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A Map of the Universe

J. Richard Gott, III, [1] Mario Jurić, [1] David Schlegel, [1] Fiona Hoyle, [2] Michael Vogeley, [2] Max Tegmark, [3] Neta Bahcall, [1] Jon Brinkmann [4]

ABSTRACT

We have produced a new conformal map of the universe illustrating recent discoveries, ranging from Kuiper belt objects in the Solar system, to the galaxies and quasars from the Sloan Digital Sky Survey. This map projection, based on the logarithm map of the complex plane, preserves shapes locally, and yet is able to display the entire range of astronomical scales from the Earth's neighborhood to the cosmic microwave background. The conformal nature of the projection, preserving shapes locally, may be of particular use for analyzing large scale structure. Prominent in the map is a Sloan Great Wall of galaxies 1.37 billion light years long, 80% longer than the Great Wall discovered by Geller and Huchra and therefore the largest observed structure in the universe.

Subject headings: clusters: cosmology, large scale structure, stars, kuiper belt objects

1. Introduction

Cartographers mapping the Earth's surface were faced with the challenge of mapping a curved surface onto a plane. No such projection can be perfect, but it can capture important features. Perhaps the most famous map projection is the Mercator projection (presented by Gerhardus Mercator in 1569). This is a conformal projection which preserves shapes locally. Lines of latitude are shown as straight horizontal lines, while meridians of longitude are shown as straight vertical lines. If the Mercator projection is plotted on an (x, y) plane, the coordinates are plotted as follows: $x = \lambda$, and $y = \ln(\tan(\frac{\pi}{4} + \frac{\phi}{2}))$ where ϕ (positive if north, negative if south) is the latitude in radians, while λ (positive if easterly, negative if westerly) is the longitude in radians (see Snyder (1993) for an excellent discussion of this and other map projections of the Earth.) This

[1]Department of Astrophysical Sciences, Princeton University, Princeton, NJ 08544

[2]Department of Physics, Drexel University, Philadelphia, PA 19104

[3]Department of Physics, University of Pennsylvania, Philadelphia, PA 19104

[4]Apache Point Observatory, 2001 Apache Point Road, P.O Box 59,Sunspot, NM, 88349

conformal map projection preserves angles locally, and also compass directions. Local shapes are good, while the scale varies as a function of latitude. Thus, the shapes of both Greenland and South America are shown well, although Greenland is shown larger than it should be relative to South America. European countries, which are generally intrinsically small and at high latitude, are conveniently shown at larger scale than countries in the tropics (a property which no doubt helped the projection's popularity in Europe). Other map projections preserve other properties. The stereographic projection which, like the Mercator projection, is conformal is often used to map hemispheres. The gnomonic map projection maps geodesics into straight lines on the flat map, but does not preserve shapes or areas. Equal area map projections like the Lambert, Mollweide, and Hammer projections preserve areas but not shapes.

Astronomers mapping the sky have also used such map projections of the sphere. Gnomonic maps of the celestial sphere onto a cube date from the 1500's. In recent times, Turner and Gott (1976) used the stereographic map projection to chart groups of galaxies (utilizing its property of mapping circles in the sky onto circles on the map.) The COBE satellite map (Smoot et al. (1992)) of the cosmic microwave background used an equal area map projection of the celestial sphere onto a cube. The WMAP satellite (Bennett et al. (2003)) mapped the celestial sphere onto a rhombic dodecahedron using the Healpix equal area map projection (Górski et al. (1999)). Its results were displayed also on the Mollweide map projection, showing the celestial sphere as an ellipse, which was chosen for its equal area property, and the fact that lines of constant declination (celestial latitude) are shown as straight lines.

de Lapparent, Geller, & Huchra (1986) pioneered use of slice maps of the universe to make flat maps. They surveyed a slice of sky, 117° long and 6° wide, of constant declination. In 3D this slice had the geometry of a cone, and they flattened this onto a plane. (A cone has zero Gaussian curvature and can therefore be constructed from a piece of paper. A cone cut along a line and flattened onto a plane looks like a pizza with a slice missing .) If the cone is at declination δ, the map in the plane will be $x = r\cos(\lambda\cos(\delta))$, $y = r\sin(\lambda\cos(\delta))$, where λ is the right ascension (in radians), and r is the co-moving distance (as indicated by the redshift of the object). This will preserve shapes. Many times a 360° slice is shown as a circle with the Earth in the center, where $x = r\cos(\lambda)$, $y = r\sin(\lambda)$. If r is measured in co-moving distance, this will preserve shapes only if the universe is flat ($k = 0$), and slice is in the equatorial plane ($\delta = 0$), otherwise, structures (such as voids) will appear lengthened in the direction tangential to the line of sight by a factor of $1/\cos(\delta)$. This correction is important for study of the Alcock-Paczynski effect, which says that structures such as voids will not be shown in proper shape if we take simply $r = z$ (Alcock and Paczynski (1979)). In fact, Ryden (1995) and Ryden & Melott (1996) have emphasized that this shape distortion in redshift space can be used to test the cosmological model in a large sample such as the Sloan Digital Sky Survey (York et al. (2000); Gunn et al. (1998); Fukugita et al. (1996)). If voids run into each other the walls will on average not have systematic peculiar velocities and therefore voids should have approximately round shapes (a proposition which can be checked in detail with N-body simulations). Therefore, it is important to investigate map projections which

will preserve shapes locally. If one has the correct cosmological model, and uses such a conformal map projection, isotropic features in the large scale structure will appear isotropic on the map.

Astronomers mapping the universe are confronted with the challenge of showing a wide variety of scales. What should a map of the universe show? It should show locations of all the famous things in space: the Hubble Space Telescope, the International Space Station, other satellites orbiting the Earth, the van Allen radiation belts, the Moon, the Sun, planets, asteroids, Kuiper belt objects, nearby stars such as α Centauri, and Sirius, stars with planets such as 51 Peg, stars in our galaxy, famous black holes and pulsars, the galactic center, Large and Small Magellanic Clouds, M31, famous galaxies like M87, the Great Wall, famous quasars like 3C273 and the gravitationally lensed quasar 0957, distant Sloan Digital Sky Survey galaxies, and quasars, the most distant known quasar and galaxy and finally the cosmic microwave background radiation. This is quite a challenge. Perhaps the first book to address this challenge was Cosmic View: The Universe in 40 Jumps by Kees Boeke published in 1957. This brilliant book started with a picture of a little girl shown at 1/10th scale. The next picture showed the same little girl at 1/100th scale who now could be seen sitting in her school courtyard. Each successive picture was plotted at ten times smaller scale. The 8th picture, at a scale of $1/10^8$, shows the entire Earth. The 14th picture, at a scale of $1/10^{14}$, shows the entire Solar system. The 18th picture, at a scale of $1/10^{18}$, includes α Centauri, The 22nd picture, at a scale of $1/10^{22}$ shows all of the Milky Way Galaxy. The 26th and last picture in the sequence shows galaxies out to a distance of 750 million light years. A further sequence of pictures labeled $0, -1, -2, ... - 13$, starting with a life size picture of the girl's hand, shows a sequence of microscopic views, each ten times larger in size, ending with a view of the nucleus of a sodium atom at a scale of $10^{13}/1$. A modern version of this book, Powers of Ten by Phillip and Phylis Morrison and the Office of Charles & Ray Eames, is probably familiar to most astronomers. This successfully addresses the scale problem, but is an atlas of maps, not a single map. How does one show the entire observable universe in a single map?

The modern Powers of Ten book described above is based on a movie, Powers of Ten, by Charles and Ray Eames which in turn was inspired by Kees Boeke's book. The movie is arguably an even more brilliant presentation than Kees Boeke's original book. The camera starts with a picture of a man lying on a picnic blanket in Chicago, and then the camera moves outward, increasing its distance from the man exponentially as a function of time. Thus, approximately every ten seconds, the view is from ten times further away and corresponds to the next picture in the book. Sometimes a movie (like Jaws by Stephen Speilberg) is even better than the book on which it is based, and Powers of Ten is one example. The movie gives one long continuous shot, which is breathtaking as it moves out. The movie is called Powers of Ten, but it could equally well be titled Powers of Two, or Powers of e, because its exponential change of scale with time, produces a reduction by a factor of two in constant time intervals, and also a factor of e in constant time intervals. The time intervals between factors of 10, factors of 2 and factors of e in the movie are related by the ratios $\ln 10 : \ln 2 : 1$. Still, this is not a single map which can be studied all at once, or which can be hung on a wall.

We want to see the large scale structure of galaxy clustering but are also interested in stars in our own galaxy and the Moon and planets. Objects close to us may be inconsequential in terms of the whole universe but they are important to us. It reminds one of the famous cartoon New Yorker cover "View of the World from 9th Avenue" by Saul Steinberg of May 29, 1976. It humorously shows a New Yorker's view of the world. The traffic, sidewalks and buildings along 9th Avenue are visible in the foreground. Behind is the Hudson river, with New Jersey as a thin strip on the far bank. Then at even smaller scale is the rest of the United States with the Rocky mountains sticking up like small hills. In the background, but not much wider than the Hudson River, is the entire Pacific ocean with China and Japan in the distance. This is, of course a parochial view, but it is just that kind of view that we want of the universe. We would like a single map that would equally well show both interesting objects in the solar system, nearby stars, galaxies in the local group, and large scale structure out to the cosmic microwave background.

2. Co-moving coordinates

Our objective here is to produce a conformal map of the universe which will show the wide range of scales encountered while still showing shapes that are locally correct.

Consider the general Friedmann metrics:

$$ds^2 = -dt^2 + a^2(t)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)), \quad k = +1 \tag{1}$$

$$ds^2 = -dt^2 + a^2(t)(d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta d\phi^2)), \quad k = 0 \tag{2}$$

$$ds^2 = -dt^2 + a^2(t)(d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta d\phi^2)), \quad k = -1 \tag{3}$$

where t is the cosmic time since the Big Bang, $a(t)$ is the expansion parameter, and individual galaxies participating in the cosmic expansion follow geodesics with constant values of χ, θ, and ϕ. These three are called co-moving coordinates. Neglecting peculiar velocities, galaxies remain at constant positions in co-moving coordinates as the universe expands. Now $a(t)$ obeys Friedmann's equations:

$$(\frac{a_{,t}}{a})^2 = -\frac{k}{a^2} + \frac{\Lambda}{3} + \frac{8\pi\rho_m}{3} + \frac{8\pi\rho_r}{3} \tag{4}$$

$$2(\frac{a_{,tt}}{a}) = \frac{2\Lambda}{3} - \frac{8\pi\rho_m}{3} - \frac{16\pi\rho_r}{3} \tag{5}$$

where $\Lambda = const.$, is the cosmological constant, $\rho_m \propto a^{-3}$, is the average matter density in the universe, including cold dark matter, $\rho_r \propto a^{-4}$ is the average radiation density in the universe, primarily the cosmic microwave background radiation. The second equation shows that the cosmological constant produces an acceleration in the expansion while the matter and radiation produce a deceleration. Per unit mass density, radiation produces twice the deceleration of normal matter because positive pressure is gravitationally attractive in Einstein's theory and radiation has a pressure in each of the three directions (x, y, z) which is 1/3rd the energy density.

We can define a conformal time η by the relation $d\eta = \frac{dt}{a}$, so that

$$\eta(t) = \int_0^t \frac{dt}{a} \tag{6}$$

Light travels on radial geodesics with $d\eta = \pm d\chi$ so a galaxy at a co-moving distance χ from us emitted the light we see today at a conformal time $\eta(t) = \eta(t_0) - \chi$. Thus, we can calculate the time t and redshift $z = \frac{a(t_0)}{a(t)} - 1$ at which that light was emitted. Conversely, if we know the redshift, given a cosmological model (i.e. values of H_0, Λ, ρ_m, ρ_r, and k today) we can calculate the co-moving radial distance of the galaxy from us from its redshift, again ignoring peculiar velocities.

The WMAP satellite has measured the cosmic microwave background in exquisite detail (Bennett et al. (2003)) and combined this data with other data (Percival et al. (2001); Verde et al. (2002); Croft et al. (2002); Gnedin & Hamilton (2002); Garnavich et al. (1998); Riess et al. (2001); Freedman et al. (2001); Perlmutter et al. (1999)) to produce accurate data on the cosmological model (Spergel et al. (2003)). We adopt best fit values at the present epoch, $t = t_0$, based on the WMAP data of:

$$\begin{aligned}
H_0 &\equiv \frac{a_{,t}}{a} = 71\,\frac{\text{km}}{\text{sec Mpc}}, \\
\Omega_\Lambda &\equiv \frac{\Lambda}{3H_0^2} = 0.73, \\
\Omega_r &= 8.35 \cdot 10^{-5}, \\
\Omega_m &= 0.27 - \Omega_r, \\
k &= 0.
\end{aligned}$$

The WMAP data implies that $w \approx -1$, suggesting that a cosmological constant is an excellent model for the dark energy, so we are simply adopting that. The current Hubble radius $R_{H_o} = cH_o^{-1} = 4222.43\,\text{Mpc}$. The cosmic microwave background is at a redshift $z = 1089$. Substituting, using geometrized units in which $c = 1$, and integrating the first Friedmann equation we find the conformal time may be calculated:

$$\eta(t) = \int_0^t \frac{dt}{a} = \int_0^{a(t)} \{-ka^2 + \frac{8\pi}{3}a^4[\rho_m(a) + \rho_r(a)] + \frac{\Lambda}{3}a^4\}^{-1/2} da \tag{7}$$

where $\rho_m \propto a^{-3}$ and $\rho_r \propto a^{-4}$. This formula will accurately track the value of $\eta(t)$, providing that this is interpreted as the value of the conformal time since the end of the inflationary period at the beginning of the universe. (During the inflationary period at the beginning of the universe, the cosmological constant assumed a large value, different from that observed today, and the formula would have to be changed accordingly. So we simply start the clock at the end of the inflationary period where the energy density in the false vacuum [large cosmological constant] is dumped in the form of matter and radiation. Thus, when we trace back to the big bang, we are really tracing back to the end of the inflationary period. After that, the model does behave just like a standard hot-Friedmann big bang model. This standard model might be properly referred to as an

inflationary-big bang model, with the inflationary epoch producing the Big Bang explosion at the start.) Now, $a(t)$ is the radius of curvature of the universe for the $k = +1$ and $k = -1$ cases, but for the $k = 0$ case, which we will be investigating first and primarily, there is no scale and so we are free to normalize, setting $a(t_0) = R_{H_0} = cH_0^{-1} = 4222.43\,\mathrm{Mpc}$. Then, χ measures co-moving distances at the present epoch in units of the current Hubble radius R_{H_o}. Thus, for the $k = 0$ case, using geometrized units, we have:

$$\eta(a) = \eta(a(t)) = \int_0^a (\frac{a}{a_0}\Omega_m + \Omega_r + (\frac{a}{a_0})^4\Omega_\Lambda)^{-1/2}\frac{da}{a_0} \tag{8}$$

where Ω_m, Ω_Λ, Ω_r are the values at the current epoch. Given the values adopted from WMAP we find:

$$\eta(a_0) = 3.38274 \tag{9}$$

That means that when we look out now at $t = t_0$ (when $a = a_0$) we can see out to a distance of

$$\chi = 3.38274 \tag{10}$$

or a co-moving distance of

$$\chi R_{H_0} = 3.38274 R_{H_0} = 14,283\,\mathrm{Mpc}. \tag{11}$$

This is the effective particle horizon, where we are seeing particles at the moment of the Big Bang. This is a larger radius than 13.7 billion light years – the age of the universe (the lookback time) times the speed of light – because it shows the co-moving distance the most distant particles we can observe now will have from us when they are as old as we are now, i.e. measured at the current cosmological epoch. We may calculate the value of η as a function of a, or equivalently as a function of observed redshift $z = a_0/a - 1$. Recombination occurs at $z_{rec} = 1089$, which is the redshift of the cosmic microwave background seen by WMAP.

$$\eta(z_{rec}) = 0.06713 \tag{12}$$

So, the co-moving radius of the cosmic microwave background is:

$$\chi R_{H_0} = (\eta_0 - \eta_{rec})R_{H_0} = 14,000\,\mathrm{Mpc} \tag{13}$$

That is the radius at the current epoch, so at recombination the WMAP sphere has a physical radius that is 1090 times smaller or about 13 Mpc.

The mean distance between bright galaxies is approximately 5.7 Mpc, so the number of bright galaxies forming within the currently observable universe is of order 64 billion. If each of these has of order 100 billion stars then the currently observable universe is home to of order 6.4×10^{21} stars.

We may compute co-moving radii $r = \chi R_{H_0}$ for different redshifts, as shown in table 1. We can also calculate the value of $\eta(t = \infty) = 4.50613$ which shows how far a photon can travel in co-moving coordinates from the inflationary big bang to the infinite future. Thus, if we wait until the infinite future we will eventually be able to see out to a co-moving distance of

$$r_{t=\infty} = 4.50613 R_{H_0} = 19,027\,\mathrm{Mpc} \tag{14}$$

This is the co-moving future visibility limit. No matter how long we wait, we will not be able to see further than this. This is surprisingly close. The number of galaxies we will eventually ever be able to see is only larger than number observable today by a factor of $(r_{t=\infty}/r_{t_0})^3 = 2.364$.

This calculation assumes the false vacuum state (cosmological constant) visible today remains unchanged. (The WMAP data is consistent with a value of $w = -1$, indicating that the vacuum state (dark energy) today is well approximated by a positive cosmological constant. This false vacuum state (with $p_{vac} = w\rho_{vac} = -1\rho_{vac}$) may decay by forming bubbles of normal zero density vacuum ($\Lambda = 0$) or even decay by forming bubbles of negative energy density vacuum ($\Lambda < 0$). If the present false vacuum is only metastable it will eventually decay by the formation of bubbles of normal or negative energy density vacuum and eventually one of these bubbles will engulf the co-moving location of our galaxy. If such a bubble forms at the epoch η_b then because the pressure inside the bubble (which is zero or positive) is greater than the outside pressure (which is negative), the bubble wall will accelerate outward, eventually expanding at nearly the speed of light. Thus, the bubble will eventually have a co-moving radius at $t = \infty$ of $\chi_b = \eta(t = \infty) - \eta_b$. If $\Lambda = 0$ inside the bubble, once the bubble has engulfed the observer, after an infinite amount of time the observer will be able to see the bubble wall at radius χ_b at $t = \infty$, and therefore anything that could be seen from that position (for a related discussion c.f. Gott (1982)). Thus, if the bubble forms at an epoch η_b and engulfs our location then in principle from our location one should, at $t = \infty$, be able to see out to a radius $\chi = \eta(t = \infty) + \chi_b = \eta(t = \infty) + \eta(t = \infty) - \eta_b$ in the most favorable direction. However, if the bubble formation rate is exponentially low per four-volume ($c^4 H_0^{-4}$) (as expected for barrier penetration) the value of η_b is likely to very near $\eta(t = \infty)$ and χ_b is likely to be small. A low bubble formation rate per four-volume ($c^4 H_0^{-4}$) is also consistent with the observed fact that we have not be engulfed by a bubble so far (for a related discussion c.f. Gott and Statler (1984)). For example if a bubble formed at $t = 10^{100}$ years from now it would have an eventual co-moving radius at $t = \infty$ of $\chi_b \sim \exp(-t_b H_0) \sim \exp(-10^{90})$ or negligibly small. If the bubble formed has a negative density vacuum ($\Lambda < 0$) then the bubble wall expands to co-moving radius $\chi_b \sim \exp(-t_b H_0) \sim exp(-10^{90})$ just as before. The observer inside the bubble within a finite time sees out to the bubble wall position at $t = \infty$, and can therefore see out to a co-moving distance of $\chi = \eta(t = \infty) + \chi_b$ just as in the case of a $\Lambda = 0$ bubble. After that, the observer sees spacelike infinity in anti-DeSitter space, and does not see any additional distance in our universe. Thus, if the bubble formation rate is low as we expect, our co-moving location will eventually be engulfed by one, but only in the far future (say 10^{100} years from now) and this will allow an observer to see only an insignificant co-moving distance χ_b further than the future visibility limit we have plotted. Linde (1990) and Garriga and Vilenkin (1998) have pointed out that if the current vacuum state is the lowest stable equilibrium then quantum fluctuations can form bubbles of high density vacuum that will start a new inflationary epoch, new baby universes growing like branches off a tree. Still as in the above case, we expect to be engulfed by such new inflating region only at late times (say 10^{100} years from now) and the observer will still be surrounded by an event horizon with a limit of future visibility in co-moving coordinates in our universe that is virtually identical with what we have plotted. Thus, although the future history of the universe will be determined by the

subsequent evolution of the quantum vacuum state (as also noted by Krauss & Starkman (2000)), in practice we expect that, although bubbles of different vacuum states may form, our universe should still continue expanding forever (forming an infinite number of bubbles). And if the bubble formation rate is low, we expect the current vacuum to stay as is for considerably longer than the Hubble time, and in many scenarios this leaves us with a limit of future visibility that is for all practical purposes just what we have plotted. If a bubble with a new vacuum state engulfs our location it well may be fatal to intelligent life at that location as the laws of physics inside the bubble will be different and the particles comprising the observer may decay.

If we send out a light signal now, by $t = \infty$ it will reach a radius $\chi = \eta(t = \infty) - \eta(t_0) = 4.50613 - 3.38274 = 1.12339$, or

$$r = 4,743\,\mathrm{Mpc} \tag{15}$$

to which we refer to as the "outward limit of reachability". We cannot reach (with light signals or rockets) any galaxies that are further away than this (Busha et al. (2003)). What redshift does this correspond to? Galaxies we observe today with a redshift of $z = 1.6876$ are at this co-moving distance. Galaxies with redshifts larger than 1.6876 today are unreachable. This is a surprisingly small redshift.

We can see many galaxies at redshifts larger than 1.6876 that we will never be able to visit or signal. In the accelerating universe, these galaxies are accelerating away from us so fast that we can never catch them. The total number of stars that our radio signals will ever pass is of order 2.4×10^{20}.

3. A Map Projection for the Universe

We will choose a conformal map that will cover the wide range of scales from the Earth's neighborhood to the cosmic microwave background. First we will consider the flat case ($k = 0$) which the WMAP data tells us is the appropriate cosmological model. Our map will be two dimensional so that it can be shown on a wall chart. de Lapparent, Geller, & Huchra (1986) showed with their slice of the universe, just how successful a slice of the universe can be in illustrating large scale structure. The Sloan Digital Survey should eventually include spectra for and accurate positions about 1 million galaxies and quasars in a 3D sample (see Strauss et al. (2002); Richards et al. (2002); Eisenstein et al. (2001); Pier et al. (2003) for further reference). But virtually complete already is an equatorial slice 4 degrees wide ($-2^\circ < \delta < 2^\circ$) centered on the celestial equator covering both northern and southern galactic hemispheres. This shows many interesting features including many prominent voids and a great wall longer than the great wall found by Geller and Huchra (1989).

Since the observed slice is already in a flat plane ($k = 0$ model, along the celestial equator) we may project this slice directly onto a flat sheet of paper using polar coordinates with $r = \chi R_{H_0}$ being the co-moving distance, and θ being the right ascension. (CMB observations from Boomerang,

DASI, MAXIMA and WMAP indicate that the case $k = 0$ is the appropriate one for the universe. For mathematical completeness we will also consider the $k = +1$ and $k = -1$ cases in an appendix.) We wish to show large scale structure and the extent of the observable universe out to the cosmic microwave background radiation including all the SDSS galaxies and quasars in the equatorial slice. In figure 1, one can see the cosmic microwave background at the surface of last scattering as a circle. Its co-moving radius is 14.0 Gpc. (Since the size of the universe at the epoch of recombination is smaller that that a present by a factor of $1 + z = 1090$, the true radius of this circle is about 12.84 Mpc.) Slightly beyond the cosmic microwave background in co-moving coordinates is the Big Bang at a co-moving distance of 14.3 Gpc.

(Imagine a point on the cosmic microwave background circle. Draw a radius around that point that is tangent to the outer circle labeled Big Bang, as shown in the figure, in other words, a circle that has a radius equal to the difference in radius between the cosmic microwave background circle and the Big Bang circle. That circle has a co-moving radius of 283 Mpc. That is the co-moving horizon radius at recombination. If the Big Bang model – without inflation – were correct we would expect a point on the cosmic microwave background circle to be causally influenced only by things inside that horizon radius at recombination. The angular radius of this small circle as seen from the Earth is $(283\,\mathrm{Mpc}/14,000\,\mathrm{Mpc})$ radians or $1.16°$. If the Big Bang model without inflation were correct we would expect the cosmic microwave background to be correlated on scales of at most $1.16°$. Inflation, by having a short period of accelerated expansion during the first 10^{-34} seconds of the universe, puts distant regions in causal contact because of the slight additional time allowed when the universe was very small. So, with inflation, we can understand why the cosmic microwave background is uniform to one part in 100,000 all over the sky. Furthermore, random quantum fluctuations predicted by inflation add a series of adiabatic fluctuations which are expected to have a peak in the power spectrum at an angular scale about half the size of the horizon radius at recombination calculated above, ~ 0.43 degrees.)

Beyond the Big Bang circle is the circle showing the future co-moving visibility limit. If we wait until the infinite future, we will be able to see out to this circle. (In other words, in the infinite future, we will be able to see particles at the future co-moving visibility limit as they appeared at the Big Bang.)

The SDSS quasars extend out about halfway out to the cosmic microwave background radiation. The distribution of quasars shows several features. The radial distribution shows several shelves due to selection effects as different spectral features used to identify quasars come into view in the visible. Several radial spokes appear due to incompleteness in some narrow right ascension intervals. Two large fan shaped regions are empty and not surveyed because they cover the zone of avoidance close to the galactic plane. These excluded regions run from approximately $3.7\,\mathrm{h} \lesssim \alpha \lesssim 8.7\,\mathrm{h}$ and approximately $16.7\,\mathrm{h} \lesssim \alpha \lesssim 20.7\,\mathrm{h}$. The quasars do not show noticeable clustering or large scale structure. This is because the quasars are so widely spaced that the mean distance between quasars is larger than the correlation length at that epoch.

The circle of reachability is also shown. Quasars beyond this circle are unreachable. Radio signals emitted by us now will only reach out as far as this circle, even in the infinite future.

The SDSS galaxies appear in as a black blob in the center. There is much interesting large scale structure here but the field is too crowded and small to show it. This illustrates the problem of scale in depicting the universe. If we want a map of the entire observable universe on one page, at a nice scale, the galaxies are crammed into a blob in the center. Let us enlarge the central circle of radius 0.06 times the distance to the Big Bang circle by a factor of 16.6 and plot it again in figure 2. This now shows a circle of co-moving radius 858 Mpc. Almost all of these points are galaxies from the galaxy and bright red galaxy samples of the SDSS. Now we can see a lot of interesting structure. The most prominent feature is a Sloan Great Wall at a median distance of about 310 Mpc stretching from 8.7h to 14h in R.A. There are numerous voids. A particularly interesting one is close in at a co-moving distance of 125 Mpc at 1.5h R.A. At the far end of this void are a couple of prominent clusters of galaxies which are recognizable as "fingers of God" pointing at the Earth. Redshift in this map is taken as the co-moving distance indicator assuming participation in the Hubble flow, but galaxies also have peculiar velocities and in a dense cluster with a high velocity dispersion this causes the distance errors due to these peculiar velocities to spread the galaxy positions out in the radial direction producing the "finger of God" pointing at the Earth. Numerous other clusters can be similarly identified. This is a conformal map, that preserves shapes – excluding the small effects of peculiar velocities. The original CfA survey in which Geller and Huchra discovered the Great Wall had a co-moving radius of only 211 Mpc, which is less than a quarter of the radius shown in figure 2. Figure 2 is a quite impressive picture, but it does not capture all of the Sloan Survey. If we displayed figure 1 at a scale enlarged by a factor of 16.6 the central portion of the map would be as you see displayed at the scale shown in figure 2 which is adequate, but the Big Bang circle would have a diameter of 6.75 feet. You could put this on your wall, but if we were to print it in the journal for you to assemble it would require the next 256 pages. This points out the problem of scale for even showing the Sloan Survey all on one page. Small scales are also not represented well. The distance to the Virgo Cluster in figure 2 is only about 2 mm and the distance from the Milky Way to M31 is only 1/13th of a millimeter and therefore invisible on this Map. Figure 2, dramatic as it is, fails to capture a picture of all the external galaxies and quasars. The nearby galaxies are too close to see and the quasars are beyond the limits of the page.

We may try plotting the universe in lookback time rather than co-moving coordinates. The result is in figure 3. The outer circle is the cosmic microwave background. It is indistinguishable from the Big Bang as the two are separated by only $380,000$ years out of 13.7 billion years. The SDSS quasars now extend back nearly to the cosmic microwave background radiation (since it is true that we are seeing back to within a billion years of the Big Bang). Lookback time is easier to explain to a lay audience than co-moving coordinates and it makes the SDSS data look more impressive, but it is a misleading portrayal as far as shapes and the geometry of space are concerned. It misleads us as to how far out we are seeing in space. For that, co-moving coordinates are appropriate. figure 3 does not preserve shapes – it compresses the large area between the SDSS

quasars and the cosmic microwave background into a thin rim. This is not a conformal map. The SDSS galaxies now occupy a larger space in the center, but there are still so crowded together that one can not see the large scale structure clearly. Figure 4 shows the central 0.2 radius circle (shown as a dotted circle in figure 3) enlarged by a factor of 5. Thus if we were to make a wall map of the observable universe using lookback time at the scale of figure 4 it would only need to be 2 feet across and would only require the next 25 pages in the journal to plot. This is an advantage of the lookback time map. It makes the interesting large scale structure that we see locally (figure 4) a factor of slightly over 3 larger in size relative to the cosmic microwave background circle than if we had used co-moving coordinates. Figure 4 looks quite similar to figure 2. At co-moving radii less than 858 Mpc, the lookback time and co-moving radius are rather similar. Still, figure 4 is not perfectly conformal. Near the outer edges there is a slight radial compression that is beginning to occur in the lookback time map as one approaches the Big Bang. This radial compression is illustrated in figure 5, where we have plotted a square grid in co-moving coordinates in terms of lookback time as would be depicted in figure 3. Each grid square would contain an equal number of galaxies in a flat slice of constant vertical thickness. This shows the distortion of space that is produced by using the lookback time. The squares become more and more distorted in shape as one approaches the edge.

Thus, it would be useful to have a conformal map projection that would show the whole SDSS survey, including galaxies, quasars and the cosmic microwave background, as well as smaller scales, covering the local supercluster, the local group, the Milky Way, nearby stars, the Sun and planets, the Moon and artificial Earth satellites. Such a map is possible.

Consider the complex plane $(u, v) = u + iv$ where $i = \sqrt{-1}$ and u and v are real numbers. Every complex number $W = u + iv$ will be represented as a point in the (u, v) plane where u and v are the usual Cartesian coordinates. The function $Z = i \ln(W)$ maps the plane (u, v) onto the plane (x, y) where $Z = x + iy$. The (u, v) plane represents a slice of the universe in isotropic coordinates (in this case co-moving coordinates since $k = 0$), and the (x, y) plane represents our map of the universe. The inverse function $W = \exp(Z/i)$ is the inverse map that takes a point in our map plane (x, y) back to the point it represents in the universe (u, v). In the universe it is useful to establish polar coordinates (r, θ) where

$$u = r \cos\theta \tag{16}$$

$$v = r \sin\theta \tag{17}$$

and $r = (u^2 + v^2)^{1/2}$ is the (co-moving) distance from the center of the Earth and $\theta = \arctan(\frac{v}{u})$ is the right ascension measured in radians. Since

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}, \quad \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \tag{18}$$

it is clear that

$$W = u + iv = r(\cos\theta + i \sin\theta) = re^{i\theta} \tag{19}$$

$$Z = i \ln(W) = i(\ln r + i\theta) = -\theta + i \ln r = x + iy \tag{20}$$

so:

$$x = -\theta \tag{21}$$

$$y = \ln r \tag{22}$$

Thus, the entire (u, v) plane, except the origin $(0, 0)$, is mapped into an infinite vertical strip of horizontal width 2π, i.e.

$$-2\pi < x \leq 0, \quad -\infty < y < \infty \tag{23}$$

(Figure 6 shows the complex plane $u + iv$ mapped onto the $x + iy$ plane by this map. One can take this map and make it into a slide rule for multiplying complex numbers. Photocopy the map on this page and cut it out. Tape the left hand edge to the right hand edge to make a paper cylinder. The θ coordinate now measures longitude angle on that cylinder. Now photocopy the map onto a transparency, and cut it out, and again tape the left hand edge to the right hand edge to make a transparent cylinder. In cutting out the left and right hand sides of the map cheat a little, cut along the outside edges of the map borderlines so that the circumference of the transparency cylinder is just a tiny bit larger than the paper cylinder and so that it will fit snugly over it. With the paper cylinder snugly inside the transparent cylinder you are ready to multiply. If you want to multiply two numbers $A = a + bi$ and $C = c + di$ all you do is rotate and slide the transparent cylinder until the transparency number 1 (i.e. $1 + 0i$) is directly over the number $a + bi$ on the paper cylinder, then look up the number $c + di$ on the transparent cylinder, below it on the paper cylinder will be the product $A \cdot B$. The logarithm of AB is equal to the sum of the logarithms of A and B. Of course, on the real axis, $\theta = 0$ $(x = 0)$, the map looks like the scale on a slide rule. Alternatively make a flat slide rule for multiplying complex numbers: make two photocopies of the map on white paper and tape them together to make two cycles in θ from right to left. Then make one photocopy of the map onto a transparency. Lay the number 1 on top of the number A in the right hand cycle of the paper map and look on the transparency for the number B, below it on the paper map will be the product AB.)

For convenience on our map of the universe, let $r = \frac{\chi R_{H_0}}{r_E}$ (co-moving cosmological distance/radius of the Earth) be measured in units of the Earth's equatorial radius $r_E = 6378$km. Thus, circles of constant radius ($r = const.$) from the center of the Earth are shown as horizontal lines ($y = const.$) in the map, and rays of constant right ascension ($\theta = const.$) are shown as vertical lines ($x = const.$) in the map. The surface of the Earth (at its equator) is a circle of unit radius in the (u, v) plane, and is the line $y = 0$ in the map. The region $y < 0$ in the map represents the interior of the Earth, so one can show the Earth's mantle and liquid and solid core. The solid inner core has a radius about $0.19\, r_E$, thus, the lower edge of the map must extend to $y = \ln(0.19) = -1.66$ to show it. The region $y > 0$ shows the universe beyond the Earth. The co-moving future observability limit at a radius of 19.027 Gpc is at 9.2×10^{19} Earth radii, and so the upper edge of the map must extend to $y = \ln(9.2 \times 10^{19}) = 45.97$ to show it. Thus the dimensions of the map are $\Delta x = 2\pi$, and $\Delta y = 47.63$. The aspect ratio for the map is $\frac{\text{height}}{\text{width}} = 7.58$. See figure 7 for a small scale version of this that will fit on one page. (A square map would have

dimensions $2\pi \times 2\pi$ and would cover a scale ratio from bottom to top of $\exp(2\pi) = 535.49$. A map with an aspect ratio $\frac{\text{height}}{\text{width}} = 7.58$ covers a scale ratio from bottom to top of $535.49^{7.58}$.)

At a scale of about 1 radian per inch for the angular scale, this would make a map about 6.28 inches wide by 47.6 inches tall, which could be easily displayed as a wall chart. We have presented the map at approximately this scale later in this article.

This is not the first time logarithmic coordinates have been used for a map of the universe. The Amoco Map of Space Mysteries (1958) plotted the curved surface of the Earth and above it altitude (from the surface, not distance from the center) marked off in equal intervals labeled 1 mile, 10 miles, 100 miles, 1,000 miles, 10,000 miles, 100,000 miles, 1 million miles, 10 million miles, and 100 million miles. In this range Solar system objects from the moon to Venus, Mars and the Sun are plotted properly. But although α Centauri, the Milky Way and M31 are shown beyond they are not shown at correct scale (even logarithmically). The Earth's surface is plotted where an altitude of 0.1 miles should have been. In any case, because of the curvature of the Earth's surface in the map, even in the range between an altitude of 10 miles and 100 million miles, the map is not conformal. In October 1999, National Geographic presented a map of the universe (that one of us (JRG) participated in producing) which was a 3D view with a spherical Earth at the center with equal width shells surrounding it like an onion with radii of 400,000 miles, 40 million miles, 4 billion miles, 4 trillion miles, 10 light years, 1,000 light years, 100,000 light years, 10 million light years, 1 billion light years, 11-15 billion light years. This map displays objects from the moon to the microwave background but is also not conformal.

The map projection we are proposing is conformal because the derivatives of the complex function $Z = i \ln W$ have no poles or zeros in the mapped region. If we want to see how a little area of the universe slice is mapped onto our slice we should do a Taylor expansion: the point $W + \Delta W$ is mapped onto the point $Z + \Delta Z = Z + \frac{dZ}{dW}\Delta W$ in the limit where $\Delta W \to 0$ providing that $\frac{dZ}{dW} \neq \infty$ so the map doesn't blow up there and $dZ/dW \neq 0$ so that the second and higher order terms in the Taylor expansion can be ignored (providing that none of the higher derivatives $\frac{d^n Z}{dW^n}$ become infinite at the point W). In this case, in the limit as $\Delta W \to 0$, the Taylor series is valid using just the first derivative term:

$$\Delta Z = \frac{dZ}{dW}\Delta W \tag{24}$$

$$\frac{dZ}{dW} = \frac{d(i \ln W)}{dW} = \frac{i}{W} \tag{25}$$

Thus for $W \neq 0$ and finite (i.e. excluding the center of the Earth and the point at infinity which are not mapped anyway) dZ/dW is neither zero nor infinity. The higher derivatives ($n \geq 2$) : $d^n Z/dW^n = i(-1)^n n! W^{-n+1}$ are also finite when W is finite and non zero. Thus, the point $W + \Delta W$ is mapped onto the point $Z + \Delta Z = Z + \frac{dZ}{dW}\Delta W$ in the limit where $\Delta W \to 0$. Characterize the point W as

$$W = r_w(\cos\theta_w + i \sin\theta_w) \tag{26}$$

Now the product of two complex numbers

$$\begin{aligned} A &= r_a(\cos\theta_a + i\sin\theta_a) \\ B &= r_b(\cos\theta_b + i\sin\theta_b) \end{aligned}$$

is

$$A \cdot B = r_a r_b(\cos(\theta_a + \theta_b) + i\sin(\theta_a + \theta_b)), \tag{27}$$

so

$$\frac{1}{W} = \frac{1}{r_w}(\cos(-\theta_w) + i\sin(-\theta_w)), \tag{28}$$

and since $i = \cos(\pi/2) + i\sin(\pi/2)$,

$$i\frac{1}{W} = \frac{1}{r_w}(\cos(\frac{\pi}{2} - \theta_w) + i\sin(\frac{\pi}{2} - \theta_w)) \tag{29}$$

and

$$\begin{aligned} \Delta Z &= r_{\Delta Z}(\cos\theta_{\Delta Z} + i\sin\theta_{\Delta Z}) \\ &= \frac{1}{r_w}(\cos(\frac{\pi}{2} - \theta_w) + i\sin(\frac{\pi}{2} - \theta_w)) \times r_{\Delta W}(\cos\theta_{\Delta W} + i\sin\theta_{\Delta W}) \\ &= \frac{r_{\Delta W}}{r_w}(\cos(\theta_{\Delta W} + \frac{\pi}{2} - \theta_w) + i\sin(\theta_{\Delta W} + \frac{\pi}{2} - \theta_w)) \end{aligned} \tag{30}$$

Thus, the vector ΔW is rotated by an angle $\frac{\pi}{2} - \theta_w$ and multiplied by a scale factor of $1/r_w$. Since any two vectors ΔW_1 and ΔW_2 at the point W will be rotated by the same amount when they are put on the map the angle between them is preserved in the map, and so the map projection is conformal. The only place the first derivative (and the higher derivatives) blow up (or go to zero) is at $W = 0$ at the center of the Earth or at the point at infinity $W = \infty$. But the Earth's center does not appear on the map at all (it is at $y = -\infty$). This is a set of measure zero. Likewise, the point at infinity $W = \infty$ is not plotted either (it is at $y = +\infty$). So the map projection is conformal at all points in the map. Shapes are preserved locally.

The conformal map projection for showing the universe presented here was developed by JRG in 1972, and he has produced various small versions of it over the years. These have been shown at various times, notably to the visiting committee of the Hayden Planetarium in 1996 and to the staff of the National Geographic Society in 1999. Recent discoveries within a wide range of scales from the solar system objects, to the SDSS galaxies and quasars have prompted us to produce and publish the map in a large scale format.

Our large scale map is shown in figures 8a through 8f. A radial vector ΔW (pointing away from the Earth's center) at the point W points in the direction $\theta_{\Delta W} = \theta_w$. This vector in the map is rotated by an angle $\frac{\pi}{2} - \theta_w$ so that $\theta_{\Delta Z} = \theta_{\Delta W} + \frac{\pi}{2} - \theta_w = \frac{\pi}{2}$, so that it points in the vertical direction. Small regions in the universe are rotated in the map so that the radial direction, away from the center of the Earth, is in the vertical direction in the map. Radial lines from the Earth's

center are plotted as vertical lines. Circles of constant radius from the center of the Earth are horizontal lines. The length of the vector ΔW is multiplied by a scale factor $1/r_w$. Thus, the scale factor at a given point on the map can be read off as the 1 over the distance of the point from the Earth, r_w. The circumference of the circle representing the surface of the Earth in figure 8a has a length of approximately 6.28 inches on the map, giving a scale of 1 inch/radian (maps at this exact scale are available in separate files on astro-ph). So the scale at the surface of the Earth is $\frac{6.28\,\text{inches}}{2\pi\cdot 6378\,\text{km}}$ which is approximately $\frac{1}{250{,}000{,}000}$. The circumference of the circle with radius 10 Earth radii has a length of approximately 6.28 inches also, so it is shown at a scale of approximately $1/2,500,000,000$. Each factor of 10 further out from Earth we go the scale factor decreased by another factor of 10. At the top of the map the scale is approximately $1/2.25 \times 10^{28}$.

Radial lines separated by an angle θ (in radians), going outward from the Earth will be plotted as parallel vertical lines separated by a horizontal distance of θ inches. Thus objects of the same angular size in the sky (as seen from the center of the Earth) will be plotted as the same size on the map. The scale is approximately 1 inch per radian. Thus, the Sun and Moon which have the same angular size in the sky ($\frac{1}{2}^\circ$) will be plotted as circles of the same size on the map (since their cross sections are circles and shapes are preserved locally in a conformal map projection).

The map gives us that Earthling's view of the universe that we want. Objects are shown at a size in the map proportional to the size they subtend in the sky. The Sun and Moon are equally large in the sky and so appear of the same size in the map. Objects that are close to us are more important to us – as depicted in that New Yorker cover. Buildings on 9th avenue may subtend as large an angle to our eye as the distant state of California. Our loved ones–important to us, are often only a few feet away and subtend a large angular scale to our eyes. A murder occurring in our neighborhood draws more of our attention than a murder of someone halfway around the globe. Plotting objects at a size equal to their angular scale makes psychological sense. Objects are shown taking up an area on the map that is proportional to the area they subtend in the sky (if they are approximately spherical – as many astronomical bodies are). The importance of the object in the map (the fraction of the map it takes up) is proportional to the chance we will see the object if we look out along a random line of sight. Indeed, if we look at the map from a constant distance, the angular size of the objects in the map will be proportional to the angular size they subtend in the sky. The visual prominence of objects in the map will be proportional to their visual prominence in the sky.

Of course this means that the Moon and Sun and other objects will be shown at their true scale relative to their surroundings (i.e. the Moon is shown in correct scale relative to the circumference of its orbit) which is small because they are small in the sky. Suppose we made an Mollweide equal area map projection of the sky at a scale to fit on a journal page: an ellipse with horizontal width 6.28 inches and vertical height of 3.14 inches. Along the equatorial plane the scale is linear at 1 inch per radian. The diameter of the Moon or Sun is $\frac{1}{2}^\circ$, or $\frac{1}{720}$th of the 360° length around the equator. Thus, on this sky map the Moon would have a diameter of $\frac{6.28\,\text{inches}}{720} = 0.0087$ inches. On our map the Moon would have the same diameter, for the scale of our map is approximately

1 radian = 1 inch. The Moon and Sun are rather small in the sky. With a printer resolution of 300 dots per inch the Sun and Moon would then be approximately 3 dots in diameter. For easier visibility we have plotted the Sun and Moon as circles 0.157 inches in diameter – or enlarged by a factor of 18. Similar enlargements of individual objects like the Sun and the Moon might appear on sky maps appearing on one page. Just as symbols for cities on world maps may be larger than the cities themselves. Still it is interesting to note what the true sizes on the map should be since it shows how much empty space in the universe there is. M31 for example subtends an angle of about 2° on the sky and so would be about 0.035 inches. If versions of the map were produced at larger scale as we shall discuss below, images of the Sun, Moon, and nearby galaxies could be displayed at proper angular scale and simply placed on the map.

The completed map is shown in figures 8a – 8f. However these are not six different maps but rather one map in 6 sheets. The reader is instructed to print the 1 inch/radian versions of each of these available on astro-ph, and tape them together to make one wall map 6.28 inches wide and 47.6 inches tall.

The map shows a complete sample of objects in the classes we are illustrating in the equatorial slice ($-2^\circ < \delta < 2^\circ$) which is shown conformally correct. These objects are shown at the correct distances and right ascensions. This we supplement with additional famous objects out of the plane which are shown at their correct distances and right ascensions. So this is basically an equatorial slice with supplements.

At the bottom, the map starts with an equatorial interior cross section of the Earth. First we see the solid inner core of the Earth with a radius of $\sim$ 1200 km. Above this is the liquid outer core (1200 km – 3480 km), and above that are the lower (3480 km – 5701 km) and upper mantle (5701 km – 6341 km). The Earth's surface has a equatorial radius of 6378 km. There is a line designated Earth's surface (& crust) which is a little thicker than an ordinary line to properly indicate the thickness of the crust. The Earth's surface (& crust) line is shown as perfectly straight, because on this scale the altitude variation in the Earth's surface is too small to be visible. The map is conformal and so shows the heights of mountains and depths of oceans at the proper scale relative to the Earth's circumference. The scale at the Earth surface line is approximately $\frac{1}{250{,}000{,}000}$. The highest altitude in our equatorial slice ($-2^\circ <$ latitude $< 2^\circ$) is Chimborazo in South America with an altitude of 6320 meters (which would be 0.001 inches) which is not visible at a printer resolution of 300 dots per inch. Likewise, the deepest ocean trench along the equator is in the mid Atlantic at -7728 meters (which would be -0.0012 inches on this scale). Thus, we have drawn the circumference of the Earth as a straight line. (Mt. Everest and the Marianas trench would have heights of 0.0015 and -0.00018 inches). The Earth's crust is of variable thickness but under the continents extends to a depth of 37 km (or 0.0058 inches) This thickness is just visible on the map at a resolution of 300 dots per inch and we have shown the maximum crust depth accordingly.

(If we had wished we could have extended the map downward to cover the entire inner solid core of the Earth down to the central neutron (or proton) in an iron atom located at the center

of the Earth. Since a neutron has a radius of approximately 1.2×10^{-13} cm or $1.9 \times 10^{-22} r_E$, the circumference of this central neutron would be plotted as a straight line at $y = -50.0$. The outer circumference of the central iron atom (atomic radius of 1.40×10^{-8}cm, Slater (1964)) would be plotted as a straight line at $y = -38.36$. Thus, including the entire inner solid core of the Earth down to the central neutron in an iron atom at the center of the Earth would require about an additional 48 inches of map, or six more journal pages, approximately doubling its length. The central atom and its nucleus would then occupy the bottom 11.6 inches of the map, giving a nice illustration of both the nucleus and all the electron orbitals. But since we are primarily interested in astronomy, and the key regions of the Earth's interior are covered in the map already, we have stopped the map just deep enough to show the extent of the inner solid core.)

We have shown the Earth's atmosphere above the Earth's surface. The ionosphere is shown which occupies an altitude range of 70 – 600 km. Below the ionosphere is the stratosphere which occupies an altitude range of 12 – 50 km. Although there was not enough space to include a label, the stratosphere on the map simply occupies the tiny space between the lower error bar indicating the bottom of the ionosphere and the "surface of the Earth" line. The troposphere (0 – 12 km) is of such small altitude that it is subsumed into the Earth's surface line thickness. The ionosphere marks the practical outer extent of the Earth's atmosphere. Aurora are a prominent and visible feature of the ionosphere. Meteors typically begin to burn up at an altitude of approximately 60 miles (100 km), near the bottom of the ionosphere. Above the ionosphere we have formally the exosphere, where the mean free path is sufficiently long that individual atoms with escape velocity can actually escape. So the top of the ionosphere effectively defines the outer extent of the Earth's atmosphere and the map shows properly just how narrow the Earth's atmosphere is relative to the circumference of the Earth.

Next we have shown all 8,420 artificial Earth satellites in orbit as of Aug 12, 2003 (at the time of full Moon 2003/08/12 04:48 UT). In fact all objects in the map are shown as of that time. This is the last full Moon before the closest approach of Mars to the Earth in 2003. The time was chosen for its placement of the Sun, Moon and Mars. We show all Earth satellites (not just those 624 in the equatorial slice). These are actual named satellites, not just space junk. Some famous satellites are designated by name. ISS is the International Space Station. HST is the Hubble Space Telescope. These are both in low Earth orbit. They are plowing through the upper part of the ionosphere (astronauts often see beautiful Auroras during their shuttle missions). Because of atmospheric drag the ISS and the HST need occasional missions to boost them in altitude to keep them in orbit. The Mir Space Station and the earlier Skylab operating at similar altitudes reentered the Earth's atmosphere before August 2003. The first Earth satellite, Sputnik, also reentered long ago. However the Vanguard 1 satellite is still up. This much maligned satellite was designed to be the U.S. answer to Sputnik – a mere 6 inches across. The first attempt to put a Vanguard satellite up was a miserable failure, the rocket rising only a few feet off the pad before falling back and exploding. Wernher von Braun, then stepped forward and successfully launched the larger Explorer I which discovered the Van Allen radiation belt. However Explorer I had an

elliptical orbit with a low perigee and so it has also reentered and burnt up long ago. Vanguard 1 was finally successfully launched into a nearly circular orbit and at the time it was stated that it should stay up in orbit for at least several hundred years. And true to that prediction, it is still up today, the earliest launched satellite still in orbit. As the map makes clear, Vanguard 1 is well clear of the ionosphere. The Chandra X-ray observatory is also shown. Here we have caught it at nearly the low point of its rather elliptical orbit. The map shows that the great majority of Earth satellites are in low Earth orbit, skimming just above the Earth's atmosphere. In fact some of the lowest satellites are on their way into reentry now. There are two main altitude layers of low Earth orbiting satellites and a scattering of them above that. There is a quite visible line of geostationary satellites at an altitude of 22,000 miles above the Earth's surface. At this altitude, the sidereal orbital period of a satellite is 23 hours 56 minutes or one sidereal day, so if the satellite is placed in equatorial orbit going east it will station keep over a fixed spot on the Earth's equator. Arthur C. Clark pointed out that this feature would be useful for communication satellites and indeed it has proven to be so. We expected to find a line of geostationary satellites at this altitude but were surprised to see how nicely spread around the globe they are. These geosynchronous satellites are nearly all in our equatorial slice. A surprise was the line of GPS (Global Positioning System) satellites at a somewhat lower altitude. These are all in nearly circular orbits at identical altitudes and so also show up as a line on the map. Although these satellites have changed everyday life (many would not go on a camping trip without a GPS hand held unit) we had not realized that there were so many of these satellites or that they would show up on the map so prominently. The inner and outer Van Allen radiation belts are also shown.

Beyond the artificial Earth satellites and the Van Allen radiation belts lies the Moon. Twelve men have walked on the Moon. At approximately 60 Earth radii (as can be seen from the logarithmic distance scale on the left), this marks the extent of direct human occupation of the universe. The Moon is full on August 12, 2003.

Behind the full Moon at approximately 4 times the distance from Earth is the WMAP satellite which has recently measured the cosmic microwave background. It is in a looping orbit about the L2 unstable Lagrange point on the opposite side of the Sun from the Earth. Therefore it is approximately behind the full Moon. The WMAP satellite station keeps at about 1.5 million km from Earth (as can be seen on the logarithmic distance scale on the right) as it and the Earth orbit the Sun.

The next object is Mars, at approximately 9,000 Earth radii, or 0.4 astronomical units (as seen on the scale on the right). Mars is near its point of closest approach (which it achieved on August 27, 2003 when it was at a center-to-center distance of 34,646,418 miles). Just beyond Mars are 3 main belt asteroids between 9h and 11h indicated by dots. Further up, past some more asteroids, are Mercury, the Sun and Venus. Venus is near conjunction with the Sun on the opposite side of its orbit. If we were to plot the orbits of Mercury and Venus on this map they would be shown in their approximate true shapes because the map is conformal. The Sun is 180° away from the Moon or halfway across the map horizontally, since the Moon is full. The Sun is 1 AU away from the

Earth. Beyond the Sun, at radii of between 1.5 and 3.5 AU are the main belt asteroids. Here, out of the total of 218,484 asteroids in the ASTORB database, we have shown only those 14,183 which are in the equatorial plane equatorial slice ($-2^\circ < \delta < 2^\circ$). If we had shown them all, it would have been totally black. By just showing the asteroids in the equatorial slice we are able to see individual dots. In addition, some famous asteroids are shown (even if off the equatorial slice) and indicated by name. Ceres, when it was discovered, was called a planet. One by one as asteroids were discovered they were added to the list of planets, until some astronomy texts announced that the total number of planets had risen to 14, as noted by Neil Tyson; then, the asteroids, because of their small size and the great number of them that were being discovered were demoted from planet status to simply be known as asteroids. The width of the main belt of asteroids is shown in proper scale relative to its circumference in the map. Since the main belt asteroids lie approximately in the ecliptic plane which is tilted at an angle of 23.5° relative to the Earth's equatorial plane, there are two dense clusters where the ecliptic plane cuts the Earth's equatorial plane and the density of asteroids is highest. One intersection is at about 12h and the other is at 24h. In addition to Ceres, main belt asteroids Eros, Gaspra, Vesta, Juno, and Pallas are labeled.

Jupiter is shown in conjunction with the Sun approximately 6 AU from the Earth. The radius of Jupiter's orbit is about 5 AU so when we add the 1 AU radius of the Earth's orbit we get 6 AU. On either side of Jupiter we can see the two swarms of Trojan asteroids trapped in the L4 and L5 stable Lagrange equilibrium points $\pm 60^\circ$ away from Jupiter along its orbit. From the vantage point of Earth, 1 AU off center opposite Jupiter in its orbit, the Trojans are a bit closer to the Earth than Jupiter and a bit closer to Jupiter in the sky on each side than 60°. The Ulysses spacecraft is visible near Jupiter. It is in an orbit far out of the Earth's equatorial plane, but we have included it anyway. Beyond Jupiter are Saturn, Uranus and Neptune.

Ever wonder what happened to Halley's comet? Here it is between the orbits of Uranus and Neptune, as of August 12, 2003. It has had close encounters with the Earth in 1835, 1910, 1986, and is now on its way out to its aphelion point. It will be back near 1 AU from the Sun in 2061.

Then there is Pluto and the Kuiper belt objects. Recently discovered Quaoar has a diameter of approximately 1250 km and a nearly circular orbit. Varuna is also shown. We are showing all 771 of the known Kuiper belt objects (rather than just those in the equatorial plane). It is surprising how many of them there are. The band of Kuiper belt objects is relatively narrow because of the selection effect that objects of a given size become dimmer approximately as the fourth power of their distance from the Earth and Sun. (The solar illumination they receive falls off like r^{-2} and so does the fraction of this light that they reflect back to Earth.) The band of Kuiper belt objects has vertical density stripes, again due to angular selection effects depending on where various surveys were conducted. A sprinkling of Kuiper belt objects extend all the way into the space between the orbit of Uranus and Saturn (the "Centaurs", of which we're only plotting the ones in the equatorial plane). The IAU still classifies Pluto as a planet, but Neil Tyson of the Hayden Planetarium has argued that Pluto should be considered as the king of the Kuiper belt objects.

Beyond the known Kuiper belt objects are the Pioneer 10, Voyager 1 and Voyager 2 spacecraft headed away from the solar system. These are on their way to the heliopause, where the solar wind meets the interstellar medium. They have not reached it yet.

Almost a hundred times further away than the heliopause is the beginning of the Oort cloud of comets which extends from about 8,000 AU to 100,000 AU. This is the reservoir of long period comets. A comet entering the inner solar system for the first time has a typical aphelion in this range. Comets in this cloud are perturbed by passing stars onto orbits that will bring them into the inner solar system. On the way in they are likely perturbed by Jupiter significantly so that they either then escape the solar system entirely or are captured into a shorter period orbit, as undoubtedly happened to Halley's comet. To fall into the inner solar system from 8,000 AU to 100,000 AU on a nearly parabolic orbit takes between approximately 50,000 to 2.4 million years. We know of no individual objects now in the Oort cloud but we have strong evidence that it exists.

Beyond the Oort Cloud are the stars. The ten brightest stars visible in the sky are shown with large star symbols. The nearest star, Proxima Centauri is shown with a small star symbol. Proxima Centauri, an M5 star, is a member of the α Centauri triple star system. α Centauri A, at a distance of a little over 1 pc (see scale on the left), and a solar type star, is one of the ten brightest stars in the sky. Sirius is the brightest star in the sky.

Stars with known confirmed planets circling them (with $M \sin i < 10 M_{\rm Jupiter}$) are shown as crosses. Of these, 66 are solar type stars whose planets were discovered by radial velocity perturbations. Some of the more famous ones like 51 Peg, 70 Vir, and ϵ Eri are labeled. The star HD 209458 had a Jupiter-mass planet which was discovered by radial velocity perturbations but was later also observed in transit. The planet around OGLE-TR-56 which lies at a distance of over 1 kpc from the Earth was discovered by transit. PSR 1267+12 is a pulsar (neutron star) with three terrestrial planets circling it which were discovered by radial velocity perturbations on the pulsar revealed by accurate pulse timing. This was the first star discovered to have planets.

The Hipparcos satellite has measured accurate parallax distances to 118,218 stars. We show only the 3,386 Hipparcos stars in the equatorial plane ($-2^\circ < \delta < 2^\circ$).

Other interesting representative objects in the galaxy are illustrated: the Pleiades open star cluster, M13 the most prominent globular cluster, the Crab nebula (M1), site of a supernova seen in 1054 and containing the crab nebula pulsar, Cygnus X-1, a binary star system containing a greater than $7 M_\odot$ black hole, the Orion Nebula (M42) a great molecular cloud and star forming region, the Dumbbell nebula and the Ring nebula, famous planetary nebulas, the Eagle nebula made famous by a beautiful Hubble Space telescope picture, the Vela pulsar, and the Hulse-Taylor binary pulsar which was used to provide sensitive tests of general relativity including its prediction of the emission of gravitational waves. At a distance of 8 kpc is the Galactic Center which harbors a 2.6 million solar mass black hole. The outermost extent of the Milky Way optical disk is shown as a dotted line.

Beyond the Milky Way are plotted 52 members of the local group of galaxies. We have included

all of these, not just the ones in the equatorial plane. These are indicated by dots or triangles. The Large and Small Magellanic clouds (LMC and SMC) at 55 and 65 kpc are satellite galaxies of the Milky Way. M31 and the Milky Way are the two principle galaxies in the local group. M31 is at a distance of 900 kpc. M31's companions M32 and NGC205 are shown as dots but not labeled since they are so close to M31. M33, another spiral galaxy, lies nearby. The other galaxies in the local group are dwarfs or irregulars or small ellipticals.

M81, is the first galaxy shown beyond the local group and is a member of the M81 – M82 group. Its distance was determined by Cepheid variables using the HST. Other famous galaxies labeled include M101 (a face-on spiral), the Whirlpool galaxy (M51), a spiral, the Sombrero galaxy, an edge-on SO galaxy, and M87, a giant elliptical galaxy in the center of the Virgo cluster. If we showed all the M objects many would crowd together in a jumble at the location of the Virgo cluster. M87 harbors in its center the largest black hole yet discovered with a mass of 3×10^9 solar masses.

The dots appearing beyond M81 in the map are the 126,594 SDSS galaxies and quasars (with $z < 5$) in the equatorial plane equatorial slice ($-2^\circ < \delta < 2^\circ$). In addition, all 31 currently known SDSS quasars with $z > 5$ are plotted, not just those in the equatorial slice. Since these large redshift quasars are shown from all over the sky, a number of them appear at right ascensions in the zone of avoidance which occurs in the equatorial slice. The upper part of our map can be compared directly with figure 1 and figure 2. The map shows clearly and with recognizable shape all the structures shown in the close-up in figure 2, while still showing all the SDSS quasars shown in the full view in figure 1. The logarithmic scale captures both scales beautifully. On the left, (at about 1.5h and 120 Mpc) we can see clearly the large circular void visible in figure 2. To the right, at a distance of 9h-14h and at a distance of 215 – 370 Mpc we can see a Sloan Great Wall in the SDSS data, longer than the Great Wall of Geller and Huchra (the CfA2 Great Wall). The blank regions are due to the zone of avoidance where the Earth's equator cuts the galactic plane and which the Sloan survey does not cover. These are fan-shaped regions as shown in figure 1 and figure 2, bounded by radial lines pointing away from the Earth, so on our map these are vertical straight lines.

The Great Attractor (which is far off the equatorial plane and toward which the Virgo super cluster has a measurable peculiar velocity) is shown.

The most prominent feature of the SDSS large scale structure seen in figure 8f is the "Sloan Great Wall". This feature was noticed early-on in the Sloan data acquisition process, and has been mentioned in passing in a couple of times in Sloan reports, accompanied by phrases such as "large" (Blanton et al. (2003)), and "striking", "wall-like", and "may be the largest coherent structure yet observed" (Tegmark et al., submitted to ApJ.). To make a quantitative comparison, we have shown the Great Wall of Geller and Huchra. This extends over several slices of the CfA2 survey (from 42° to −8.5° declination). Rather than plotting points for it here, which would be confused with SDSS survey galaxies we have plotted density contours averaging over all the CfA2 slices from

42° to −8.5° declination. This volume extends far above the equatorial plane, and since we are plotting it in right ascension correctly, it is not presented conformally, but is being lengthened in the tangential direction by a factor of as much as $\frac{1}{\cos(42°)} = 1.34$. Note that since the CfA2 Great Wall is a factor of approximately 2.5 closer to us than the Sloan Great Wall so it is depicted at scale that is 2.5 times larger. So although the CfA2 Great Wall stretches from 9h to 16.7h (or 7.7 hours of right ascension), as compared with the SDSS Great Wall which stretches from 8.7h to 13.7h (or 5 hours of right ascension) its real length in co-moving coordinates relative to the CfA2 Great Wall is, by this simple analyses, $2.5 \times \frac{5}{7.7} \times 1.34 = 2.17$ times as long. This is apparent in the comparison figure supplied in figure 9, where both are shown at the same scale. To make a fair comparison, since the Great Wall is almost a factor of 3 closer than the Sloan Great Wall, we have plotted a 12° wide slice from the CfA2 survey to compare with our 4° wide slice in the Sloan, so that both slices have approximately the same width at each wall.

Of course, the walls are not perfectly aligned with the x axis of our map, so one has to measure their length along the curve. The Sloan Great Wall is at a median distance of 310 Mpc. It's total length in co-moving coordinates is 450 Mpc as compared with the total length of the Great Wall of Geller and Huchra which is 240 Mpc long in co-moving coordinates. This indicates the sizes the two walls would have at the current epoch. But the Great Wall is at a median redshift of $z = 0.02918$ so it's true size at the epoch we are observing it is smaller by a factor of $1 + z$ giving it an observed length of 232.64 Mpc (or 758 million light years). The Sloan great wall is at a redshift of $z = 0.0734$ so it's true observed length is 419 Mpc (or 1,365 million light years). For comparison, the CMB sphere has an observed diameter of $\frac{2 \cdot 14{,}000}{1090} = 25.7$ Mpc. The observed length of the Sloan Great Wall is thus 80% greater than the Great Wall of Geller and Huchra.

Since we have numerous studies that show that the 3D topology of large scale structure is sponglike (Gott, Dickinson, & Melott (1986); Vogeley et al. (1994); Hikage et al. (2002)) it should not be surprising that as we look at larger samples we should find examples of larger connected structures. Indeed, we would have had to have been especially lucky to have discovered the largest structure in the observable universe in the initial CfA survey which has a much smaller volume than the Sloan survey. Simulated slices of the Sloan using flat lambda models show great walls and great wall complexes that are quite impressive (Colley et al. (2000)). Cole, Hatton, Weinberg, & Frenk (1998) for example had a great wall in their $\Omega_m = 0.4$, $\Omega_\Lambda = 0.6$ Sloan simulation which is 8% longer than the Great Wall of Geller and Huchra; and so it could be said that the existence of a Great Wall in the Sloan longer than the Great Wall of Geller and Huchra was predicted in advance. Visual inspection of the 275 PThalos simulations reveals similar structures to the Sloan Great Wall in more than 10% of the cases (Tegmark et al, submitted to ApJ.). Thus, it seems reasonable that the Sloan Great Wall can be produced from random phase Gaussian fluctuations in a standard flat-lambda model, a model that also predicts a spongelike topology of high density regions in 3D. Notably, our quantitative topology algorithm applied to the 2D Sloan Slice identifies the Sloan Great Wall as one connected structure (Hoyle et al. (2002)). Figure 2 in Hoyle et al. (2002) clearly shows this as one connected structure at the median density contour when smoothed

at $5h^{-1}$ Mpc in a volume limited sample where the varying thickness and varying completeness of the survey in different directions are accounted for. It is perhaps no accident that both the Sloan Great Wall and the Great Wall of Geller and Huchra are seen roughly tangential to the line of sight. Great Walls tangential to the line of sight are simply easier to see in slice surveys. A Great Wall perpendicular to the line of sight would be more difficult to see because the near end would be lost due to thinness of the slice and the far end would be lost due to the lack of galaxies bright enough to be visible at great distances. Redshift space distortions on large scales, i.e. infall of galaxies from voids onto denser regions enhances contrast for real features tangential to the line of sight. Fingers of God also make tangential structures more noticable by thickening them. When Park (1990) first simulated a volume large enough to simulate the CfA survey, a Great Wall was immediately seen in the 3D data. When a slice to simulate the CfA slice seen from Earth – which gets wider as it gets further from the Earth – was made, the Great Wall in the simulation was pretty much a dead ringer for the Great Wall of Geller and Huchra – equal in length, shape, and density. This was an impressive success for N-body simulations. The Sloan Great Wall and the CfA Great Wall have been found in quite similar circumstances, each in a slice of comparable thickness, and as illustrated in figure 9, both are qualitatively quite similar except that the Sloan Great Wall is simply larger. The CfA Great Wall is as large a structure as could have fit in the CfA sample, but the Sloan Great Wall is smaller than the size of the Sloan survey, showing the expected approach to homogeneity on the very largest scales.

The 2dF survey (Colless et al. (2001)), of similar depth to the Sloan, completed two slices, an equatorial slice ($9^h : 50^m < \alpha < 14^h : 50^m, -7.5^\circ < \delta < 2.5^\circ$), and a southern slice ($21^h : 40^m < \alpha < 03^h : 40^m, -37.5^\circ < \delta < -22.5^\circ$). This survey was thus not appropriate for our logarithmic map of the universe. The southern slice was not along a great circle in the sky, and therefore would be streched if plotted in our map in right ascension. The equatorial slice was of less angular extent than the corresponding Sloan Slice and so the Sloan with its greater coverage in a flat equatorial slice was used to plot large scale structure in our map. Indeed, the 2dF survey, because of its smaller coverage in right ascension, missed the western end of the Sloan Great Wall and so the wall did not show up as prominantly in the 2dF survey as in the Sloan. The Sloan Great Wall contains a number of Abell clusters (including, for example, A1238, A1650, A1692 and A1750 for which redshifts are known). The spongelike nature of 3D topology means that clusters are connected by filaments or walls but if extended far enough, walls should show holes allowing the voids on each side to communicate.

Indeed, in our map we can see some remnants of the CfA2 Great Wall (a couple of clumps or "legs") extending into the equatorial plane of the Sloan Sample. As shown in Vogeley et al. (1994), if extended to the south the Great Wall develops holes that allows the foreground and background voids to communicate leading to a spongelike topology of the median density contour surface in 3D.

In the center of the Great Wall is the Coma Cluster, one of the largest clusters of galaxies known. In its center are two giant cD galaxies orbiting each other.

The quasar 3C273 is shown as a cross, the first quasar to have its redshift measured (by M. Schmidt). Note how much further away this is than the Coma Cluster and the famous galaxies in the Virgo cluster like M87.

The gravitational lens quasar 0957 is shown as well as the lensing galaxy producing the multiple image. The lensing galaxy is along the same line of sight but at about one third the co-moving distance.

The gamma ray burster GRB990123 is shown - for a brief period this was the most luminous object in the observed universe.

The redshift $z = 0.7551$ is shown as a line which marks the epoch that divides the universe's decelerating and accelerating phase. Objects closer than this line are observed at an epoch when the universe's expansion is accelerating, while objects further away than this line are observed at an epoch when the universe's expansion is decelerating.

The unreachable limit is shown at $z = 1.6876$. Because of the acceleration of the expansion of the universe, photons sent from Earth now will never reach objects beyond this line. Galaxies beyond this line will never hear our current TV signals. Spaceships from Earth, traveling slower than light will also find the territory beyond this line unreachable. This redshift is surprisingly low. It is interesting that we can see many objects today that are so far away that we can never get to them.

SDSS quasars in the equatorial plane ($-2^\circ < \delta < 2^\circ$) are shown as points out to a redshift of $z = 5.0$ using redshifts determined from the SDSS survey spectra. For quasars with $z > 5.0$ we have shown all 31 quasars from the SDSS with $z > 5$ regardless of declination. There are now a surprisingly large number of quasars with $z > 5$ known. Because a number of these are at high declination, they occur also in the zone of avoidance for the equatorial plane. Each is shown at its proper right ascension and distance, including to the largest redshift one with $z = 6.42$ which is labeled (M. Strauss 2003, private communication). This is the largest redshift quasar known.

The largest redshift galaxy, with $z = 6.68$ is also shown. This was discovered in the Subaru Deep Field (Kodaira et al. (2003)).

The first stars are shown as a dashed line. The WMAP satellite has found that the first stars appear about 200 million years after the Big Bang and the map therefore indicates the distance out to which stars could be seen in principle.

Finally there is the cosmic microwave background radiation discovered by Penzias and Wilson in 1965. CMB photons from this surface arrive directly from an epoch only 380,000 years after the Big Bang.

A line showing the co-moving radius back to the Big Bang is also shown. This represents seeing back to the epoch just after inflation. The co-moving distance between the cosmic microwave background and the Big Bang is shown in correct proportion to the circumference. In comparing

with figure 1, we note that in that map (a circle with diameter of approximately 6.28 inches) the circumference is a factor of π larger. So in our map, which shows the 360° circumference of the cosmic microwave background as approximately 6.28 inches, the scale at that point is about a factor of π smaller than in figure 1 and the cosmic microwave background and the Big Bang are closer to each other (by a factor of π) than in figure 1 as expected.

Last is shown the co-moving future visibility limit. If we wait until the infinite future we will eventually be able to see the Big Bang at the co-moving future visibility limit. Stars and galaxies that lie beyond this co-moving future visibility limit are forever hidden from our view. Because of the de Sitter expansion produced by the cosmological constant the universe has an event horizon which we cannot see over no matter how long we wait.

It is remarkable how many of the features shown in this map have been discovered in the current astronomical generation. When one of us (JRG) began studing astronomy at age 8 (in 1955), on astronomical maps there were no artificial satellites, no Kuiper belt objects, no other stars with planets, no pulsars, no black holes, no non-solar X-ray sources, no gamma ray bursters, no great walls, no great attractors, no quasars, no gravitational lenses, and no observation of the cosmic microwave background.

4. Applications

This map shows large scale structure well. Fingers of God are vertical which makes removing them for large scale structure purposes particularly easy. A test for roundness of voids as proposed by Ryden (1995) – the Alcock-Paczynski isotropy test – could be done on this map as well as on the co-moving map. The ability of this test to differentiate between cosmological models can be deduced by measuring void isotropy as a function of cosmological model in the plane of parameters (Ω_m, Ω_Λ). For the correct cosmological model the void pictures will be isotropic (since the Earth is not in a special position in the universe). To do this test we need conformal maps for various cosmological models, and so we need conformal maps for the $k = +1$ and $k = -1$ cases as well as the $k = 0$ case so that statistical comparisons can be made. The formulas for these projections are given in the appendix.

A Fourier analysis of large scale structure modes in the map (k_x, k_y) can provide information on the parameter $\beta = \frac{\Omega_m^{0.6}}{b}$ where b is the bias parameter, as we will show in a separate paper (Jurić, and Gott 2003, in preparation). For fluctuations in the linear regime in redshift space $d\rho/\rho \propto (1 + \beta\mu^2)$ where $\mu = \cos\theta$ where θ is the angle between the normal to the wave and the line of sight in 3D (Kaiser (1987)). Waves tangential to the line of sight have a larger amplitude in redshift space than waves parallel to the line of sight because the peculiar velocities induced by the wave enhance the amplitude of the wave when peculiar radial velocities are added to Hubble positions as occurs when galaxies are plotted using redshift at the Hubble flow positions assuming peculiar velocities are zero. Imagine a series of waves isotropic in 3D. Then one can show that

the average value of μ^2 in 3D of those waves with an orientation $\mu' = \cos(\phi)$ in the equatorial slice is $\langle \mu^2 \rangle = \frac{2}{3}\mu'^2$. Thus, we expect approximately, for waves observed in our equatorial slice $\delta\rho/\rho \propto (1 + \frac{2}{3}\beta\mu'^2)$. Waves in our map with constant (k_x, k_y) represent global logarithmic spiral modes with constant inclination relative to the line of sight. Of course this is an oversimplified treatment, since we must consider the power spectrum of fluctuations when relating the modes seen at a particular wavelength in the plane and in 3D and the fingers of God must be eliminated by some friend-of-friend algorithm. But in general, we expect that modes that are radial ($k_x = 0$) will have higher amplitudes than modes that are tangential ($k_y = 0$) and this effect can be used empirically, in conjunction with N-body simulations, to provide an independent check on the value of β. One simply adopts a cosmological model, checks with N-body simulations the relative amplitude of map modes as a function of μ' in the logarithmic map and compares with the observations assuming the same cosmological model when plotting the logarithmic map: if the cosmological model $(\Omega_m, \Omega_\Lambda)$ is correct, the results should be similar. We plan to investigate this check empirically using N-body simulations, and the SDSS data in a later paper (Jurić, and Gott 2003, in preparation). Our map projection will be used to do this Fourier analysis in global logarithmic spiral modes.

Since logarithmic spirals appear as straight lines in our map projection, it may prove useful for mapping spiral galaxies. Take a photograph of a face-on spiral galaxy, place the origin of the coordinate system in the center of the galaxy and then construct our logarithmic map of this 2D planar photograph. The spiral arms (which approximate logarithmic spirals) should then appear as straight lines on the conformal map. We have tried this on a face-on spiral galaxy photograph given to us by James Rhoads and the results were very satisfying. The spiral arms indeed were beautiful straight lines, and from their slope one could easily measure their inclination angle. Star images in the picture were still circular in the map because the map is conformal.

5. Conclusions

The map presented here is appropriate for use as a wall map. It is convenient to print out in six sheets. A version at twice the scale, 12.56 inches by 95.2 inches tall would also be appropriate for a wall chart and would run nearly from floor to ceiling in a normal room with an 8ft ceiling. If one wanted to show individual objects at 60× scale, the Moon and Sun would be 1 inch across, M31 would be 4 inches across, and Mars would be 0.0145 inches in diameter and Jupiter 0.0272 inches. Alternately the Sun and Moon and Messier objects could be shown a 60× scale with the planets at 600× scale to illustrate their usual appearance in small telescopes. A version at a scale of 2.7 inches by 20.5 inches tall could be printed in a newspaper.

Consider some possible (and some fanciful) ways this map of the universe might be presented for educational use.

We have presented the map on the internet in color on astro-ph. In principle it would be easy to have such a map on the internet automatically continuously updated to track the current

positions of the satellites, Moon, Sun, asteroids, planets, and Kuiper belt objects as a function of time, in fact we have plotted them as of a particular date and time using such programs. New objects could be added to the map as they were discovered. Click on an object, and a 60x enlarged view of it would appear. Two clicks, and a 3600x enlarged view would appear, and so forth – until the highest resolution picture available was presented. Individual images of all the SDSS galaxies and quasars shown on the map could be accessed in this way, as well as M objects . When an individual object was selected, helpful internet links to sites telling more about it would appear.

Since the left and right hand edges of the map are identical, the map could be profitably shown as a cylinder. The floor to ceiling wall chart described above could be rolled up into a cylinder and wrapped around a 4 inch diameter pole. In cylindrical form, the map at the approximate scale and detail of figure 7 is in perfect proportion to be used on a pencil with the Earth's surface at the eraser end and the Big Bang at the point end. In fact, hemispherical eraser could represent the interior of the Earth (with the Earth's center at the center of the hemisphere, using an inverse Mercator projection from our map onto the hemisphere) and at the other end, the map could end with the cosmic microwave background and the point could symbolically represent the Big Bang. No matter which direction you look you always get back to the Big Bang (at the end of the inflationary period). The map might naturally be displayed around the cylindrical surface of a long focal length refracting telescope tube (of $f/24$ focal ratio). Perhaps the best cylindrical form for the map would be a cylinder on the interior of an elevator shaft for a glass elevator. If the cylindrical glass elevator was about 8.5 feet in diameter, the shaft would be about 202.3 feet or 20 floors. Standing at the center of the glass elevator you could see 360° around and you would see each object in its proper azimuth as seen from the Earth. The Moon would be 0.4 inches across and would be seen at the correct angular scale as seen by the naked eye. Every floor you went up you would be looking at objects that were 10 times further away than the preceding floor. This elevator trip could be simulated virtually by simply putting it up in a cylindrical room with a 360° video screen covering its perimeter. Alternatively, this could be done in a planetarium show, with the cylindrical map being projected onto the dome showing the view from the elevator as it rose.

We do plan to put our map up on Princeton's flat video wall which is about 22 feet wide. This wall has a horizontal resolution of 4096 pixels. From top to bottom of the video wall is about 1600 pixels, so the scale change in the map from top to bottom is about a factor of 10. This shows a small portion of the entire map. We will then scan this in real time moving steadily upward from the Earth to the cosmic microwave background and the Big Bang. Thus, as Charles and Ray Eames made a movie of an atlas of maps of the universe, we can make a movie of our single map! This will produce a virtual map 22 feet wide and 167 feet tall. With laser beams it would be easy to paint a large version of the map on the side of a building.

The map could be produced on a carpet 6 feet wide 45.5 feet long for a hallway in an astronomy department. Then every step you took down the hallway would take you a factor of about ten in distance further from the Earth.

The fact that the objects depicted in our map are small relative to the size of the map itself can be turned to advantage. Charles Allen, former President of the Astronomical League often gave a famous lecture where he brought small illuminated astronomical models into a large empty room to illustrate the scales in the universe. A 1 foot globe and a tennis ball Moon placed 30 feet away illustrated the distance to the Moon. A pea-sized Earth and a bb-sized Mars placed 90 feet away illustrated the distance to Mars. Models of the Milky Way and M31 the size of dinner plates were placed to represent the local group. Precisely because the models were small, the presentation was inexpensive to produce because the space in the large room was free. Likewise for us: a very large temporary version of our map of the universe could be set up in a park or at a star party by simply planting markers for the salient objects. If the dimensions were 360 feet by a half a mile, a picture of the Moon 6 inches across could be shown, a 2 foot wide picture of M31, and so forth. If the object depicted was above the horizon at the time a telescope could be placed at that location to look at it. People could choose to track a satellite, take a look at the crescent Moon, see a bright star, the globular cluster M13, a galaxy like M81, or even try for a quasar like 3C273. Perhaps a radio telescope could even be set up to pick up some static from the cosmic microwave background. One could start at the line representing the surface of the Earth and work one's way outward. The circumference of the Earth's surface at the equator could be a line 360 feet long giving a scale of 1 foot per degree. A picture of Mt. Everest 1 inch tall could be placed next to this line, showing its proper size relative to the circumference of the Earth. The Hubble Space telescope marker would be 5.4 feet away from this line. Every 20 steps (40 feet) you walk away from the line representing the Earth's surface would double your distance from Earth. The Moon would be 235 feet away, and so forth, with the cosmic microwave background a half a mile away. The placement of the object markers and the telescopes would remind people of how far out in the universe they were looking.

Maps can change the way we look at the world. Mercator's map presented in 1569 was influential not only because it was a projection that showed the shapes of continents well but because for the first time we had pretty accurate contours for North America and South America to show. The Cosmic View and the Powers of Ten alerted people to the scales in the universe we had begun to understand. De Lapparent, Geller, and Huchra showed how a slice of the universe could give us an enlightening view of the universe in depth. Now that astronomers have arrived at a new understanding of the of the universe from the solar system to the cosmic microwave background, we hope our map will provide in some small way a new visual perspective on these exciting discoveries.

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The SDSS is managed by the Astrophysical Research Consortium (ARC) for the Participating Institutions. The Participating Institutions are The University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, The Johns Hopkins University, Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, University of Pittsburgh, Princeton University, the United States Naval Observatory, and the University of Washington.

A. $k = +1$, $k = 0$ and $k = -1$ cases

Although the observations suggest the $k = 0$ case is appropriate for the universe, for mathematical completeness we consider the general Friedmann metrics:

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta \cdot d\phi^2)], \quad k = +1, \tag{A1}$$

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \chi^2(d\theta^2 + \sin^2\theta \cdot d\phi^2)], \quad k = 0, \tag{A2}$$

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta \cdot d\phi^2)], \quad k = -1 \tag{A3}$$

Define the conformal time by:

$$\eta(t) = \int_0^t \frac{dt}{a} = \int_0^{a(t)} (-ka^2 + \frac{8\pi}{3}a^4\{\rho_m(a) + \rho_r(a)\} + \frac{\Lambda}{3}a^4)^{-1/2} da \tag{A4}$$

If we are currently at the epoch t_0, then the current conformal time is $\eta(t_0)$. When we look back to a redshift z we are seeing an epoch when $a_0/a(t) = 1 + z$, and out to a co-moving distance χ given by:

$$\chi(z) = \eta(t_0) - \eta(t) = \int_{\frac{a(t_0)}{1+z}}^{a(t_0)} (-ka^2 + \frac{8\pi}{3}a^4\{\rho_m(a) + \rho_r(a)\} + \frac{\Lambda}{3}a^4)^{-1/2} da \tag{A5}$$

As we have noted, in the flat case $k = 0$, we are free to adopt a scale for a, so we set $a(t_0) = R_{H_0}$.

For the $k = +1$ case, a two dimensional slice through the universe is a sphere. So we need to make a conformal projection of the sphere onto a plane, and then we can apply our conformal logarithmic projection as before to produce our map. The stereographic map projection is such a conformal projection of a sphere onto a plane. Adopt coordinates on the sphere of χ, θ, where the metric on the sphere is given by:

$$ds^2 = a^2(t_0)(d\chi^2 + \sin^2\chi \cdot d\theta^2) \tag{A6}$$

Where the angle θ is now a longitude (Thus, θ is equivalent to ϕ in metric A2 above – where in metric A2 we are considering the equatorial slice with $\theta = const. = \frac{\pi}{2}$. So to get the metric above

from metric A2 we set $\theta = const. = \frac{\pi}{2}$, and replace ϕ with θ). Now make a stereographic conformal projection of this sphere (χ, θ) onto a plane with polar coordinates (r, θ):

$$r = 2a(t_0)\tan(\frac{\chi}{2}) \tag{A7}$$

$$\theta = \theta \tag{A8}$$

This is a projection from the north pole onto a plane tangent to the south pole. The Earth would be at the south pole of the sphere (where $\chi = 0$). A line from the south pole to the plane through the point (χ, θ) on the sphere will be at an angle $\alpha = \chi/2$ relative to a normal to the plane, and also at an angle $\alpha = \chi/2$ relative to a normal to the surface of the sphere at the point (χ, θ), thus the foreshortening that occurs along the ray from the north pole as it crosses the surface of the sphere is exactly the same as the foreshortening that occurs when it crosses the surface of the plane. Thus, shapes in the surface are locally mapped without distortion locally onto the plane. The map is conformal. Now we have a conformal map of the sphere and we apply our logarithmic conformal map to the planar map to get our map of the universe:

$$x = -\theta \tag{A9}$$

$$y = \ln\frac{r}{r_E} = \ln\frac{2a(t_0)\tan(\frac{\chi}{2})}{r_E} \tag{A10}$$

This provides a conformal mapping from (χ, θ) to (x, y). It is in fact a Mercator projection of the spherical section of the universe with the Earth located at the south pole of the sphere! We are used to seeing the Mercator projection for the surface of the Earth cut off at the Antarctic circle, so we do not see that if it were extended much nearer to the south pole (in the limit as $\chi \to 0$) it would approximate our logarithmic map of a plane in the region of the south pole. (This projection would be useful for models that were slightly closed. For example, Luminet et al. (2003) have recently suggested that the universe is a topologically multiply connected dodecahedron with positive curvature. In this case the universe covers $1/120^{th}$ of the volume of a 3-Sphere universe; a planar slice of it is a sphere with metric (A1). They propose $\Omega_m + \Omega_\Lambda = 1.013$, consistent with the WMAP error bars. In this case the big bang is at radius $\chi_H = 0.38$, which is plotted on our conformal map using equation A10, while the dodecahedron's in-radius is $\chi = 0.31$, and its out-radius is $\chi = 0.39$. Beyond our dodecahedral cell, we would see duplicate images. But as is apparent from comparing the ratio 0.31/0.38 and figure 1, none of the Sloan galaxy and quasar images are duplicate images (i.e. objects with other images nearer Earth). Also since the future visibility limit in figure 1 is about 1.36 horizon radii away, and this is less than twice the in-radius, the acceleration of the universe will make it impossible for us to ever see a distant image of the early Milky Way. The acceleration of the universe would make it impossible to circumnavigate the universe and return to the Milky Way. The horizon sphere self-intersects in six pairs of circles of angular radius 35°. This model strives to explain the low quadrupole in the WMAP data, and can be checked by looking for the pairs of rings in the WMAP data. Our map of this model would be quite similar to what we have presented. There would be a small difference in the scaling of y as a function of z for large-scale structure, using equation A10, but the lower portions of the map would be virtually identical to what we have presented.)

For the $k = -1$ case, a 2D slice of the universe is a negatively curved pseudosphere with metric:

$$ds^2 = a^2(t_0)(d\chi^2 + \sinh^2\chi(d\theta^2)), \quad k = -1 \tag{A11}$$

Where χ is the co-moving radius, and as before we have set $\theta = const. = \frac{\pi}{2}$, in metric A3 above to look at an equatorial slice, and we have replaced ϕ in metric A3 with θ, so that θ is now a longitude. We can conformally project the pseudosphere onto plane with a map projection that is an analogue of the stereographic projection for the sphere. The metric above for a pseudosphere of radius $a(t_0)$ is the metric on the hyperboloid surface:

$$x^2 + y^2 - t^2 = -a^2(t_0) \tag{A12}$$

where $t > 0$ in a three dimensional Minkowski space with metric:

$$ds^2 = -dt^2 + dx^2 + dy^2 \tag{A13}$$

define coordinates on this surface (χ, θ) by

$$t = a(t_0)\cosh\chi \tag{A14}$$
$$y = a(t_0)\sinh\chi\sin\theta \tag{A15}$$
$$x = a(t_0)\sinh\chi\cos\theta \tag{A16}$$

with the definitions above it is easy to see that $x^2 + y^2 - t^2 = -a^2(t_0)$, since $a^2(t_0)\sinh^2\chi\cos^2\theta + a^2(t_0)\sinh^2\chi\sin^2\theta - a^2(t_0)\cosh^2\chi = -a^2(t_0)$. Connect each point (χ, θ) on the surface with the origin $(x, y, t) = (0, 0, 0)$ via a straight worldline. This worldline has a velocity relative to the t axis of $v = \frac{\sinh\chi}{\cosh\chi} = \tanh\chi$. So this worldline has a boost of χ relative to the t axis. This worldline is normal to the surface at the point (χ, θ), because that boost takes the t axis to the worldline in question and leaves the hyperbolic surface invariant. Let the intersection of that worldline with the tangent plane $t = a(t_0)$ be the gnomonic map projection of the point (χ, θ) onto the plane with polar coordinates (r, θ). Then

$$r = a(t_0)\tanh\chi \tag{A17}$$
$$\theta = \theta \tag{A18}$$

This gnomonic projection maps the pseudosphere $0 \leq \chi \leq \infty$ into a disk of radius $r = a(t_0)$. Why? Because in a time $t = a(t_0)$ the worldline which has a velocity $v = \tanh\chi$ travels a distance of $r = vt = a(t_0)\tanh\chi$. This gnomonic projection is not conformal, because the worldline is normal to the hyperboloid surface but not normal to the plane $t = a(t_0)$. This gnomonic projection maps geodesics on the negatively curved hyperboloid onto straight lines on the plane $t = a(t_0)$ because any plane in the space (x, y, t) passing through the origin intersects the surface in a geodesic and this plane will intersect the tangent plane $t = a(t_0)$ in a straight line.

A conformal projection, like the stereographic projection of the sphere is provided for the pseudosphere by connecting each point on the surface (χ, θ), to the point $(x, y, t) = (0, 0, -a(t_0))$,

and letting this worldline intersect the plane $t = a(t_0)$ at a point (r, θ). Now that worldline has a velocity $v = \frac{\sinh \chi}{(\cosh \chi + 1)} = \tanh(\chi/2)$ (using a half angle trigonometric identity). Thus, this world line has a boost relative to the t axis of $\chi/2$ and therefore a boost of $\chi/2$ relative to the normal to the plane $t = a(t_0)$. When it intersects the hyperbolic surface at the point (χ, θ), the normal to the hyperbolic surface at that point has a boost of χ relative to the t axis in the same plane. Thus, the worldline has a boost of $\chi/2$ relative to the normal of the tangent plane and also a boost of $\chi/2$ relative to the hyperbolic surface, so it has an equal boost (and observes equal Lorentz contractions) relative to both (giving identical foreshortenings as in the stereographic projection of the sphere) and therefore the map projection is conformal. This worldline connecting the point $(x, y, t) = (0, 0, -a(t_0))$ to the point (χ, θ) on the surface thus intersects the tangent plane $t = a(t_0)$ at the point (r, θ) given by

$$r = 2a(t_0) \tanh(\frac{\chi}{2}) \tag{A19}$$

$$\theta = \theta \tag{A20}$$

This maps the pseudosphere $0 \leq \chi < \infty$ into a disk of radius $r = 2a(t_0)$. Now we have a conformal map projection of the negatively curved $k = -1$ universe onto a plane. This conformal map is the one illustrated in Escher's famous angels and devils print which is often used to illustrate this cosmology (cf. Gott (2001)). Next we take this conformal planar map (r, θ) and apply our conformal logarithmic mapping function to it. Thus, we produce a conformal map of the universe with coordinates:

$$x = -\theta \tag{A21}$$

$$y = \ln \frac{2a(t_0) \tanh(\chi/2)}{r_E} \tag{A22}$$

Note the similarity with the formula for a spherical $k = +1$ universe; in the $k = -1$ case "tanh" simply replaces "tan".

We can make maps for all three cases. These are useful for comparison purposes. For example suppose we develop an isotropy measure for voids as suggested by Barbara Ryden. We could apply this to the voids on our conformal universe map since it preserves shapes locally and the voids are small. The shapes of the voids depend on the assumed cosmological model. This is the Alcock–Paczynski test. If we have the right cosmological model the voids will be approximately round as suggested by Ryden, and this can be tested. Suppose, as WMAP suggests, the flat $k = 0$ case is correct with $\Omega_m + \Omega_\Lambda = 1$, Then we can plot the same redshift data but conformally assuming slightly closed $\Omega_m + \Omega_\Lambda > 1$, $k = +1$ models or slightly open $\Omega_m + \Omega_\Lambda < 1$, $k = -1$ models and check isotropy of the voids in each case. We can thus check how sensitive the isotropy test is in limiting the location of the model in the $(\Omega_m, \Omega_\Lambda)$ parameter plane.

B. Sources

In applying the logarithmic map to the problem of showing the Universe, we have used a multitude of both online sources, personal communication and data published in journals. To illustrate the wealth and variety of data depicted in the Map, we have chosen to list the sources in this section.

- Moon phase
  United States Naval Observatory Astronomical Almanac
  (http://aa.usno.navy.mil/data/docs/MoonPhase.html)

- Earth geological data
  Allen's Astrophysical Quantities, 4th edition, Cox, A. N., AIP press, Springer, 2000.

- Artificial satellites
  Mike McCants' Satellite Tracking web pages (alldat.tle file). The file includes orbital elements supplied by OIG (NASA/GSFC Orbital Information Group) and data on other satellites obtained primarily by amateur visual observations.
  (http://oig1.gsfc.nasa.gov/)
  (http://users2.ev1.net/˜mmccants/tles/index.html)

- Asteroid catalog
  Lowell Observatory Asteroid Database (ASTORB), 2003/04/19 snapshot, with ephemeris calculated for Aug 12, 2003
  ftp://ftp.lowell.edu/pub/elgb/astorb.html

- Minor bodies' ephemeris
  Ephemeris computed using an adapted version of the OrbFit software package.
  OrbFit is written by the OrbFit consortium: Dept. of Mathematics, Univ. of Pisa (Andrea Milani, Steven R. Chesley), Astronomical Observatory of Brera, Milan (Mario Carpino), Astronomical Observatory, Belgrade (Zoran Knežević), CNR Institute for Space Astrophysics, Rome (Giovanni B. Valsecchi).
  (http://newton.dm.unipi.it/˜neodys/astinfo/orbfit/)

- Quaoar information
  (http://www.gps.caltech.edu/˜chad/quaoar/)

- Comet, Sun, Moon and planet ephemeris
  Generated using JPL HORIZONS On-line Solar System Data and Ephemeris Computation Service.
  (http://ssd.jpl.nasa.gov/horizons.html)

- Space probes
  Voyager 1, Voyager 2, Pioneer 10, Ulysses positions obtained from NASA HeloWeb webpage

at NSSDC (National Space Science Data Center)
(http://nssdc.gsfc.nasa.gov/space/helios/heli.html)

- Heliopause
  Distance taken to be approximately 110 AU in the direction of approximately 18h RA.

- Oort Cloud
  Taken to extend from 8,000 AU to 100,000 AU

- 10 brightest stars
  Source for positions and parallaxes: SIMBAD Reference database, Centre de Donnees astronomiques de Strasbourg
  (http://simbad.u-strasbg.fr/sim-fid.pl)

- Extra-solar planets
  IAU "Working Group on Extrasolar Planets" list of planets, the "Extrasolar Planets Catalog" of the "Extrasolar Planets Encyclopedia" maintained by Jean Schneider at CNRS – Paris Observatory
  (http://www.ciw.deu/IAU/div3/wgesp/planets.shtml)
  (http://www.obspm.fr/encycl/encycl.html)
  (http://cfa-www.harvard.edu/planets/OGLE-TR-56.html)

- Hipparcos stars
  ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200
  (http://tdc-www.harvard.edu/software/catalogs/hipparcos.html)

- Selected Messier objects
  Distances and common names taken from the Students for the Exploration and Development of Space (SEDS) Messier Catalog pages. Positions taken from the SIMBAD reference database
  (http://www.seds.org/messier/)

- Milky Way
  Disk radius taken to be 15kpc. Distance to galactic center taken to be 8kpc.

- Local Group
  Data taken from a list of all Local Group Member Galaxies maintained by SEDS
  (http://www.seds.org/~spider/LG/lg.html)

- Great Attractor
  Location data from SIMBAD, redshift distance: $cz = 4,350\,\mathrm{km\,s^{-1}}$

- Great Wall
  Contours based on CfA2 redshift Catalog, subset CfA2. The contours are based on galaxies satisfying $-8.5° < \delta < 42.5°$, $120° < \alpha < 255°$, and $0.01 < z < 0.05$ comprising the CfA2's

first 6 slices (Geller and Huchra (1989)).
(http://cfa-www.harvard.edu/˜huchra)

- SDSS data
  Plotted from raw SDSS spectroscopy data obtained using David Schlegel's SPECTRO pipeline – spALL.dat file – dated 2003/01/15 with a $z < 5$ inclusion cut applied.
  SDSS quasar data with $z > 5$ provided by Michael Strauss, private communication.
  (http://spectro.princeton.edu/)

- Plotting
  Plotted using SM software by Robert H. Lupton.
  (http://astro.princeton.edu/˜rhl/sm)

- Individual quasar data
  SIMBAD database

- Cosmological data
  WMAP Collaboration publications (Bennett et al. (2003))

- WMAP location in space at the time of the map
  Dale Fink, Gary Hinshaw, Hiranya Peiris (2003), personal communication

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Table 1. Co-moving radii for different redshifts

z	$r(z)$ (Mpc)	Note
∞	14,283	Big Bang (end of inflationary period)
3233	14,165	Equal matter and radiation density epoch
1089	14,000	Recombination
6	8,422	
5	7,933	
4	7,305	
3	6,461	
2	5,245	
1	3,317	
0.5	1,882	
0.2	809	
0.1	413	

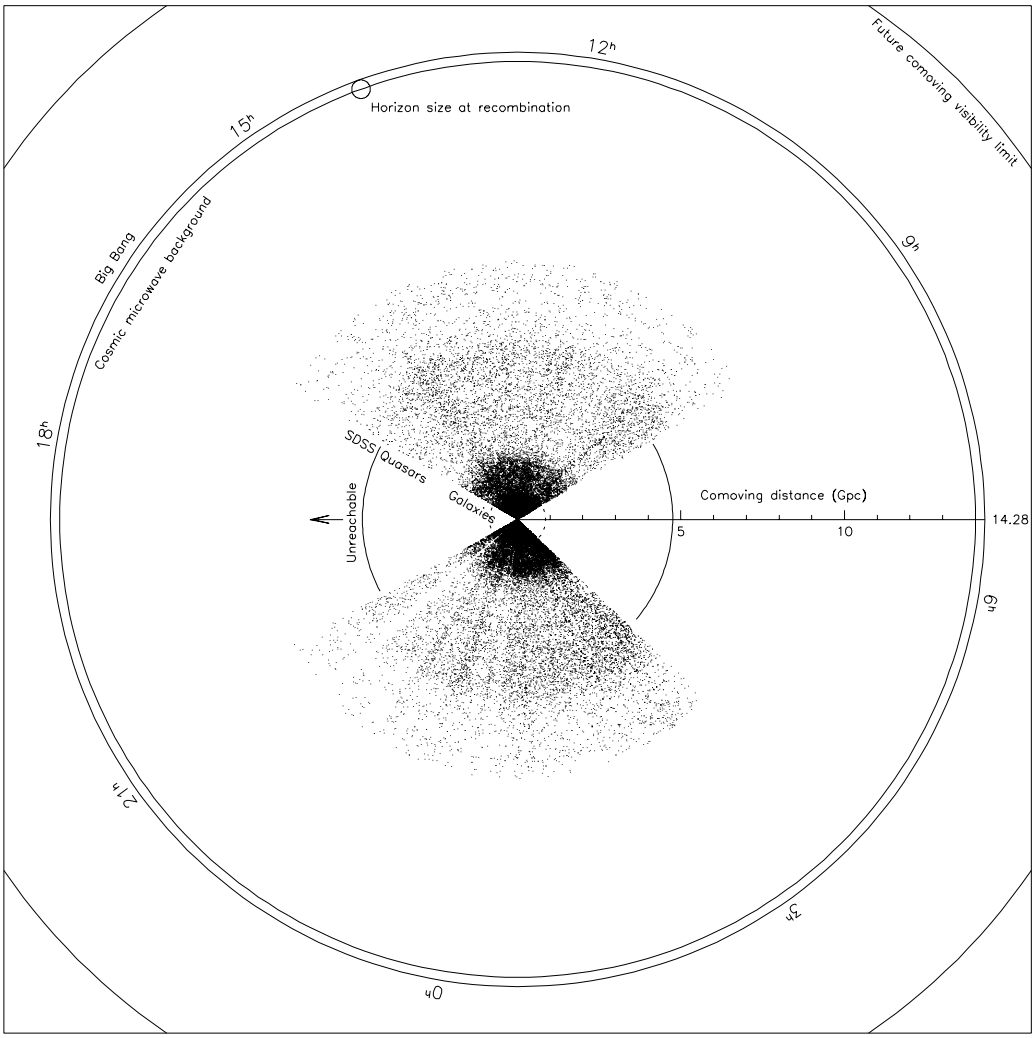

Fig. 1.— Co-moving co-ordinates out to the horizon. The dotted circle marks the outer limit of figure 2.

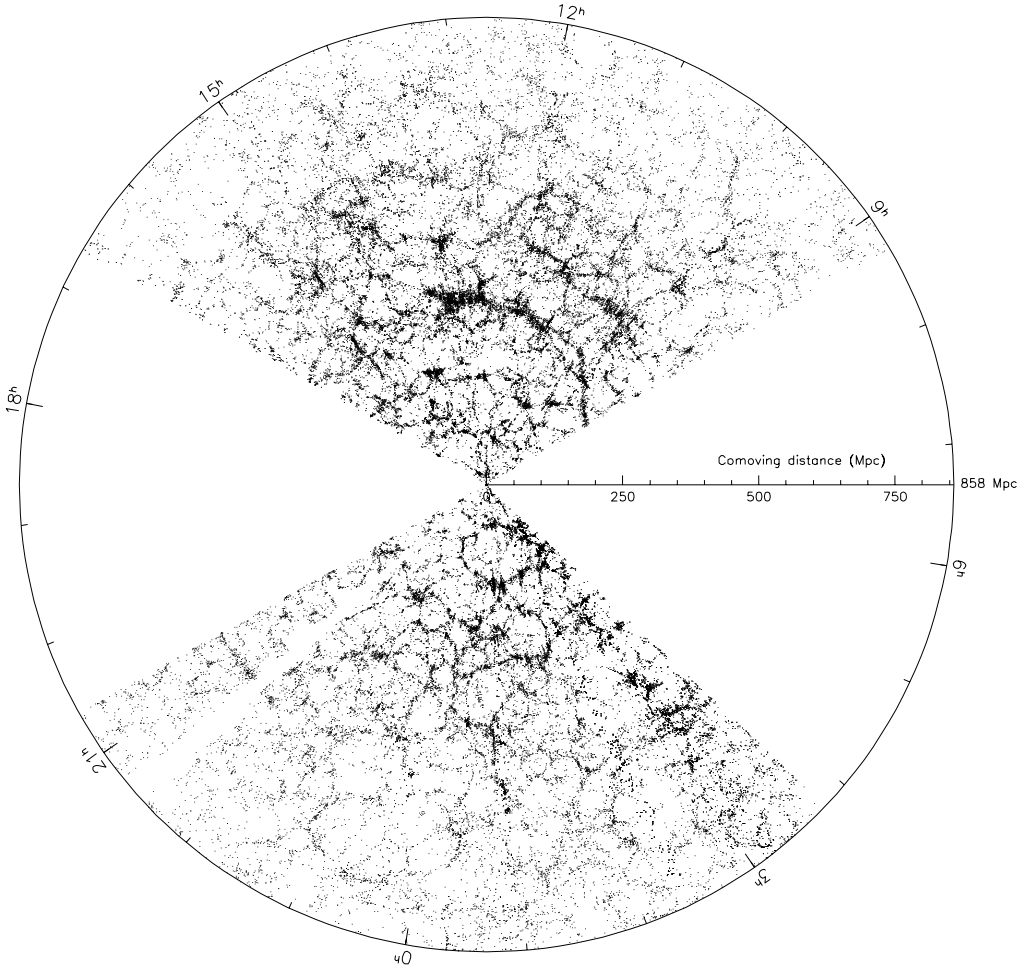

Fig. 2.— Co-moving co-ordinates out to 0.06 $r_{\rm horizon}$.

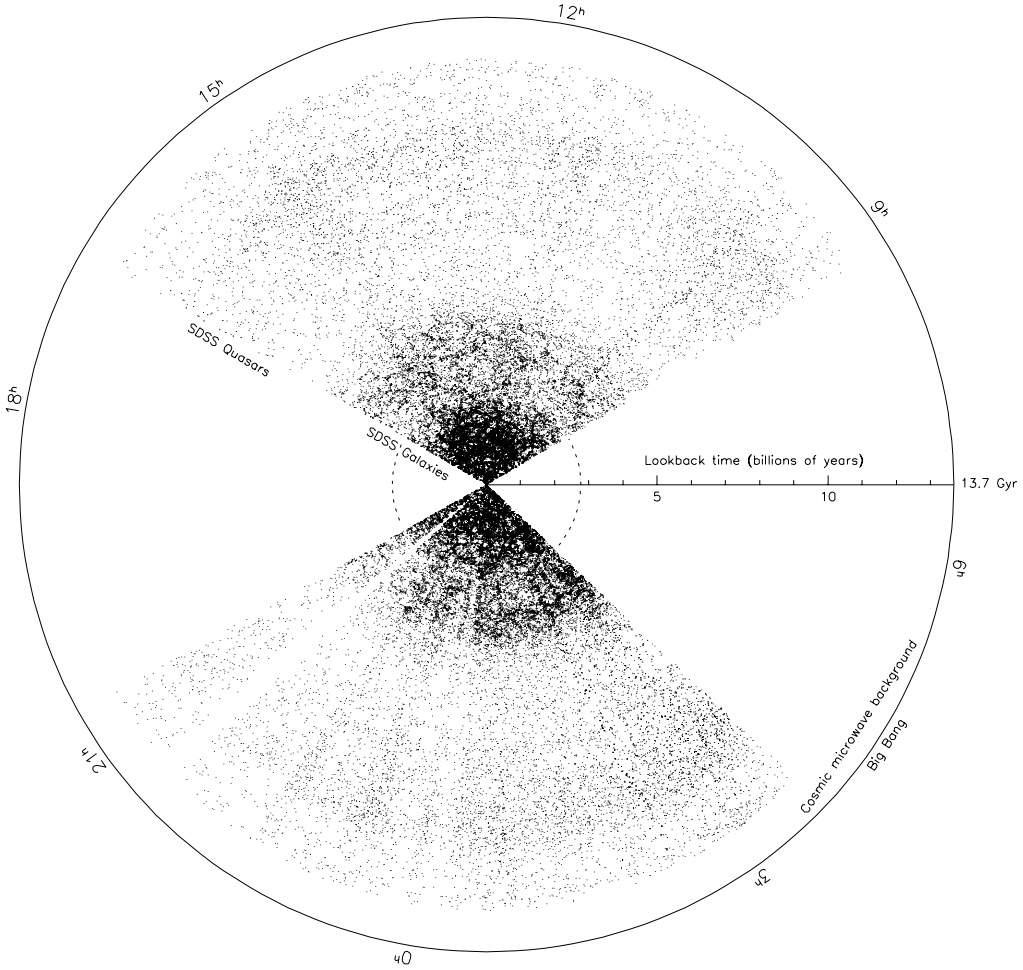

Fig. 3.— Lookback time out to the horizon. The dotted circle marks the outer limit of figure 4.

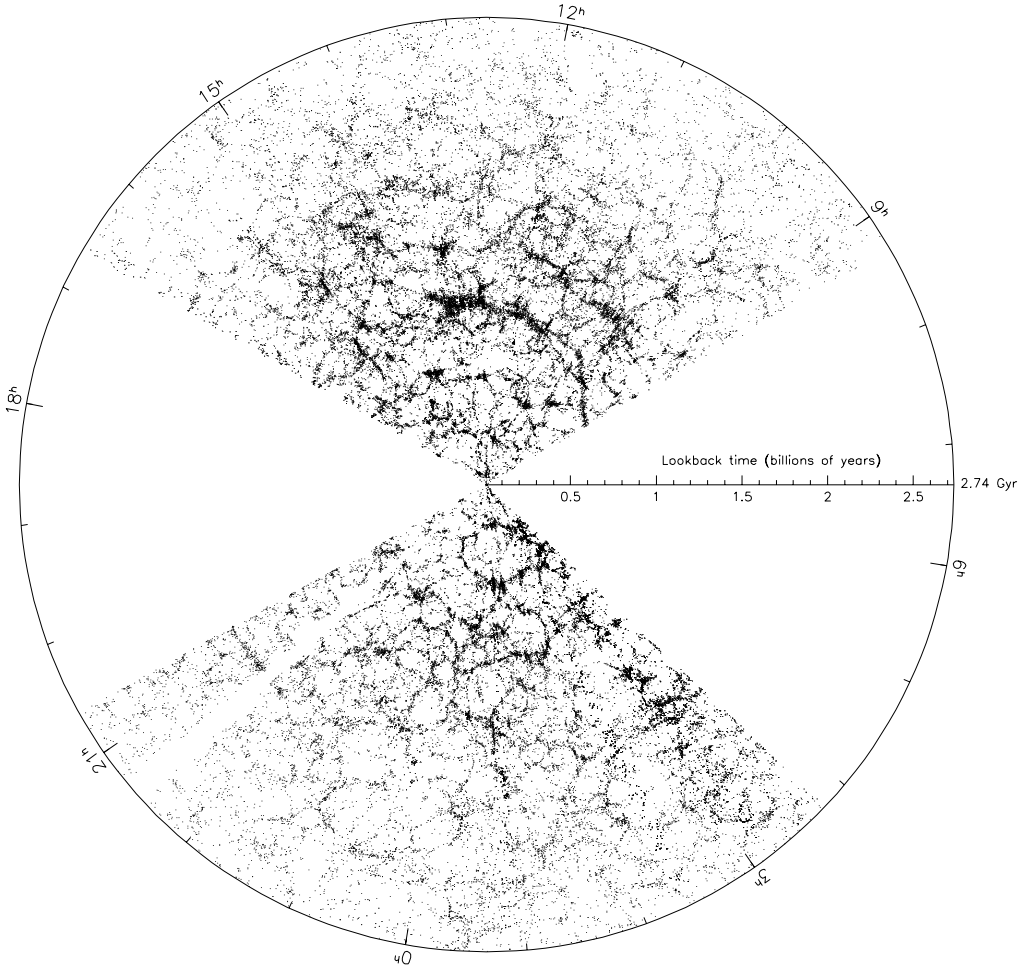

Fig. 4.— Lookback time out to 0.2 $t_{\rm horizon}$

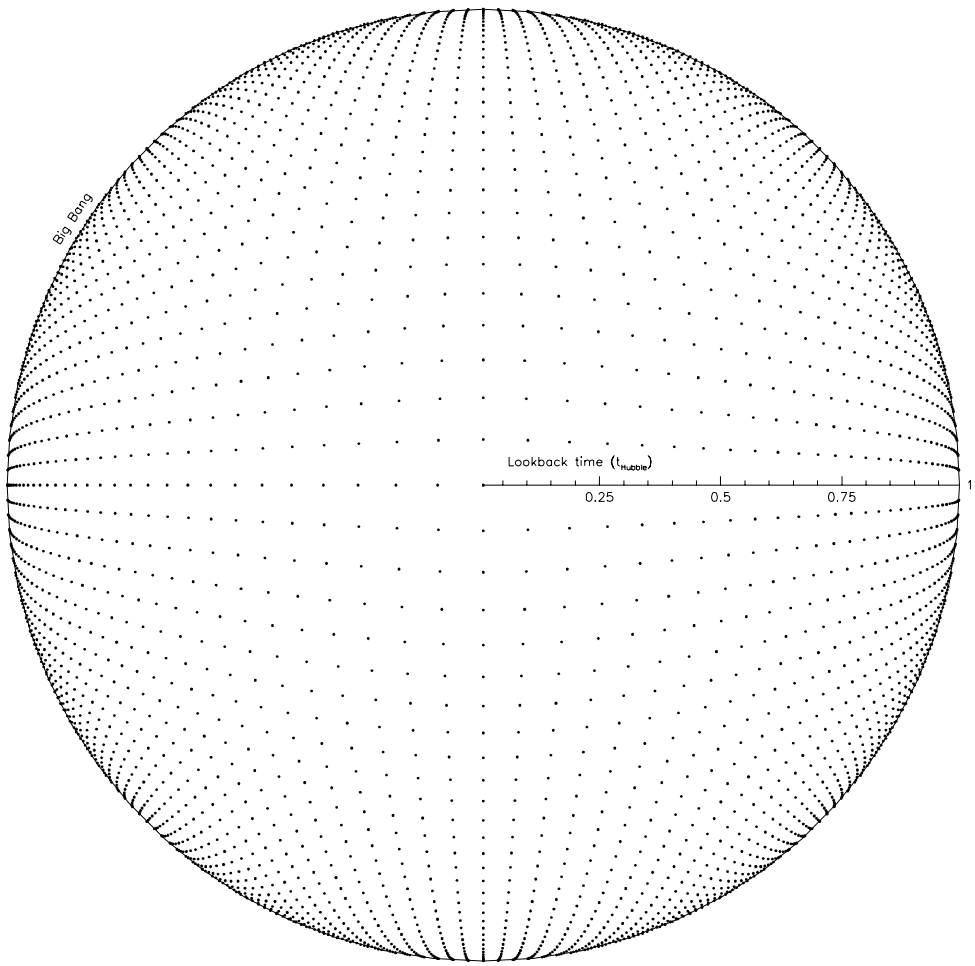

Fig. 5.— Square comoving grid shown in lookback time. Grid spacing is $0.1R_{H_0} = 422.24$ Mpc

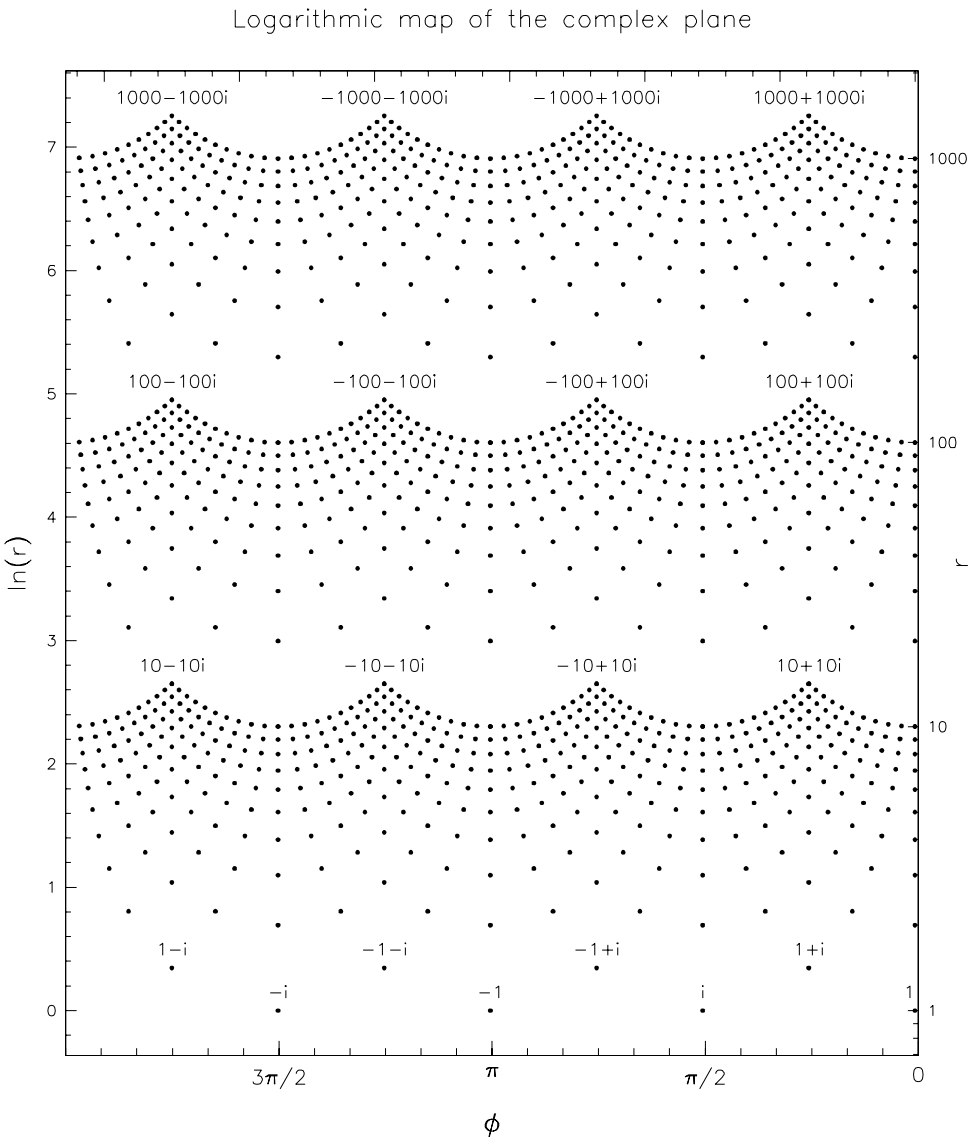

Fig. 6.— Logarithmic map of the complex plane.

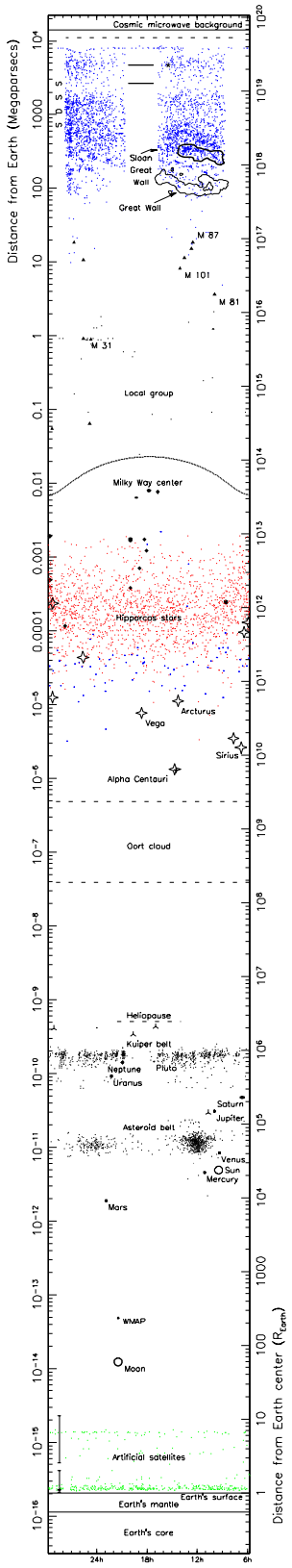

Fig. 7.— Pocket map of the universe

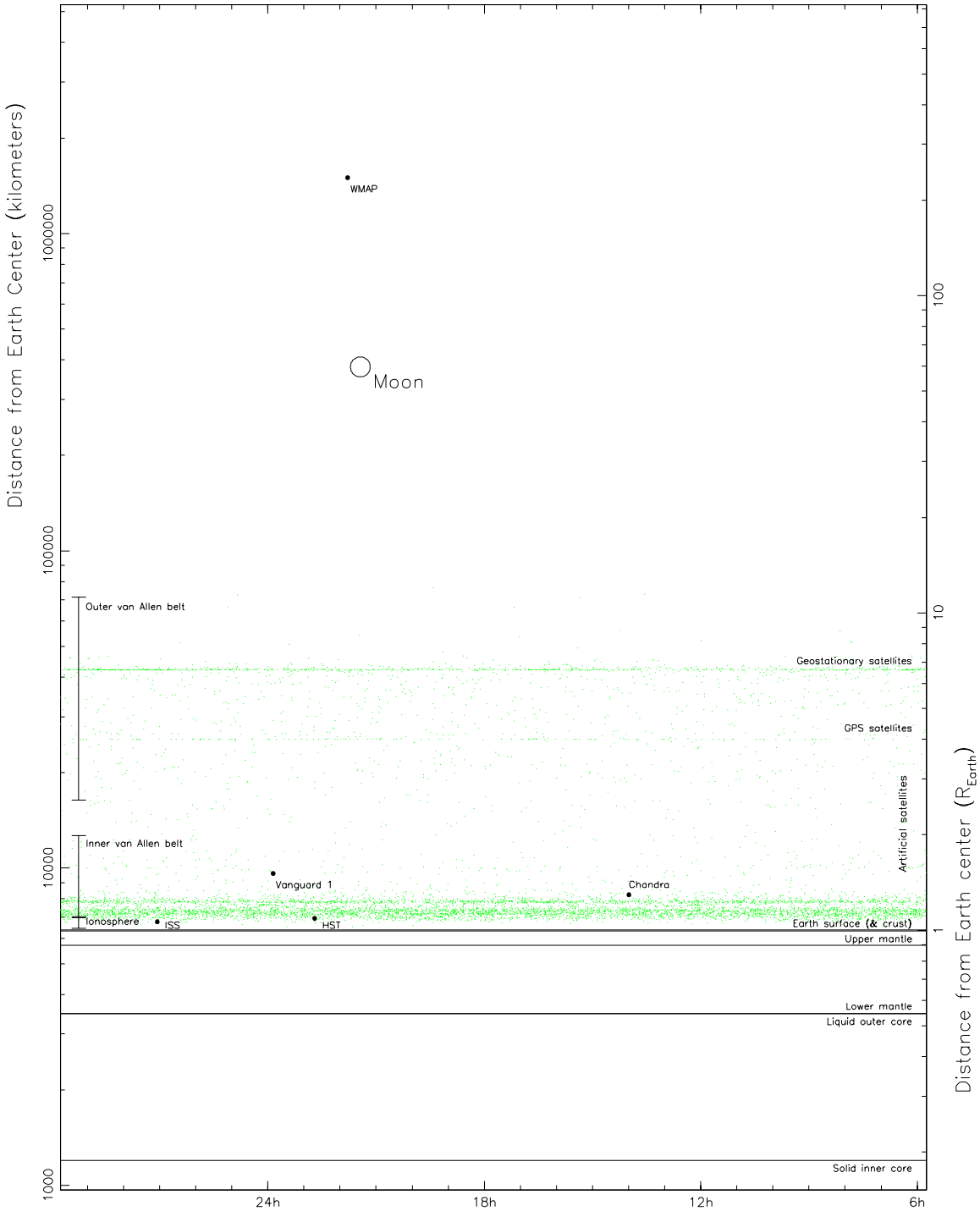

Fig. 8a.— Map of the Universe 1/6 - Near Earth Space. (Figures 8a-8f are available in 1 inch/radian scale as separate files on astro-ph.)

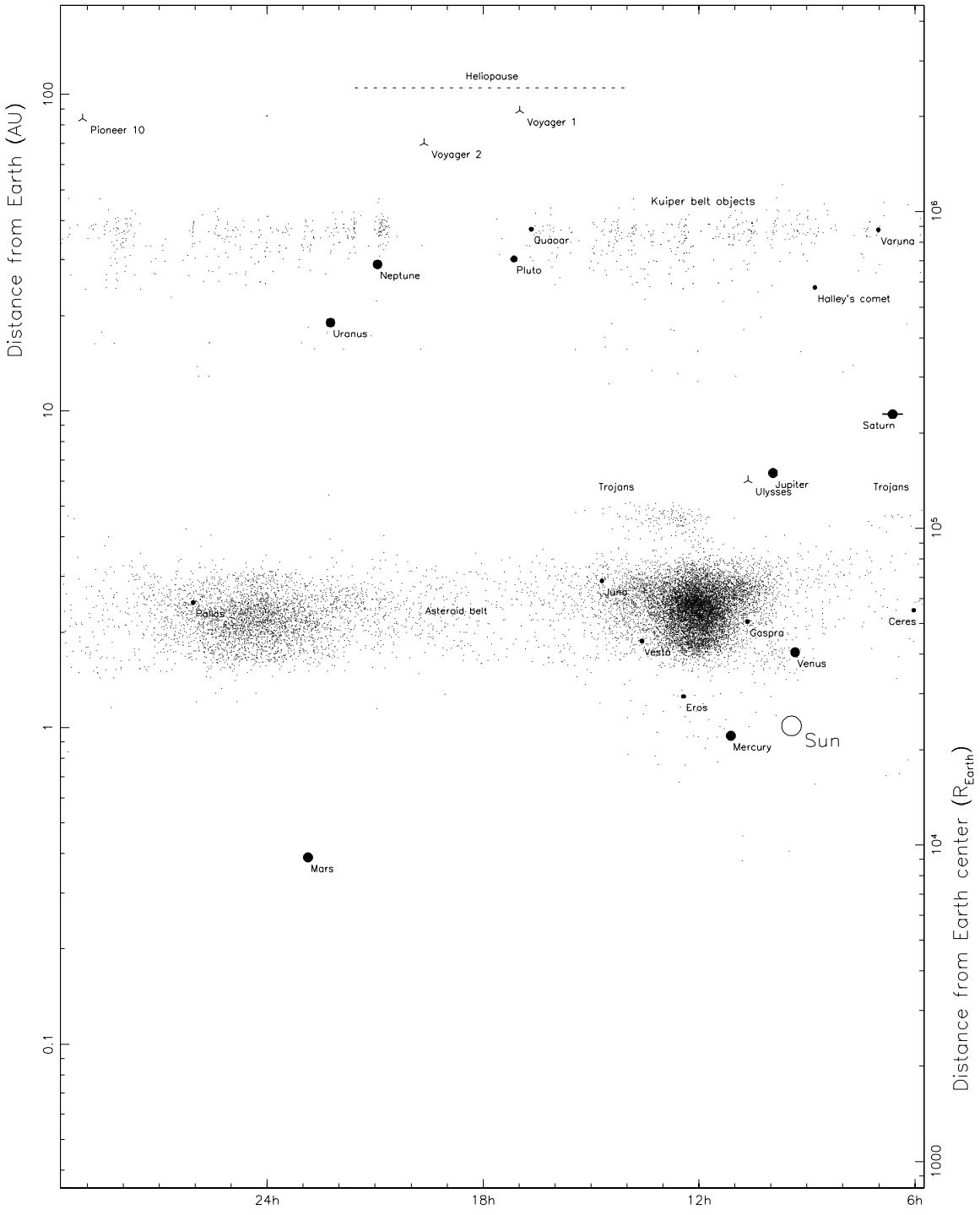

Fig. 8b.— Map of the Universe 2/6 - The Solar System

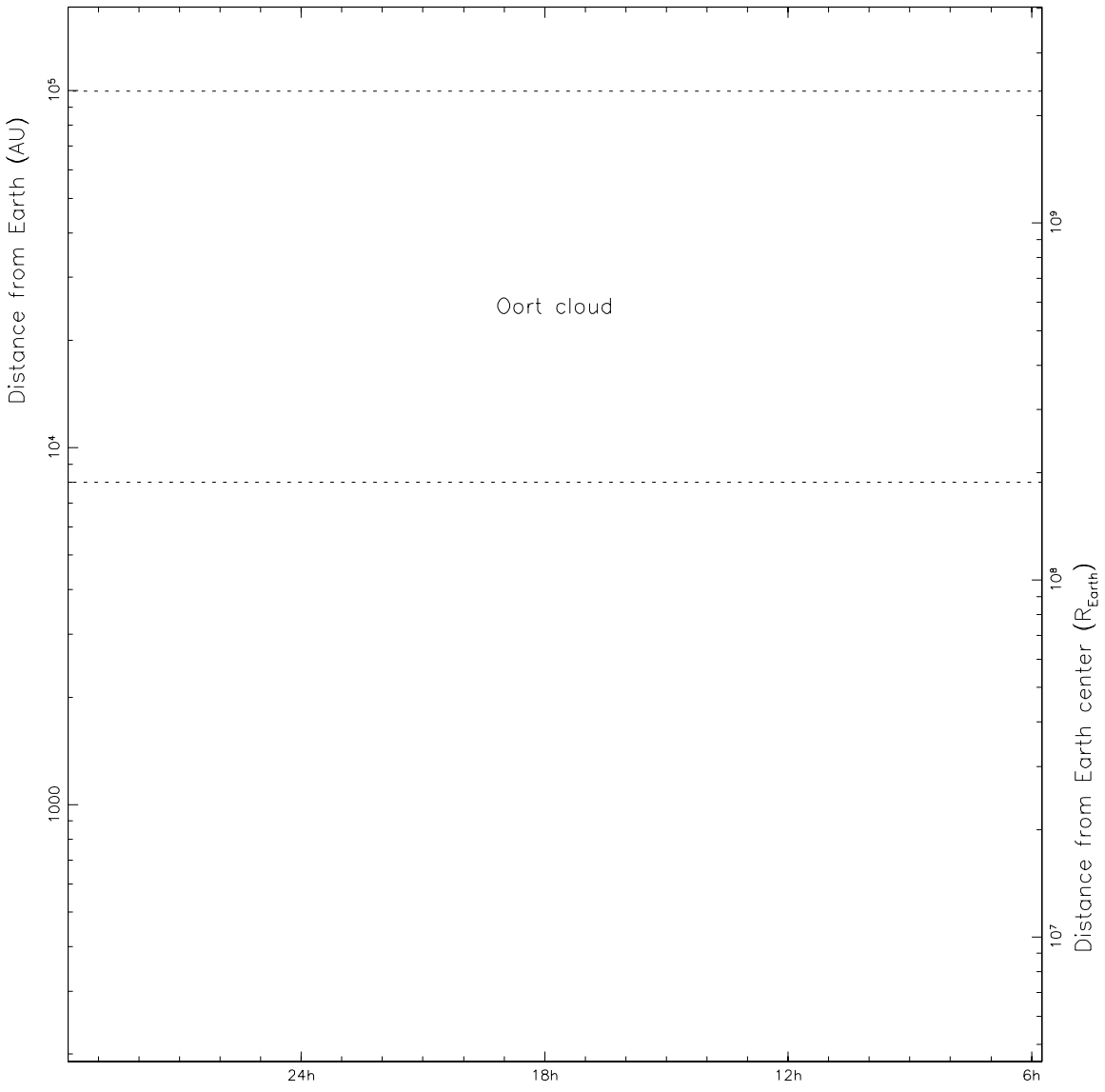

Fig. 8c.— Map of the Universe 3/6 - The Oort Cloud

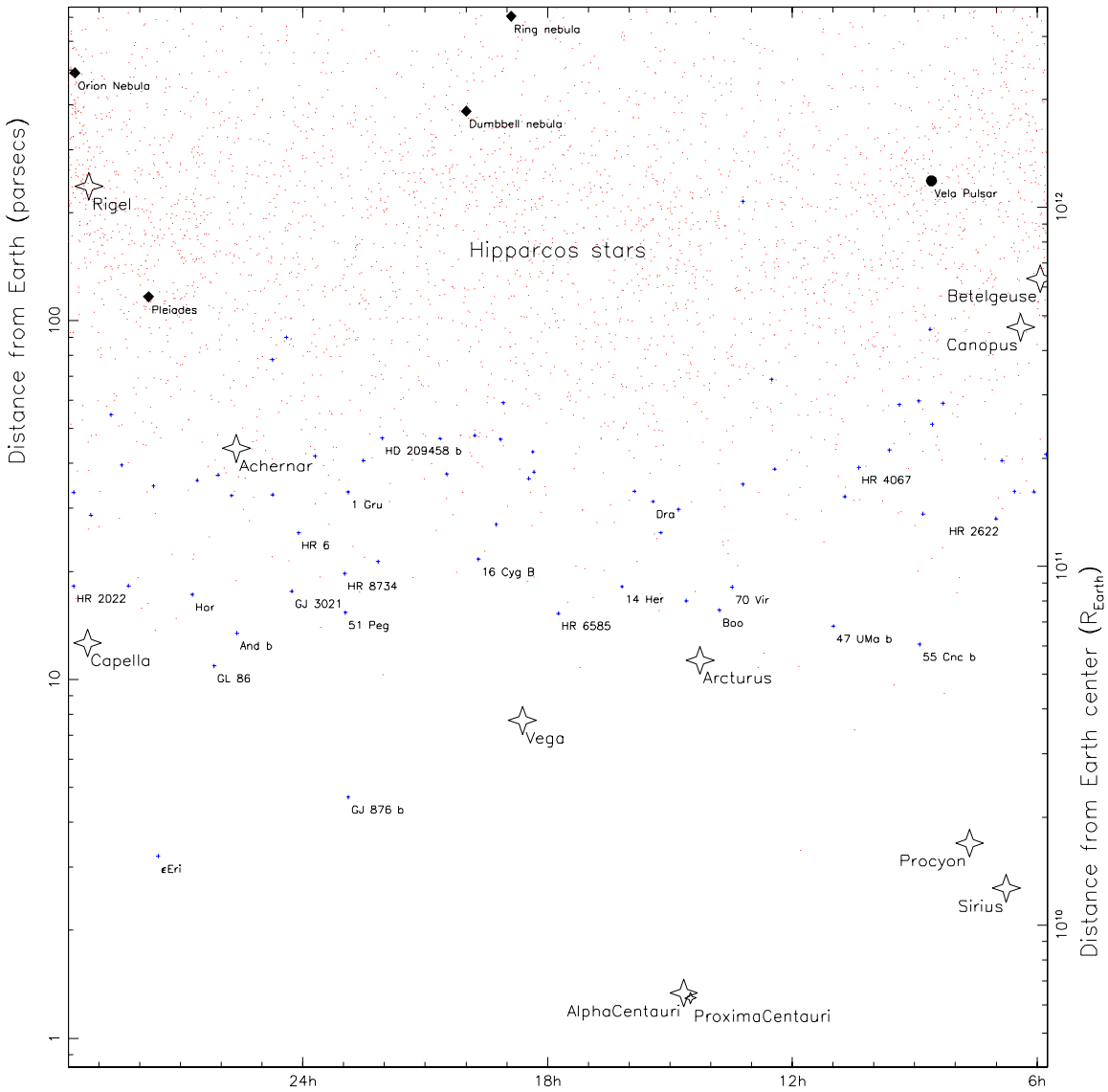

Fig. 8d.— Map of the Universe 4/6 - Stars in the Milky Way

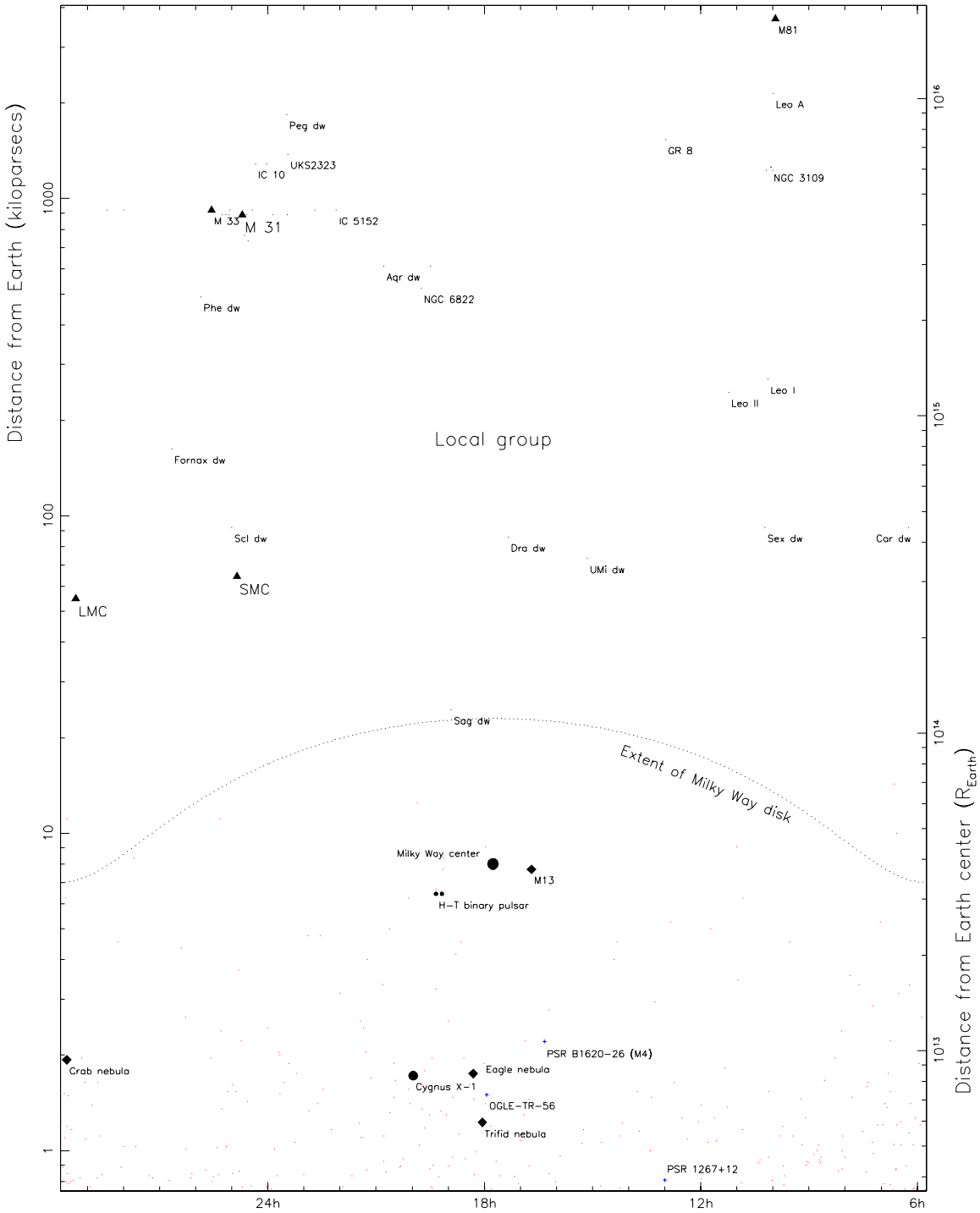

Fig. 8e.— Map of the Universe 5/6 - The Local Group

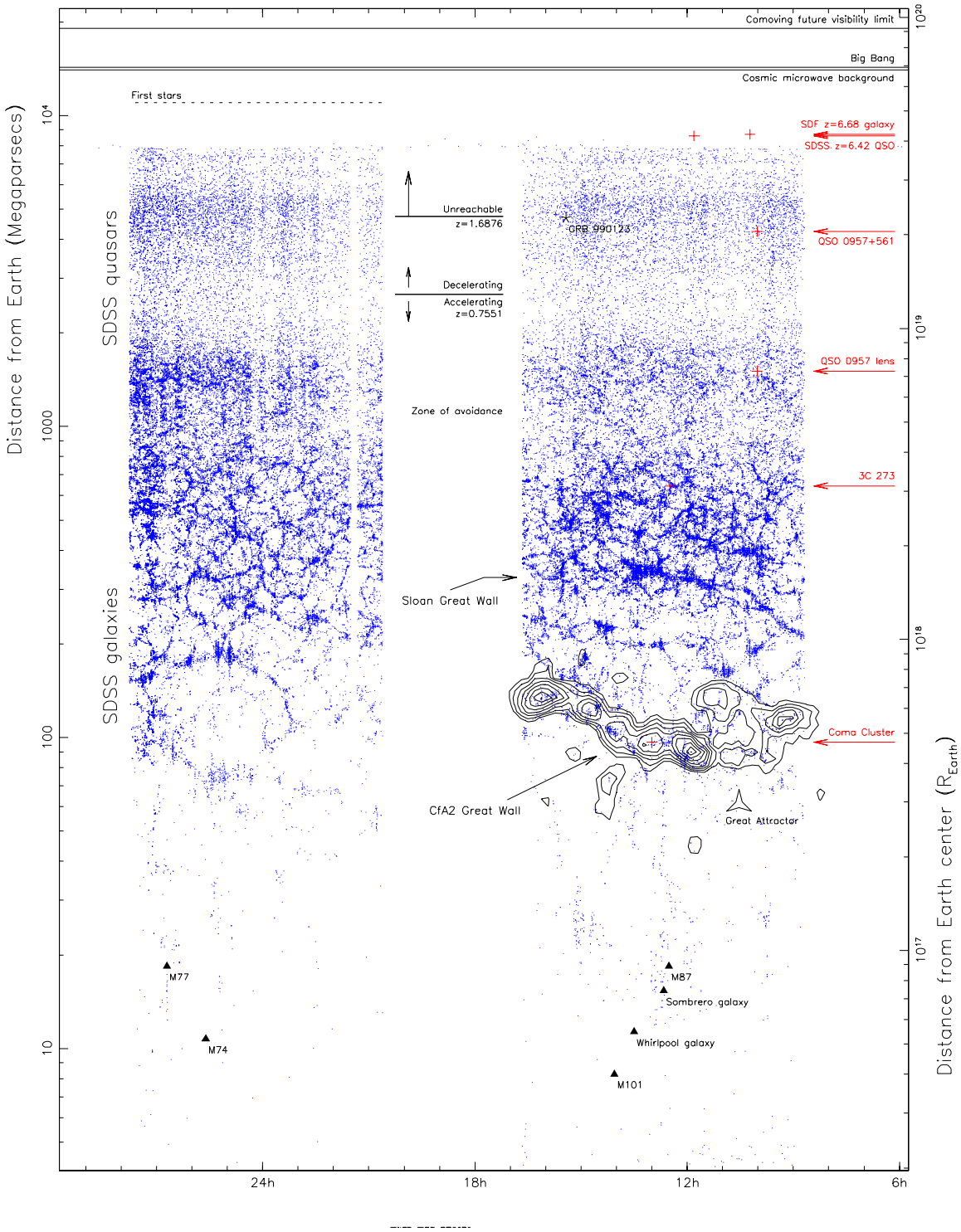

Fig. 8f.— Map of the Universe 6/6 - The Large Scale Structure

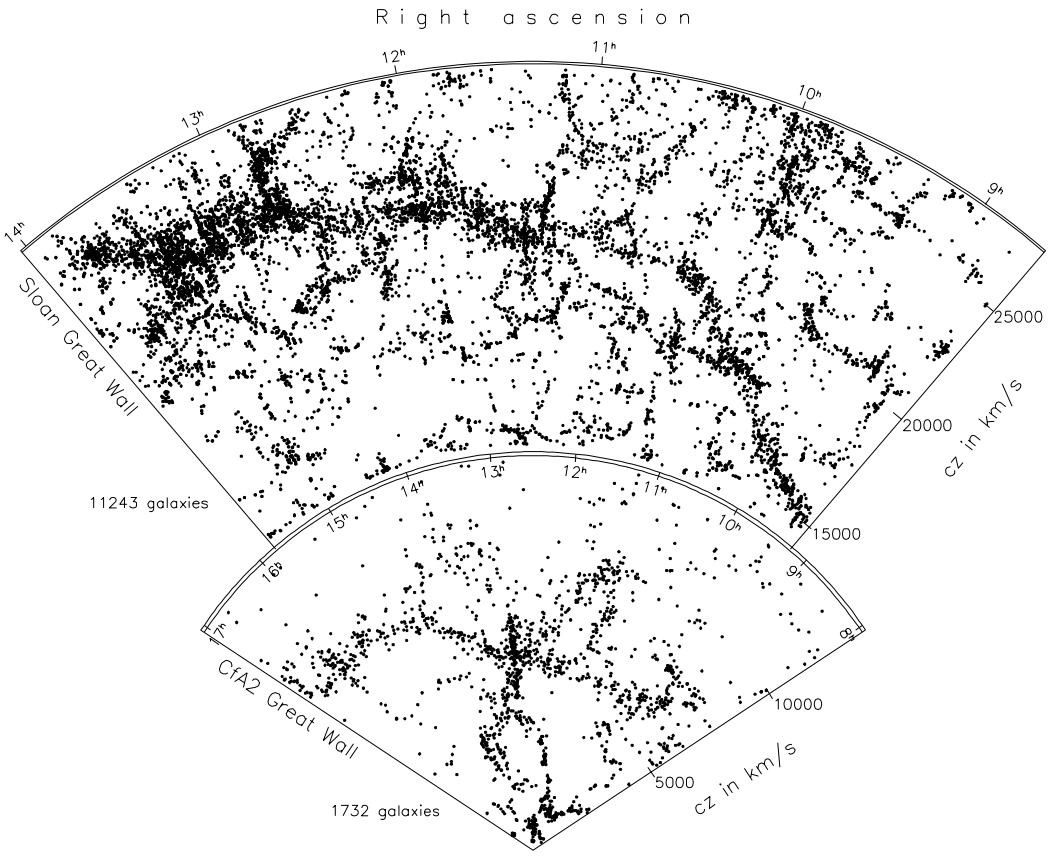

Fig. 9.— Sloan Great Wall compared to CfA2 Great Wall at the same scale in co-moving coordinates. Equivalent redshift distances *cz* are indicated. The Sloan slice is 4° wide, the CfA2 slice is 12° wide to make both slices approximately the same physical width at the two walls. The Sloan Great Wall extends from 14h to 9h. It consists of one strand at the left, which divides to form two strands between 11.3h and 9.8h, which come back together to fom one strand again (like a road that becomes a divided highway for a while). The CfA2 Great Wall (which includes the Coma cluster in the center), has been plotted on a cone and then flattened onto a plane. Total numbers of galaxies shown in each slice are also indicated.