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# A Physically Based Numerical Approach for Modeling Fracture-Matrix Interaction in Fractured Reservoirs

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## ABSTRACT

Modeling fracture-matrix interaction within a multiple-phase flow system is a key issue for fractured reservoir simulation. Commonly used mathematical models for dealing with such interactions employ dual- or multiple-continuum concepts, in which fractures and matrix are represented as overlapping, different, but interconnected continua, described by parallel sets of conservation equations. The conventional single-point upstream weighting scheme is most commonly used to estimate flow mobility for fracture-matrix flow. However, such a scheme may have serious limitations or flaws, which lead to unphysical solutions or significant numerical errors. To overcome the limitations of the conventional upstream weighting scheme, this paper presents a physically based modeling approach for estimating physically correct relative permeability in calculating multiphase flow between fractures and the matrix, using continuity of capillary pressure at the fracture-matrix interface. The proposed approach has been implemented into two multiphase reservoir simulators and verified using analytical solutions and laboratory experimental data. The new method is demonstrated to be accurate, numerically efficient, and easy to implement in dual- or multiple-continuum reservoir simulators.

## 1. INTRODUCTION

Since the 1960s, significant progress has been made in numerical modeling of flow and transport processes in fractured rock. Research efforts, driven by the increasing need to develop petroleum and geothermal reservoirs, other natural underground resources, and to resolve subsurface contamination problems, have developed many numerical modeling approaches and techniques (Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985).

Mathematical modeling approaches in general rely on continuum approaches and involve developing conceptual models, incorporating the geometrical information of a given fracture-matrix system, setting up mass and energy conservation equations for fracture-matrix domains, and then solving discrete nonlinear algebraic equations. The key issue for simulating flow in fractured rock, however, is how to handle facture-matrix interaction under different conditions (involving multiple phase flow). This is because the fracture-matrix interaction distinguishes the flow through *fractured* porous media from the flow through heterogeneous *single-porosity* porous media.

To model fracture-matrix interaction, researchers have developed and applied many different conceptual models and modeling approaches (Berkowitz, 2002). Commonly used mathematical methods include: (1) an explicit discrete-fracture and matrix model (e.g., Snow, 1969; Stothoff and Or, 2000), (2) the dual-continuum method, including double- and multiporosity, dual-permeability, or the more general "multiple interacting continua"' (MINC) method (e.g., Barenblatt et al., 1960; Warren and Root, 1963; Kazemi, 1969; Pruess and Narasimhan, 1985), and (3) the effective-continuum method (ECM) (e.g., Wu, 2000a).

The explicit discrete-fracture approach is, in principle, a rigorous model. However, the actual application of this method is currently limited because of the computational intensity involved, as well as the lack of detailed knowledge of fracture and matrix geometric properties and their associated spatial distributions at a given site. On the other hand, the dual-continuum method is conceptually simpler and computationally much less demanding than the discrete-fracture approach, and is able to handle fracture-matrix interaction more easily than the discrete-fracture model.

Dual-continuum approaches include the classical *double-porosity* model (Barenblatt et al., 1960; Warren and Root, 1963), the dual-permeability concept, and the more rigorous dual-continuum generalization of the MINC (Pruess and Narasimhan, 1985) for modeling flow in fractured porous media. In the double-porosity model, a flow domain is composed of matrix blocks with low permeability, embedded in a network of interconnected fractures. Global flow and transport in the formation occur only through the fracture system, conceptualized as an effective continuum. This model treats matrix blocks as spatially distributed sinks or sources to the fracture system without accounting for global matrix-matrix flow. Because of its computational efficiency and its ability to match many types of laboratory- or field-observed data simultaneously (e.g., Kazemi, 1979; Wu et al., 1999), the dual-continuum model has perhaps been the most widely used method in petroleum and geothermal engineering and groundwater hydrogeology, and it has also been implemented in many commercially available reservoir simulators.

In numerical modeling of flow through fractured reservoirs, one of the critical issues is how to estimate flow mobility at the fracture-matrix interface. In conventional simulation practice, especially in petroleum reservoir simulation, the fully upstream weighting scheme (or simply *upstream weighting* or *upwinding*) is routinely used (e.g., Aziz and Settari, 1979). As a result, the fracture relative permeability is commonly selected in estimating the mobility when local flow is towards the matrix. However, this scheme is physically incorrect, because of the inherent anisotropy of the fracture-matrix medium at this scale. The fracture relative permeability functions are properties for flow along fractures, determined independently from matrix flow–for example, by laboratory studies (e.g., Persoff and Pruess,

1995). In general, fracture-matrix flow (or interaction) occurs perpendicular to fracture planes, which is controlled mainly by matrix flow properties. Therefore, the fracture relative permeability, if used in this case, may lead to unphysical solutions or significant numerical errors.

The objective of this study is to develop a physically based upstream weighting scheme for determining relative permeability functions that can be generally applicable to calculating multiphase flow between fractures and the rock matrix using a dual-continuum concept. Specifically, the proposed mobility-weighting approach has been implemented into two multiphase reservoir simulators to demonstrate its application. In addition, we attempt to verify the proposed weighting scheme by using an analytical solution and published laboratory results. The new method is shown to be accurate, numerically efficient, and easy to implement in existing dual- or multiple-continuum models in reservoir simulators.

## 2. MATHEMATICAL MODEL

In the dual-continuum approach, multiphase flow processes in fractured rock are described separately, using a doublet of governing equations for the two fracture and matrix continua. This conceptualization results in a set of partial differential equations for flow in either continuum, which are in the same form as that for a single porous medium. In this work, the multiphase flow system, assumed in an isothermal, fractured porous formation, consists of three phases: gas (air), water, and NAPL (or oil), in which two-phase flow or the Richards' equation (1931) is considered as a special case. Although each of the three phases contains a number of components, they are treated here as a single "pseudo-component" with averaged properties of the fluids. In addition, the three fluid components (gas, water, and NAPL) are assumed to be present only in their associated phases. Each phase flows in response to its pressure gradients, gravitational forces, and capillary forces, according to the multiphase extension of Darcy's law.

In an isothermal system containing three mass components, three mass balance equations are needed to describe flow and transport in the fracture and matrix blocks. For flow of phase β (β =g for gas, β = w for water, and β = o for NAPL),

$$\frac{\partial}{\partial t}\left(\phi S_\beta \rho_\beta\right) = -\nabla \bullet \left(\rho_\beta \mathbf{v}_\beta\right) + q_\beta \qquad (2\text{-}1)$$

where the Darcy velocity of phase β is defined by:

$$\mathbf{v}_\beta = -\frac{k k_{r\beta}}{\mu_\beta}\left(\nabla P_\beta - \rho_\beta g \nabla D\right) \qquad (2\text{-}2)$$

In Equations (2-1) and (2-2), $\rho_\beta$ is the density of phase β under reservoir conditions; $\phi$ is the effective porosity of the medium; $\mu_\beta$ is the viscosity of phase β; $S_\beta$ is the saturation of phase β; $P_\beta$ is the pressure of phase β; $q_\beta$ is the sink/source term of phase (component) β per unit volume of formation; g is gravitational acceleration; k is the absolute/intrinsic permeability of the formation; $k_{r\beta}$ is relative permeability to phase β; and D is depth from a datum.

The governing equation of mass balance for three-phase fluids, Equation (2-1), needs to be supplemented with constitutive equations, which express all the secondary variables and parameters as functions of a set of primary variables of interest. In particular, the relationships include relative permeability and capillary pressure functions as well as other constitutive data. In addition, the initial and boundary conditions of the system are also needed to complete the description of multiphase flow through fractured or porous media.

## 3. NUMERICAL FORMULATION

### 3.1 Discrete Equations

Multiphase flow equations, as discussed in Section 2, have been implemented into a general-purpose multiphase code TOUGH2 (Pruess et al., 1999) and a three-phase reservoir simulator MSFLOW (Wu, 2000b). As implemented numerically, Equation (2-1) is discretized in space using an integral finite-difference or control-volume scheme for a porous and/or fractured medium. The time discretization is carried out with a backward, first-order, finite-difference scheme. The discrete nonlinear equations for water, NAPL, and gas flow at node i are written as follows:

$$\left\{\left(\phi S_\beta \rho_\beta\right)_i^{n+1} - \left(\phi S_\beta \rho_\beta\right)_i^{n}\right\}\frac{V_i}{\Delta t} = \sum_{j\in\eta_i} F_{\beta,ij}^{n+1} + Q_{\beta i}^{n+1} \qquad (3\text{-}1)$$

(for β = g, w and o).

where superscript n denotes the previous time level; n+1 is the current time level; $V_i$ is the volume of element i (porous or fractured block); Δt is time step size; $\eta_i$ contains the set of neighboring elements (j) (porous or fractured) to which element i is directly connected; $F_{\beta,ij}$ is the mass flow term for phase β between elements i and j; and $Q_{\beta i}$ is the mass sink/source term at element i, of phase β.

The "flow" term ( $F_{\beta,ij}$ ) in Equation (3-1) for single-phase, Richards', or multiphase flow is described by a discrete version of Darcy's law. This mass flux of fluid phase β along the connection is given by

$$F_{\beta,ij} = \lambda_{\beta,ij+1/2}\gamma_{ij}\left[\psi_{\beta j} - \psi_{\beta i}\right] \qquad (3\text{-}2)$$

where $\lambda_{\beta,i\,j+1/2}$ is the mobility term to phase β, defined as

$$\lambda_{\beta,i\,j+1/2} = \left(\frac{\rho_\beta k_{r\beta}}{\mu_\beta}\right)_{ij+1/2} \qquad (3\text{-}3)$$

and subscript ij+1/2 denotes a proper averaging or weighting of properties at the interface between two elements i and j (discussed in the sections below), and $k_{r\beta}$ is the relative permeability to phase β. In Equation (3-2), $\gamma_{ij}$ is transmissivity and is defined differently for finite-difference or finite-element discretization. If the integral finite-difference scheme (Pruess et al., 1999) is used, the transmissivity is calculated as

$$\gamma_{ij} = \frac{A_{ij} k_{ij+1/2}}{d_i + d_j} \qquad (3\text{-}4)$$

where $A_{ij}$ is the common interface area between connected blocks or nodes i and j; $d_i$ is the distance from the center of block i to the interface between blocks i and j;

and $k_{ij+1/2}$ is an averaged (such as harmonic weighted) absolute permeability along the connection between elements i and j,

The flow potential term in Equation (3-2) is defined as

$$\psi_{\beta i} = P_{\beta i} - \rho_{\beta,ij+1/2}\, g\, D_i \qquad (3\text{-}5)$$

where $D_i$ is the depth to the center of block i from a reference datum.

Discrete Equation (3-1) has the same form regardless of the dimensionality of the model domain, i.e., it applies to one-, two-, or three-dimensional analyses of flow through fractured or porous media. In our numerical model, Equation (3-1) is written in a residual form and is solved using the Newton/Raphson iteration.

### 3.2 Handling Fractured Media

The technique used in this work for handling multiphase flow through fractured rock follows the dual-continuum methodology (Warren and Root, 1963; Pruess and Narasimhan, 1985). This method treats fracture and matrix flow and interactions using a multicontinuum numerical approach, including the double- or multiporosity method, the dual-permeability method, and the more general MINC method (Pruess and Narasimhan, 1985). It can be shown that the same continuum concept is also applicable to multiphase flow though a discrete fracture network.

The multiphase flow formulation, Equations (2-1) and (3-1), is applicable to both single-continuum and multicontinuum media. Using the dual-continuum concept, Equations (2-1) and (3-1) can be used to describe multiphase flow both in fractures and inside matrix blocks, as well as fracture-matrix interaction. However, special attention needs to be paid to treating fracture-matrix flow. The flow between fractures and the matrix is still evaluated using Equation (3-2); however, the transmissivity for the fracture-matrix flow is given by

$$\gamma_{ij} = \frac{A_{FM} k_M}{l_{FM}}, \qquad (3\text{-}6)$$

where $A_{FM}$ is the total interfacial area between fractures and the matrix of elements i and j (one of them is a fracture and the other a matrix block); $k_M$ is the matrix absolute permeability along the fracture-matrix connection; and $l_{FM}$ is a characteristic distance for flow crossing fracture-matrix interfaces, which can be determined for idealized 1-D, 2-D and 3-D dimensional rectangular matrix blocks when using the double-porosity model (Warren and Root, 1963).

### 3.3 Mobility Weighting Scheme

The appropriate spatial weighting scheme for averaging flow properties, such as the mobility of Equation (3-3) in a heterogeneous formation, has been much debated in reservoir simulation and groundwater modeling literature (Peaceman, 1977; Huyakorn and Pinder, 1983). Single-point or fully upstream weighting has been the exclusive approach for averaging mobility or relative permeability in calculating flow terms, using a discrete Darcy's law for multiphase flow in heterogeneous petroleum reservoirs (Aziz and Settari, 1979). The reasons behind the early application of the conventional upstream weighting scheme for relative permeability were based on several physical arguments, such as the need for upstream weighting to initialize imbibition into completely dry rock. In addition, the upstream weighting approach was found to be necessary to avoid incorrect solutions in two-phase immiscible displacement (hyperbolic) problems (Aziz and Settari, 1979).

Recently, several theoretical studies (Forsyth et al., 1995; Forsyth and Kropinski, 1997) have shown that the upstream weighting scheme, if used with the control-volume discretization of the Richards' equation, will satisfy monotonicity conditions regardless of time step or mesh size. It will guarantee that converged numerical solutions are physically correct, while other weighting schemes, such as central weighting, may converge to an incorrect, unphysical solution (Forsyth and Kropinski, 1997). However, determining flow along fracture-matrix connections (i.e., flow across fracture-matrix interfaces in the direction perpendicular to fracture planes) is different from fracture-fracture flow, and the conventional upstream weighting scheme may no longer be applicable. This is because fracture relative permeability functions are fracture flow properties describing flow along fractures, determined independently from matrix flow. Conversely, fracture-matrix flow or interaction normally occurs along the directions perpendicular to fractures, and is largely controlled by matrix properties or by flow resistance within the matrix block. The physical inconsistency in selecting fracture relative permeability for calculating fracture-matrix flow may lead to unphysical solutions or significant numerical errors.

To overcome these limitations, this paper presents a modified upstream weighting scheme for fracture-matrix interaction. This new scheme is based on the principle that the capillary pressure is continuous at the fracture-matrix interface, and on the assumption that there is instantaneous local equilibrium in pressure for each phase on the matrix surface within fracture and matrix systems. This should hold true for most subsurface fractured reservoirs, because fracture aperture is normally very small and fracture lateral boundaries are defined by matrix surfaces. Any dynamic changes in fractures, such as capillary pressures, could be instantaneously equilibrated locally with those at contacted matrix surfaces. As a result, the matrix relative permeability at the matrix surface can be readily determined as a function of fracture capillary pressure or the matrix saturation corresponding to that fracture capillary pressure. Therefore, the new scheme, when the upstream direction for fracture-matrix flow is in the fractures, uses the matrix relative permeability function (instead of the fracture relative permeability function, as in the conventional upstream weighting scheme) to calculate the mobility. Physically, this is equivalent to evaluating flow through the fracture-matrix interface into the matrix with the effective matrix permeability at that interface, obviously a more reasonable approach. The proposed weighting scheme is still dependent on the upstream fracture condition, and therefore does not lose the advantages of upstream schemes. In addition, in case fracture-matrix flow is from matrix to fractures, such as in a situation of drainage or flow between globally connected fractures or along global or local matrix-matrix connections, the conventional upstream weighting scheme should still be used. We call this hybrid scheme capillary pressure-based weighting or physically based upstream weighting.

Within the context of the dual-continuum concept, the proposed approach can be applied to different matrix

discretizations, such as double-porosity, dual-permeability, or MINC grids. We have implemented the proposed physical upstream weighting scheme into two multidimensional reservoir simulators and conducted a series of numerical experiments with all the commonly used dual-continuum models. In all test cases, this new weighting scheme is found to work efficiently, similarly to using the traditional single-point upstream weighting, without numerical difficulties. Note that this new scheme should be applicable for discrete fracture-network models as well.

## 4. VALIDATION AND APPLICATION EXAMPLES

To examine and verify the proposed mobility-weighting scheme in this section, we present two validation and application examples. The proposed physical upstream weighting scheme has been implemented in the two reservoir simulators TOUGH2 (Pruess et al., 1999; Wu et al., 1996) and MSFLOW (Wu, 2000b), which are used in the following application examples. The first example compares numerical simulation results with analytical solutions for Richards' equation (Wu and Pan, 2004). The second case matches published laboratory experiment results of water imbibition and oil displacement conducted on fractured cores (Kazemi, 1979).

### 4.1 Comparison with the Analytical Solution

This example attempts to validate the new weighting scheme using an analytical solution (Wu and Pan, 2004). This solution is based on specially correlated relative permeability and capillary functions, and describes the transient unsaturated fracture-matrix interaction for radial flow through a uniform, radially infinite fractured formation, consisting of many cubic matrix blocks and fracture sets.

This verification problem addresses transient flow from a fully penetrating injection well into a partially saturated, 10 m thick horizontal fractured formation. The fractured formation consists of identical $1 \times 1 \times 1$ m$^3$ cubes of matrix blocks, surrounded by a uniform, 3-D fracture network. The geological model considered is identical to the Warren and Root (1963) conceptual model. Initially, both fracture and matrix are at dry conditions, with both initial saturations equal to their residual values, respectively. A constant saturation of $S_W = 0.279$ is imposed at the wellbore as the inner boundary condition in the beginning, and then water is sucked into the formation from the well immediately.

In the numerical model, a finite reservoir ($r_e = 100$ m) of 10 m thickness is used, and the disk-type formation is discretized into a one-dimensional radially symmetric (primary) grid. The radial domain from $r_w = 0.1$ m to distance $r_e = 100$ m is subdivided into 500 intervals, with $\Delta r = 0.005$ m for the first 200 elements; and the rest of the domain is subdivided into 300 gridblocks following a logarithmic scale. A double-porosity numerical grid is generated and is used for simulations with both the traditional and new upstream weighting schemes.

For this problem, fracture-matrix rock and fluid properties are given in Table 4-1. Numerical simulations with two different weighting schemes are performed using the TOUGH2 code. Two numerical simulations were completed for this problem using the double-porosity grid with two alternative mobility-weighting schemes, i.e., the traditional single-point upstream weighting and the physical upstream weighting proposed here.

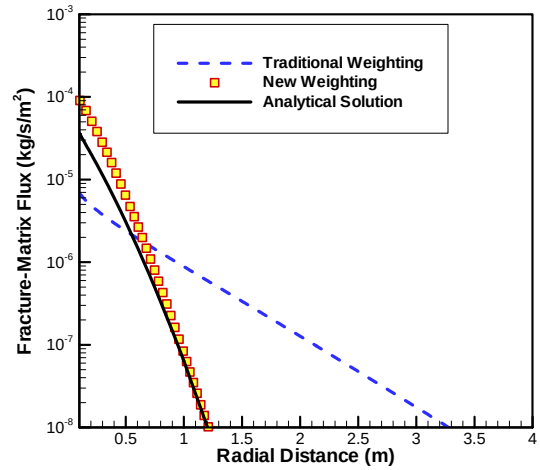


**Figure 1: Comparison of fracture-matrix mass fluxes along the radial distance at 1 day, calculated using the analytical solution and numerical simulations using the double-porosity model with two different mobility-weighting schemes.**

Figure 1 presents fracture-matrix flux distributions along the radial distance at one day, simulated by the analytical and three numerical modeling results. Note that the physical process simulated in this example is extremely nonlinear and dynamic. The initial liquid saturations are at residual values for both fracture and matrix systems. At the beginning, the boundary saturation for fractures at the wellbore jumps to a constant (= 0.279) (flow rate at the well thus becomes infinitely large). Once imbibed into the fractures near the well, the liquid will be drawn by two forces in two different directions, flowing along fractures away from the well, and simultaneously imbibing into dry matrix blocks.

Figure 1 clearly shows that the traditional upstream weighting scheme, by selecting a fracture relative permeability function to calculate fracture-matrix flow, cannot match the analytical results, while the new weighting scheme provides a good approximation.

Figure 2 shows a comparison between cumulative fracture-matrix mass exchanges from the analytical solution and from the two weighting schemes, calculated by integrating fracture-matrix fluxes over time. The traditional weighting is seen to lead to significant errors, while the new scheme agrees well with the analytical solution.

Many more numerical experiments and comparisons (using different parameter sets, and boundary and initial conditions) have been carried out. All the tests and comparisons, similar to those shown in Figures 1 and 2, indicate that the proposed new weighting scheme is able to closely match analytical solutions for unsaturated radial flow problems, while in general the traditional weighting method produces large errors.

### 4.2 Comparison with Laboratory Experimental Results

Kazemi (1979) presented a series of laboratory experimental results of water imbibition into fractured matrix cores to displace oil. The laboratory tests were conducted on three sets of artificial fractured cores using cylindrical and rectangular blocks, with one fracture along

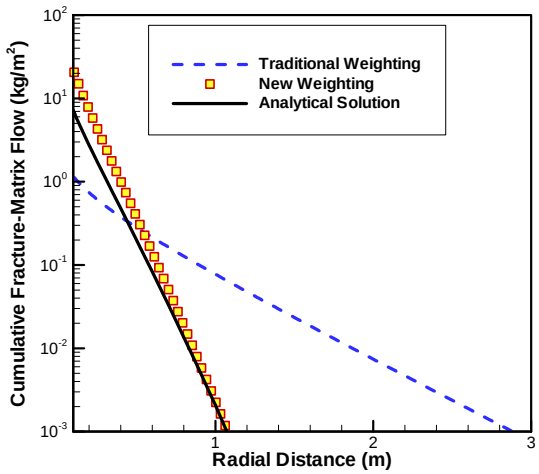


**Figure 2: Comparison of fracture-matrix mass fluxes along the radial distance at 1 day, calculated using the analytical solution and numerical simulations using the double-porosity model with two different mobility-weighting schemes.**

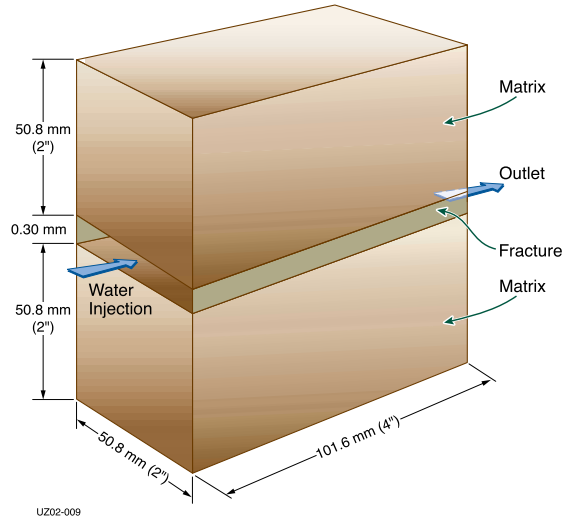


**Figure 3: Schematic of fractured cores used the experimental studies (Kazemi, 1979).**

the long axis for each set. The cylindrical and rectangular matrix blocks were cut from Berea sandstone. The laboratory model we consider here consists of a fractured core with two brick-type matrix blocks. Each matrix block has a brick shape with dimension of width, height, and length (50.8 × 50.8 × 101.6 mm) as shown in Figure 3. The fracture formed between the two matrix cores has an aperture of 0.30 mm. The experimental data used in this study were from Test 38423 (Kazemi, 1979). In the experiment, flow channels were left open only at the inlet and outlet ends of the fracture (i.e., for water injection and for oil and water outflow), and side fracture and matrix surfaces were sealed. Initially, the fracture and matrix system was fully saturated uniformly with oil (diesel), and then water was injected with a constant rate at the inlet (Figure 3) to displace the oil.

Basic model experimental and modeling parameters are listed in Table 4-2. The relative permeability and capillary pressure curves used are shown in Figures 4 and 5, respectively. Note that relative permeability curves for both fracture and matrix, shown in Figure 4, were estimated using the equations given in Kazemi (1979), and the matrix capillary pressure curve was taken from the capillary-pressure curve on the Berea core of Figure 12 in Kazemi (1979). However, several important parameters were not provided in Kazemi (1979), including residual water saturation, residual oil saturation, and fracture capillary pressure curves. Actual values used for these missing parameters were determined in this work by model calibration, with the final estimates given in Table 4-2 and Figures 4 and 5.

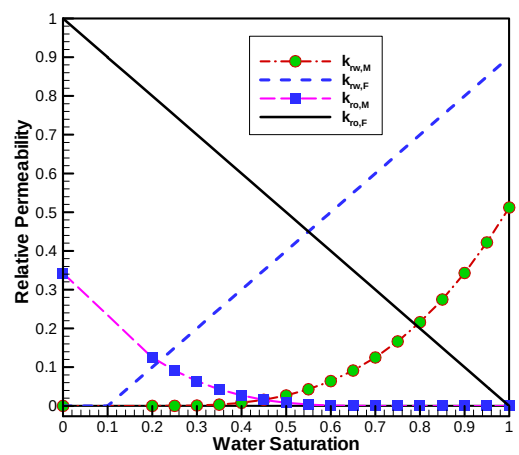


**Figure 4: Relative permeability curves for fractures and matrix used in matching laboratory experimental results.**

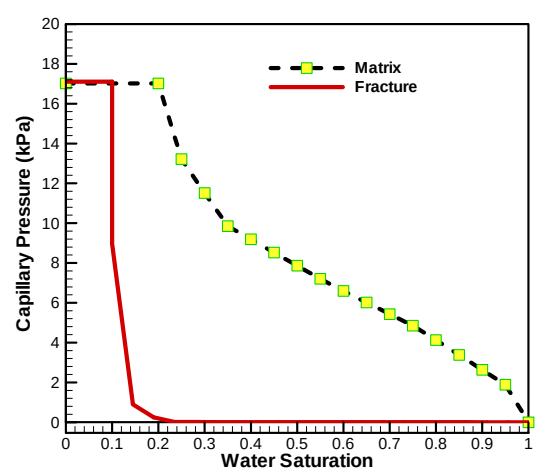


**Figure 5: Capillary pressure curves for fractures and matrix used in matching laboratory experiment results.**

Here, this test is analyzed using a double-porosity approach (equivalent to the explicit-fracture model in this case) to examine the numerical scheme for handling fracture-matrix interaction under multiphase flow conditions. The fracture-matrix set of Figure 3 is treated as a 2-D system along the longitudinal (x) direction (from inlet to outlet). Because of the symmetry, only half of the 2-D model domain (one matrix block and half the fracture) is discretized into a double-porosity grid, using a 1-D parallel fracture concept, with one (actually half) fracture element corresponding to one matrix element in the transverse direction (perpendicular to the fracture plane). Along the x direction, a uniform linear grid of 10 elements is generated for both the fracture and the matrix block, with a uniform grid spacing of $\Delta x = 10.16$ mm.

We have performed several model calibration analyses to estimate the missing model parameters. The final simulation results using the proposed physical upstream weighting scheme are compared with the laboratory experimental data in Figure 6. Figure 6 shows excellent agreement between measured and simulated volumetric fractional oil recovery versus pore volume of water injected. This result indicates that the proposed new upstream mobility-weighting scheme is able to capture the main factors that control fracture-matrix interaction during the oil-water displacement for this test problem.

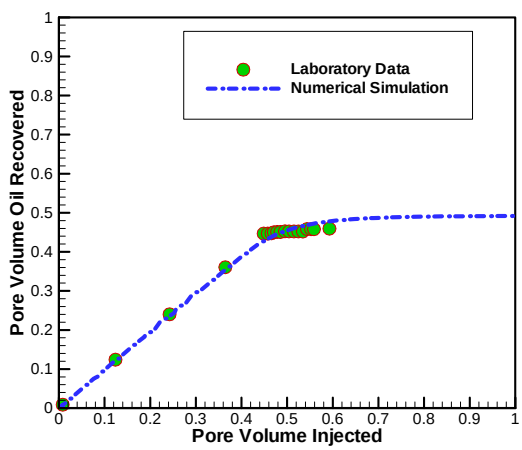


**Figure 6: Comparison of simulation results with experimental data (Kazemi, 1979)**

## 5. SUMMARY

We have presented a physically based upstream weighting scheme for modeling multiphase fracture-matrix flow and interaction. This new approach is based on continuity of capillary pressure at the fracture-matrix interface in estimating physically correct relative permeability for multiphase flow between fractures and the matrix. This new conceptual model overcomes a serious flaw that exists in most current simulation practice when estimating flow mobility for fracture-matrix flow terms using the conventional upstream weighting method. Numerically, the new scheme uses exactly the same dual-continuum grids as the traditional modeling approaches, without requiring an additional computational burden or using refined grids, to achieve not only accurate but also physically correct results for fracture-matrix interaction.

To verify the proposed mobility-weighting scheme, we first used the new mobility-weighting method to simulate unsaturated flow in a radially fractured formation with fully transient fracture-matrix interaction. In this case, the proposed physical upstream weighting scheme is found to provide accurate simulation results when compared with analytical solutions. It is also found that the conventional weighting scheme will result in significant errors in estimating fracture-matrix flow in modeling the same physical processes. In the second case, we sought to match published laboratory results for oil-water displacement through a fractured core. Our proposed approach proved able to match these laboratory experimental results.

In this work, we have demonstrated that the proposed physical upstream weighting method is accurate, numerically efficient, and easy to implement into existing dual- or multiple-continuum reservoir simulators. Although the test cases reported here involved isothermal flow only, application of the method to nonisothermal flow with or without phase change is straightforward.

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**Table 4-1: Parameters for the comparison problem with radial unsaturated flow**

| Parameter | Matrix | Fracture | UNIT |
|---|---|---|---|
| Matrix dimension | B = 1 | | m |
| Porosity | $\phi_M = 0.30$ | $\phi_F = 0.001$ | |
| Permeability | $k_M = 1.0 \times 10^{-15}$ | $k_F = 1.0 \times 10^{-12}$ | $m^2$ |
| Residual/initial saturation | $S_{Mr} = 0.2$ | $S_{Fr} = 0.2$ | |
| Coefficient of [1] relative permeability | $C_{kM} = 1.0$ | $C_{kF} = 0.2$ | |
| Coefficient of [2] capillary pressure | $C_{pM} = 1.0 \times 10^4$ | $C_{pF} = 1.0 \times 10^3$ | Pa |
| Saturation at well | $S_0 = 0.279$ | | |
| Fluid viscosity | $\mu_w = 1.0 \times 10^{-3}$ | | Pa•s |
| Fluid density | $\rho_w = 1{,}000$ | | $kg/m^3$ |
| Wellbore radius | $r_w = 0.1$ | | m |

[1], [2] Note that coefficients for relative permeability and capillary pressure are constants for the specially correlated functions (Wu and Pan, 2003)

**Table 4-2: Parameters used in the comparison with laboratory testing results (Kazemi, 1979)**

| Parameter | Value | Unit |
|---|---|---|
| Fracture aperture | b = 0.0003 | m |
| Fracture porosity | $\phi_F = 1.0$ | |
| Matrix porosity | $\phi_M = 0.21$ | |
| Absolute fracture permeability | $k_F = 1 \times 10^{-11}$ | $m^2$ |
| Absolute matrix permeability | $k_M = 4{,}23 \times 10^{-13}$ | $m^2$ |
| Water density | $\rho_w = 1{,}000$ | $kg/m^3$ |
| Water viscosity | $\mu_w = 1 \times 10^{-3}$ | Pa • s |
| Oil (diesel) density | $\rho_w = 828$ | $kg/m^3$ |
| Oil (diesel) viscosity | $\mu_w = 4.6 \times 10^{-3}$ | Pa • s |
| Residual fracture water saturation | $S_{wr,F} = 0.10$ | |
| Residual matrix water saturation | $S_{wr,M} = 0.20$ | |
| Residual fracture oil saturation | $S_{or,F} = 0.0001$ | |
| Initial fracture water saturation | $S_{wi,F} = 0.00$ | |
| Initial matrix water saturation | $S_{wi,M} = 0.00$ | |
| Water injection rate | $q = 2.568 \times 10^{-5}$ | $m^3/d$ |