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Physics Division

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Dynamical Instability of Brane Solutions with a Self-Tuning Cosmological Constant

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Dynamical Instability of Brane Solutions with a Self-Tuning Cosmological Constant

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Abstract

A five-dimensional solution to Einstein's equations coupled to a scalar field has been proposed as a partial solution to the cosmological constant problem: the effect of arbitrary vacuum energy (tension) of a 3-brane is cancelled; however, the scalar field becomes singular at some finite proper distance in the extra dimension. We show that in the original model with a vanishing bulk potential for the scalar, the solution is a saddle point which is unstable to expansion or contraction of the brane world. We construct exact time-dependent solutions which generalize the static solution, and demonstrate that they do not conserve energy on the brane; thus they do not have an effective 4-D field theoretic description. When a bulk scalar field potential is added, the boundary conditions on the brane cannot be trivially satisfied, raising hope that the self-tuning mechanism may still give some insight into the cosmological constant problem in this case.

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1 Introduction

The idea that our world is a 3-brane embedded in extra spatial dimensions has been widely discussed as a solution to the weak-scale hierarchy problem [1, 2]. More recently, attention has been focused on the possibilities for understanding the cosmological constant problem within this setting [3–23] (see [24] for an earlier treatment). A partial solution was proposed in [7, 8] (ADKS–KSS), where the addition of a bulk scalar, ϕ, plays a crucial role (see also [25–27] for a discussion of the physics of a scalar field coupled to gravity). The scalar is a free field in the bulk, but has nontrivial couplings to fields living on the brane. Ref. [7] found static solutions of Einstein's equations with the property that ϕ becomes singular at a finite distance in the extra dimension, and the warp factor for the metric vanishes at the singularity. If one assumes that the extra dimension terminates at the singularity, or that the warp factor remains integrably small beyond it, then gravity appears to be four dimensional on large distance scales. Most importantly, the scalar field is supposed to adjust itself to any arbitrary value of the tension on the brane, which represents the four-dimensional vacuum energy–or at least that part of it which comes from nongravitational vacuum fluctuations. The fact that the metric is static means that the effective cosmological constant observed on the 3-brane is zero, regardless of the size of the brane tension. This could constitute significant progress toward the solution of the cosmological constant problem.

The self-tuning mechanism is incomplete in several ways. In its original form in [7,8], the orbifold solution requires a very particular exponential coupling of ϕ to the matter fields on the brane, $e^{\pm\kappa_5\phi}$, requiring just the right coefficient κ_5 in the exponent, where κ_5 is related to the 5-D gravity scale M_5 by $\kappa_5^2 = M_5^{-3}$. As understood by ADKS, and explicitly realized in refs. [9], different choices of the coupling function $f(\phi)$ give de Sitter or anti-de Sitter branes in a $\mathbb{Z}_2$ bulk. Furthermore, the scalar potential in the bulk was assumed to vanish. Ref. [8] extended the analysis to non-vanishing potentials, and [28] gave the procedure for finding solutions with arbitrary potentials in the bulk (see [29] for an analytic solution associated to a bulk cosmological constant and see [30–32] for a discussion of exponential potentials which can be associated with Neveu–Schwarz dilaton tadpoles in non-supersymmetric string theories [33]). Actually, as recently pointed out in Ref. [34], the $\mathbb{Z}_2$ symmetric and 4D Poincaré invariant solution is unstable under bulk quantum corrections: indeed with a conformal coupling allowing flat solution with a vanishing bulk potential, the brane becomes curved as soon as a bulk potential is turned on and the jump equations relate the curvature of the brane to the value of the potential on the brane by $R_{4\text{D}} = \kappa_5^2 V(\phi_0)$. Supersymmetry in the bulk may prevent from such instability. Another difficulty anticipated by [7,10,11], and explicitly shown by [34,35], is that any procedure which regularizes ("resolves") the singularity in the solutions causes the reintroduction of the fine-tuning which self-tuning is supposed to avoid, unless some more explicit dynamical mechanism which automatically relaxes the effects of the brane tension can be demonstrated. A further possible problem is the claim that when normal matter is added to the brane tension, the brane remains static, in contradiction with cosmology [36] (however, see [37] for a recent tentative to recover usual cosmology).

In this letter we demonstrate a shortcoming which is more severe than the foregoing ones; namely, starting from the very same Lagrangian which gives the static self-tuned solutions,

there also exist dynamical solutions, which either begin or end with a singularity as time evolves. In section 2 we will review the static solution, and discuss the conformal symmetry which allows construction of the dynamical solutions. These constitute a family of solutions, of which the static one is a special example. In section 3 we emphasize that, even starting arbitrarily close to the static solution within this family, the brane world inevitably collapses to a singularity or else expands starting from one, with a Hubble parameter of $H \sim \pm 1/(4t)$ as $t \to \mp\infty$. The interpretation is that the static solution is a saddle point, unstable to small perturbations. The solution on the brane is shown to violate the positive energy condition, reflecting the loss of energy from the brane into the bulk via the scalar field. These remarks apply in the case when the scalar bulk potential, $V(\phi)$, vanishes. In section 4 we show that for $V(\phi) \neq 0$, our construction cannot be trivially applied to generate dynamical solutions. This gives further motivation for studying the stability of self-tuning solutions with a nonvanishing scalar bulk potential.

The solutions we constructed were independently found by Horowitz, Low and Zee in Ref. [38] and were interpreted as describing a phase transition. We will argue in the final section that they actually rather signal an instability of the static solution.

2 Dynamical Self-Tuning Solutions

We will consider solutions arising from the action[1]

$$S = \int d^5x \sqrt{|g_5|}\left(\alpha R - \beta \nabla_M \phi \nabla^M \phi - \gamma V(\phi)\right) - \int d^4x \sqrt{|g_4|} f(\phi_0)\, T, \tag{1}$$

where g_5 and g_4 are, respectively, the determinants the 5-D metric g_{MN} and the 4-D metric induced on the brane, $g_{\mu\nu}$, and T is the bare tension. The brane is supposed to couple to the bulk in a conformal way defined by the function $f(\phi)$. The physical tension is thus given by

$$\tilde{T} = f(\phi_0) T, \tag{2}$$

where ϕ_0 is the value of the scalar field on the brane. There are many conventions for the normalization of the terms in the bulk part of the action; to facilitate comparison with other papers we will leave α, β, γ unspecified. We will be primarily concerned with the case of vanishing bulk potential, $V(\phi) = 0$, but we shall also consider nonzero $V(\phi)$ below. Einstein's equations and the equation of motion for the scalar field read

$$\alpha G_{MN} = \beta \nabla_M \phi \nabla_N \phi - \tfrac{1}{2}\left(\beta (\nabla\phi)^2 + \gamma V(\phi)\right) g_{MN} - \tfrac{1}{2} f(\phi) T g_{\mu\nu} \delta^\mu_M \delta^\nu_N \frac{\sqrt{|g_4|}}{\sqrt{|g_5|}}\, \delta(y)\ ; \tag{3}$$

$$2\beta \frac{1}{\sqrt{|g_5|}} \nabla_M \left(\sqrt{|g_5|} g^{MN} \nabla_N \phi\right) - \gamma \frac{dV}{d\phi} - \frac{df}{d\phi} T \frac{\sqrt{|g_4|}}{\sqrt{|g_5|}}\, \delta(y) = 0\ , \tag{4}$$

[1]Our conventions correspond to a mostly positive Lorentzian signature $(- + \ldots +)$ and the definition of the curvature in terms of the metric is such that a Euclidean sphere has positive curvature. Bulk indices will be denoted by capital Latin indices and brane indices by Greek indices: x^μ are coordinates on the brane (τ or t will be the time coordinate and x^i the spatial ones), and y (or z, if the metric is conformally flat) is the coordinate along the fifth dimension such that the brane is located at $y = 0$ (or $z = 0$).

and, for a conformally flat metric with the form $ds^2 = \Omega^2(z)(-d\tau^2 + dx_i^2 + dz^2)$, the jump conditions for the derivatives of the fields at the brane are

$$\left[\Omega^{-2}\,\frac{d\Omega}{dz}\right]_{z=0^-}^{z=0^+} = -\frac{T}{6\alpha} f(\phi_0) \quad \text{and} \quad \left[\Omega^{-1}\,\frac{d\phi}{dz}\right]_{z=0^-}^{z=0^+} = \frac{T}{2\beta}\frac{df}{d\phi}(\phi_0). \tag{5}$$

For a vanishing scalar bulk potential, the self-tuning solution of [7,8] with a $\mathbb{Z}_2$–symmetric bulk orbifold is given by

$$ds^2 = \Omega^2(y)\,(-d\tau^2 + dx_i^2) + dy^2 = \Omega^2(z)\,(-d\tau^2 + dx_i^2 + dz^2)\ ; \tag{6}$$

$$\phi = \phi_0 \pm \sqrt{\tfrac{3\alpha}{4\beta}}\ln(1 - |y|/y_c) = \phi_0 \pm \sqrt{\tfrac{4\alpha}{3\beta}}\ln(1 - |z|/z_c), \tag{7}$$

where y is the proper distance coordinate, z is the conformal coordinate for the bulk and

$$\Omega(y) = (1 - |y|/y_c)^{1/4}\,, \quad \Omega(z) = (1 - |z|/z_c)^{1/3}\,. \tag{8}$$

However, this solution satisfies the jump conditions (5) only if the conformal coupling is an exponential function with the particular form

$$f(\phi) = \exp(\mp\sqrt{\tfrac{4\beta}{3\alpha}}\,\phi). \tag{9}$$

Then the integration constant, y_c or z_c, is related to the brane tension by

$$y_c = \tfrac{3}{4}z_c = 3\alpha\tilde{T}^{-1} = 3\alpha e^{\pm\sqrt{\frac{4\beta}{3\alpha}}\,\phi_0}T^{-1} \tag{10}$$

and ϕ_0 remains unconstrained—which is important for what follows. A notable peculiarity of this conformal coupling is that it satisfies the following equations everywhere in the bulk:

$$f(\phi(y)) = -\frac{12\alpha}{T}\,\mathrm{sgn}(y)\,\Omega^{-1}(y)\frac{d\Omega}{dy} \quad \text{and} \quad \frac{df}{d\phi}(\phi(y)) = \frac{4\beta}{T}\,\mathrm{sgn}(y)\,\frac{d\phi}{dy} \tag{11}$$

or, in conformal coordinates,

$$f(\phi(z)) = -\frac{12\alpha}{T}\,\mathrm{sgn}(z)\,\Omega^{-2}(z)\frac{d\Omega}{dz} \quad \text{and} \quad \frac{df}{d\phi}(\phi(z)) = \frac{4\beta}{T}\,\mathrm{sgn}(z)\,\Omega^{-1}(z)\frac{d\phi}{dz}. \tag{12}$$

Hence, the relation between the field derivatives and ϕ required by the jump conditions at the brane are satisfied not just there, but everywhere in the bulk.

There is a simple procedure for transforming static bulk solutions into dynamical ones. As emphasized in [39], bulk symmetries can be used to construct new solutions involving a singularity interpreted as a brane. Starting from any regular static solution to the bulk equations of motion, written for simplicity in conformal coordinates,

$$\phi(z) \quad \text{and} \quad ds^2 = \Omega^2(z)\,\eta_{MN}dx^M \otimes dx^N, \tag{13}$$

we obtain a physically equivalent solution by applying a diffeomorphism

$$\tilde{\phi}(\tilde{x}^\mu, \tilde{z}) = \phi(z(\tilde{x}^\mu, \tilde{z})) \quad \text{and} \quad ds^2 = \tilde{g}_{MN}(\tilde{x}^\mu, \tilde{z})\, d\tilde{x}^M \otimes d\tilde{x}^N. \tag{14}$$

However if we orbifold the new solution in the $\tilde{z}$ direction, it becomes singular and describes a brane located at $\tilde{z} = 0$. The orbifold projection and diffeomorphisms do not commute: in general an orbifold solution constructed from (14) is not equivalent under a change of coordinates to an orbifold solution constructed from (13). The difficulty consists in finding a diffeomorphism such that the singularities introduced in the right hand side of the equations of motion by the orbifold projection can be associated with the brane components deduced from the action (1).

In order to preserve the geometry of the brane embedded in the bulk, we want to restrict ourselves to diffeomorphisms (14) that keep the metric diagonal,

$$ds^2 = -\tilde{n}^2(\tilde{x}^\mu, \tilde{z})\, d\tilde{\tau}^2 + \tilde{a}^2(\tilde{x}^\mu, \tilde{z})\, d\tilde{x}_i^2 + \tilde{b}^2(\tilde{x}^\mu, \tilde{z})\, d\tilde{z}^2. \tag{15}$$

A particular subgroup is provided by the 5-D conformal transformations under which the metric remains conformally flat

$$ds^2 = \tilde{\Omega}^2(\tilde{x}^\mu, \tilde{z})\, \eta_{MN}\, d\tilde{x}^M \otimes d\tilde{x}^N. \tag{16}$$

The infinitesimal conformal transformations are generated by the Killing vectors

$$\xi^M = a^M + a^{[MN]} x_N + \lambda x^M + (x^P x_P\, \eta^{MN} - 2x^M x^N) k_N \tag{17}$$

where the parameters a^M, $a^{[MN]}$, λ, k_N correspond respectively to translations, Lorentz transformations, dilations and special conformal transformations.

In the present case, a combination of a boost in the z direction and a dilation provides a suitable diffeomorphism that will lead to a dynamical solution also satisfying the boundary conditions on the brane. If $\phi(z)$ and $\Omega(z)$ is a regular solution in the bulk, then it is simple to show that

$$\tilde{\phi}(z, \tau) = \phi(|z| + z_c h\tau) \quad \text{and} \quad \tilde{\Omega}(z, \tau) = \frac{\Omega(|z| + z_c h\tau)}{\sqrt{1 - z_c^2 h^2}} \tag{18}$$

is a $\mathbb{Z}_2$-symmetric solution to the bulk equations of motion. This can be checked from the explicit form of the bulk part of the action (1), which looks like

$$S_{\text{bulk}} = \int d^5x \, \left[4\alpha(\Omega(\partial\Omega)^2 + 2\Omega^2\partial^2\Omega) - \beta\Omega^3(\partial\phi)^2 - \gamma\Omega^5 V(\phi)\right]. \tag{19}$$

Here the contractions of ∂_M are performed with the Minkowski space metric. This action has the additional symmetry that, for any constant ζ, leaves the equations of motion invariant:

$$\Omega \to \zeta\,\Omega; \quad \phi \to \phi; \quad V \to \zeta^{-2} V. \tag{20}$$

In the case of a vanishing bulk potential, this symmetry implies that

$$\tilde{\phi}(z, \tau) = \phi(|z| + z_c h\tau) \quad \text{and} \quad \tilde{\Omega}(z, \tau) = \Omega(|z| + z_c h\tau) \tag{21}$$

is a solution in the bulk. Applying the procedure to the regular solution corresponding to (6)–(7), we obtain

$$ds^2 = (1 - |z|/z_c - h\tau)^{2/3} \left(-d\tau^2 + dx_i^2 + dz^2\right) ; \tag{22}$$

$$\phi = \phi_0 \pm \sqrt{\tfrac{4\alpha}{3\beta}} \ln(1 - |z|/z_c - h\tau). \tag{23}$$

The nontrivial step is to satisfy the jump equations at the brane, which read

$$\tilde{\Omega}^{-2} \left.\frac{d\tilde{\Omega}}{dz}\right|_{z=0^+} = -\frac{T}{12\alpha} \left. f(\tilde{\phi})\right|_{z=0^+} ; \quad \tilde{\Omega}^{-1} \left.\frac{d\tilde{\phi}}{dz}\right|_{z=0^+} = \frac{T}{4\beta} \left.\frac{df}{d\phi}(\tilde{\phi})\right|_{z=0^+} . \tag{24}$$

These equations are more difficult to satisfy when the solutions are dynamical, because they must remain true for all conformal times τ. However, in the present case of a vanishing bulk potential, we notice that, due to eqs. (11)–(12), the conformal coupling f of the scalar to the brane satisfies the relations (12) for any value of z, which ensures that the dynamical solution we construct satisfies the jump equations for any τ, as can also be explicitly verified. So from the same Lagrangian which gives the static self-tuned solutions, there also exist dynamical solutions for which the induced metric on the brane exhibits time dependence, as will be discussed in the next section. The original 4-D Poincaré invariant solution corresponds to a very particular value of the parameter h that characterized our more general family of solutions. This evades the no-go result of [7,9] which excluded the possibility of de Sitter or anti-de Sitter branes in this case.

3 Physical Interpretation

The dynamical solution (22)–(23) represents a singularity which is either approaching or receding from the brane, depending on the sign of the continuous parameter h. *Assuming the extra dimension is simply truncated at the singularity*, the strength of gravity is therefore time-dependent because of the growth or collapse of the extra dimension. The 4-D Planck mass M_p is related to the 5-D analogue M_5 by

$$M_p^2 = 2M_5^3 \int_0^{z_c(1-h\tau)} \Omega^3(z,\tau)\, dz = M_5^3 z_c(1 - h\tau). \tag{25}$$

Moreover, an observer on the brane will see that his universe, although spatially flat, does not have 4-D Poincaré invariance, but it is growing or shrinking with a scale factor given by $\Omega(z = 0, \tau)$. In FRW time, $dt = \Omega d\tau$, the scale factor and the corresponding Hubble parameter are given by

$$a(t) = (1 - h\tau)^{1/3} = (1 - \tfrac{4}{3}ht)^{1/4}; \qquad H = \frac{\dot{a}}{a} = -\frac{h}{3(1 - \frac{4}{3}ht)}. \tag{26}$$

The universe either begins or ends in a singularity, depending on whether $h < 0$ or $h > 0$. For the case $h = 0$, the static solution of ADKS is recovered. Therefore one can interpret h

as the parameter determining how far away from the unstable saddle point solution one is, in the space of all solutions.

The situation is qualitatively similar to a 4-D field theory analogy, in which a cosmological "constant" Λ is coupled to scalar fields through the Lagrangian [2]

$$\mathcal{L} = a^3(t)\left[\tfrac{1}{2}(\dot{\phi}_1^2 + \dot{\phi}_2^2) - \phi_1(\Lambda - \phi_2)\right]. \tag{27}$$

This system also has a saddle point solution, at $\phi_2 = \Lambda$ and $\phi_1 = 0$, which could be construed as a self-tuning of the cosmological constant to zero. However, it is not a good solution to the cosmological constant problem because it is unstable against small perturbations. In figure 1(a) we show the time dependence of the Hubble parameter for both the 5-D self-tuning solution and the 4-D toy model, in the case of a collapsing universe, which is the generic outcome for the 4-D model. Although $H(t)$ looks rather similar in the two cases, for the 4-D model H starts positive and crosses zero, while for the 5-D solution it is always negative. The differences are more clearly seen in figure 1(b), showing the scale factors $a(t)$ in FRW time.

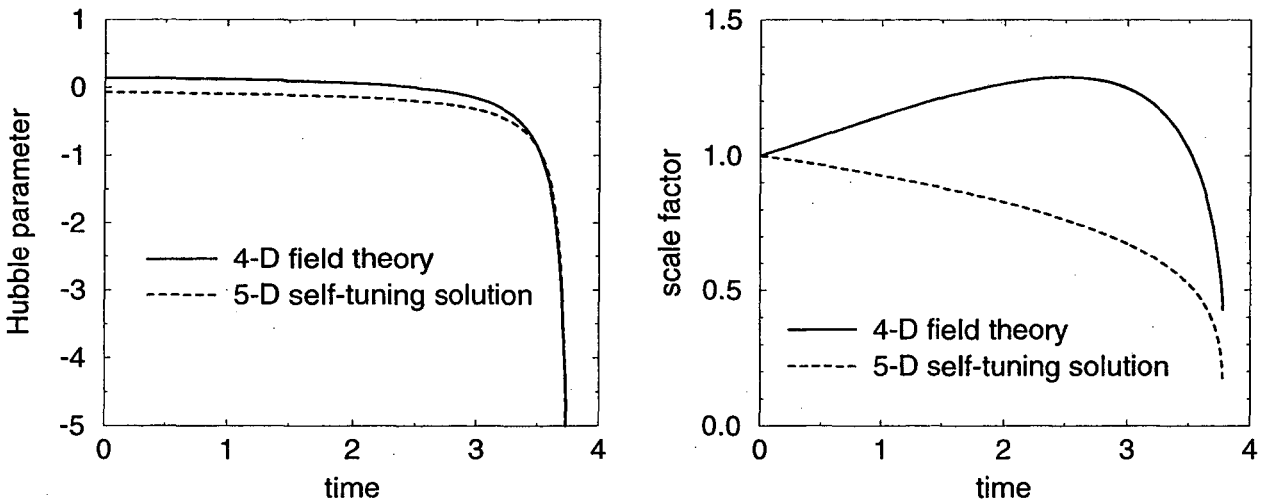

Figure 1. (a) Comparison of the Hubble parameter as a function of time for the 5-D self-tuning solution and the 4-D toy model (27), for the case of a collapsing brane-world. (b) Same, but showing the respective scale factors versus time.

A further difference between the 5-D solution and any attempt to describe it in a 4-D effective theory is that the 5-D solution appears to violate energy conservation when viewed from the 3-brane. If we were in 4 dimensions, the scale factor dependence in eq. (26) would correspond to a 4-D stress energy tensor

$$T^{\mu}_{\nu} = \frac{\rho_0}{a^8(t)} \begin{pmatrix} -1 & & & \\ & \frac{5}{3} & & \\ & & \frac{5}{3} & \\ & & & \frac{5}{3} \end{pmatrix} \tag{28}$$

[2]We thank Nima Arkani-Hamed for making us aware of this concept, through a related example.

where $\rho_0 = M_p^2 h^2/3$. This has the equation of state $p = (5/3)\rho$, which implies that there exists a null vector ξ_μ which, when contracted with T^μ_ν, gives a spacelike vector, contrary to the requirement of positivity of T^μ_ν. This is due to the nonconservation of energy on the brane, which can be explicitly demonstrated in the 5-D theory, by computing the singular part of the divergence of the 5-D stress energy tensor[3]

$$\dot{\rho} + 3H(\rho + p) = \tfrac{1}{4}(\rho - 3p)\frac{f'}{f}\dot{\phi} \tag{29}$$

In this context, $\rho - 3p = 4\tilde{T}$, and in the dynamical solutions, as long as h is non-vanishing, $\dot{\phi}$ is nonzero on the 3-brane, so the right hand side of (29) is nonzero. More simply put, since the physical tension $\tilde{T} = f(\phi)T$ is time-dependent,[4] energy is not conserved on the brane.

4 Nonvanishing Bulk Potential

An interesting question is whether our procedure can be generalized to the case when the bulk potential is nonvanishing. We do not have a definitive answer, but we can argue that, if dynamical instabilities of the static solution exist, they are much more difficult to find than when $V(\phi) = 0$. We are interested in orbifold solutions that can be constructed from a regular bulk solution, $\Omega(z)$ and $\phi(z)$, which satisfies the jump equations (5) at $z = 0$. An important property of the vanishing potential solution that facilitated finding dynamic solutions was that the jump equations were actually satisfied for any z, not just at $z = 0$. This property cannot be maintained when the scalar potential is turned on, while still having a self-tuning solution. Indeed, it is easy to verify that if the relations (5) are satisfied for any arbitrary values of z, the bulk equations of motion imply the relation

$$V(\phi) = \frac{2}{\gamma}\left(\frac{T^2}{32\beta}\left(\frac{df}{d\phi}\right)^2 - \frac{T^2}{24\alpha}f^2(\phi)\right) \tag{30}$$

between the bulk scalar field and the conformal coupling. This relation is incompatible with the self-tuning mechanism unless the bulk potential vanishes; otherwise either $V(\phi)$ or $f(\phi)$ would have to depend on T, rather than ϕ_0. One way to overcome this difficulty might be to consider diffeomorphisms $\tau, z \to \tilde{\tau}, \tilde{z}$ such that $\tilde{z} = 0$ implies $z = 0$ independently of $\tilde{\tau}$. But it seems difficult to find such a diffeomorphism that leaves the metric diagonal. Another possibility would consist in relaxing the $\mathbb{Z}_2$ symmetry in the bulk since a naive count of parameters [28] shows that an integration constant remains unconstrained by the jump equations and could be promoted to a time-dependent function. However the continuity of the solution on the brane is no longer guaranteed at each time. These observations may

[3]In this equation ρ and p are the physical energy density and pressure describing the matter living on the brane and conformally coupled to the scalar field.

[4]Curiously, if all the parameters of the Lagrangian (1) as well as the parameter h are of the order of the 4-D Planck scale, with the age of the Universe estimated to fifteen billions years, today the physical tension would be of the order of $(5\,\mathrm{TeV})^4$, close to the electroweak scale.

indicate that the self-tuned solutions in the case of a nonvanishing bulk potential do not suffer from the kind of instability we have found with the ADKS-KSS solution whose pathology comes from the massless and unstabilized scalar field.

5 Conclusion

We have shown that the ADKS-KSS self-tuning solution is unstable against eternal expansion or singular collapse of the brane-world. Interestingly, the dynamics on the brane cannot be represented by a 4-D field theory, since energy flows off the brane into the bulk and thus appears not to be conserved on the brane. We have suggested that these problems may not occur in the presence of a potential energy in the bulk for the scalar field. If this hypothesis is correct, then further indirect evidence would consist in perturbing the brane with matter or radiation, and checking whether it behaves according to normal 4-D cosmology. We expect, in analogy to brane models without a stabilized radion, that a nonstandard Friedmann equation on the brane, such as $H \propto \rho$ [40], will be a diagnostic of instabilities in the extra dimension, if they exist. Work along these lines is currently in progress [41].

In a closely related study, Horowitz, Low and Zee recently presented in [38] a general class of plane wave solutions where the metric is parametrized as $ds^2 = e^{2A(t,y)}(-dt^2 + dy^2) + e^{2B(t,y)}dx_i^2$, and $B(t,y) = (1/3)\ln(f(t-y) + g(t+y))$ for arbitrary functions f and g. The corresponding expressions for $A(t,y)$ and $\phi(t,y)$ are generally more complicated, but it is possible to find solutions where $A(t,y) = B(t,y)$ and f and g are both linear functions, which are the same as our solutions.[5] Interestingly, [38] interpret these solutions as describing a phase transition claiming that it is possible to start with the static ADKS-KSS solution, at times $t < 0$, and smoothly match it to the dynamical solutions for $t > 0$, provided there is a sudden change ΔT in the brane tension at time $t = 0$. This would have demonstrated a physical mechanism for triggering the instability: an arbitrarily small change in the brane tension, such as would occur during a first order phase transition. However, it does not seem possible to have continuous time derivatives of the fields when such a gluing of the two kinds of solutions is attempted.[6] Such a solution would require that the tension on the brane be proportional to $\delta(t)$, rather than simply having a discontinuous change. On the other hand, the property of having dynamical solutions evolving to or from a Big Crunch or Bang with the *same* value of the brane tension translates into an instability of the static solution with respect to initial time derivatives since a small perturbation in ϕ_0' drives the solution to a nonvanishing value of h, and then unavoidably leads to a singularity in the time evolution.

[5] When the warp factor is normalized to one on the brane at $t = 0$, the relation between the integration constants are $z_c = y_0/(\xi - 2)$, $h = \xi/y_0$ and $\phi_0 = d + \epsilon \ln|y_0|$.

[6] This can be seen in (V.18) of [38], where $y_*''(t)$ must have a Dirac delta function at $t = 0$ if y_* goes from being a constant to being linear in t at $t = 0$.

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