A COMPACT HIGH GRADIENT PULSED MAGNETIC QUADRUPOLE*

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Abstract

A design for a high gradient, low inductance pulsed quadrupole magnet is presented. The magnet is a circular current dominated design with a circular iron return yoke. Conductor angles are determined by a method of direct multipole elimination which theoretically eliminates the first four higher order multipole field components. Coils are fabricated from solid round film-insulated conductor, wound as a single layer "non-spiral bedstead" coil having a diagonal leadout entirely within one upturned end. The coils are wound and stretched straight in a special winder, then bent in simple fixtures to form the upturned ends.

GENERAL REQUIREMENTS

Heavy ion accelerators (drivers) for inertial confinement fusion (ICF), and near term high energy density physics experiments (HEDP) require the transport of space charge dominated high current ion beams during acceleration and final focusing on the target [1]. These space charge dominated beams require much higher beam focussing in both field strength and frequency (high lattice occupancy) than for non space charge dominated beams. Both quadrupole and solenoidal beam focussing schemes are feasible [2][3], and experiments are underway to determine optimum parameters for both [4][2]. For quadrupole focusing, electrostatic quadrupoles are most effective at energies below several MeV, for higher energies magnetic quadrupoles become effective. For ICF, side-by-side pole-connected magnet arrays [9] are needed. The high fields $\sim 3-6\,T$ required usually require current dominated magnets. Acceleration is typically provided by induction cores which surround the magnets; compact magnet designs significantly reduce induction core costs.

Although superconducting (SC) quadrupoles are preferred in a fusion driver, the pulsed nature of a fusion driver allows pulsed conventional magnets to be a feasible alternative in certain sections, such as in a final focus section [5] where high radiation exists, or in a low energy electrostatic to magnetic focussing transition section, where compactness is a primary criterion. They are also cost effective for proposed ICF experiments such as the High Current Transport Experiment (HCX), to which this effort is directed [2]. HCX requires a compact, low cost, high gradient quadrupole capable of single channel use. Fig. 1 shows a simplified model of the proposed quadrupole inside a vacuum shell (partially removed for clarity) designed to mount inside the bore of an induction cell (not shown).

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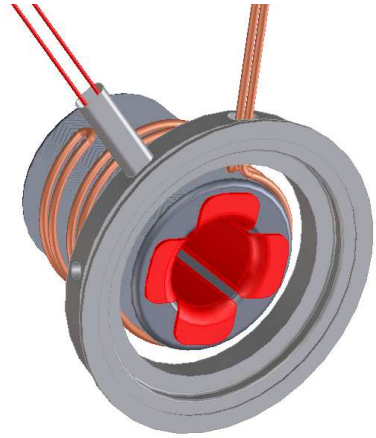

Figure 1: Pulsed Quadrupole Magnet for HCX-II

SPECIFIC REQUIREMENTS

HCX currently operates at a beam energy of 1 MeV Na^+ ions, and utilizes a magnetic transport FODO section of 4 quads (2 FODO cells). These quads, built for a different experiment, are longer than optimum, resulting in poor beam matching from the preceding short period electrostatic focus section. Beam current is limited to 0.2 A, at 10% beam loss. A short period magnetic focus section of the same period length of the electrostatic section would maximize the transport potential of HCX, with an expected 3x current improvement with $< 1\%$ beam loss, and an upgrade is planned (HCX-II), with this magnet design.

Table 1: HCX Pulsed Quadrupole Requirements

Parameter		Units
Beam Passage Time	8	μsec
Field Flattop Variation	0.1	%
Beam Stay Clear Radius	3.5	cm
Gradient Range	20-65	T/m
Good Field Radius	2.3	cm
2D Design Field Quality	< 5	units @ GFR
Magnetic Length	15	cm
Total Length	16.5	cm

MAGNETIC DESIGN

Pulsed Multipole Magnets

Although the above requirements could be met with an iron dominated quadrupole by using vanadium-permendur laminations, we desired a design scalable to higher gradi-

ents, and to be pole-connectable for use in an array. In quadrupole arrays, individual channel alignment is not feasible by moving the main coils. Current dominated designs allow the addition of independent steering (crossed dipoles plus a skew quad) and/or higher order multipole coils (pulsed or DC) inside the bore, e.g. with a flexible printed circuit material.

For pulsed magnets in this frequency range, the primary concern is insulation breakdown from high inductive voltages; reducing the magnet inductance is thus a primary design criterion. This means using as few turns as possible in each coil, and reducing electric field stress where possible, such as by using round conductors and radiused iron yokes. Reducing inductance allows shorter current pulses with less coil heating and allows smaller diameter conductors, with less radial buildout. For current dominated iron yoked magnets using standard electrical steel, peak field can be as high ~ 2.3T.

Direct Multipole Elimination

It is well known that a $\cos(n\theta)$ sheet current distibution on a circle (in free space, or on an unsaturated iron boundary) yields a pure multipole B field of $2n$ poles. However, finite numbers of discrete conductors arranged to approximate this $\cos(n\theta)$ current distribution are slow to converge to a pure single multipole field with increasing numbers of conductors. This is because all higher order field multipoles (HOMs) are reduced (essentially in a random fashion) by increasing the number of conductors approximating the $\cos(n\theta)$ current distribution, instead of only the lowest HOMs being reduced. Since the HOMs have a strong radial dependence ($\vec{B}_n \propto r^{n-1}$), it makes sense to reduce the lowest HOMs first.

The multipole coefficients of the B field from a single filamentary conductor at r_w , θ for $r > r_w$ are [6]:

$$B_n + iA_n = \frac{\mu_0 I}{2\pi r^n}(\cos(n\theta) + i\sin(n\theta))\ ; n = 1, 2, ...\infty$$

For a quadrupole, each angle represents an octuplet of conductors reflected about symmetry planes, and the nonzero multipole components (fundamental plus HOMs) produced by each octuplet are [6] :

$$B_n = \frac{4\mu_0 I}{\pi r^n}\cos(n\theta)\ ; n = 2, 6, ...\infty$$

For a multipole magnet composed of N filamentary conductors per unit sector (quadrant=dipole, octant=quadrupole, etc.) located at the same radius, conductor angles θ_i can be chosen to eliminate the first N allowed HOMs [7] [8]. For a quadrupole, the HOMs are $m = 6, 10...\infty$. This is done by solving N equations (in N unknowns) of the form:

$$\sum_{i=1,2,...}^{N} \cos(m\theta_i) = 0\ ; \text{for } m = 6, 10, ..., (4N+2) \quad (1)$$

These equations are easily solved, for example in MathCAD [11], using a Levenberg-Marquardt algorithm, starting with an initial guess; we have found solutions for up to 10 conductors. One can also choose to eliminate only $n < N$HOMs, giving at least one extra degree of freedom that could be used, e.g. to maximize conductor to conductor spacing, or some other criteria. Brady, [7] using an optimization method from J. Laszlett based on Lagrangian multipliers for producing a cos 2θ-like spacing, has produced solutions for the elimination of up to 10 HOMs (N=20) which, for $n < N$ eliminated HOMs, give well-spaced conductor angles. Brady's solutions for $n = N$ are given for $N < 5$ and match those given here. The design chosen here is the quadrupole equivalent of one such solution, for 5 conductors, with the first 4 HOMs reduced to a (theoretical) ppb level. Table 2 show some solutions for N conductor angles (deg.) that eliminate n lowest HOMs, with the last row showing the angles chosen for this design.

Table 2: Conductor angles (deg.)

N, n	θ_1	θ_2	θ_3	θ_4	θ_5
$1, 1$	15				
$2, 2$	6	24			
$3, 3$	5.835	13.468	28.028		
$4, 4$	0.429	12.429	17.571	30.429	
$4, 4$	$3/7$	$87/7$	$123/7$	$213/7$	
$5, 5$	2.865	10.703	17.549	27.947	43.566
$5, 4$	2.484	8.475	15.224	21.057	31.832

These angles can be doubled to give equivalent dipole conductor angles, or multiplied by 2/3 to give sextupole angles, etc. Note the rational 4 turn solution; the other solutions are also rational but with very large integers. Solutions for $N = 10$ are not unique, as two solutions have been found; it is not known if solutions for $N < 10$ are unique. Solutions for $N > 6$ typically have one conductor angle slightly greater than 45 degrees (pole axis); this is physically realizable as a negative current mirrored about the pole axis, which can be built into a real coil. Many solutions have conductors very close to the midplane (0 deg.) or pole axis (45 deg.) and are not possible to build with thick conductors. Pulsed magnets can potentially use smaller cross-section conductors, so these solutions are of value. Formula 1 can be generalized to include conductor position radius, e.g. as a function of angle so as to arrange conductors on an elliptical or rectangular boundary. Magnet array or iron boundaries can be simulated by adding dependent "reflected", or "image" conductors into the summations, respectively. Blocks of conductors can be simulated as the results are accurate for realistic finite dimensioned conductors; this greatly expands the number of possible solutions.

The physical accuracy of conductor placement limits the effectiveness of such HOM reduction. Taking the total

derivative of equation (1) for a single HOM:

$$\delta B_m = \frac{1}{N} \sum_{i=1,2,\ldots}^{N} m \sin(m\theta_i)\delta\theta_i \qquad (2)$$

To reach HOM levels below one unit at $r = 0.6r_w$ conductor centroid accuracy must be better than 0.01 deg. Indeed, these solutions may be of most utility for benchmarking field solver codes. Increasing the number of conductors relaxes this accuracy requirement, at the cost of higher inductance. And, of course, end effects (pseudo-multipoles) for short magnets are not negligible, though beyond the scope of this paper. The solver method can be used in conjunction with a field solver code in a "feedback mode" to compensate for saturation, eddy currents, etc. by solving for the (negative of the) multipoles which result from a first pass solution. Eddy currents in the conductors, though well below their maximum at the peak magnet current, have the effect of shifting the current centroid in both r and θ. In this design, the shift is less than 0.1mm (0.1 deg) and will be included in the final coil form dimensions, even though it is on the order of the mechanical tolerances.

In this design, the peak B field in the steel reachs as high as 2.2 T, at 80 T/m. The outer radius is adjusted so that, as the poletips saturate, some saturation also occurs in the return leg, slowing the growth of lower HOMs with increasing current. This is only effective if the tolerance on the outer radius dimension is less than 1 mm. A concern here is that saturation results in a stray field beyond the yoke outer radius, which may affect induction cores surrounding the magnet.

PHYSICAL DESIGN

Fig 2 shows the magnet octant cross section. Coils are

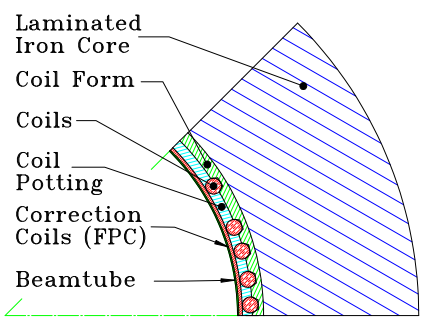

Figure 2: Octant Cross Section

first wound (as flat coils) and stretched straight on a special winding machine which was used successfully for several earlier magnets [9],[10]. The conductors are stretched significantly past the copper yield point, but well below the polymer film insulation rupture strain, maintaining insulation integrity. The straightening process allows the conductors to seat well into the coil form, yielding consistently accurate conductor placement. Coils are a single layer only, with a diagonal leadout [10] which compensates the spiral asymmetry of the coil, simulating an "endless loop" coil. The coils are collapsed to a flat pancake (with some application of pressure here, due to the relatively large conductors which interfere) and the ends bent 90 deg. in a brake press-like bending machine (not shown) to form the upturned "bedstead" ends. The coil will then "uncollapse" to fit onto a coil form which has previously been cast of heat conducting epoxy to the yoke bore. They are then potted in place with heat conducting epoxy.

The diagonal leadout, crossing over the conductors, is located entirely out of the magnet bore to minimize radial buildout, which reduces maximum field, iron saturation, and current for a given gradient. The flat upturned ends also minimize axial space and equalize the conductor lengths. For a pole connected magnet array the upturned bedstead end would interfere; in this case ends would be formed in an arc, as was done for the NTX quadrupoles [10].

Table 3: HCX Pulsed Quad Parameters

Parameter		Units
Winding radius, r_w	3.56	cm
Current, I	7.2	kA (@65 T/m)
Voltage, V	2.8	kV (@65 T/m)
Pulse duration, t_p	200	μsec
Conductor diameter, r_c	2.3	mm
Stored Energy, U	780	J (@65 T/m)
Ohmic Loss, Q	150	J (@65 T/m)

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