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Very Intense Pulse in the Groundwater Flow in Fissurized-Porous Stratum

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Abstract. An asymptotic solution is obtained corresponding to a very intense pulse: a sudden strong increase and fast subsequent decrease of the water level at the boundary of semi-infinite fissurized-porous stratum. This flow is of practical interest: it gives a model of a groundwater flow after a high water period or after a failure of a dam around a collector of liquid waste.

It is demonstrated that the fissures have a dramatic influence on the groundwater flow increasing the penetration depth and speed of fluid penetration into the stratum. A characteristic property of the flow in fissurized-porous stratum is the rapid breakthrough of the fluid at the first stage deeply into the stratum via a system of cracks, feeding of porous blocks by the fluid in cracks, and at a later stage feeding of advancing fluid flow in fissures by the fluid, accumulated in porous blocks.

1 Introduction

As is well known (see ([1,2,3])) a gently sloped groundwater flow in porous stratum supported by a horizontal impermeable bed can be described by the Boussinesq equation:

$$\partial_t h = \kappa \partial_{xx}^2 h^2 \qquad [1.1]$$

Here $h(x,t)$ is the water level (Figure 1(a)), x is the coordinate reckoned from the vertical boundary of the stratum, t is time, $\kappa = \rho g k / 2m\mu$ is a coefficient assumed to be a constant for the case of a homogeneous rock in the stratum. Furthermore, k is the rock permeability, m its porosity, g the acceleration of gravity, ρ and μ the fluid properties (its density and dynamic viscosity). The horizontal extent of the stratum is considered to be large, so that the flow region is considered to be semi-infinite: $0 \leq x < \infty$.

The boundary condition for the problem of a very intense pulse, which will be considered here for fissurized-porous rock (Figure 1(b)), is formulated as follows: At the initial moment $t = -\tau$, the water level at the vertical boundary $x = 0$ starts to increase and quickly reaches a level h_0 much higher than the initial groundwater level in the stratum. After a short time τ, i.e. at $t = 0$, the water level at the boundary $x = 0$ returns to the initial value. The solution to this problem describes groundwater flow in a river or channel bank after a short flood or after the breakthrough of a dam separating a channel or river from a reservoir of liquid waste.

Thus the boundary condition at the vertical boundary of the stratum $x = 0$ takes the form

$$h(0,t) = h_0 f(\theta) \ , \qquad \theta = t/\tau \qquad [1.2]$$

where $f(\theta)$ is a dimensionless function equal to zero at $\theta = -1$, and at $\theta > 0$, and non-negative at $-1 < \theta < 0$.

Furthermore, we consider the initial water level in the stratum as negligible in comparison with h_0, and so we assume the initial condition in the form

$$h(x, -\tau) \equiv 0, \qquad 0 \leq x < \infty \ . \qquad [1.3]$$

An accurate description of the function $f(\theta)$ is in fact not needed, because we are interested in the flow at large times, $t >> \tau$, i.e., at $\theta >> 1$.

For a *purely porous* stratum the asymptotic behavior of the groundwater flow under consideration is given by the "dipole" self-similar solution, obtained in ([4]) (see also ([5,3])):

$$h = \left(\frac{Q}{\kappa t}\right)^{\frac{1}{2}} \Phi(\zeta) \; , \qquad \zeta = \frac{x}{x_f} \; , \qquad \Phi(\zeta) = \frac{\sqrt{5}}{3} \zeta^{\frac{1}{2}} (1 - \zeta^{3/2}) \; , \qquad 0 \le \zeta \le 1, \;\; h \equiv 0, \;\; \zeta \ge 1. \tag{1.4}$$

Here x_f (see Figure 1(a)) is the longitudinal extent of the groundwater "dome", equal to

$$x_f = 2(5Qt)^{\frac{1}{4}} \; , \tag{1.5}$$

and Q is the "dipole moment" of the initial water height distribution, equal to

$$Q = \int_0^\infty x h(x, 0) dx \; . \tag{1.6}$$

It can be easily shown that the dipole moment of the water height distribution in the dome for porous stratum

$$Q(t) = \int_0^\infty x h(x, t) dx \tag{1.7}$$

remains time invariant so that $Q(t) = Q$ at arbitrary time $t > 0$.

A generalization of this dipole problem taking into account capillary retention of groundwater and forced drainage was proposed in ([6]). Numerical computations for this extended dipole problem were performed in ([7]). In the present Note we qualitatively answer the question of practical importance: What is the effect of cracks always present in the rock on the groundwater "dome" evolution? We will show that fissurization of the rock has a dramatic effect on the evolution of the groundwater dome, so that predictions based on a model of purely porous stratum are completely inadequate even for a tiny degree of fissurization.

2 Basic model

We use, in application to the groundwater flow, the basic idea of the "double-porosity" model ([8,3]). According to this model the rock is considered to consist of two mutually embedded porous media with a fluid exchange between them. The first porous medium is the ordinary medium of the porous blocks, and the second is the medium in which the cracks play the role of pores and the blocks play the role of grains. At every x and t we introduce not a single, but *two water levels*, h_B and h_C: mean water levels in porous blocks and in fissures around the plane under consideration: $h_B = h_B(x, t)$; $h_C = h_C(x, t)$. Generally speaking these levels are different. Indeed, the system of cracks occupies only a small part (in comparison with pores) of the void volume of rocks. At the same time, the cracks are much wider than pores,

so that the fluid mobility in cracks is essentially larger. The difference between water levels in pores and cracks stimulates the water exchange between cracks and porous blocks, and this exchange is not instantaneous, it requires a certain time. We assume naturally that this exchange between two components of double-porous media is a quasi-steady one, i.e. that its intensity is proportional to the difference of squares of water levels in both media. The balance of groundwater in both media gives us the following system of equations

$$m\partial_t h_B = \kappa_B m \partial^2_{xx} h_B^2 - \alpha(h_B^2 - h_C^2) \ , \qquad [2.1]$$

$$m\varepsilon\partial_t h_C = \kappa_C m\varepsilon \partial^2_{xx} h_C^2 + \alpha(h_B^2 - h_C^2) \ . \qquad [2.2]$$

Here α is an exchange coefficient, assumed to be constant, and the coefficients κ_B and κ_C appear in the same form as in the derivation of the Boussinesq equation

$$\kappa_B = \frac{\rho g\, k_B}{2m\mu} \ , \qquad \kappa_C = \frac{\rho g\, k_C}{2m\varepsilon\mu} \ , \qquad [2.3]$$

where k_B is the permeability of porous blocks, k_C is the permeability of the system of cracks, and ε is the ratio of "crack porosity" (relative volume of cracks) to the porosity of the porous blocks, usually a very small quantity. Therefore $\varepsilon << 1$, and $\kappa_C >> \kappa_B$.

We consider for the system [2.1]–[2.2] the same problem of a very intense pulse. Therefore the boundary and initial conditions assume the form

$$h_B(0,t) = h_C(0,t) = h_0\, f(\frac{t}{\tau}) \qquad [2.4]$$

where h_0 is the maximum level, τ is the duration of pulse, and the function $f(\theta) \equiv 0$ at $\theta > 0$, $f(-1) = f(0) = 0$, $f(\theta) > 0$ at $-1 < \theta < 0$. Furthermore

$$h_B(x,-\tau) = h_C(x,-\tau) \equiv 0 \ , \qquad 0 \le x < \infty \ . \qquad [2.5]$$

Condition [2.5] reflects the fact that the initial water level in the blocks and fissures is negligible in comparison with h_0. Under the condition [2.5] the system [2.1]–[2.2] becomes a degenerate one, and this leads to a finite speed of the dome extension. The usual condition at infinity should be added:

$$h_B(\infty,t) = h_C(\infty,t) \equiv 0 \ . \qquad [2.6]$$

The system [2.1]–[2.2], together with the boundary and initial conditions [2.4], [2.5], [2.6] at $t > 0$, has an integral of the same "dipole" type:

$$\int_0^\infty x[h_B(x,t) + \varepsilon h_C(x,t)]dx \equiv \text{const} = Q \ \text{ at } t \ge 0. \qquad [2.7]$$

Here Q is the value of the integral at $t = 0$.

To derive this integral we add equations [2.1]and [2.2], multiply the resulting equation by x and integrate from $x = 0$ to $x = \infty$. Integrating by parts the terms $x\partial^2_{xx}h^2_C$ and $x\partial^2_{xx}h^2_B$, and using the boundary conditions [2.4] and [2.6] for $t \geq 0$ we obtain

$$\frac{d}{dt}\int_0^\infty x[h_B(x,t) + \varepsilon h_C(x,t)]dx = 0 \qquad [2.7]$$

from which the conservation law [2.7] follows immediately.

3 Computational experiment

By introducing the dimensionless variables

$$\theta = \frac{t}{\tau}\,, \qquad \xi = \frac{x}{\sqrt{\kappa_C\, h_0 \tau}}\,, \qquad H_B = \frac{h_B}{h_0}\,, \qquad H_C = \frac{h_C}{h_0}\,, \qquad \beta = \frac{\alpha h_0 \tau}{m} \qquad [3.1]$$

we reduce the basic system of equations and boundary and initial conditions to a convenient dimensionless form:

$$\begin{aligned} \partial_\theta H_B &= \tfrac{\kappa_B}{\kappa_C}\partial^2_{\xi\xi} H_B^2 - \beta(H_B^2 - H_C^2)\,, \\ \partial_\theta H_C &= \partial^2_{\xi\xi} H_C^2 + \tfrac{\beta}{\varepsilon}(H_B^2 - H_C^2)\,. \end{aligned} \qquad [3.2]$$

and

$$\begin{aligned} H_B(0,\theta) &= H_C(0,\theta) = f(\theta)\,; \\ H_B(\xi,-1) &= H_C(\xi,-1) \equiv 0\,, \\ H_B(\infty,\theta) &= H_C(\infty,\theta) = 0\,. \end{aligned} \qquad [3.3]$$

We remind that $f(\theta) \equiv 0$ at $\theta \geq 0$. The system [3.2] should be solved for $0 \leq \xi < \infty$ and $\theta > -1$. The total dipole momentum can be represented in the dimensionless form:

$$\int_0^\infty \xi(H_B + \varepsilon H_C)d\xi = \frac{Q}{\kappa_C\, h_0^2 \tau} = M\,. \qquad [3.4]$$

The system [3.2] was integrated numerically under the boundary and initial conditions [3.3] for several sets of realistic values of parameters.

In Figures 2–4 we present the results of numerical experiments for typical values of parameters

$$\kappa_B/\kappa_C = 10^{-4}\,, \qquad \beta = 10^{-2}\,, \qquad \varepsilon = 10^{-4}\,. \qquad [3.5]$$

The function $f(\theta)$ was assumed to have a piecewise linear shape:

$$f(\theta) = \frac{\theta+1}{\theta_*+1}\,, \quad \text{at } -1 < \theta < \theta_* < 0\,; \qquad f(\theta) = \frac{\theta}{\theta_*}\,, \quad \text{at } \theta_* < \theta < 0. \qquad [3.6]$$

Figure 2(a) shows the distributions of H_B and H_C for $\theta = 10.00$; $\theta = 100.00$ and $\theta = 1000.00$. The insert shows in more detail the distribution of groundwater levels in cracks and blocks near the front $x = x_f$ for $\theta = 10.00$. In fact this Figure gives a good check of the accuracy of the numerical method. Indeed, the dimensionless velocity of the fluid tongue extension rate $V = d\xi_f/d\theta$ can be considered as a constant at a short time interval near $\theta = \theta_0$ when $\xi_f = \xi_{f0}$, and the distribution of groundwater levels near $\xi = \xi_{f0}$ as a steady one, so that

$$H_B = H_B(\zeta)\ , \qquad H_C = H_C(\zeta)\ , \qquad \zeta = \xi - V(\theta - \theta_0) - \xi_{f0}\ .$$

so that $\zeta = 0$ corresponds to $\xi = \xi_f = \xi_{f0} + V(\theta - \theta_0)$. Then equations [3.2] assume the form

$$\begin{aligned} V\,\frac{dH_B}{d\zeta} + \frac{\kappa_B}{\kappa_C}\,\frac{d^2H_B^2}{d\zeta^2} - \beta(H_B^2 - H_C^2) = 0 \\ V\,\frac{dH_C}{d\zeta} + \frac{d^2H_C^2}{d\zeta^2} + \frac{\beta}{\varepsilon}(H_B^2 - H_C^2) = 0\ . \end{aligned} \tag{3.1}$$

Near $\zeta = 0$ the first equation of [3.8] takes the form

$$V\,\frac{dH_B}{dz} + \beta\,H_C^2 = 0$$

because the terms $\kappa_B/\kappa_C\, d^2H_B^2/d\zeta^2$ and βH_B^2 can be neglected. Near $\zeta = 0$ the behavior of the function $H_C(\zeta)$ is a linear one, $H_C = -A\zeta$, where A is a certain positive constant. Then [3.9] gives $H_B = -(\beta A^2/3V)\zeta^3$, so that

$$H_B^{\frac{1}{3}} = (\beta/3AV)^{\frac{1}{3}}\,H_C\ .$$

The insert in Figure 2(a) demonstrates this proportionality of $H^{\frac{1}{3}}$ to H_C with the proper value of the proportionality constant. Furthermore, Figure 3 demonstrates the evolution of the dipole moment both by components and as a whole. We see that the dipole moment quickly becomes concentrated in the blocks, although the fluid propagation and groundwater dome extension is mostly due to the flow in cracks.

Figure 4(a) represents the evolution of the total mass of fluid in porous blocks and cracks. Figure 4(b) represents the extension of the groundwater dome, i.e., the function $x_f(t)$ (curve 1), and the function $x^*(t)$ (curve 2) — the extension of the zone where the groundwater level in blocks is higher than in cracks. For comparison, the extension of the groundwater dome in purely porous stratum under the same conditions is presented in Figure 4(b) (curve 3). Remarkably this comparison demonstrates that the basic amount of fluid is moving like it would in purely porous stratum. However, the flow in fissures plays the role of a long precursor forerunning the basic flow.

4 Conclusions

The mathematical model proposed in the present Note allowed us to come to the following conclusions.

1. The fissures influence the flow dramatically: the amount of groundwater entering the stratum is larger, as are the depth and speed of penetration of fluid.

2. At any time the groundwater dome is divided into two regions: a *blocks-dominated* one, adjacent to the boundary where the groundwater level in blocks is larger than that in fissures, and *fissures-dominated*region, adjacent to the front where the groundwater level in the fissures is larger than that in blocks.

3. The groundwater enters the stratum via the fissures and at any fixed place the fissures at first feed the blocks. Later the porous blocks start to feed the fissures supporting the fluid advancement to the depth of the stratum via the fissures.

4. Rather early the total amount of fluid available in the stratum becomes concentrated in blocks. However the forerunning flow in fissures ahead of the basic mass of fluid can be a dangerous agent of contamination. The correct evaluation of contamination should take into account fissurization of rocks.

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Figure Captions

Figure 1. (a) Dome extension in porous stratum.

(b) Dome extension in fissurized-porous stratum. The water levels in porous blocks h_B and cracks h_C are different.

Figure 2. (a) Fluid levels in porous blocks and cracks at $t = 10.00$.

(b) Fluid levels in porous blocks and cracks at $t = 100.00$.

(c) Fluid levels in porous blocks and cracks at $t = 1000.00$.

Figure 3. Dipole moments in porous blocks and cracks. A major part of the dipole moment is concentrated in porous blocks.

Figure 4. (a) Evolution of the bulk fluid mass in fissurized porous stratum. A major part of the fluid is contained in porous blocks.

(b) Comparison of the groundwater dome extension in fissurized porous and purely porous strata. The curve (1) corresponds to $x_f(t)$ in fissurized porous stratum, the curve (3) corresponds to $x_f(t)$ in purely porous stratum. The boundary $x^*(t)$ (curve (2)) corresponds to $h_B(x^*, t) = h_C(x^*, t)$.

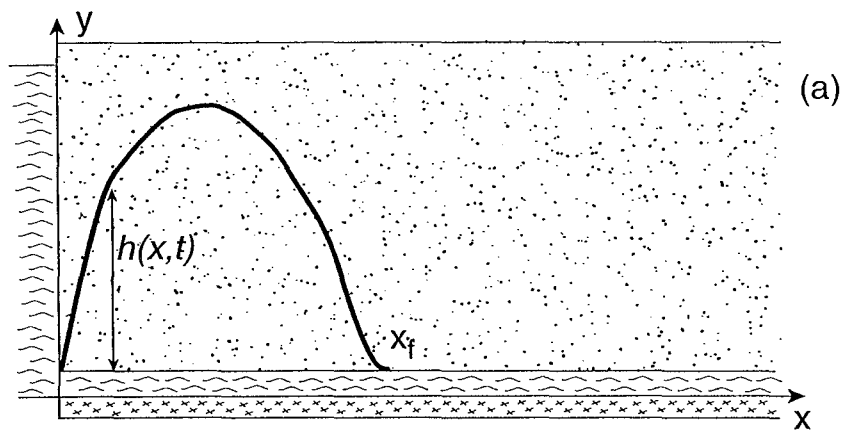

Figure 1(a)

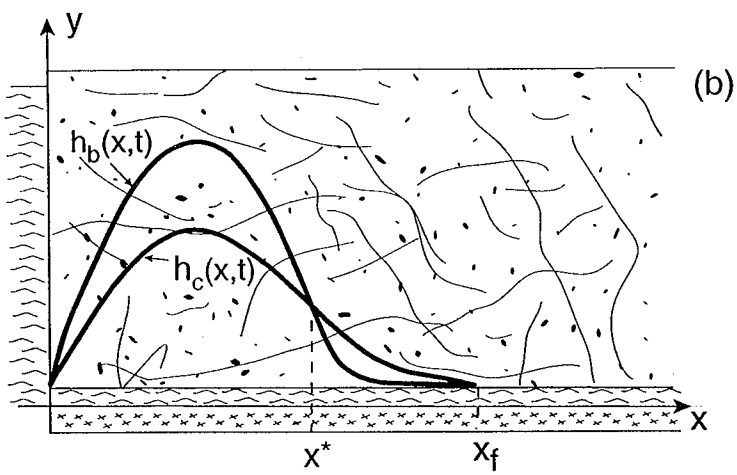

Figure 1(b)

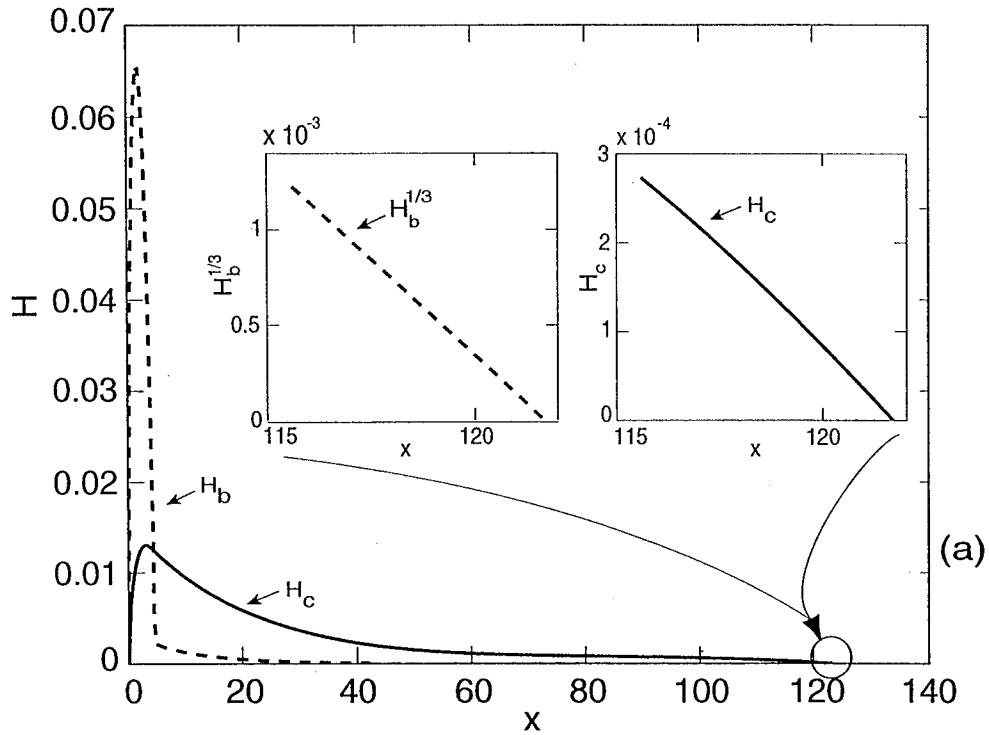
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Figure 2(a)

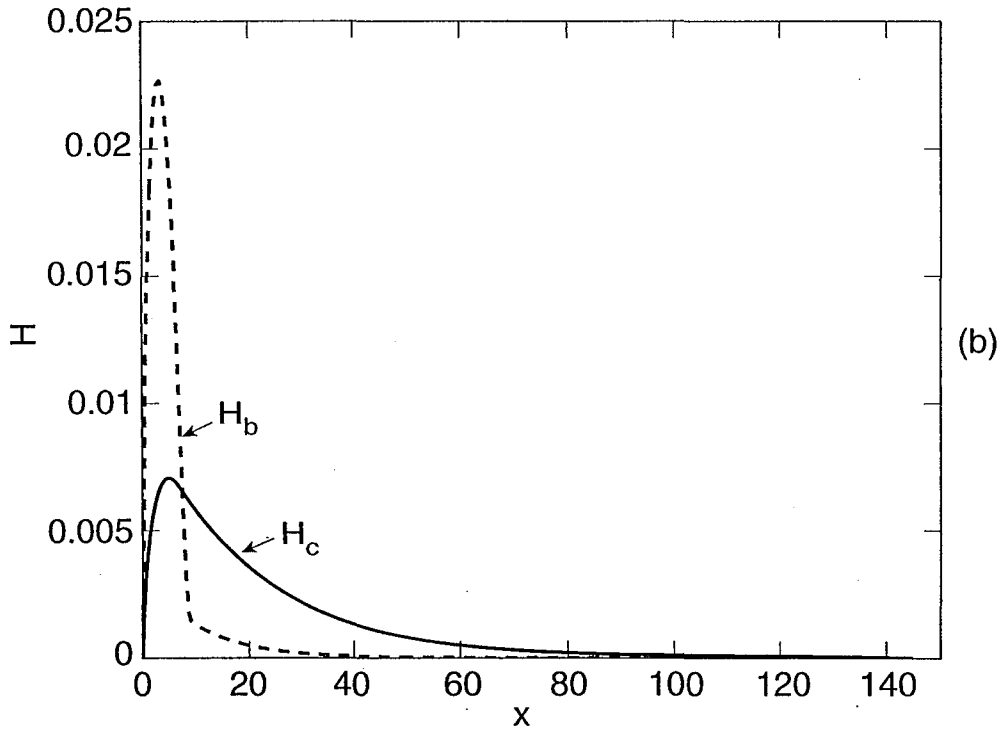
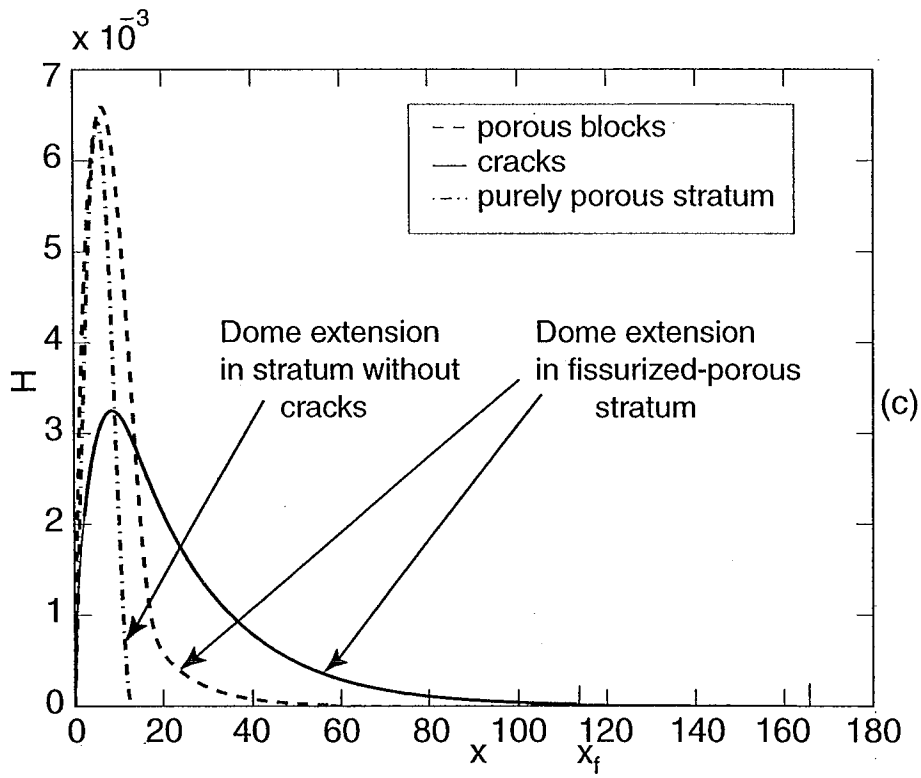


Figure 2(b)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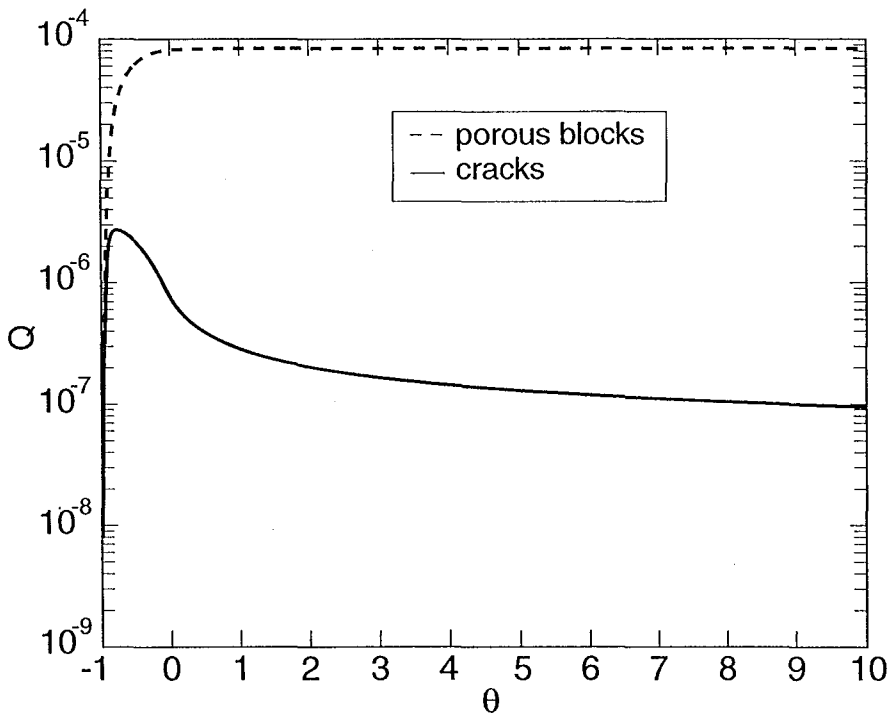
porous blocks
cracks
purely porous stratum
Dome extension in stratum without cracks
Dome extension in fissurized-porous stratum
(c)

Figure 2(c)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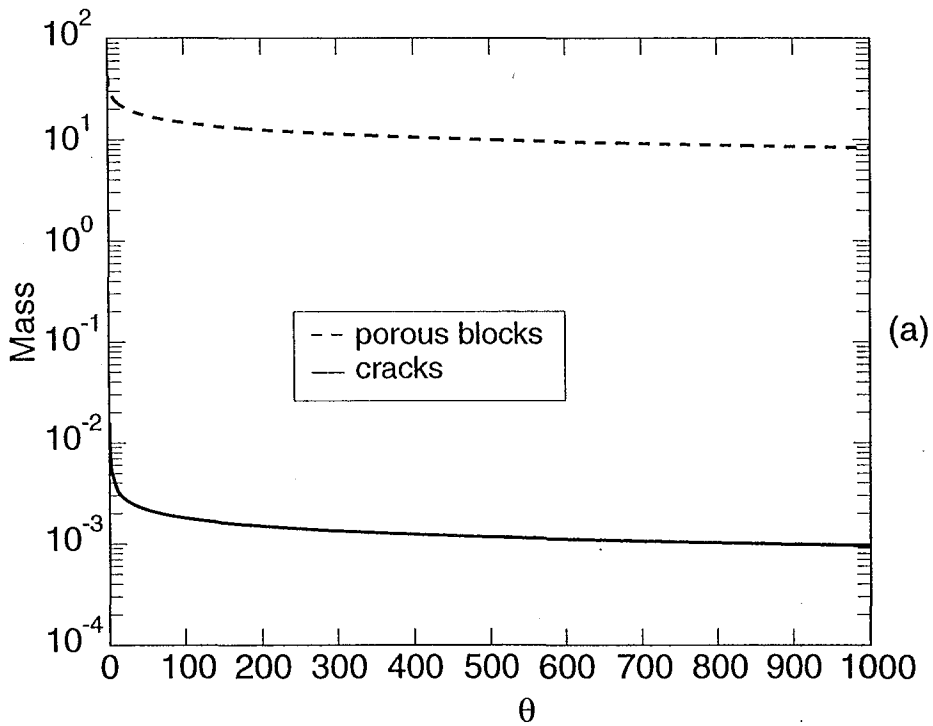
porous blocks
cracks

Figure 3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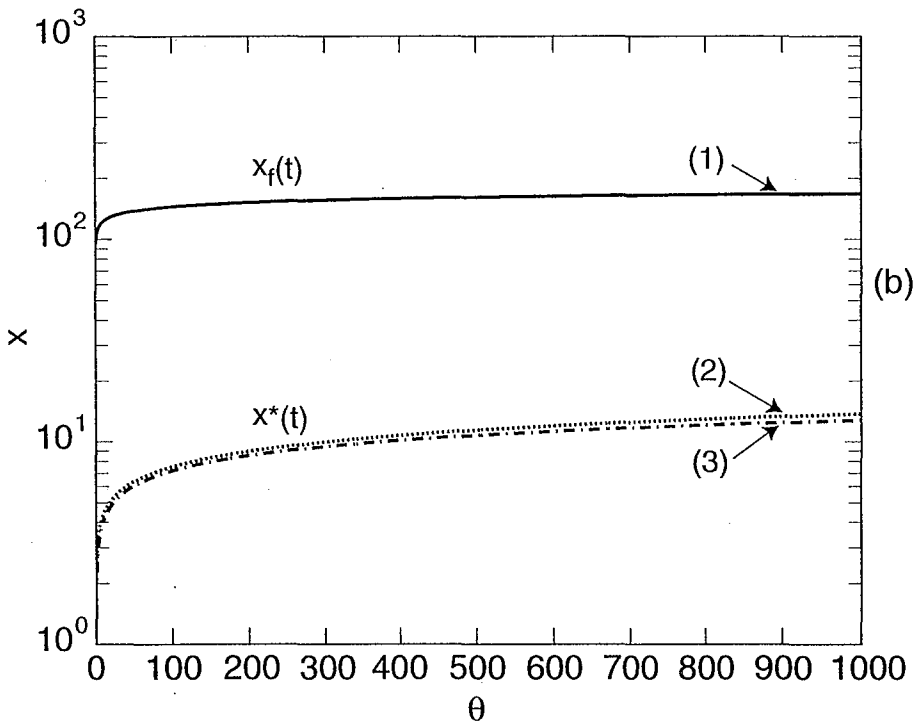
porous blocks
cracks
(a)

Figure 4(a)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x
$x_f(t)$
(1)
(b)
(2)
$x^*(t)$
(3)
0 100 200 300 400 500 600 700 800 900 1000
θ

Figure 4(b)