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Chiral Suppression of Scalar Glueball Decay

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Abstract

Because glueballs are $SU(3)_{Flavor}$ singlets, they are expected to couple equally to u, d, and s quarks, so that equal coupling strengths to $\pi^+\pi^-$ and K^+K^- are predicted. However, we show that chiral symmetry implies the scalar glueball amplitude for $G_0 \to \overline{q}q$ is proportional to the quark mass, so that mixing with $\overline{s}s$ mesons is enhanced and decays to K^+K^- are favored over $\pi^+\pi^-$. Together with evidence from lattice calculations and from experiment, this supports the hypothesis that $f_0(1710)$ is the ground state scalar glueball.

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1. Introduction. — The existence of gluonic states is a quintessential prediction of Quantum Chromodynamics (QCD). The key difference between Quantum Electrodynamics (QED) and QCD is that gluons carry color charge while photons are electrically neutral. Gluon pairs can then form color singlet hadronic bound states, "glueballs," like mesons and baryons, which are color singlet bound states of valence quarks.[1] Because of formidable experimental and theoretical difficulties, it is frustrating, though not surprising, that this simple, dramatic prediction has resisted experimental verification for more than two decades. Quenched lattice simulations predict that the mass of the lightest glueball, G_0, a scalar, is near $\simeq 1.65$ GeV,[2] but the prediction is complicated by mixing with $\overline{q}q$ mesons that require more powerful computations. Experimentally the outstanding difficulty is that glueballs are not easily distinguished from ordinary $\overline{q}q$ mesons, themselves imperfectly understood. This difficulty is also exacerbated if mixing is appreciable.

The most robust identification criterion, necessary but not sufficient, is that glueballs are extra states, beyond those of the $\overline{q}q$ meson spectrum. This is difficult to apply in practice, though ultimately essential. In addition, glueballs are expected to be copiously produced in gluon rich channels such as radiative J/ψ decay, and to have small two photon decay widths. These two expectations are encapsulated in the quantitative measure "stickiness,"[3] which characterizes the relative strength of gluonic versus photonic couplings.

Another popular criterion is based on the fact that glueballs are $SU(3)_{\text{Flavor}}$ singlets which should then couple equally to different flavors of quarks. However we show here that the amplitude for the decay of the ground state scalar glueball to quark-antiquark is proportional to the quark mass, $\mathcal{M}(G_0 \to \overline{q}q) \propto m_q$, so that decays to $\overline{s}s$ pairs are greatly enhanced over $\overline{u}u + \overline{d}d$, and mixing with $\overline{s}s$ mesons is enhanced relative to $\overline{u}u + \overline{d}d$. We exhibit the result at leading order and show that it holds to all orders in standard QCD perturbation theory.

The result has a simple nonperturbative physical explanation, similar, though different in detail, to the well known enhancement of $\pi \to \mu\nu$ relative to $\pi \to e\nu$. For $m_q = 0$ chiral symmetry requires the final q and $\overline{q}$ to have equal chirality, hence unequal helicity, so that in the G_0 rest frame with z axis in the quark direction of motion, the total z component of spin is nonvanishing, $|S_Z| = 1$. Because the ground state G_0 gg wave function is isotropic ($L = S = 0$), the $\overline{q}q$ final state is pure s-wave,[2] $L = 0$. The total angular momentum is zero, and since there is no way to cancel the nonvanishing spin contribution, the amplitude vanishes. With one power of $m_q \neq 0$, the q and $\overline{q}$ have unequal chirality and the amplitude is allowed.

The enhancement is substantial, since m_s is an order of magnitude larger than m_u and

[2]Integrating over the gluon direction to project out the s-wave gg wave function is equivalent to integrating over the final quark direction with the initial gluon direction fixed.

m_d.[4] But for scalar glueballs of mass $\simeq 1.5 - 2$ GeV, $\Gamma(G_0 \to \overline{s}s)$, is suppressed of order $(m_s/m_{G_0})^2$, so that it may be smaller than the nominally higher order $G_0 \to \overline{q}qg$ process, which *is* $SU(3)_{\rm Flavor}$ symmetric. We find that the soft and collinear quark-gluon singularities of $G_0 \to \overline{q}qg$ vanish for $m_q = 0$, as they must if $G_0 \to \overline{q}q$ is to vanish at one loop order for $m_q = 0$. Unsuppressed, flavor-symmetric $G_0 \to \overline{q}qg$ decays are dominated by configurations in which the gluon is well separated from the quarks, which hadronize predominantly to multi-body final states. The enhancement of $\overline{s}s$ relative to $\overline{u}u + \overline{d}d$ is then most strongly reflected in two body decays: we expect K^+K^- to be enhanced relative to $\pi^+\pi^-$, while multibody decays are more nearly flavor symmetric.

Glueball decay to light quarks cannot be computed reliably in any fixed order of perturbation theory. However, the predicted ratio, $\Gamma(G_0 \to \overline{s}s)/\Gamma(G_0 \to \overline{u}u + \overline{d}d) \gg 1$, is credible since it follows from an analysis to all orders in perturbation theory and, in addition, from a physical argument that does not depend on perturbation theory. The implication that $\Gamma(G_0 \to K^+K^-) \gg \Gamma(G_0 \to \pi^+\pi^-)$ is less secure and is best studied on the lattice. Remarkably, it is supported by an early quenched study of G_0 decay to pseudoscalar meson pairs for two "relatively heavy" $SU(3)_{\rm Flavor}$ symmetric values of m_q, corresponding to $m_{\rm PS} \simeq 400$ and $\simeq 630$ MeV.[5] Linear dependence on m_q implies quadratic dependence on $m_{\rm PS}$,[6] which is consistent at 1σ with the lattice computations.[5] Chiral suppression could then be the physical basis for the unexpected and unexplained lattice result. With subsequent computational and theoretical advances in lattice QCD, it should be possible today to verify the earlier study and to extend it to smaller values of m_{PS}, nearer to the chiral limit and to the physical pion mass. If the explanation is indeed chiral suppression, then the couplings of higher spin glueballs should be approximately flavor symmetric and independent of $m_{\rm PS}$, a prediction which can also be tested on the lattice.

Enhanced strange quark decay changes the expected experimental signature and supports the hypothesis that $f_0(1710)$ is predominantly the ground state scalar glueball. This identification was advocated by Sexton, Vaccarino, and Weingarten,[5] and is even more compelling today in view of recent results from J/ψ decay obtained by BES[7, 8] — see [9] for an overview of the experimental situation.

In section 2 we compute $\mathcal{M}(G_0 \to \overline{q}q)$ at leading order for massive quarks, with the expected linear dependence on m_q. In section 3 we show that $\mathcal{M}(G_0 \to \overline{q}q) \propto m_q$ to any order in α_S. In section 4 we describe the infrared singularities of $\mathcal{M}(G_0 \to \overline{q}qg)$. We conclude with a brief discussion, including experimental implications.

2. $G_0 \to \overline{q}q$ at leading order. — Consider the decay of a scalar glueball G_0 with mass M_G to a $\overline{q}q$ pair[3] with quark mass m_q. The effective glueball-gluon-gluon coupling is

[3]Elastic scattering, $gg \to gg$, contributes to the glueball wave function, not to the decay amplitude.

parameterized by

$$\mathcal{L}_{eff} = f_0 G_0 G_{a\mu\nu} G_a^{\mu\nu} \tag{1}$$

where G_0 is an interpolating field for the glueball, $G_{a\mu\nu}$ is the gluon field strength tensor with color index a, and f_0 is an effective coupling constant with dimension $1/M$ that depends on the G_0 wave function. The $gg \to \overline{q}q$ scattering amplitude can be written as

$$\mathcal{M}(gg \to \overline{q}q) = \epsilon_{1\mu}\epsilon_{2\nu}\mathcal{M}^{\mu\nu}(gg \to \overline{q}q) \tag{2}$$

where $\epsilon_{i\mu} = \epsilon_\mu(p_i, \lambda_i)$ with $i = 1, 2$ are the polarization vectors for massless constituent gluons with four momentum p_i and polarization λ_i. Using equations (1) and (2), summing over the polarizations λ_i, and averaging over the gluon direction in the G_0 rest frame to project out the s-wave, we obtain

$$\mathcal{M}(G_0 \to \overline{q}q) = \frac{f_0}{4\pi}\int d\Omega X^{\mu\nu}\mathcal{M}_{\mu\nu}(gg \to \overline{q}q), \tag{3}$$

where $X^{\mu\nu} = 2p_2^\mu p_1^\nu - M_G^2 g^{\mu\nu}$ projects out the $|(++) \ + \ (--)>$ helicity state that couples to $G_{a\mu\nu}G_a^{\mu\nu}$ in equation (1).

From the lowest order Feynman diagrams we obtain

$$X^{\mu\nu}\mathcal{M}_{\mu\nu} = -\frac{32\pi\sqrt{2}\alpha_S}{3}\ \frac{m_q}{1-\beta^2\cos^2\theta}\ \overline{u}(p_3, h_3)v(p_4, h_4)\delta_{ij} \tag{4}$$

where u_3, v_4 are the $q, \overline{q}$ spinors for quark and antiquark with center of mass momenta p_3, p_4, helicities h_3, h_4, color indices i, j and center of mass velocity β. Equation (4) includes a color factor from the color singlet gg wave function,

$$C_{ij} = \frac{\delta_{a,b}}{\sqrt{8}}\frac{\lambda_{ik}^a}{2}\frac{\lambda_{kj}^b}{2} = \frac{\sqrt{2}}{3}\delta_{ij} \tag{5}$$

Performing the angular integration, the decay amplitude is

$$\mathcal{M}(G_0 \to \overline{q}q) = -f_0\alpha_S\ \frac{16\pi\sqrt{2}}{3}\ \frac{m_q}{\beta}\ \log\frac{1+\beta}{1-\beta}\ \overline{u}_3 v_4 \delta_{ij}. \tag{6}$$

Squaring equation (6), summing over quark helicities and color indices, and performing the phase space integration, the decay width is

$$\Gamma(G_0 \to \overline{q}q) = \frac{16\pi}{3}\ \alpha_S^2 f_0^2 m_q^2 M_G \beta\ \log^2\frac{1+\beta}{1-\beta}. \tag{7}$$

The dissociation process, $gg \to \overline{q}q + \overline{q}q$, in which each gluon makes a transistion to a color-octet $\overline{q}q$ pair, is kinematically forbidden for $m_q \neq 0$; an additional gluon exchange is required to allow it to proceed on-mass-shell, which is therefore of order g^4 in the amplitude.

Notice that an explicit factor m_q appears in the $gg \to \overline{q}q$ amplitude, equation (4), which is not averaged over the initial gluon direction and which clearly has contributions from higher partial waves, $J > 0$. It may then appear that chiral suppression applies not just to spin 0 glueballs but also to glueballs of higher spin. However when equation (4) is squared and the phase space integration is performed, a factor $1/m_q^2$ results from the t and u channel poles, which cancels the explicit factor m_q^2 in the numerator, so that the total cross section $\sigma(gg \to \overline{q}q)$ does not vanish in the chiral limit, because of the $J > 0$ partial waves.

3. $G_0 \to \overline{q}q$ to all orders. — We now show that $\mathcal{M}(G_0 \to \overline{q}q)$ vanishes to all orders in perturbation theory for $m_q = 0$. Consider the Lorentz invariant amplitude

$$\mathcal{M}_X(p_1, p_2, p_3, p_4) = X^{\mu\nu}\mathcal{M}_{\mu\nu} \tag{8}$$

where $\mathcal{M}_{\mu\nu}$ is defined in equation (2) and $X^{\mu\nu}$ below (3). The perturbative expansion for $\mathcal{M}_X$ is a sum of terms arising from Feynman diagrams with arbitrary numbers of loops. After evaluation of the loop integrals, regularized as necesary, $\mathcal{M}_X$ is a sum of terms,

$$\mathcal{M}_X = \sum_i \overline{u}(p_3, \chi_3)\Gamma_i u(p_4, \chi_4), \tag{9}$$

where u_3, u_4 are respectively massless fermion and antifermion spinors[10] of chirality χ_3, χ_4. The Γ_i are 4×4 matrices, each a product of n_i momentum-contracted Dirac matrices,

$$\Gamma_i = \not{k}_{i1}\ \not{k}_{i2} \ldots \not{k}_{in_i} \tag{10}$$

where each k_{ia} is one of the external four-momenta, p_1, p_2, p_3, p_4.

Chiral invariance for $m_q = 0$ implies that the number of factors, n_i, in equation (10) is always odd. Since all external momenta vanish and the spinors obey $\not{p}_3 u_3 = \not{p}_4 u_4 = 0$, by suitably anticommuting the $\not{k}_{ia}$, each term in equation (9) can be reduced to a sum of terms linear in $\not{p}_1$ and $\not{p}_2$, which we choose to be symmetric and antisymmetric,

$$\overline{u}(p_3, \chi_3)\Gamma_i u(p_4, \chi_4) = \overline{u}(p_3, \chi_3)[S_i(s,t,u)(\not{p}_1 + \not{p}_2) + A_i(s,t,u)(\not{p}_1 - \not{p}_2)]u(p_4, \chi_4). \tag{11}$$

The coefficients A_i, S_i are Lorentz invariant functions of the Mandelstam variables s, t, u. Since $p_1 + p_2 = p_3 + p_4$, the symmetric term vanishes and equation (9) reduces to

$$\mathcal{M}_X = A(s,t,u)\ \overline{u}(p_3, \chi_3)(\not{p}_1 - \not{p}_2)u(p_4, \chi_4). \tag{12}$$

where $A(s,t,u) = \sum_i A_i(s,t,u)$.

Next consider the integration over the gluon direction, equation (3). In the G_0 rest frame with the z-axis chosen along the quark direction of motion, $\hat{z} = \hat{p}_3$, we integrate over $d\Omega = d^2\hat{p}_1 = d\phi_1 d\cos\theta_1$. The Mandelstam variables are then $s = M_G^2$ and $u, t =$

$-\frac{1}{2}M_G^2(1 \pm \cos\theta_1)$. Since the color and helicity components of the G_0 wave function are symmetric under interchange of the two gluons, Bose symmetry requires $A(s,t,u)$ to be odd under $p_1 \leftrightarrow p_2$. In our chosen coordinate system A is a function only of $\cos\theta_1$, and must therefore be odd, $A(-\cos\theta_1) = -A(\cos\theta_1)$. But evaluating $\overline{u}_3(\not{p}_1 - \not{p}_2)u_4$ explicitly[10] we find

$$\overline{u}_3(\not{p}_1 - \not{p}_2)u_4 = M_G^2\, e^{-i\phi_1}\, \sin\theta_1 \tag{13}$$

which is *even* in $\cos\theta_1$, while the azimuthal factor, $e^{-i\phi_1}$, provides the required oddness under $p_1 \leftrightarrow p_2$: $e^{-i\phi_1} \to e^{-i(\phi_1+\pi)} = -e^{-i\phi_1}$. Consequently the integral $\int d\cos\theta_1 \mathcal{A}$ vanishes, and $\mathcal{M}(G_0 \to \overline{q}q) = 0$ to all orders in the chiral limit. In fact, because of our choice of axis, $\hat{z} = \hat{p}_3$, the integral over ϕ_1 also vanishes. For other choices of $\hat{z}$ the azimuthal and polar integrals do not vanish separately, but the full angular integral, $\int d^2\hat{p}_1$, vanishes in any case.

For nonvanishing quark mass, $m_q \neq 0$, chirality-flip amplitudes contribute. With one factor of m_q from the fermion line connecting the external quark and antiquark, the Γ_i matrices in equation (9) include products of even numbers of Dirac matrices, i.e., n_i in equation (10) may be even. Beginning in order m_q there are then nonvanishing contributions to $\mathcal{M}(G_0 \to \overline{q}q)$, like the leading order term shown explicitly in equation (6).

The vanishing azimuthal integration for $\hat{z} = \hat{p}_3$ reflects the physical argument given in the introduction. The factor $e^{-i\phi_1}$ corresponds to $S_Z = 1$ from the aligned spins of the q and $\overline{q}$, while the absence of a compensating factor in $\mathcal{A}$ is due to the projection of the orbital s-wave by the $\int d^2\hat{p}_1$ integration and the absence of spin-polarization in the initial state.

4. Infrared singularities of $G_0 \to \overline{q}qg$. — Although it is of order α_S^3, $\Gamma(G_0 \to \overline{q}qg)$ is not chirally suppressed and may therefore be larger than $\Gamma(G_0 \to \overline{s}s)$, which is of order $\alpha_S^2 \times m_s^2/m_G^2$. Setting $m_q = 0$ we evaluated the 13 Feynman diagrams using the helicity spinor method[10] with numerical evaluation of the 9 dimensional integral:

$$\begin{aligned}\Gamma(G_0 \to \overline{q}qg) &= \textstyle\sum_{h_3,h_4,\lambda_5} \int_{\rm PS} |\mathcal{M}(G_0 \to \overline{q}_3 q_4 g_5)|^2 \\ &= \tfrac{f_0^2}{16\pi^2} \textstyle\sum_{h_3,h_4,\lambda_5} \int_{\rm PS} \int d\Omega_1 \int d\Omega_1'\ \epsilon^*_{5\alpha} X_{\mu\nu} \mathcal{M}^{\mu\nu\alpha}(g_1 g_2 \to \overline{q}_4 q_3 g_5) \\ &\qquad \times \epsilon_{5\beta} X'_{\sigma\rho} \mathcal{M}^{\sigma\rho\beta}(g_1' g_2' \to \overline{q}_4 q_3 g_5)^*.\end{aligned} \tag{14}$$

Details will be presented elsewhere.[11] We focus here on the infrared singularities, which provide a consistency check at one loop order that $\mathcal{M}(G_0 \to \overline{q}q)$ vanishes in the chiral limit.

In general there could be soft IR divergences for $E_q, E_{\overline{q}}, E_g \to 0$ and collinear divergences for $\theta_{qg}, \theta_{\overline{q}g}, \theta_{\overline{q}q} \to 0$. In fact, only the $\overline{q}q$ collinear divergence occurs, as can be seen from the distributions in figure 1, obtained by imposing only the cut $\theta_{\overline{q}q} > 0.1$ in the G_0 rest frame: neither dN/dE_g nor dN/dE_q diverge at low energy, and only $dN/\theta_{\overline{q}q}$ diverges at $\theta_{\overline{q}q} \to 0$. Instead dN/dE_g diverges at the *maximum* energy, $E_g = m_{G_0}/2$, and $dN/d\theta_{qg}$ diverges for $\theta_{qg} \to \pi$. Both of these divergences are kinematical reflections of the collinear singularity at

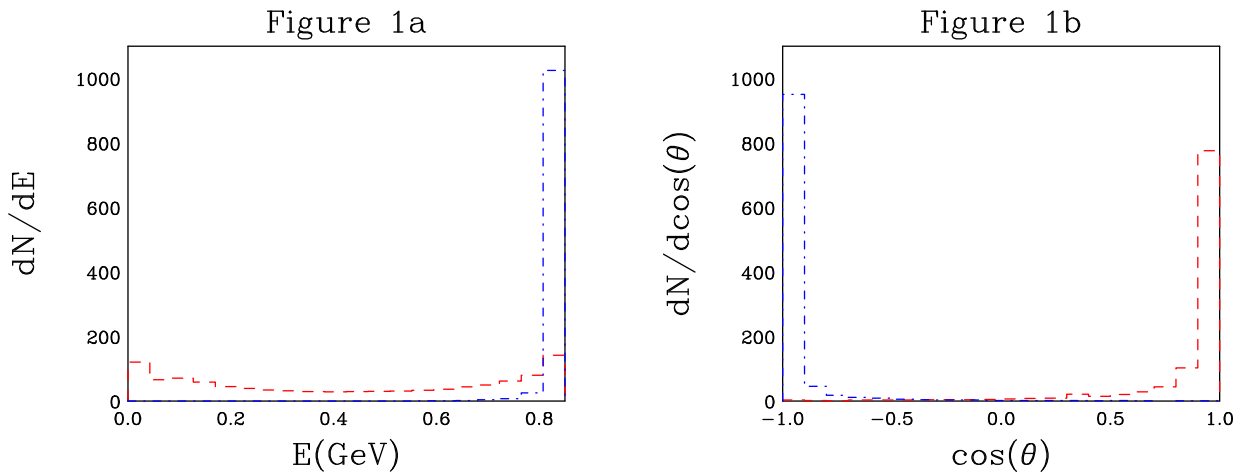

Figure 1: Distributions for $G_0 \to \overline{q}qg$ in arbitrary units. In figure (1a) the dot-dashed line is dN/dE_g and the dashed line is dN/dE_q. In figure (1b) the dot-dashed line is $dN/\cos\theta_{qg}$ and the dashed line is $dN/d\cos\theta_{\overline{q}q}$.

$\theta_{\overline{q}q} \to 0$, for which the $\overline{q}q$ pair with $m_{\overline{q}q} = 0$ recoils with half of the available energy against the gluon in the opposite hemisphere.

This is precisely the pattern of divergences required if $G_0 \to \overline{q}q$ is chirally suppressed to all orders and, in particular, at one loop. For if there were soft divergences in any of $E_q, E_{\overline{q}}, E_g$ or collinear divergences in θ_{qg} and $\theta_{\overline{q}g}$, then the resulting singularities at $m_{qg}, m_{\overline{q}g} \to 0$ would have to be cancelled by virtual corrections to $G_0 \to \overline{q}q$, such as gluon self energy contributions to the quark propagator. The absence of these singularities is a consistency check (i.e., a necessary condition) that $G_0 \to \overline{q}q$ is chirally suppressed at one loop order. The collinear divergence for $\theta_{\overline{q}q} \to 0$ is cancelled by quark loop contributions to the $gg \to gg$ amplitude, which in the present context are one loop corrections to the G_0 wave function.

5. Discussion. — We have shown to all orders in perturbation theory and with a simple, nonperturbative physical argument that the ground state $J = 0$ glueball has a chirally suppressed coupling to light quarks, $\mathcal{M}(G_0 \to \overline{q}q) \propto m_q$, with corrections of higher order in m_q/m_G. From equation (7) with m_u, m_d, m_s varied within 1σ limits,[4] $\Gamma(G_0 \to \overline{s}s)$ dominates $\Gamma(G_0 \to \overline{u}u + \overline{d}d)$ by a factor between 20 and 100. Flavor symmetry is reinstated for $G_0 \to \overline{q}qg$ when the gluon is well separated from the q and $\overline{q}$. For sufficiently heavy m_G one can test this picture by measuring strangeness yield as a function of thrust or sphericity, with enhanced strangeness in high thrust or low sphericity events, but it is unclear if this is feasible for $m_G \simeq 1.7$ GeV. It is more feasible for the ground state pseudoscalar glueball, which is expected to be heavier than the scalar and which we also expect to be subject to chiral suppression.

For light scalar glueballs, the best hope to see strangeness enhancement is the two

body decays, $G_0 \to K^+K^-/\pi^+\pi^-$. Since $G_0 \to \overline{q}qg$ is not chirally suppressed, naive power counting suggests $\Gamma(G_0 \to \overline{q}qg) \geq \Gamma(G_0 \to \overline{s}s)$, so that $\Gamma(G_0 \to \overline{q}qg)$ is probably the dominant mechanism for multiparticle production. Then $\overline{K}K$ will dominate two body decays while multiparticle final states are approximately $SU(3)_{\rm Flavor}$ symmetric, up to phase space corrections favoring nonstrange final states.

Chiral suppression has a major impact on the experimental search for the ground state scalar glueball. Candidates cannot be ruled out because they decay preferentially to strange final states, especially $\overline{K}K$, and mixing with $\overline{s}s$ mesons may be enhanced. This picture of a chirally suppressed G_0 fits nicely with the known properties of the $f_0(1710)$ meson. It is copiously produced in radiative ψ decay in the $\psi \to \gamma\overline{K}K$ channel[7] and in the gluon-rich central rapidity region in pp scattering,[12] has a small $\gamma\gamma$ coupling,[13] has a mass consistent with the prediction of quenched lattice QCD,[2] and has a strong preference to decay to $\overline{K}K$, with $B(\pi\pi)/B(\overline{K}K) < 0.11$ at 95% CL.[8] As a rough estimate of the stickiness,[3] we combine the $\gamma\gamma$ 95% CL upper limit with central values for ψ radiative decay[7], with the result $S(f_0(1710)) : S(f_2'(1525)) : S(f_2(1270)) \simeq (> 36) : 12 : 1$. A more complete discussion of the experimental situation will be given elsewhere[11] — see also [9].

The interpretation of $f_0(1710)$ as the chirally suppressed scalar glueball can be tested both theoretically and experimentally. Lattice QCD can test the prediction that $G_0 \to \overline{K}K$ is enhanced for the ground state $J = 0$ glueball but not for $J > 0$. With an order of magnitude more J/ψ decays than BES II, experiments at BES III and CESR-C will extend partial wave analysis to rarer two body decays and to multiparticle decays. They could confirm $B(f_0 \to \overline{K}K)/B(f_0 \to \pi\pi) \gg 1$ and, if, as is likely, the rate for multiparticle decays is big, the lower bound on $B(\psi \to \gamma f_0(1710)$ will increase beyond its already appreciable value from $\overline{K}K$ alone. A large inclusive rate $B(\psi \to \gamma f_0)$, a large ratio $B(f_0 \to \overline{K}K)/B(f_0 \to \pi\pi)$, and approximately flavor symmetric couplings to multiparticle final states would support the identification of $f_0(1710)$ as the chirally suppressed, ground state scalar glueball.

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