is to require $L_\pm(z)$ to satisfy the Virasoro algebra

$$[L_\pm(z), L_\pm(z')] = \mp i\delta'(z-z')\left(L_\pm(z) + L_\pm(z')\right) + \frac{ic}{2\pi}(\partial_z^3 + \partial_z)\delta(z-z'), \tag{12}$$

given that the currents satisfy the commutation relations of eq.(7). It can be shown that [4] this leads to the following equation for the constants c_{ij}:

$$c_{ij} = 2\kappa c_{ik}c_{kj} - f_{kli}f_{k'l'j}c_{kk'}c_{ll'} - f_{kk'l}f_{ll'j}c_{ik}c_{k'l'} - f_{l'k'l}f_{kli}c_{kl'}c_{k'j}, \qquad (13)$$

with a similar equation for c'_{ij}. A large number of solutions to eq.(13) with real and symmetric c's are known [4]. Any one of them would be satisfactory for our purposes.

Another commutation relation that will be needed in the future is

$$[T_{0,\pm}(z), L_{\pm}(z')] = \pm i\delta(z-z')L'_{\pm}(z) \mp 2i\delta'(z-z')L_{\pm}(z'), \qquad (14)$$

which follows from free field commutation relation after the use of eqs.(8) and (11).

The use of the same symbols c_{ij} and c'_{ij} in both the above equation and in eq.(3) was not accidental; from now on we will fix the c's as well as the c''s that appear in the interaction term in eq.(3) to be a real and symmetric solution to their respective master equations. For the time being, g and g' are arbitrary; later, they will also be fixed.

4. The Interaction

After having fixed the constants in the interaction term, we are going to study the model in the interaction representation. Our goal is to establish the conformal invariance of the model. Since the model is translation invariant, one can easily construct the translation operators P^0 and P^1 by the usual Noether procedure. However, since the model is not manifestly Lorentz invariant, this stress tensor is not symmetrical, and the existence of the generators of the Lorentz group, let alone the conformal group, is problematic. Since we can no longer employ to the Noether construction, we will instead show that the stress tensor can be determined uniquely by appealing to the following principles:

a) The stress tensor should be local function of the coordinates.

b) It should be symmetric and traceless.

c) The components

$$T_{\pm} = \frac{1}{2}(T^{0,0} \pm T^{0,1})$$

should satisfy the conservation equations (9).

d) The energy and momentum operators should be given by the standard

expressions

$$P^{\pm} = \frac{1}{2}(P^0 \pm P^1) = \int d\sigma T^{\pm}. \tag{15}$$

The conditions we have listed above refer to operators in the Heisenberg picture. However, for technical reasons, we have found it convenient to go through the intermediate step of the interaction representation. The great advantage of this picture is that all the manipulations involve only free fields.

We remind the reader of a few well known facts about the interaction representation. In this picture, the field ψ is taken to be the free field ψ_0, and the states satisfy the Schroedinger equation

$$i\partial_\tau|\tau\rangle = H_I(\tau)|\tau\rangle, \tag{16}$$

where the interaction Hamiltonian H_I is given in terms free currents by

$$\begin{aligned} H_I(\tau) &= -\int d\sigma\Big(g\, c_{ij} \,:\, J^i_{0,+}(\tau-\sigma)J^j_{0,+}(\tau-\sigma) : \\ &+ g'\, c'_{ij} \,:\, J^i_{0,-}(\tau+\sigma)J^j_{0,-}(\tau+\sigma) : \Big) \\ &= -\int d\sigma\, (g\, L_+(\tau-\sigma) + g'\, L_-(\tau+\sigma)) \\ &= -\int d\sigma\, (g\, L_+(\sigma) + g'\, L_-(\sigma))\,. \end{aligned} \tag{17}$$

Actually, we are interested in the fields expressed in the Heisenbeg picture, but we find it advantageous to rewrite them in terms free fields of the interaction picture. For this purpose, we need the Dyson operator $U(\tau, 0)$, which governs the time development of the states in the interaction representation:

$$|\tau\rangle = U(\tau, 0)|\tau = 0\rangle.$$

From its definition, this operator satisfies

$$\begin{aligned} i\partial_\tau U(\tau, 0) &= H_I(\tau)U(\tau, 0), \\ U(\tau = 0, 0) &= 1, \end{aligned} \tag{18}$$

where the Heisenberg and the interaction pictures are taken to coincide at time $\tau = 0$. In our case, the above equation is easily integrated since $H_I(\tau)$ given by eq.(17) is τ independent:

$$U(\tau, 0) = \exp(-i\tau H_I). \tag{19}$$

A general field operator $\phi(\tau,\sigma)$ in the Heisenberg picture can be expressed in terms of the same operator $\phi_I(\tau,\sigma)$, in the interaction picture by the equation

$$\phi(\tau,\sigma) = U^{-1}(\tau,0)\phi_I(\tau,\sigma)U(\tau,0). \tag{20}$$

ϕ_I is either a free field, or for a composite operator like the currents or the stress tensor, it is a product of free fields. In what follows, we will specify various operators of interest in the interaction picture in terms of free fields, and attach to them an index I in order to distinguish them from the Heisenberg operators, which will be free of this index. The latter can then be constructed explicitly through eqs.(19) and (20).

Having gotten these preliminaries out of way, we are ready to specify the combination of components $T_I^{\pm}$ in the interaction picture. The unique solution that is local and that satisfies the energy and momentum conditions of eq.(15) is,

$$\begin{aligned} T_{I,+} &= T_{0,+}(\sigma-\tau) - \frac{g}{2}L_+(\sigma-\tau) - \frac{g'}{2}L_-(\sigma+\tau), \\ T_{I,-} &= T_{0,-}(\sigma+\tau) - \frac{g}{2}L_+(\sigma-\tau) - \frac{g'}{2}L_-(\sigma+\tau). \end{aligned} \tag{21}$$

As usual, the + components of free fields depend only on the variable $\sigma-\tau$ and the − components on the variable $\sigma+\tau$. The remaining components of T_I can then be solved for using the symmetry and the zero trace condition, and therefore, these conditions are automatically satisfied.

Now that we have the stress tensor in the interaction picture, we can translate it into the Heisenberg picture. It is easy to carry out the calculation explicitly. For example,

$$\begin{aligned} i\partial_\tau \left(U^{-1}(\tau,0)L_+(z)U(\tau,0)\right) &= U^{-1}(\tau,0)[L_+(z),H_I]U(\tau,0) \\ &= igL'_+(z), \end{aligned} \tag{22}$$

where, in the last step, the expression for H_I in terms of $L_\pm$ (eq.(17)) and the commutation relations (12) were used. This equation, and a similar one for $L_-(z)$, have the solutions

$$\begin{aligned} U^{-1}(\tau,0)L_+(z)U(\tau,0) &= L_+(z+g\tau), \\ U^{-1}(\tau,0)L_-(z)U(\tau,0) &= L_-(z-g'\tau). \end{aligned} \tag{23}$$

Proceeding in the same fashion, we have,

$$i\partial_\tau \left(U^{-1}(\tau,0)T_{0,+}(z)U(\tau,0)\right) = ig\, U^{-1}(\tau,0)L'_+(z)U(\tau,0) = ig\, L'_+(z+g\tau). \tag{24}$$

This equation, and the corresponding one for $T_{0,-}$, have the solutions

$$\begin{aligned} U^{-1}(\tau,0)T_{0,+}(z)U(\tau,0) &= L_+(z+g\tau) - L_+(z) + T_{0,+}(z), \\ U^{-1}(\tau,0)T_{0,-}(z)U(\tau,0) &= L_-(z-g'\tau) - L_-(z) + T_{0,-}(z). \end{aligned} \tag{25}$$

Putting everything together, we can convert the stress tensor in the interaction picture given by eq.(21) into the Heisenberg picture:

$$\begin{aligned} T_+(\tau,\sigma) &= (1-\frac{g}{2})L_+(\sigma-\tau+g\tau) - L_+(\sigma-\tau) - \frac{g'}{2}L_-(\sigma+\tau-g'\tau) \\ &+ T_{0,+}(\sigma-\tau), \\ T_-(\tau,\sigma) &= (1-\frac{g'}{2})L_-(\sigma+\tau-g'\tau) + L_-(\sigma+\tau) - \frac{g}{2}L_+(\sigma-\tau+g\tau) \\ &+ T_{0,-}(\sigma+\tau). \end{aligned} \tag{26}$$

According to the conservation equations, T_+ should be a function of only $\sigma-\tau$ and T_- should be a function of only $\sigma+\tau$. This requirement fixes the coupling constants to be the following four combinations:

$$g = 0, 2 \quad g' = 0, 2. \tag{27}$$

The zero values for the coupling constants correspond to the trivial free field solutions. We exhibit below the solution $g = g' = 2$:

$$\begin{aligned} T_+(\tau,\sigma) &= T_{0,+}(\sigma-\tau) - L_+(\sigma-\tau) - L_-(\sigma-\tau), \\ T_-(\tau,\sigma) &= T_{0,-}(\sigma+\tau) - L_-(\sigma+\tau) - L_+(\sigma+\tau). \end{aligned} \tag{28}$$

As a further confirmation of conformal invariance, one can easily show that $T_\pm$ satisfy the Virasoro algebra (eq.(12)). It is also easy to check that the model has Poincare invariance. The single Lorentz generator is given by the standard expression

$$M(\tau) = \int d\sigma\, ((\sigma-\tau)T_+ + (\sigma+\tau)T_-), \tag{29}$$

and, using the Virasoro algebra, the Poincare commutation relations

$$[M, P^\pm] = \pm iP^\pm,$$

are easily verified. Here we have the interesting situation of the restauration of Lorentz symmetry as a result of quantum effects in a model that violates this symmetry classically. This is a non-perturbative phenomenon that happens only for certain fixed values of the coupling constants.

We have just seen that the stress tensor has a simple exact expression in terms of free fields, even though the model is interacting. This simplification only occurs at the conformal points, with c's fixed by the master equation (13) and g's fixed by (27). Another set of fields that are exactly calculable in terms of free fields are $L_\pm$ (see eq.(23)). However, as far as we know, no other fields enjoy this property even when the model is conformal.

5. Lorentz Transformations

It is of some interest to find the transformation properties of the fermion fields and the currents under Lorentz transformations. In what follows, we will set $g = g' = 2$ and focus on the + chirality fields; the calculation for the − chirality fields is entirely analogous. To find the transformation law of ψ_+, for example, one has to compute its commutator with the Lorentz generator M of eq.(29). It is easiest to do this calculation at $\tau = 0$; at this point, $U = 1$, and the Heisenberg and the interaction pictures coincide. One can then carry out the computation in the interaction picture using free fields. The equal time commutator of the fermion field with the stress tensor has the form

$$\begin{aligned}&[\psi_{0,+}(\tau,\sigma), T_{I,+}(\tau,\sigma')] = \delta(\sigma-\sigma')A(\tau,\sigma)\\&+\delta'(\sigma-\sigma')\left(B(\tau,\sigma)+B(\tau,\sigma')\right),\end{aligned} \tag{30}$$

where A and B will be calculated below. Given this result, the commutators of ψ_+ with the Poincare generators at $\tau = 0$ are easily found to be

$$\begin{aligned}[\psi_{0,+}(0,\sigma), P^+] &= [\psi_{0,+}(0,\sigma), \int d\sigma' T_{I,+}(0,\sigma')]\\&= A(0,\sigma) + \partial_\sigma B(0,\sigma),\\[\psi_{0,+}(0,\sigma), M(0)] &= [\psi_{0,+}(0,\sigma), \int d\sigma'\sigma' T_{I,+}(0,\sigma')]\\&= \sigma\, A(0,\sigma) + B(0,\sigma) + \sigma\,\partial_\sigma B(0,\sigma),\end{aligned} \tag{31}$$

leading to the result

$$[\psi_{0,+}(0,\sigma), M(0)] = \sigma\,[\psi_{0,+}(0,\sigma), P^+] + B(0,\sigma). \tag{32}$$

The first term on the right comes from the transformation of the coordinates, since

$$\frac{i}{2}(\partial_\tau \mp \partial_\sigma)\psi = [\psi, P^\pm].$$

The second term, B, is then the spin transformation term. This is also the term that determines the conformal weight of ψ.

It remains to find what A and B are. From eq.(21), we see that we need the commutators of ψ_+ with $T_{0,+}$ and L_+. The first commutator contributes the following terms to A and B:

$$A_1 = -\frac{3i}{4}\partial_\sigma \psi_{0,+}, \quad B_1 = -\frac{i}{4}\psi_{0,+}. \tag{33}$$

Next, we need the commutator of ψ_+ with L_+. This part of the computation is a bit more involved, since the expression for L_+ given by (11) has to be regularized, and for this purpose, we found it convenient to use the operator product expansion. We will first calculate the OPE of $\psi_{0,+}(z)$ with $L_+(z')$, where $z = \sigma - \tau$, and then convert the result into the equivalent result for the commutator. In this approach, it is natural to regulate L_+ by point splitting. We let

$$L_+(z) \to c_{ij}\, J^i_{0,+}(z+\epsilon) J^j_{0,+}(z-\epsilon), \tag{34}$$

subtract the term singular in ϵ, and let $\epsilon \to 0$ at the end. In this case, because of the symmetry of c_{ij} in i and j, the singular term does not contribute, so one can forget about it.

Starting with the basic OPE

$$\psi_{0,+}(z)\, J^i_{0,+}(z') \cong \frac{1}{8\pi i}\frac{1}{z-z'}\left(\lambda^i \psi_{0,+}(z) + \lambda^i \psi_{0,+}(z')\right), \tag{35}$$

from(34), we have,

$$\begin{aligned}\psi_{0,+}(z)\, L_+(z') \cong{}& \frac{c_{ij}}{8\pi i}\frac{1}{z-z'-\epsilon}\lambda^i\left(\psi_{0,+}(z) + \psi_{0,+}(z'+\epsilon)\right) J^j_{0,+}(z'-\epsilon) \\ &+ \frac{c_{ij}}{8\pi i}\frac{1}{z-z'+\epsilon} J^i_{0,+}(z'+\epsilon)\lambda^j\left(\psi_{0,+}(z) + \psi_{0,+}(z'-\epsilon)\right).\end{aligned} \tag{36}$$

One has to apply the OPE (35) once more to the products of the form $J_{0,+}\psi_{0,+}$ on the right hand side of this equation. After that, the limit $\epsilon \to 0$ can be

taken without encountering any singularities:

$$\psi_{0,+}(z)\, L_+(z') = \frac{1}{2}\frac{c_{ij}}{(8\pi i)^2}\frac{1}{z-z'}\lambda^i\lambda^j\left(\psi'_{0,+}(z)+\psi'_{0,+}(z')\right)$$
$$+\frac{3c_{ij}}{(8\pi i)^2}\frac{1}{(z-z')^2}\lambda^i\lambda^j\left(\psi_{0,+}(z)+\psi_{0,+}(z')\right)$$
$$+\frac{c_{ij}}{4\pi i}\frac{1}{z-z'}\left(:J^i_{0,+}(z)\lambda^j\psi_{0,+}(z):+:J^i_{0,+}(z')\lambda^j\psi_{0,+}(z')\right). \qquad (37)$$

The double dots around the composite operators indicate that these terms have been regulated by subtracting the short distance singularity. By applying the usual translation

$$\frac{1}{2\pi i}\frac{1}{z-z'} \to \delta(z-z'),$$

The contributions from the commutator of $\psi_{0,+}$ with L_+ to A and B are

$$\begin{aligned} A_2 &= c_{ij}\left(\frac{i}{32\pi}\lambda^i\lambda^j\partial_\sigma\psi_{0,+} - :J^i_{0,+}\lambda^j\psi_{0,+}:\right), \\ B_2 &= -\frac{3i}{32\pi}c_{ij}\,\lambda^i\lambda^j\psi_{0,+}. \end{aligned} \qquad (38)$$

The total values of A and B are obtained by adding up these results:

$$A = A_1 + A_2, \quad B = B_1 + B_2,$$

and we finally have the following transformation law for ψ_+:

$$[\psi_+(\tau,\sigma), M(\tau)] = \frac{i}{2}(\sigma-\tau)(\partial_\tau - \partial_\sigma)\psi_+$$
$$-\frac{i}{4}\left(1+\frac{3}{\pi}c_{ij}\,\lambda^i\lambda^j\right)\psi_+. \qquad (39)$$

A similar calculation for the current J^k_+ gives

$$[J^k_+(\tau,\sigma), M(\tau)] = \frac{i}{2}(\sigma-\tau)(\partial_\tau - \partial_\sigma)J^k_+$$
$$-iJ^k_+ + \frac{i}{\pi}\left(4\kappa\, c_{km} - 3c_{ij}\, f_{ikl}\, f_{jml}\right)J^m_+. \qquad (40)$$

These equations show that both the fundamental fermion and the current transform linearly, but they have anamolous spin terms. This is equivalent

to the existence of anomalous conformal dimensions. Fields with definite transformation properties are obtained by diagonalizing the matrices that appear in these equations.

6. Conclusions

We have presented a simple model in 1+1 dimensions with a four fermion interaction term. Classically, the interaction term seemed to violate Lorentz invariance. We have shown that, quantum mechanically, for values of the coupling constants satisfying the master equation, the model is not only Lorentz invariant, but it is conformally invariant as well. Apart from their intrinsic interest of these models, this opens the possibility of utilizing them for string compactification.

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