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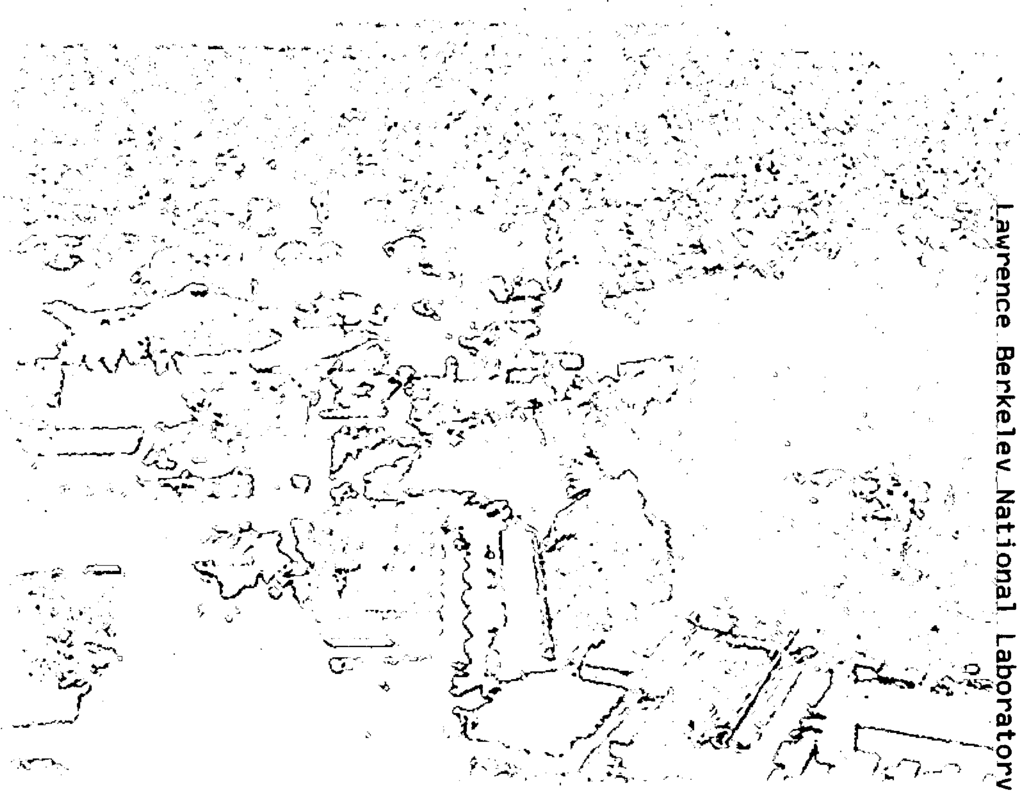

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Neutrino Mass Anarchy

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What is the form of the neutrino mass matrix which governs the oscillations of the atmospheric and solar neutrinos? Features of the data have led to a dominant viewpoint where the mass matrix has an ordered, regulated pattern, perhaps dictated by a flavor symmetry. We challenge this viewpoint, and demonstrate that the data are well accounted for by a neutrino mass matrix which appears to have random entries.

1 Neutrinos are the most poorly understood among known elementary particles, and have important consequences in particle and nuclear physics, astrophysics and cosmology. Special interests are devoted to neutrino oscillations, which, if they exist, imply physics beyond the standard model of particle physics, in particular neutrino masses. The SuperKamiokande data on the angular dependence of the atmospheric neutrino flux provides strong evidence for neutrino oscillations, with ν_μ disappearance via large, near maximal mixing, and $\Delta m^2_{atm} \approx 10^{-3}$ eV^2 [1]. Several measurements of the solar neutrino flux can also be interpreted as neutrino oscillations, via ν_e disappearance [2]. While a variety of $\Delta m^2_\odot$ and mixing angles fit the data, in most cases $\Delta m^2_\odot$ is considerably lower than Δm^2_{atm}, and even in the case of the large angle MSW solution, the data typically require $\Delta m^2_\odot \approx 0.1 \Delta m^2_{atm}$ [3]. The neutrino mass matrix apparently has an ordered, hierarchical form for the eigenvalues, even though it has a structure allowing large mixing angles.

All attempts at explaining atmospheric and solar neutrino fluxes in terms of neutrino oscillations have resorted to some form of ordered, highly structured neutrino mass matrix [4]. These structures take the form $M_0 + \epsilon M_1 + ...$, where the zeroth order mass matrix, M_0, contains the largest non-zero entries, but has many zero entries, while the first order correction terms, ϵM_1, have their own definite texture, and are regulated in size by a small parameter ϵ. Frequently the pattern of the zeroth order matrix is governed by a flavor symmetry, and the hierarchy of mass eigenvalues result from carefully-chosen, small, symmetry-breaking parameters, such as ϵ. Such schemes are able to account for both a hierarchical pattern of eigenvalues, and order unity, sometimes maximal, mixing. Mass matrices have also been proposed where precise numerical ratios of different entries lead to the desired hierarchy and mixing.

In this letter we propose an alternative view. This new view selects the large angle MSW solution of the solar neutrino problem, which is preferred by the day to night time flux ratio at the 2σ level [2]. While the masses and mixings of the charged fermions certainly imply regulated, hierarchical mass matrices, we find the necessity for an ordered structure in the neutrino sector to be less obvious. Large mixing angles would result from a random, structureless matrix, and such large angles could be responsible for solar as well as atmospheric oscillations. Furthermore, in this case the hierarchy of Δm^2 need only be an order of magnitude, much less extreme than for the charged fermions. We therefore propose that the underlying theory of nature has dynamics which produces a neutrino mass matrix which, from the viewpoint of the low energy effective theory, displays *anarchy*: all entries are comparable, no pattern or structure is easily discernable, and there are no special precise ratios between any entries. Certainly the form of this mass matrix is not governed by approximate flavor symmetries.

There are four simple arguments against such a proposal

- The neutrino sector exhibits a hierarchy with $\Delta m^2_\odot \approx 10^{-5} - 10^{-3}$eV^2 for the large mixing angle solution, while $\Delta m^2_{atm} \approx 10^{-3} - 10^{-2}$eV^2,
- Reactor studies of $\overline{\nu}_e$ at the CHOOZ experiment have indicated that mixing of ν_e in the 10^{-3}eV^2 channel is small [5], requiring at least one small angle,
- Even though large mixing would typically be expected from anarchy, *maximal* or near maximal mixing, as preferred by SuperKamiokande data, would be unlikely,
- ν_e, ν_μ and ν_τ fall into doublets with e_L, μ_L and τ_L, respectively, whose masses are extremely hierarchical ($m_e : m_\mu : m_\tau \approx 10^{-4} : 10^{-1} : 1$).

By studying a sample of randomly generated neutrino mass matrices, we demonstrate that each of these arguments is weak, and that, even when taken together, the possibility of neutrino mass anarchy still appears quite plausible.

2 We have performed an analysis of a sample of random neutrino matrices. We investigated three types of neutrino mass matrices: Majorana, Dirac and seesaw. For the Majorana type, we considered 3×3 symmetric matrices with 6 uncorrelated parameters. For the Dirac type, we considered 3×3 matrices with 9 uncorrelated parameters. Lastly, for the seesaw type, we considered matrices of the form $M_D M_{RR}^{-1} M_D^T$ [6], where M_{RR} is of the former type and M_D is of the latter. We ran one million sample matrices with independently generated elements, each with a uniform distribution in the interval $[-1, 1]$ for each matrix type: Dirac, Majorana and seesaw.

To check the robustness of the analysis, we ran smaller sets using a distribution with the logarithm base ten uniformly distributed in the interval $[-1/2, 1/2]$ and with random sign. We further checked both of these distributions but with a phase uniformly distributed in $[0, 2\pi]$. Introducing a logarithmic distribution and phases did not significantly affect our results (within a factor of two), and hence we discuss only matrices with a linear distribution and real entries.

We make no claim that our distribution is somehow physical, nor do we make strong quantitative claims about the confidence intervals of various parameters. However, if the basic prejudices against anarchy fail in these simple distributions, we see no reason to cling to them.

In each case we generated a random neutrino mass matrix, which we diagonalized with a matrix U. We then investigated the following quantities:

$$R \equiv \Delta m_{12}^2 / \Delta m_{23}^2, \tag{1}$$

$$s_C \equiv 4|U_{e3}|^2(1 - |U_{e3}|^2), \tag{2}$$

$$s_{atm} \equiv 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2), \tag{3}$$

$$s_\odot \equiv 4|U_{e2}|^2 |U_{e1}|^2, \tag{4}$$

where Δm_{12}^2 is the smallest splitting and Δm_{23}^2 is the next largest splitting. What ranges of values for these parameters should we demand from our matrices? We could require they lie within the experimentally preferred region. However, as experiments improve and these regions contract, the probability that a random matrix will satisfy this goes to zero. Thus we are instead interested in mass matrices that satisfy certain *qualitative* properties. For our numerical study we select these properties by the specific cuts

- $R < 1/10$ to achieve a large hierarchy in the Δm^2.
- $s_C < 0.15$ to enforce small ν_e mixing through this Δm^2.
- $s_{atm} > 0.5$ for large atmospheric mixing.
- $s_\odot > 0.5$ for large solar mixing.

The results of subjecting our 10^6 sample matrices, of Dirac, Majorana and seesaw types, to all possible combinations of these cuts is shown in Table I. First consider making a single cut. As expected, for all types of matrices, a large percentage (from 18% to 21%) of the random matrices pass the large mixing angle solar cut, and similarly for the large mixing angle atmospheric cut (from 59% to 71%). Much more surprising, and contrary to conventional wisdom, is the relatively large percentage passing the individual cuts for R (from 10% to 64%) and for s_C (from 12% to 18%). The distribution for R is shown in Figure 1. Naively, one might expect that this would peak at $R = 1$, which is largely the case for Dirac matrices, although with a wide peak. In the Majorana case there is an appreciable fraction ($\sim 20\%$) that have a splitting $R \leq 1/10$, while in the seesaw scenario the *majority* of cases ($\sim 64\%$) have a splitting $R \leq 1/10$ — it is not at all unusual to generate a large hierarchy.

We can understand this simply: first a splitting of a factor of 10 in the Δm^2's corresponds to only a factor of 3 in the masses themselves if they happen to be hierarchically arranged. Secondly, in the seesaw scenario,

Dirac	no cuts	s_{atm}	$s_\odot$	$s_{atm} + s_\odot$
no cuts	1,000,000	671,701	184,128	135,782
s_C	145,000	97,027	66,311	45,810
R	106,771	78,303	17,538	14,269
$s_C + R$	12,077	9,067	5,656	4,375

Majorana	no cuts	s_{atm}	$s_\odot$	$s_{atm} + s_\odot$
no cuts	1,000,000	709,076	200,987	164,198
s_C	121,129	91,269	70,350	56,391
R	200,452	149,140	37,238	31,708
$s_C + R$	21,414	16,507	12,133	10,027

seesaw	no cuts	s_{atm}	$s_\odot$	$s_{atm} + s_\odot$
no cuts	1,000,000	594,823	210,727	133,800
s_C	186,684	101,665	86,511	49,787
R	643,394	390,043	132,649	86,302
$s_C + R$	115,614	64,558	53,430	31,547

TABLE I. Mass matrices satisfying various sets of cuts for the real linear Dirac, Majorana and seesaw scenarios.

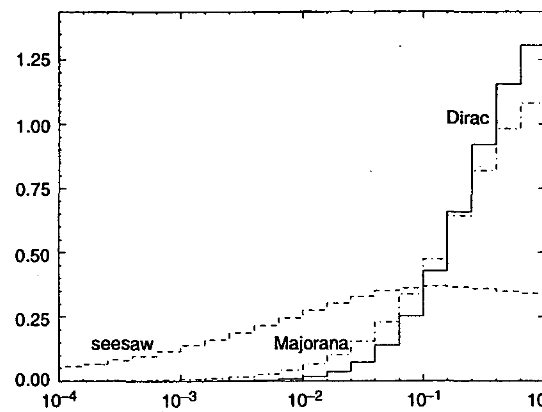

FIG. 1. The distribution of $\Delta m_\odot^2 / \Delta m_{atm}^2$ for Dirac (solid) Majorana (dot-dashed) and seesaw (dashed) scenarios.

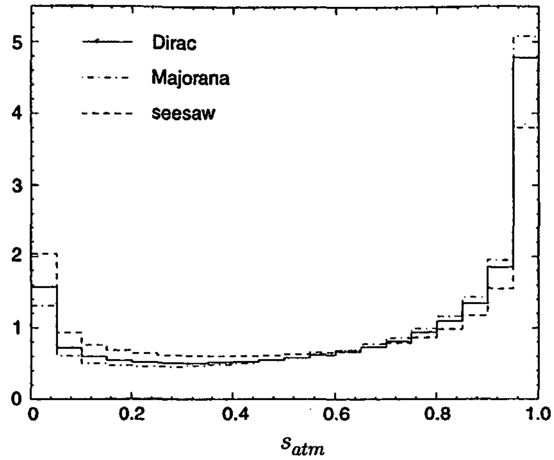

FIG. 2. Plots of the normalized, binned distributions of s_{atm} for Dirac, Majorana and seesaw cases. Contrary to intuition, the distributions actually peak at large s_{atm}.

taking the product of three matrices spreads the Δm^2 distribution over a wide range.

While one would expect random matrices to typically give large atmospheric mixing, is it plausible that they would give near-maximal mixing, as required by the SuperKamiokande data? In Figure 2 we show distributions of s_{atm}, which actually *peak* in the $0.95 < s_{atm} < 1.0$ bin. We conclude that it is not necessary to impose a precise order on the mass matrix to achieve this near-maximal mixing. Finally, we consider correlations between the various cuts. For example, could it be that the cuts on R and s_C selectively pass matrices which accidentally have a hierarchical structure, such that s_{atm} and $s_\odot$ are also small in these cases? From Table I we see that there is little correlation of s_{atm} with s_C or R: the fraction of matrices passing the s_{atm} cut is relatively insensitive to whether or not the s_C or R cuts have been applied. However, there is an important anticorrelation between $s_\odot$ and s_C cuts; for example, in the seesaw case roughly half of the matrices satisfying the s_C cut satisfy the $s_\odot$ cut, compared with 20% of the original set. This anticorrelation is shown in more detail in Figure 3, which illustrates how the s_C cut serves to produce a peak at large mixing angle in the $s_\odot$ distribution.

For random matrices we expect the quantity

$$s_C + s_\odot = 4(|U_{e1}U_{e2}|^2 + |U_{e1}U_{e3}|^2 + |U_{e2}U_{e3}|^2) \quad (5)$$

to be large, since otherwise ν_e would have to be closely aligned with one of the mass eigenstates. Hence, when we select matrices where s_C happens to be small, we are selecting ones where $s_\odot$ is expected to be large.

3 We have argued that the neutrino mass matrix may follow from complete anarchy, however the electron, muon, tau mass hierarchies imply that the charged fermion mass matrix has considerable order and regularity. What is the origin for this difference? The only answer which we find plausible is that the lepton doublets, $(\nu_l, l)_L$, appear

cuts	none	s_{atm}	$s_\odot$	$s_{atm} + s_\odot$
none	1,000,000	537,936	221,785	126,914
s_C	222,389	102,178	99,050	50,277
R	643,127	345,427	142,789	81,511
$s_C + R$	143,713	65,875	63,988	32,435

TABLE II. Mass matrices satisfying various sets of cuts for the real linear seesaw scenario, with additional mixing from the charged lepton sector.

randomly in mass operators, while the lepton singlets, l_R, appear in an orderly way, for example, regulated by an approximate flavor symmetry. This idea is particularly attractive in SU(5) grand unified theories where only the 10-plets of matter feel the approximate flavor symmetry, explaining why the mass hierarchy in the up quark sector is roughly the square of that in the down quark and charged lepton sectors. Hence we consider a charged lepton mass matrix of the form

$$M_l = \hat{M}_l \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix} \quad (6)$$

where $\lambda_{e,\mu,\tau}$ are small flavor symmetry breaking parameters of order the corresponding Yukawa couplings, while $\hat{M}_l$ is a matrix with randomly generated entries. We generated one million neutrino mass matrices and one million lepton mass matrices, and provide results for the mixing matrix $U = U_l^\dagger U_\nu$, where U_ν and U_l are the unitary transformations on ν_l and l_l which diagonalize the neutrino and charged lepton mass matrices. We find that the additional mixing from the charged leptons does not substantially alter any of our conclusions – this is illustrated for the case of seesaw matrices in Table II. The mixing of charged leptons obviously cannot affect R, but it is surprising that the distributions for the mixings $s_{atm,\odot,C}$ are not substantially changed.

4 All neutrino mass matrices proposed for atmospheric and solar neutrino oscillations have a highly ordered form. In contrast, we have proposed that the mass matrix appears random, with all entries comparable in size and no precise relations between entries. We have shown, especially in the case of seesaw matrices, that not only are large mixing angles for solar and atmospheric oscillations expected, but $\Delta m^2_\odot \approx 0.1 \Delta m^2_{atm}$, giving an excellent match to the large angle solar MSW oscillations, as preferred at the 2σ level in the day/night flux ratio. In a sample of a million random seesaw matrices, 40% have such mass ratios and a large atmospheric mixing. Of these, about 10% also have large solar mixing while having small ν_e disappearance at reactor experiments. Random neutrino mass matrices produce a narrow peak in atmospheric oscillations around the observationally preferred case of maximal mixing. In contrast to flavor

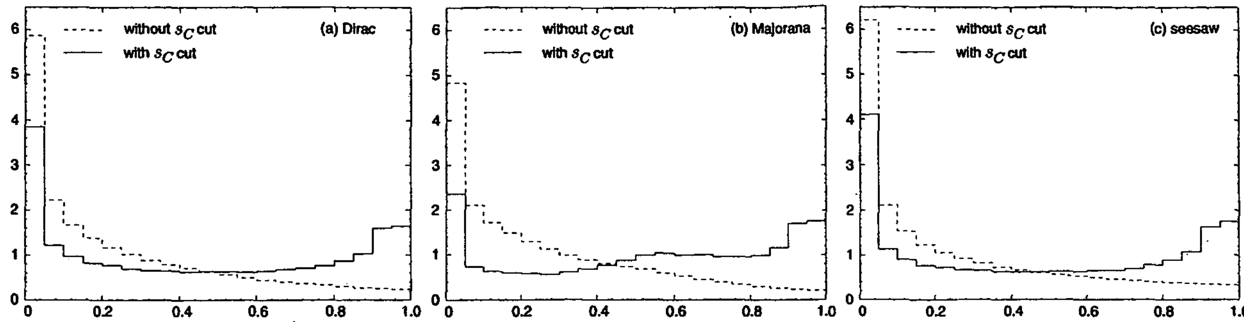

FIG. 3. Plots of the normalized, binned distributions of $s_\odot$ for Dirac (a), Majorana (b), and seesaw (c) cases. The distribution after imposing the s_C cut (solid) shows a greater preference for large $s_\odot$ compared with the original distribution (dashed).

symmetry models, there is no reason to expect U_{e3} is particularly small, and long baseline experiments which probe Δm^2_{atm}, such as K2K and MINOS, will likely see large signals in $\overline{\nu}_e$ appearance. If Δm^2_{atm} is at the lower edge of the current Superkamiokande limit, this could be seen at a future extreme long baseline experiment with a muon source. Furthermore, in this scheme $\Delta m^2_\odot$ is large enough to be probed at KamLAND, which will measure large $\bar{\nu}_e$ disappearance.

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