Strong-strong beam-beam simulation on parallel computer

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1. Introduction

The beam-beam interaction puts a strong limit on the luminosity of the high energy storage ring colliders. At the interaction points, the electromagnetic fields generated by one beam focus or defocus the opposite beam. This can cause beam blowup and a reduction of luminosity. An accurate simulation of the beam-beam interaction is needed to help optimize the luminosity in high energy colliders.

Macroparticle tracking provides an invaluable tool for the study of beam-beam interactions. In this approach, a number of simulation particles ("macroparticles") are used with the same charge-to-mass ratio as the real particles. Outside the interaction region, the macroparticles are transported through the simulated lattice using transfer maps associated with external elements, radiation damping, and quantum excitation. At the interaction point, the electromagnetic fields from the beams are calculated and applied to the particles of the opposing beam.

The soft Gaussian approximation is sometimes used to obtain the electromagnetic fields of the beams at the collision point [1-3]. While this approximation has the advantage of computational speed, it is not self-consistent because it assumes a Gaussian distribution for the macroparticles even when the actual distribution might differ substantially from the Gaussian shape. To take into account the effects of the beam distribution self-consistently, one has to solve the Poisson equation numerically during each collision for the actual macroparticle distribution at that instant. A number of methods have been used to solve the Poisson equation. The five-point finite difference method with Fourier analysis and cyclic reduction (FACR) has been used by Krishnagopal [4] and Cai et al. [5]. This method solves the Possion equation efficiently with finite domain boundary conditions. For the open boundary conditions, which are appropriate in typical beam-beam simulations, the method requires finding an effective boundary condition on the problem boundary; this can be computationally expensive. In addition, this method is not efficient to handle the case with two widely separated beams, where the domain of the source particles (particle domain) and the domain of the electric field (field domain) are different. Another method based on the fast multipole expansion has been used by Herr et al. [6] to solve the Poission equation. In this method, the computational cost scales linearly with the number of particles or with the number of total mesh points for the open boundary condition. The efficiency of this method is independent of the distribution of the source particles and the field domain, which makes it suitable to handle the situation with two separated beams. However, this method is an approximate algorithm in a sense that the accuracy of the expansion depends on the radius of convergence. The computational speed depends on the number of polynomials required in the multipole expansion.

The most widely used method to solve the Poisson equation in beam-beam simulations is the Green function method with fast Fourier transform (FFT). This method uses an FFT to calculate the cyclic summation on a doubled computational grid [7-11]. By defining a new shifted integrated Green function, this method can handle the separated beams, and beams with large aspect ratio. During the beam-beam interaction, when the bunch length is large compared with the beta function value or the beam-beam forces are strong, finite bunch length effects are not negligible. In this case, a multiple slice model has to be used. The computational cost scales as the square of the number of slices. For a hadron collider with tiny radiation damping, it is required to track the beams for many millions of turns to study the dynamics on the time scale of the lifetime of the beams. To study the beam-beam interaction fully self-consistently for both beams (i.e. a "strong-strong" formulation), and to include all the physical processes of long range off-centroid interactions, finite beam bunch length effects, and crossing angle collisions, requires computational resources far beyond the capability of current serial computers. A parallel beam-beam simulation model, with both weak-strong and strong-strong capabilities, that can simulate these physical processes accurately using high performance computers has been developed at Lawrence Berkeley National Laboratory [12], and is briefly described below along with a few applications.

2. Computational Model

In the computational model of strong-strong beam simulation, each charged particle is characterized by its charge, mass, and phase space coordinates (x, p_x, y, p_y, Δz, $\Delta p / p_0$). Here, the independent variable, s, is the arc length along a reference trajectory inside the accelerator, $p_{\{x,y\}}$ is the transverse momentum normalized by the total momentum of a reference particle ($p_0 = E_0/c$), $\Delta z = s - ct(s)$ with c the speed of light, $\Delta p = |p| - p_0$ with p_0 the absolute momentum value of the reference particle. The beam-beam forces, under the relativistic limit, can be obtained from the solution of the two dimensional Poisson equation. The effect of finite bunch length is included using a multiple slice model. In this model, each beam bunch is divided into a number of slices along the longitudinal direction in the moving frame. Each slice contains nearly the same number of particles at different longitudinal locations z. The collision point between two opposite slices is determined and the particles are moved to the collision point through a drift. At the collision point, the slopes of the particles are updated using the beam-beam electromagnetic forces at the collision point following

$$p_{x_2}^{new} = p_{x_2} + \frac{2q_1 q_2 N_1}{\gamma_2 4\pi\varepsilon_0 m_2 c^2} E_{x_1} \tag{1}$$

$$p_{y_2}^{new} = p_{y_2} + \frac{2q_1 q_2 N_1}{\gamma_2 4\pi\varepsilon_0 m_2 c^2} E_{y_1} \tag{2}$$

In the above equations, the subscripts 1 and 2 refer to each of the two beams. The corresponding equations for the other beam are obtained from the above by exchanging 1 and 2. The other symbols have the following meaning: $\gamma = 1/(1-\beta^2)^{1/2}$, $\beta_i = v_i/c$, $i = x,y,z$, c is the speed of light, ε_0 is the vacuum permittivity, q is the charge of the particle, m is the rest mass of the particle, N is the number of particles in a bunch, and E_x and E_y are the transverse electric fields generated by the opposite moving beam.

The solution of Poisson's equation can be written as

$$\phi(x,y) = \frac{1}{2\pi\varepsilon_0}\int G(x,\bar{x},y,\bar{y})\rho(\bar{x},\bar{y})\, d\bar{x}\bar{y} \tag{3}$$

where G is the Green function and ρ is the accumulated charge density distribution within a slice. For the case of transverse open boundary conditions, the Green function is given by:

$$G(x,\bar{x},y,\bar{y}) = -\frac{1}{2}\ln((x-\bar{x})^2 + (y-\bar{y})^2) \tag{4}$$

The calculation of the above convolution using a direct summation is not efficient and scales as the square of the total number of computational mesh points. Fortunately, the direct summation can be replaced by a cyclic summation in a double-gridded computational domain. The cyclic summation can be done very efficiently using a discrete fast Fourier transform (FFT) [13].

In the original FFT-based algorithm, the particle source domain and the electric field domain are contained in the same computational domain. Here, the particle domain is the configuration space containing the charged particles, and the field domain is the space where the electric field is generated by the charged particles. In the beam-beam interaction, the two opposite moving beams might not overlap with each other, e.g. in the case of parasitic collisions. To apply Hockney's algorithm directly requires the computational domain to contain both the particle domain and the field domain, i.e. both beams must be fully contained in the common domain. Since there is a large empty space between two separated beams, containing both beams in one computational domain results in poor spatial resolution. This is also computationally inefficient because the electric fields in the empty space between the two beams are not used. To avoid this problem, we have defined a shifted Green function as

$$G_s(x,\bar{x},y,\bar{y}) = -\frac{1}{2}\ln((x_c + x - \bar{x})^2 + (y_c + y - \bar{y})^2) \tag{5}$$

where x_c and y_c are the center coordinates of the field domain. Using the shifted Green function, the computational domain contains the larger one of the two separated beams. This saves the computational cost and also improves the numerical accuracy of the solution.

When the colliding beams have large horizontal (x) to vertical (y) aspect ratio, the straightforward use of the Green function at each mesh point is not efficient since it requires a large number of mesh points along the longer direction in order to get sufficient resolution for the Green function along that direction. An alternative way is to define an effective Green function as

$$G_s^{eff} = \int_{x_{i'}+h_x/2}^{x_{i'}-h_x/2} dx' \int_{y_{j'}+h_y/2}^{y_{j'}-h_y/2} dy'\, G_s(x_i - x', y_j - y') \tag{6}$$

The use of the effective Green function has been found to be valuable for modeling high aspect ratio beams [8,12]

Lastly, the effects of external focusing fields can be simulated, in the small-amplitude approximation, by a one-turn linear map. In our case, we include in this map the effects from linear machine chromaticity. The effects of radiation damping and quantum excitation can be represented using a localized stochastic map [14]. When two beams

collide at a finite angle, a transformation is used to change the crossing angle collision in the laboratory frame into a head-on collision in the boosted moving frame [15-16].

3. Parallel Implementation

In the parallel implementation, we have used a particle-field decomposition method. In this approach, each processor possesses the same number of particles and the same number of computational grid points, i.e, the same size of spatial subdomain. Fig. 1 shows a schematic plot of the particle-field decomposition among three processors.

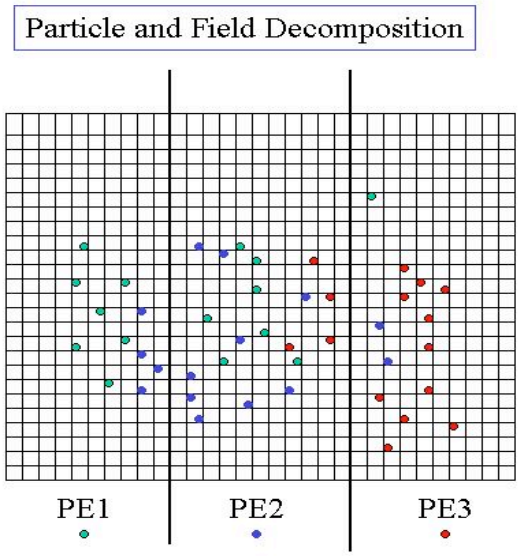

Fig. 1: A schematic plot of particle-field decomposition

In this approach, the global computational mesh is uniformly distributed among the processors. Each processor also has the same number of particles. The spatial coordinates of the particles on each processor might not stay within the spatial mesh domain of that processor. In the process of solving the Poisson equation, the particles are deposited onto the computational grid to obtain the charge density distribution. For the particles with spatial positions outside the local subdomain, an auxiliary computational grid is used to store the charge density. After the deposition, the charge density stored on the auxiliary grid is sent to the processor containing that subdomain. With charge density local to each processor, the Poisson equation is solved in parallel on a local subdomain using a Green function method. Since each processor contains the same number of computational grid points, the workload is well balanced among all processors. The solution of the electric potential on the local subdomain is sent to all processors. With the electric potential on each processor, the electric field is calculated on the grid and interpolated onto individual particles of the opposite beam. The particles are advanced using the electromagnetic field and the external maps. Since each processor contains the same number of particles, the work of this process is also well balanced among processors. In the particle-field decomposition approach, the volume of communication is proportional to the number of

computational grid points. In the domain decomposition approach, it is proportional to the number of particles that cross domain boundaries; this number can be close to the total number of particles due to the particle movement associated with the one-turn map. Since, in the study of beam-beam interactions, the number of particles is much larger than the number of computational grid points, e.g. typically10^6 versus 10^4, the particle-field decomposition approach can significantly reduce the communication cost in the simulation. This algorithm is more scalable than domain decomposition or particle decomposition when applied to beam-beam simulations [12].

4. Applications

The computational program described in the preceding sections has been applied to the study of beam-beam interactions at RHIC, LHC, Tevatron, PEP-II, and KEKB [10, 17-18]. In the following, we will present two examples of the strong-strong simulations, one for LHC and one for PEP-II.

A "sweeping beam detector" has been proposed as a device to monitor and to optimize the luminosity at the LHC. In this scheme, one of the beams is deliberately made to rotate about a fixed axis as it collides with the opposite beam. Previous beam-beam simulations, carried out up to 100,000 turns, suggest that there is little or no emittance growth for the nominal design parameters [10]. This corresponds to about nine seconds of machine operation time. In order to be measurable in real machine operation, in this example, we show the emittance evolution up to one million turns from the strong-strong beam-beam simulation corresponding to about 90 seconds of machine time. For such a long time simulation, the emittance growth driven by the numerical collisionality is no longer negligible. To quantify this purely computational effect, we have varied the number of macroparticles per beam from $2x10^5$ to $2x10^6$. Fig. 1 shows the averaged emittance growth after one million turns as a function of number of macroparticles. The real emittance growth after one million turns can be estimated from the extrapolation, which gives about 0.05% emittance growth. It can be seen that for a quarter million macroparticles, the numerical emittance growth is much higher than the real emittance growth.

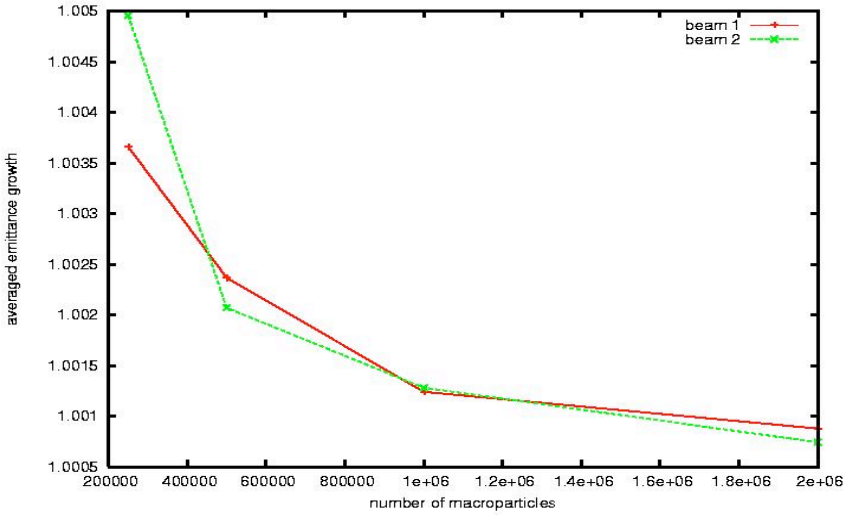

Fig. 2: Emittance growth after one million as a function of the number of macroparticles for the LHC sweeping luminosity monitor.

In the strong-strong beam-beam simulation, when the beam-beam forces are weak and the bunch length is small compared with the beta function at the collision point, finite bunch length effects can be neglected and the simulation can be done using a single slice for each beam. In most lepton colliders, the beam-beam forces are relatively strong and the bunch length is not small compared with the beta function value. In this case, a multiple slice model has to be used for each beam in the strong-strong beam-beam simulation. Fig. 3 shows luminosity as a function of turns using a single slice and 20 slices for a strong-strong beam-beam simulation for PEP-II. It can be seen that there is a significant difference in the simulated luminosity after several damping times. This suggests that multiple slices should be used in order to accurately predict the luminosity of a lepton machine like PEP-II.

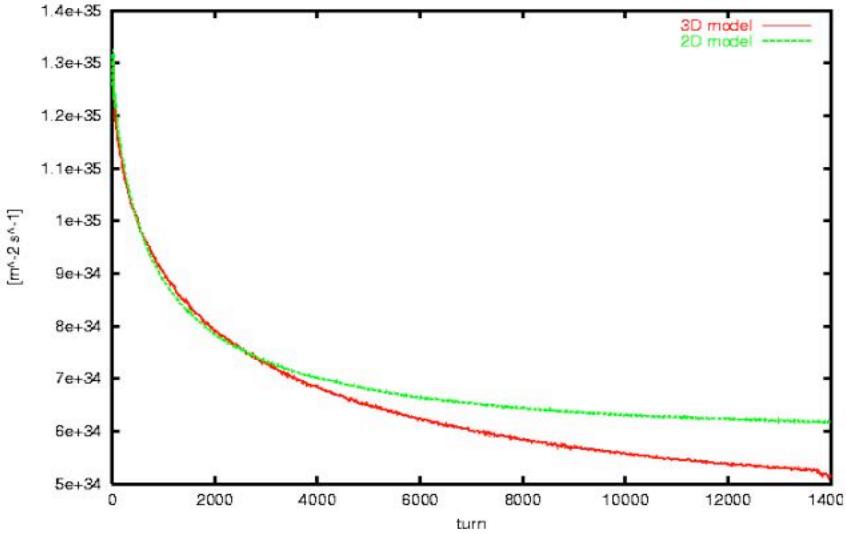

Fig. 3: Single collision luminosity as a function of turns at PEP-II predicted using 1 slice (2d model) and 20 slices (3d model).

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