BEAM IMPEDANCE CALCULATION AND ANALYSIS OF HIGHER ORDER MODES (HOMS) DAMPED RF CAVITIES USING MAFIA IN TIME DOMAIN*

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Abstract

A time domain method using MAFIA code has been developed to calculate narrow band beam impedance in RF cavities over a wide range of frequency spectrum. The impedance is obtained through Fast Fourier Transformation (FFT) of computed wakefields by the MAFIA. Analysis of the calculated impedance spectrum will be presented. Application of the method to a known RF cavity design (PEP-II cavity) has shown good agreements with bench and beam measurements. The method has been applied to the RF cavity design of of Damping Rings for Next Linear Collider (NLC).

1 INTRODUCTION

It is well known that higher order modes (HOMs) in RF cavities has been one of the main sources of longitudinal and transverse impedance which could contribute to coupled bunched beam instability in storage rings and synchrotron light sources, and become a limit factor for beam intensity. Properly damping the HOMs while leaving the fundamental one intact has been main efforts for cavity designs for decades for high intensity electron/positron storage rings and light sources[1]. HOMs exist in any RF cavity, once they are excited (for instance by beam under acceleration), they oscillate and eventfully decay with their natural frequencies and time constants which are determined by the cavity geometry, surface resistance and couplings to surroundings. In addition to interaction with fundamental mode in the cavity, the beam bunch interacts with the HOMs as well. Subsequently two things may happen, 1) the beam excites HOMs within its spectrum; 2) the resulting *EM* fields, which is called wakefield, from the HOMs act back on the beam. If the natural time constant of a HOM is long (or in another word, Q value is high), the *EM* field of this mode will last for a long time and act back on the trailing beam bunch, usually harms the beam. It is important, but not easy to damp HOMs efficiently (keep the Q below a certain value) while leaving the fundamental mode intact. Ways to damp HOMs include, for example, adding damping ports of HOMs on the cavity body or putting antennas inside the cavity, or on the beam pipe (superconducting cavity). Discussions on the damping mechanism can be found in many literatures. In this paper, we discuss how to calculate beam impedance exhibited by the HOMs in a RF cavity using the MAFIA in the time domain. Traditionally HOMs are computed in the frequency domain, HOMs impedance are studied and conducted experimently on cold-test cavities, which is time-consuming, expensive and limited to a few measurable modes. Simulating a cavity with HOMs couplers in the frequency domain has been difficult due to limited computational capability of available codes. With the development of advanced 3-D computer codes in recent years, for instance the MAFIA code, broadband waveguide boundary conditions become available in the time domain, many of these *experiments* now can be carried out on computers. We developed a method using the MAFIA in the time domain to calculate narrow band beam impedance in a wide frequency range. The method has been applied to a RF cavity design of the Damping Ring (DR) for the Next Linear Collider (NLC), and cavity designs for light sources [2].

2 THE TIME DOMAIN METHOD

The beam impedance is a description of the wakefield in the frequency domain. In principle the beam impedance of the HOMs can be calculated in the frequency domain. However, once a cavity has HOM coupling ports, couplings of these ports with outside make the boundary conditions on the port interfaces difficult to treat mathematically in the frequency domain. Hence without broadband waveguide boundary conditions, the frequency domain simulation results are good and limited only to trapped or nearly trapped modes. For a heavily HOMs damped cavity in particular (PEP-II cavity for instance), the frequency domain modeling simply does not represent the real physics in the cavity. Special method has been developed successfully for calculating external coupling [3] using the MAFIA in the frequency domain before the *waveguide* boundary condition became available in the time domain. We developed a method using the simulation results of the MAFIA in the time domain where the external coupling is dealt by the waveguide boundary condition. The wakefield in the cavity is excited by a Gauss distribution beam, and computed and recorded. This computed wakefield is then being Fast Fourier Transformed (FFT) to obtain its spectrum in the frequency domain. The beam impedance of the cavity is yielded by normalizing the wakefield spectrum to the spectrum of the excitation beam. The Gauss distribution beam

* This Research Work is supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, of the U.S. Department of Energy, under Contract No. DE-AC03-76SF00098

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(line charge) used in the calculation is given by,

$$\rho(s) = \frac{\mathcal{Q}}{\sqrt{2\pi}\sigma_s} e^{\frac{(s-s_0)^2}{2\sigma_s^2}} \tag{1}$$

where $\mathcal{Q}$ is total charge of the beam bunch, σ_s is bunch length.

The resulting wake function [1] may be expressed as a sum of all modes being excited in the cavity, where each mode is represented by an index of n

$$W(s) = \sum_{n=1}^{\infty} -\frac{\omega_n}{2}\left(\frac{R_n}{Q_n}\right) e^{j\frac{\omega_n}{c}s} e^{-\frac{\omega_n}{2Q_n}s} \tag{2}$$

where $W(s)$ represents the wake function, $\omega_n = 2\pi f_n$ with f_n as resonant frequency of mode n; ${R_n}^2$ and Q_n are shunt impedance and quality factor of mode n, respectively; $s = vt$ with v as the speed of beam bunch and s starts from the head of the beam. The beam impedance is obtained by the FFT of the wake function,

$$Z(\omega) = \frac{1}{c}\int_0^{\infty} W(s)\, e^{-j\frac{\omega}{c}s} \tag{3}$$

where we have denoted the beam impedance with $Z(\omega)$ and assumed $v = c$. Apparently the beam impedance depends on the frequency ω. For a RF cavity, the beam impedance can be expressed as a sum of all narrow band shunt impedance which it can be deduced directly from Equation (3),

$$Z = \sum_{n=1}^{\infty} z_n = \sum_{n=1}^{\infty} R_n. \tag{4}$$

It is worth to point out that the wakefield simulation is always computed and recorded in a finite time period τ or length $s_{\max} = c\tau$, the integral in Equation (3) does not extend to ∞. We therefore yield a calculated beam impedance z_n for mode n,

$$z_n = R_n\left(1 - e^{-\frac{\tau}{\tau_n}}\right) \tag{5}$$

where $\tau_n = \frac{2Q_n}{\omega_n}$ is the natural time constant of mode n.

The calculated beam impedance does not always equal to the shunt impedance of the cavity as it should be. This is due to the truncation of the wakefield calculation. Therefore the beam impedance calculated based on the FFT of the calculated wakefield may or may not give the correct beam impedance depending on the mode is resolved (no truncation) or not resolved (truncated) during the finite time period of the wakefield calculation. This can be further categorized as the following,

$$Z_n \approx \begin{cases} R_n & \text{if } \frac{\tau}{\tau_n} >> 1 \\ \frac{\tau}{2}\omega_n\left(\frac{R_n}{Q_n}\right) & \text{if } \frac{\tau}{\tau_n} << 1 \\ \text{needs two runs} & \text{if } \frac{\tau}{\tau_n} \sim 1 \end{cases} \tag{6}$$

A single time domain simulation of the wakefield can not get the beam impedance of high Q modes which is quantified in Equation (6) by τ_n. The simplest solution is to make two simulations with the second simulation doubling the time record of the first wakefield calculation. Two calculated beam impedance $z_n(\tau_1)$ and $z_n(\tau_2)$ ($\tau_2 = 2\tau_1$) are then obtained. We define a parameter $\xi = \frac{z_n(\tau_1)}{z_n(\tau_2)}$, which is the beam impedance ratio of the two simulations. It is evident that a mode is fully resolved if $\xi \approx 1$; not resolved if $\xi \approx 0.5$ (e.g., fundamental mode). At $\xi \approx 0.5$, external coupling is very weak, thus the frequency domain results are good and can be used directly (e.g., $R, Q,$ and $\left(\frac{R}{Q}\right)$). At $0.5 \le \xi \le 1$, the mode is partially resolved, both $z_n(\tau_1)$ and $z_n(\tau_2)$ are needed for calculating R_n. It is not difficult to find that R_n is given by,

$$R_n = \frac{\xi}{2\xi - 1} z_n(\tau_1) = \frac{z_n(\tau_1)^2}{2z_n(\tau_1) - z_n(\tau_2)}. \tag{7}$$

Typically $z_n(\tau_1)$ and $z_n(\tau_2)$ can be measured quite accurately from the calculated beam impedance spectrum, but sometimes need to be fitted and measured carefully due to limited resolution of the spectrum in the frequency domain. Better resolution can always be gained by increasing the calculated wakefield length or decreasing the time step in sacrifice of the CPU time and computer memory.

3 SIMULATION PROCEDURES

We list the procedures to compute the beam impedance using the MAFIA in the time domain,

- generate 3-D model in x-y-z coordinate (many built-in functions in the MAFIA post processor do not support the cylindrical coordinate)
- based on the 3-D model, slice out 2-D waveguide ports with the same mesh configuration in the mesh generator
- use the 2-D eigen-value solver to solve for the waveguide modes (TW) at each port
- load the calculated waveguide mode solutions to each port for using in the waveguide boundary conditions in the time domain solver (T3)
- define an excitation source as either beam or current, and calculate record the induced wake fields in T3
- post process (FFTs, etc.) the calculated wakefields to obtain the beam impedance in the post-processor (P3)

[1] The wake function can be obtained mathematically or numerically by computer codes by,

$$\vec{W}(s) = \frac{1}{\mathcal{Q}}\int_0^{\infty}\left(\vec{E}(\vec{r}, z, \frac{z+s}{v}) + \vec{v}\times\vec{B}(\vec{r}, z, \frac{z+s}{v})\right) dz,$$

where $\vec{E}$ and $\vec{B}$ are excited by charge $\mathcal{Q}$

[2] R_n is defined as $\left(\frac{V_t^2}{P}\right)_n$

4 SIMULATION RESULTS AND DISCUSSIONS

we use simulation results conducted for the PEP-II RF cavity as an example to detail the discussions on the beam impedance calculation. The PEP-II cavity is shown in Figure 1.

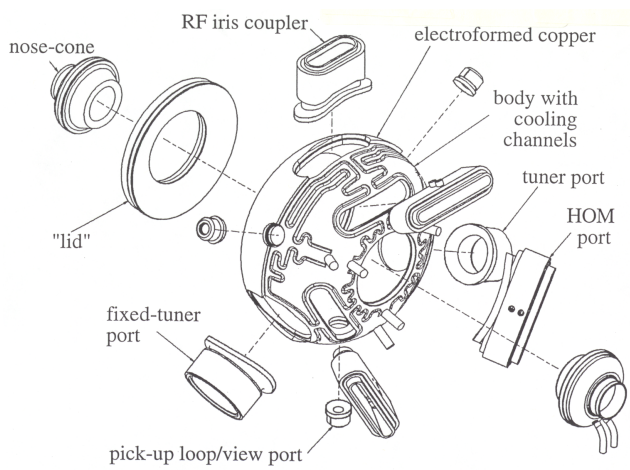

Figure 1: The PEP-II RF cavity: three HOM ports are equally spaced on the cavity body

A 3-D MAFIA model is established for the PEP-II cavity, good agreements between the MAFIA time domain simulations, measured and beam induced spectrum are achieved and reported in [1]. To illustrate the time domain method, only the calculated beam impedance of the dipole modes is shown in Figure 2 where the wakefields are computed and recorded for 100, 200, 400 and 800 meters long, respectively. The beam impedance is plotted versus frequency in Figure 2.

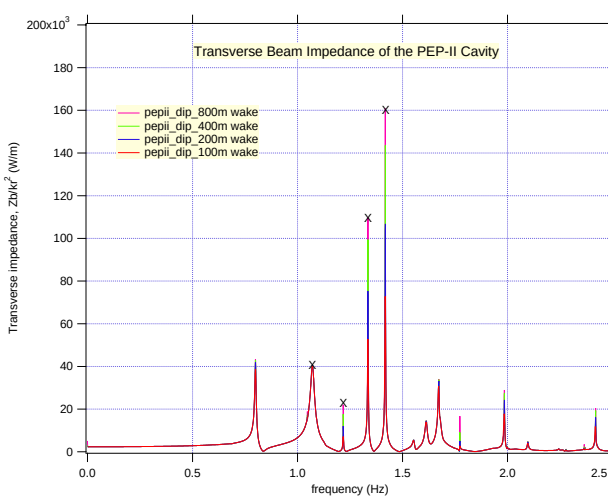

Figure 2: Calculated beam impedance of the dipole modes in the PEP-II cavity is plotted versus frequency.

Notation **X** in Figure 2 denotes the beam impedance calculated using Equation (7) from the z_n of 100 and 200 meters long wakefields. As a cross-check, the beam impedance is also calculated using the z_n from 200 and 400 meters long wakefields. They agree very well with the former calculation. As indicated in Figure 2, some of the modes are fully resolved, and some of them are partial resolved and their amplitudes increase with the time record of the wakefield. A close view at a mode frequency near 1.2 GHz is shown in Figure 3 exhibiting this amplitude changes explicitly,

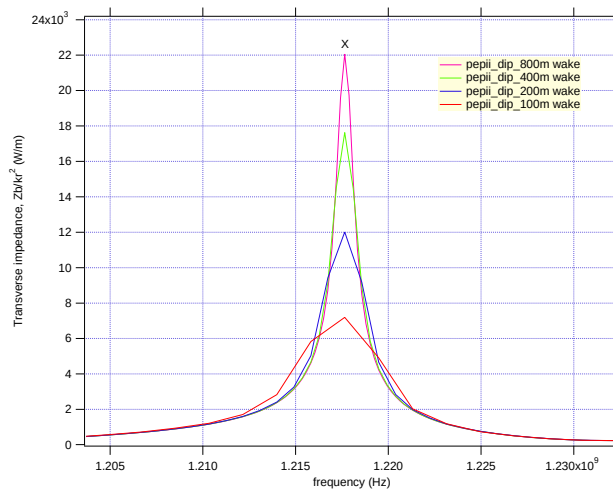

Figure 3: A close view of the beam impedance of the PEP-II RF cavity near 1.2 GHz

where the calculated z_n is indicated again by **X** which has slightly higher value than that obtained from 800 meters long wakefield implying that this mode is nearly resolved by the 800 meter wakefield.

5 CONCLUSION

The time domain method has been verified with the beam induced HOM spectrum measurement and bench measurement of the PEP-II cavity, and used successfully for the NLC DR RF cavity design and RF cavity designs in light sources. It can be easily extended for applications of any coupler design of RF cavities.

6 REFERENCES

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