Application of joint inversion for mapping fluid parameters

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ABSTRACT

A two-dimensional joint inversion technique, based on a least-squares criterion of the data misfit and model smoothness, has been developed using electromagnetic (EM) and seismic traveltime data to assess the feasibility of directly inverting for hydrological parameters, such as fluid electrical conductivity, porosity, and saturation. This is accomplished by relating hydrological parameters to geophysical properties with the help of the empirical Archie's law and the Wyllie time average equation. While the latter links the underground seismic wave velocity and subsurface media porosity, the former relates the bulk formation conductivity to hydrological parameters such as fluid conductivity and porosity. Direct joint inversion using various geophysical data also reduces the non-uniqueness of the problem since common parameters are involved, as is the porosity related to both seismic traveltime and magnetic field.

This newly developed joint inversion algorithm has been applied to a set of crosshole seismic and EM field data provided by Chevron as part of the Lost Hills CO_2 pilot project in Southern California. Both EM and seismic pre- and post-injection data were evaluated in terms of hydrological parameters using general empirical relationships derived from logging data. The results show that the injection has decreased the water saturation and bulk conductivity in the whole inter-well section. Layered structure in the region is clearly displayed and major changes in water saturation and bulk conductivity are also observed.

Key Words: joint inversion, conductivity, velocity, Pososity

INTRODUCTION

Accurate mapping of hydrological parameters such as permeability, porosity, fluid saturation and clay content is required to characterize the subsurface distribution and the flow of fluids, including gas. However, these parameters are currently measured or inferred from well tests or core samples. Results interpolated from the well data may be inaccurate and misleading. Among subsurface physical properties, acoustic wave velocity and electrical conductivity depend to a varying degree on formation porosity, saturation, pore fluid, temperature, etc. The non-uniqueness problem, however, makes individual geophysical inversion techniques to directly resolve the hydrological properties impossible. Joint analysis/inversion using different geophysical data along with available information about the relationships between geophysical and hydrological parameters may allow us to achieve the objective.

Direct joint inversion for hydrological properties using both EM and seismic methods could not be found in the literature yet. Hering et al. (1995) showed that improved interpretations can be achieved through joint inversion if multitudes of data are available. In their case, Schlumberger, radial-dipole and two-electrode sounding data were jointly inverted for better resistivity images, and Love and Rayleigh group slowness data were both used to obtain improved near-surface velocity structures. However, the EM and seismic interpretations were done separately.

To assess the feasibility of deriving hydrological properties directly, we have developed a joint inversion technique using EM and seismic traveltime data. The objective is to derive fluid conductivity, saturation, and rock porosity of the medium between two boreholes. Archie's law and the Wyllie time average relations are used to relate geophysical parameters to two of the hydrological variables: rock porosity and fluid conductivity. The inversion is based on a least-square criteria that minimizes the misfit between the observed data (synthetic data in this study) and that of the inverted hydrological model. A smoothness constraint is used to reduce the non-uniqueness. To begin with, the inversion is tested for a two-dimensional model. For the EM method, the model is axially symmetric about the transmitter borehole, and numerical simulation is carried out with the algorithm developed by Alumbaugh and Morrison (1995). Bulk electrical conductivity used for the EM simulation is estimated using Archie's law. Straight ray paths are assumed for the seismic method and traveltime data are calculated using the simplified Wyllie equation. The newly developed joint inversion algorithm has been applied to a set of crosshole seismic and EM field data from a CO_2 pilot project.

THEORY AND NUMERICAL ALGORITHM

The magnetic field due to an EM transmitter is a function of the bulk formation conductivity (σ_b), which, in turn, is a function of hydrological parameters: porosity (ϕ), pore fluid conductivity (σ_{fl}), and water saturation (S_w). They can be related with the empirical Archie's law (1942):

$$\sigma_b = c\sigma_{fl} S_w^n \phi^m , \qquad (1)$$

where c, n, and m are constants specific to the formation. In the inversion process using EM data alone to obtain the three constituents, it is difficult to separate the three parameters in (1) since they are lumped together to give the value of the bulk conductivity. The uncertainty, or non-uniqueness, can be reduced either by applying heavy smoothness constraint in the inversion process or by incorporating other independent data. For this purpose we include the seismic traveltime data, which is a function of seismic wave velocity. Seismic velocity depends on media porosity, density, pressure, etc. For simplicity, we chose the Wyllie time average equation to express seismic wave slowness (the reciprocal of velocity) as:

$$S = S_{ma} + (S_{fl} - S_{ma})\phi . \qquad (2)$$

Consequently, seismic traveltime is now linked to the porosity, ϕ, the pore fluid slowness, S_{fl}, and rock matrix slowness, S_{ma}. If we further assume both S_{fl} and S_{ma} are known, then the traveltime data is a function of the porosity only. Based on (1) and (2), one can jointly invert for the medium porosity, pore fluid conductivity, and/or water saturation using EM and seismic data.

If we assume that porosity and fluid conductivity are the desired hydrological parameters and the water saturation is known, a joint inversion scheme may be developed based on a least-square criteria that minimizes a cost function, Φ, defined as:

$$\begin{aligned}\Phi = &\left\{\left\| \underline{\underline{\mathbf{W}}}_d \left(\underline{\mathbf{d}}(\underline{\sigma}_{fl}, \underline{\phi}) - \underline{\mathbf{d}}_{obs}\right)\right\|^2 - \chi^2 \right\} \\ &+ \lambda_\phi \left\| \underline{\underline{\mathbf{W}}}_\phi \cdot \underline{\phi} \right\|^2 + \lambda_{fl} \left\| \underline{\underline{\mathbf{W}}}_{fl} \cdot \underline{\sigma}_{fl} \right\|^2\end{aligned} . \qquad (3)$$

The one-dimensional vectors $\underline{\mathbf{d}}$ and $\underline{\mathbf{d}}_{obs}$ are, respectively, the calculated and measured data; $\underline{\underline{\mathbf{W}}}_d$ is a square weighting matrix that assigns a relative importance to each data point. The number χ^2 is the estimated square-error in the observed data. To reduce the non-uniqueness to obtain a plausible solution, the inversion is constrained by the *a priori* information of the porosity and fluid conductivity using the two matrixes $\underline{\underline{\mathbf{W}}}_\phi$ and $\underline{\underline{\mathbf{W}}}_{fl}$ in the last two terms of (3). For this study, we use smoothness constraints for both parameters. The degree of smoothness is controlled by the two independent Lagrange multipliers λ_ϕ and λ_{fl}. The larger they are, the more smoothness is emphasized, and, consequently, the lower the resolution. Upon minimizing the functional with respect to the fluid conductivity and porosity, a system matrix equation is solved iteratively to derive the two unknowns: ϕ and σ_{fl}.

Evaluating the sensitivity function (Jacobian) is an essential part of the inversion processes. Since the magnetic field, **H**, is a function of rock conductivity, which in turn is a function of porosity and fluid conductivity, its perturbation due to the constituents is:

$$\delta\mathbf{H} = \frac{\partial \mathbf{H}}{\partial \sigma_b}\frac{\partial \sigma_b}{\partial \phi}\delta\phi + \frac{\partial \mathbf{H}}{\partial \sigma_b}\frac{\partial \sigma_b}{\partial \sigma_{fl}}\delta\sigma_{fl} . \qquad (4)$$

Here, the derivative terms in (4) can be obtained from traditional EM sensitivity analysis and Archie's law described in (1). Similarly, the traveltime data, **T**, is affected by the change in porosity in a way that can be expressed as:

$$\frac{\partial \mathbf{T}}{\partial \phi} = l\left(S_{fl} - S_{ma}\right) .$$

Here, l is the length of the ray passing through a certain model cross section.

In deriving the data misfit and sensitivity functions, forward modeling is necessary. For the EM method, a cylindrically symmetric geometry with the transmitter borehole located at the coordinate center is assumed. Vertical magnetic fields in the other borehole are calculated with an algorithm developed by Alumbaugh and Morrison (1995), based on the extended Born approximation (Habashy et al., 1993). For the seismic data, a straight-ray method is used for calculating the traveltime and the resulting slowness is related to the porosity based on the time average equation.

A synthetic crosshole EM and seismic data set for a simulated model is used to validate the algorithm. A simplified Archie's law is applied in (1): $c = 1$, $m = 2$, and the media is fully saturated, i.e., $S_w = 1$. As sketched in Figure 1, the transmitter and receiver boreholes are separated by 20 m in a half space with a pore fluid conductivity of 0.1 S/m and a porosity of 0.1. The P-wave velocities are assumed to be 5490 m/s and 1690 m/s for the matrix and fluid, respectively. Three horizontal thin square anomalous zones centered at the source borehole are located at 15, 30, and 45 m from the surface, and extend 12, 8, and 10 m toward the receiver borehole, respectively. In the three zones, the fluid conductivities are 1, 0.5, and 0.00625 S/m, respectively, and their corresponding porosities are 0.1, 0.4, and 0.4. The top two zones are anomalies compared to the bulk conductivity of the background, while the bulk velocities of the two bottom bodies are different from the host. Twenty-three source positions for both methods are distributed at 2.5 m intervals between the depths of 5 and 60 m. As many receivers are located in the receiver borehole. For the EM method, the source is a 10 kHz vertical magnetic dipole and vertical magnetic fields are calculated at the receivers using an algorithm SHEETS developed by Zhou (1989). P-wave traveltime data is obtained with the time average equation for straight ray paths between the transmitter and the receiver locations.

The initial model for the inversion is a half space with the same fluid conductivity and porosity as the host. We have been able to derive the bulk conductivity successfully with EM data only, but simultaneous reconstruction for both porosity and fluid conductivity with EM data only was not satisfactory due to the fact that both parameters are lumped together and equivalence problem could not be resolved. However, with seismic traveltime data added, the joint inversion is able to recover the two hydrological parameters. Figures 2(a) and (c) are the inverted rock porosity and fluid conductivity, respectively, between the two boreholes. The locations of the anomalous bodies and their sizes are well recovered. This result was obtained by inverting for the two parameters alternately. A similar image can be obtained as well if both parameters are inverted simultaneously. However, considering the performance of the code in terms of computing time and memory usage, obviously the alternating scheme is preferred.

GENERAL EMPIRICAL RELATIONSHIPS AND FIELD APPLICATION

General empirical relationships can be used for the joint inversion to comply with logging data for a specific site. Our joint inversion algorithm has been applied to a set of crosshole field data provided by Chevron as part of the Lost Hills CO_2 pilot project in Southern California. The separation between the two vertical boreholes is 24 m and the CO_2 injection well is near the center of the section defined by the two wells. Both EM data and seismic data for the pre- and post-injection were available for interpretation.

To characterize the hydrological structures in terms of rock porosity and water saturation with the field data, Archie's law should be modified to comply with the logging data and the time average equation is no longer appropriate for this case since the fluid and the rock matrix slownesses are unknown. Therefore, a multi-regression is performed to find the best constants for Archie's law to relate these parameters based on logging data. The solid line In Figure 3(a) displays the resistivity log to the transmitter borehole and the crosses are the calculated bulk conductivity using the revised formulation:

$$\sigma_b = 4.57 \cdot S_w^{1.81} \phi^{1.8} . \qquad (5)$$

The fluid conductivity is assumed fixed and subsumed into the leading constant in Archie's law. Similarly, the relationship between the bulk rock porosity vs. P-wave velocity (Figure 3(b)) is described by a polynomial

$$S_b = 0.00288\phi^2 - 0.00205\phi + 0.000891 . \qquad (6)$$

Here, the regression is based on the logging data of porosity and seismic wave velocity to a well near by. Based on these two general relationships obtained from logging data, the rock porosity and water saturation were inverted using the pre-injection EM and seismic data, while only the magnetic field was used to derive the post-injection water saturation since the porosity of the formation is assumed to be constant throughout the injection. Both EM and seismic crosshole data cover the section in the depth between 1400 and 2100 ft. The calculated pre-injection porosity and water saturation distributions in this section are displayed in Figures 4(a) and (c). Figure 4(e) is the post-injection results for the water saturation. The associated velocity and bulk conductivity are displayed in Figures 4(b), (d), and (f). The pre-injection porosity, water saturation, and bulk conductivity logging data to the source borehole are displayed in corresponding panels for comparison.

The inversion results show the layered structure of the area. The anomalous zone with higher porosity, water saturation and bulk conductivity between 1650 and 1710 ft roughly matches the porosity log as shown in Figure 4(a). However, this zone does not exist in the saturation and conductivity logging data. Two other layers, one between 1900 and 1980 ft, and the other one below 2030 ft, show higher water saturation and bulk conductivity, which correspond well to the logging data at the same depth as in Figures 4(c) and (d). Figures 5(a) and (b) show the change in water saturation and bulk conductivity due to the CO_2 injection. Obviously, the injection has decreased the water saturation and bulk conductivity in the whole section. Major changes in both properties can be observed in three zones centered at the depths of 1530, 1680, 1900 ft and one below 2030 ft. Also presented in Figure 5(c) is the change in bulk conductivity inverted by Hoversten of LBNL using the same EM data with a three-dimensional (3-D) algorithm. Compared to Figure 5(b), only the changes around 1700 and 1900 ft can be correlated. The discrepancy may be caused by the geometry (2-D vs. 3-D) assumed in the two algorithms, data quality, and the empirical relationships applied. However, the extent of the effects of these factors on the inversion results is still not clear.

CONCLUSIONS

Based on the preliminary results, we have demonstrated that hydrological parameters can be obtained directly with a joint inversion of two geophysical survey methods. The non-uniqueness problem can be alleviated by adding appropriate data associated with common parameters, as is the porosity related to both seismic traveltime and magnetic field. The optimum result, in terms of match with simulated model and computing efficiency, of the joint inversion has been obtained by alternately inverting the selected parameters. However, the empirical relationships among the geophysical and hydrological parameters should be studied further to obtain appropriate and more complicated links among them. To obtain correct interpretations from various geophysical techniques, efficient numerical routine for each individual method, such as full 3-D waveform seismic and EM inversions, is necessary for complex hydrological structures.

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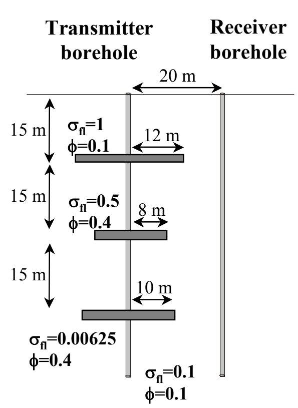

Archie's law:

$$\sigma_b = \sigma_{fl}\phi^2$$

Intrinsic P-wave velocity:

V_{ma}: 5486 m/s
V_{fl}: 1692 m/s

$$S_b = S_{ma}(1-\phi) + S_{fl}\phi$$

	Bulk Velocity (m/s)	Bulk Conductivity (S/m)
Background	4500	0.001
Anomaly 1	4500	0.01
Anomaly 2	2900	0.08
Anomaly 3	2900	0.001

EM frequency: 10 kHz
Data: For both methods, 25 Tx & 25 Rx from 1 m to 60 m at 2.5 m intervals

Figure 1. Simulated model for verifying the joint inversion algorithm.

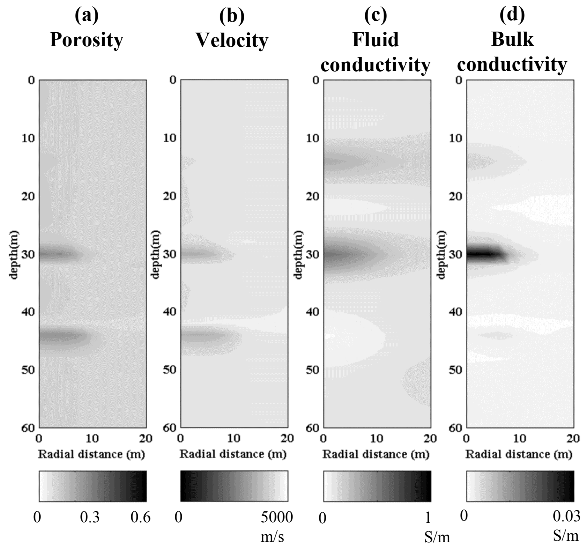

Figure 2. Results derived from the joint inversion: (a) porosity, (b) calculated P-wave velocity, (c) fluid conductivity, and (d) calculated bulk conductivity.

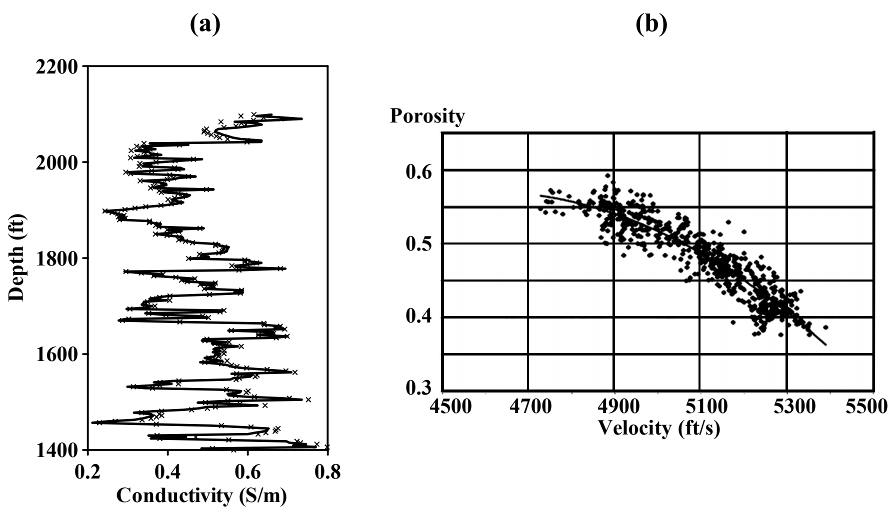

Figure 3. Well logging data. (a) Resistivity data (crosses) vs. calculated resistivity (solid line) based on the revised Archie's law. (b) Porosity vs. P-wave velocity with the best-fit relationship represented by the solid line.

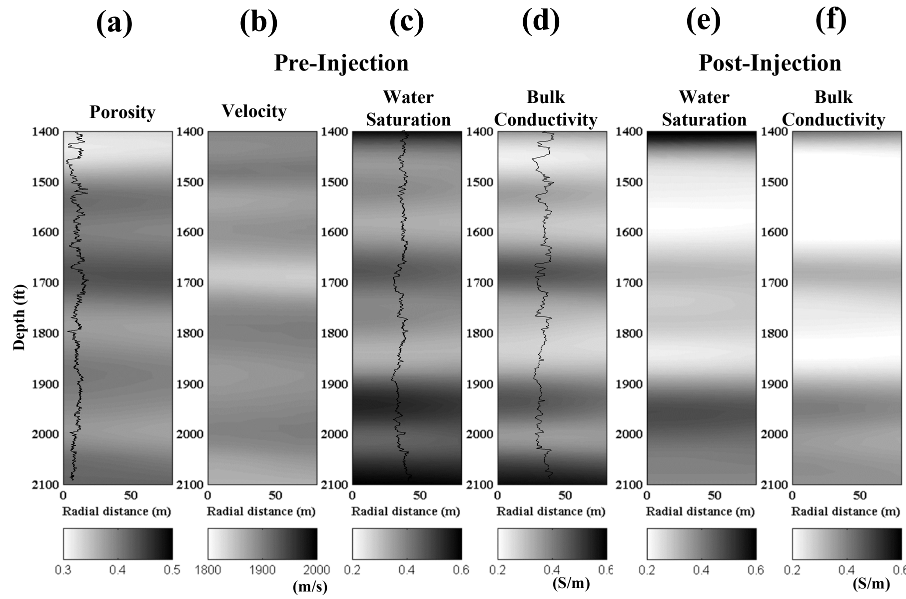

Figure 4. Inverted hydrological parameters using field data. Pre-injection results: (a) porosity, (b) velocity, (c) water saturation, and (d) calculated bulk conductivity. Post-injection results: (e) water saturation, and (f) calculated bulk conductivity.

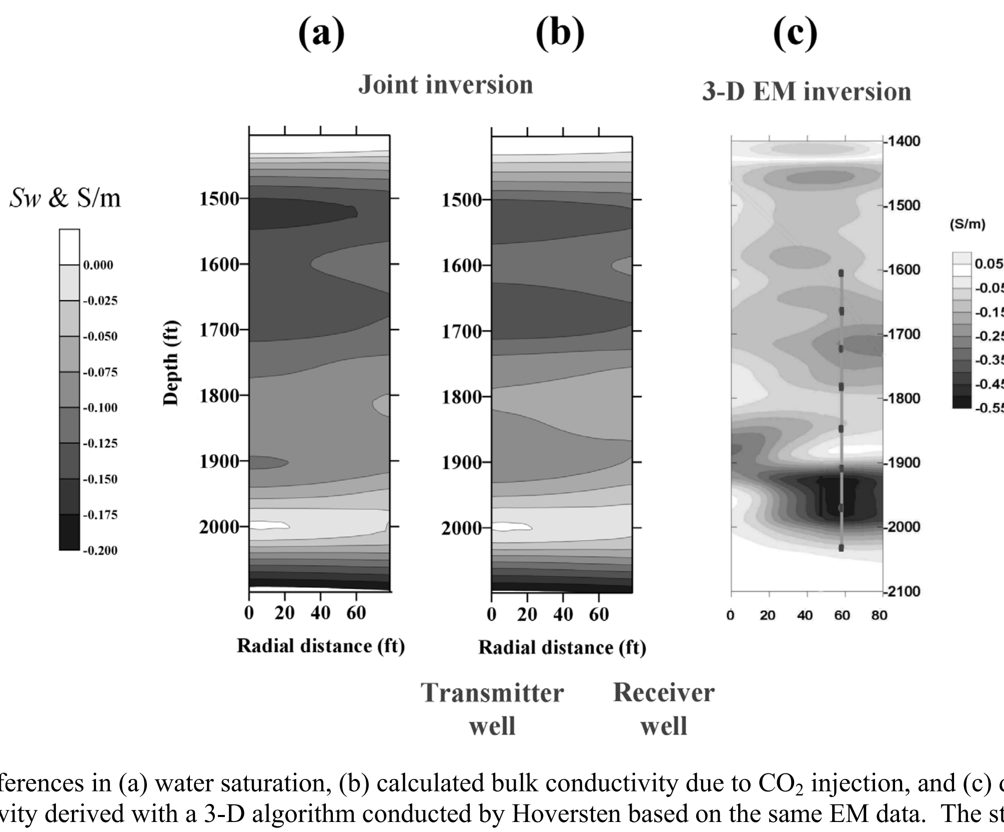

Figure 5. Differences in (a) water saturation, (b) calculated bulk conductivity due to CO_2 injection, and (c) difference in bulk conductivity derived with a 3-D algorithm conducted by Hoversten based on the same EM data. The straight line in (c) is the projection of the injection well onto the cross-hole section plane.