# CPT Tests: Kaon vs Neutrinos[*]

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CPT violation has an impressive limit in the neutral kaon system $|m(K^0)-m(\overline{K}^0)| < 10^{-18} m_K = 0.50\times 10^{-18}$ GeV. However, if viewed as a constraint on the mass-squared, the bound appears weak, $|m^2(K^0)-m^2(\overline{K}^0)| < 0.25$ eV$^2$. We point out that neutrino oscillation offers better limits on CPT violation in this case. The comparison of solar and rector neutrino results puts the best limit on CPT violation by far, $|\Delta m^2_\nu - \Delta m^2_{\bar\nu}| < 1.3\times 10^{-3}$ eV$^2$ (90% CL).

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The CPT theorem is one of the few solid predictions of the relativistic local quantum field theory [1]. In particular, it states that a particle and its anti-particle must have the same mass and lifetime. It is based on three reasonable assumptions:

- Lorentz invariance,
- Hermiticity of the Hamiltonian,
- Locality.

If CPT is found violated, the implication to the fundamental physics is enormous, as at least one of the three assumptions above must be violated. One way to prove the CPT theorem is by defining $S$-matrix elements by analytic continuation of the Euclidean correlation functions. The CPT transformation is then achieved by the Euclidean rotation that changes the sign of the (imaginary) time and all spatial coordinates, and hence is a symmetry of the $S$-matrix elements. String theory is normally argued to be CPT-conserving, as its $S$-matrix elements are defined precisely in this fashion. However, it does not exclude the possibility of a spontaneous violation of the CPT symmetry depending on the details of the low-energy limit. Kostelecky and collaborators have a series of papers on possible CPT violation based on this point of view [2]. It was also argued that it may be possible to break CPT in a field theory by giving up locality but not the other two [3] (see, however, Ref. [4] for criticisms).

Recently, a possible CPT violation which allows different masses for particles and anti-particles has attracted attention in neutrino oscillation phenomenology. If three indications for the neutrino oscillation, solar [5, 6, 7, 8, 9, 10], atmospheric [11], and LSND [12], are all correct, we have to accommodate three mass-squared differences of quite different orders of magnitudes, which is not possible within the three generations of neutrinos. A fourth kind of neutrino is usually invoked to explain the data. It has to be "sterile" so that it does not violate the data from $Z^0$ decay at Large Electron Positron collider (LEP): $N_\nu = 2.994 \pm 0.012$ [13]. However, recent data from SNO requires $\nu_e$ oscillation into an active (non-sterile) neutrino [9], while the SuperKamiokande prefers $\nu_\mu$ oscillation into $\nu_\tau$ [14], leaving little room for a sterile neutrino. Further combined with older data from CDHSW [15], Bugey [16], even the extension with a sterile neutrino does not help explain the data very much [17]. Yanagida and the author [18] have pointed out that we can explain all data consistently allowing different mass spectra for neutrinos and anti-neutrinos, because the solar neutrino oscillation is purely in neutrinos while statistically significant evidence for oscillation at LSND is in anti-neutrinos. This observation was partially motivated by the consistency between the LSND and SN1987A data. This possibility of CPT-violating neutrino mass spectra was elaborated further by a series of works by Barenboim *et al* [19]. (Indirect constraints are important only for the Majorana case [20].)

Phenomenologically, a stringent limit exists on the CPT violation in the neutral kaon system. Thanks to the mixing between $K^0$ and $\overline{K}^0$, the limit on the possible mass difference between them is exceptionally strong [13]:

$$|m(K^0)-m(\overline{K}^0)| < 10^{-18} m_K = 0.50\times 10^{-9}\ \text{eV}. \quad (1)$$

Given such a stringent limit, there does not appear much window for CPT violation or improved tests [21].

We point out that the strength of the CPT limit from the neutral kaon system may be misleading. In lack of a concrete theory of CPT violation, the limit Eq. (1) may be looked at as a limit on the difference in *mass squared* rather than the masses. In fact, a local Lagrangian field theory always has mass squared as a natural parameter for bosons. Also in relativistic kinematics, mass squared is the natural parameter in Einstein's relation $E^2 = \vec{p}^2c^2 + m^2c^4$ rather than the mass itself. If reinterpreted as a limit on the possible difference in mass squared, it reads

$$|m^2(K^0)-m^2(\overline{K}^0)| < 0.25\ \text{eV}^2. \quad (2)$$

It is intriguing that the possible violation of CPT in quantum gravity suppressed by the Planck scale may lead to an order of magnitude $\langle v\rangle^2/M_{Pl} \sim 10^{-5}$ eV, which is well within the above bound.

On the other hand, the neutrino oscillation experiments always measure $\Delta m^2$, and cannot measure the

masses themselves. Yet, limits on the difference $\delta \equiv \Delta m_\nu^2 - \Delta m_{\bar\nu}^2$ can be obtained. The SuperKamiokande collaboration has studied the possible difference in neutrino and anti-neutrino $\Delta m^2$ in atmospheric neutrino oscillations. Their current limit is [23]

$$-7.5\times 10^{-3}\mathrm{eV}^2 < \delta < 5.5\times 10^{-3}\mathrm{eV}^2. \tag{3}$$

This limit is much better than that from the kaon system. We have to note, however, that this limit assumes the same maximal mixing for both neutrinos and anti-neutrinos. The limit may be considerably worse if this assumption is relaxed [22].

We find that the best limit comes from the comparison of the solar neutrino data and the recent KamLAND result [24]. We analyze the data within the two-flavor oscillation framework. However, we emphasize that we cannot naively use the result of the global fit to compare the preferred values of $\Delta m^2$ between solar and reactor data. It is because global fits are based on the $\Delta\chi^2$ relative to the minimum and hence defines only the relative probability, while throwing away information on which region of the parameter space is excluded on the basis of the absolute probability. We have to find a way to obtain an absolute limit on the parameter.

KamLAND has recently reported its initial result of a significant deficit in the reactor anti-neutrino flux [24]. It demonstrated a deficit in the reactor anti-neutrino flux, which we interpret as neutrino oscillation. Then we can speak of $\Delta m_{\bar\nu}^2$. Combined with the previous reactor experiments CHOOZ [25] and Palo Verde [26], we have a range of $\Delta m_{\bar\nu}^2$ not excluded by the data:

$$1.9\times 10^{-5}\mathrm{eV}^2 < \Delta m_{\bar\nu}^2 < 1.1\times 10^{-3}\mathrm{eV}^2 \tag{4}$$

at 90% CL *independent of the mixing angle* [27]. We emphasize that both ends of the inequality are the exclusion limits, rather than the "preferred range" from the $\Delta\chi^2$ analysis. Therefore, this statement has an absolute meaning: the probability that a value of $\Delta m_{\bar\nu}^2$ outside this range would fluctuate and produce the observed data is less than 10%.

As for the solar neutrino data, currently the Large Mixing Angle (LMA) solution is the most preferred, while the LOW solution or Vacuum oscillation (VAC) solution may exist at a higher confidence level. From the analysis in [28], the goodness-of-fit is not necessarily bad even for these solutions or Small Mixing Angle (SMA) solution. It is not clear we can set a lower limit on $\Delta m_\nu^2$. Fortunately for our purpose, it will suffice to have only an upper bound on the $\Delta m_\nu^2$.

SNO [9] convincingly established that the survival probability of $^8$B neutrinos is about a third. By naively combining the reported numbers on solar neutrino fluxes with the charged-current reaction $\phi_{CC} = 1.76^{+0.06+0.09}_{-0.05-0.09}$ and with the neutral-current reaction $\phi_{NC} = 5.09^{+0.44+0.46}_{-0.43-0.43}$, we find $P_{\rm surv} = \phi_{CC}/\phi_{NC} = 0.346\pm 0.048$. The upper bound at 90% CL is $P_{\rm surv} < 0.425$. It is important that it is less than a half. If the neutrinos oscillate purely in the vacuum, the deficit would be at most a half in the case of the maximal mixing [29]. The deficit of two thirds is explained only by the presence of the matter effect. In order for the matter effect to be important relative to the mass difference, $\Delta m_\nu^2$ is bounded from above. We would like to obtain a quantitative upper limit on $\Delta m_\nu^2$ using this piece of information.

The Hamiltonian of the two-flavor neutrinos is

$$H = \frac{\Delta m_\nu^2}{4p}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix}. \tag{5}$$

In this expression we dropped terms that are proportional to the identity matrix as they are not important for the consideration of the survival probabilities. This Hamiltonian is time-dependent as the electron number density $n_e$ changes in the course of neutrino propagation. The time evolution of the neutrino states is adiabatic for high $\Delta m_\nu^2$, and hence we only need to study the eigenstates of the Hamiltonian at the point of production ($n_e \approx 100 N_A/\mathrm{cm}^3$) and the detection ($n_e = 0$) [35]. In the vacuum, the eigenstates are given simply by

$$H\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix} = -\frac{\Delta m_\nu^2}{4p}\begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}, \tag{6}$$

$$H\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} = \frac{\Delta m_\nu^2}{4p}\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}. \tag{7}$$

We choose the convention that $\Delta m_\nu^2 > 0$ without a loss of generality, while the mixing angle is varied $0 < \theta < \pi/2$ [30, 31]. On the other hand, in the core of the sun,

$$H\begin{pmatrix} \cos\theta_m \\ -\sin\theta_m \end{pmatrix} = E_-\begin{pmatrix} \cos\theta_m \\ -\sin\theta_m \end{pmatrix},$$

$$H\begin{pmatrix} \sin\theta_m \\ \cos\theta_m \end{pmatrix} = E_+\begin{pmatrix} \sin\theta_m \\ \cos\theta_m \end{pmatrix}, \tag{8}$$

with

$$E_\pm = \frac{\sqrt{2}G_F n_e}{2}\left(1 \pm \sqrt{1 - 2\Delta\cos 2\theta + \Delta^2}\right) \tag{9}$$

and

$$\Delta = \frac{\Delta m_\nu^2}{2p\sqrt{2}G_F n_e}. \tag{10}$$

The mixing angle in the presence of matter is

$$\tan\theta_m = \frac{1 - \Delta\cos 2\theta + \sqrt{1 - 2\Delta\cos 2\theta + \Delta^2}}{\Delta\sin 2\theta}. \tag{11}$$

Because two different mass eigenstates decohere on the averaging over the energy and the production region for this range of $\Delta m_\nu^2$, we can obtain a very simple expression for the survival probability

$$\begin{aligned} P_{\rm surv} &= \cos^2\theta\cos^2\theta_m + \sin^2\theta\sin^2\theta_m \\ &= \frac{1}{2}\left(1 - \frac{\cos 2\theta(1 - \Delta\cos 2\theta)}{\sqrt{1 - 2\Delta\cos 2\theta + \Delta^2}}\right). \end{aligned} \tag{12}$$

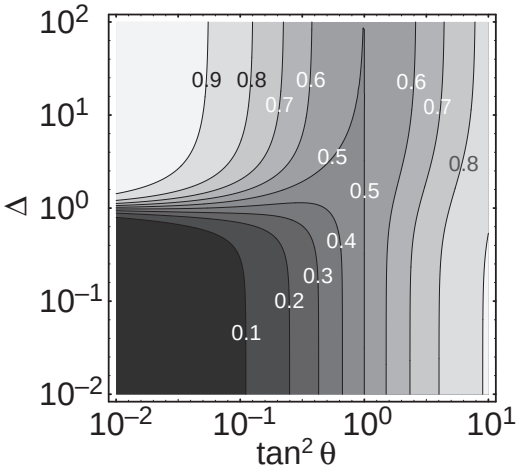


FIG. 1: The contour plot of $P_{\rm surv}$ in Eq. (12).

The point here is that one cannot explain the reduction of the electron neutrino flux to less than a half if $\Delta$ is too large. One can show that the 90% CL upper limit $P_{\rm surv} < 0.425$ translates to $\Delta < 1.31$. Therefore,

$$\Delta m_\nu^2 < 1.31 \times 2p\sqrt{2}G_F n_e. \qquad (13)$$

To be conservative, we use $n_e = 100 N_A {\rm cm}^{-3}$ at the core, even though the production region of $^8$B neutrino is spread over about a tenth of the solar radius. We also conservatively take $p \simeq 10$ MeV, the higher end of the $^8$B spectrum. We then find numerically,

$$\Delta m_\nu^2 < 2.0 \times 10^{-4} {\rm eV}^2. \qquad (14)$$

Now we combine Eqs. (4) and (14) to obtain a limit on possible CPT violation. We, however, allow for the possibility that the definition of $\Delta m^2$ may be different between neutrinos and anti-neutrinos and hence they may have a different sign. Given this, the limit is

$$|\delta| \equiv |\Delta m_\nu^2 - \Delta m_{\bar\nu}^2| < 1.3 \times 10^{-3} {\rm eV}^2 \qquad (90\%{\rm CL}). \quad (15)$$

Indeed this constraint is the world best bound on CPT violation in mass-squared parameters so far.

The situation on the LSND evidence for neutrino oscillation remains unresolved. Naively, the consistency between solar neutrino data and KamLAND seems to exclude the possibility of explaining LSND together with other data using CPT violation within three generations. However, the authors of Ref. [22] argued that the anti-neutrinos are subdominant in atmospheric neutrino data and hence $\Delta m^2$ as large as that of LSND is allowed for anti-neutrinos. This point had been criticized in [32]. If LSND data stands, we may either need more than one sterile neutrino [33] or lepton number violating muon decay [34]. In the latter case, Mini-BooNE data will not neither confirm or verify LSND data and the situation may remain ambiguous.

In summary, we argued that the limit on CPT violation from the neutral kaon system is not as strong as it appears once viewed as a constraint on the mass-squared difference between kaon and anti-kaon. Compared to the kaon constraint, neutrino oscillation data provide much stronger limits. We derived a limit on $\delta = \Delta m_\nu^2 - \Delta m_{\bar\nu}^2$ quantitatively from SNO and KamLAND data, with an emphasis on using the absolute probability rather than relying on the $\Delta\chi^2$ analysis. The obtained bound $|\delta| \equiv |\Delta m_\nu^2 - \Delta m_{\bar\nu}^2| < 1.3 \times 10^{-3}{\rm eV}^2$ (90% CL) is currently the best limit on the possible CPT violation in mass-squared of particles and anti-particles.

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