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Is the Universal String Axion the QCD Axion?*

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Abstract

We consider the class of effective supergravity theories from the weakly coupled heterotic string in which local supersymmetry is broken by gaugino condensation in a hidden sector, with dilaton stabilization achieved through corrections to the classical dilaton Kähler potential. If there is a single hidden condensing (simple) gauge group, the axion is massless (up to contributions from higher dimension operators) above the QCD condensation scale. We show how the standard relation between the axion mass and its Planck scale coupling constant is modified in this class of models due to a contribution to the axion-gluon coupling that appears below the scale of supersymmetry breaking when gluinos are integrated out. In particular there is a point of enhanced symmetry in parameter space where the axion mass is suppressed. We revisit the question of the universal axion as the Peccei-Quinn axion in the light of these results, and find that the strong CP problem is avoided in most compactifications of the weakly coupled heterotic string.

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1 Introduction

As observed by Banks and Dine [1], in a supersymmetric Yang Mills theory with a dilaton chiral superfield that couples universally to Yang-Mills fields there is a residual R-symmetry in the effective theory for the condensates of a strongly coupled gauge sector, provided that there is a single condensation scale governed by a single β-function, there is no explicit R-symmetry breaking by fermion mass terms in the strongly coupled sector, and the dilaton S has no potential. The latter requirement is met in effective supergravity from string theory, and explicit realizations of this scenario have been constructed [2, 3] in the context of the weakly coupled heterotic string. Since the axion is massless above the QCD condensation scale in these scenarios, it is a natural candidate for the Peccei-Quinn axion.

In [2, 3] gaugino and matter condensation was studied for gauge sectors that have no dimension-two gauge invariant operators. In these effective theories, two-condensate models have a point of enhanced symmetry where the condensing gauge sectors $\mathcal{G}_a$ have the same beta-function coefficients b_a, and the axion mass is proportional to $|b_1 - b_2|$. Here we are interested in the case where one condensing gauge group $\mathcal{G}_Q$ is $SU(N_c)$ with a $U(N)$ flavor symmetry for quark supermultiplets. In the following section we construct supersymmetric models for this case. This has the advantages that all the symmetries are manifest and the effective Lagrangian is highly constrained by supersymmetry. In Section 2.1 we extend the methods of [2] to the case of $\mathcal{G}_Q$ and show that in the rigid supersymmetry limit $m_P \to \infty$ we recover known results [4]. In Section 2.2 we include a condensing hidden sector with a gauge group $\mathcal{G}_c$ of the class studied in [2] and show that the point of enhanced symmetry in this case corresponds to

$$b_c = \frac{N_c}{8\pi^2}. \tag{1.1}$$

Although this is not a realistic model for QCD, where the condensation scale is far below the scale of supersymmetry breaking, we recover the effective potential for light pseudoscalars by identifying the $q\bar{q}$ pseudoscalar bound states with the F-components of the quark condensate superfields, and show that the presence of the symmetric point (1.1) is reflected in the axion mass.

In Section 3 we consider a more realistic model in which the QCD gauge and matter degrees of freedom are unconfined at the supersymmetry-breaking scale. We show that R-symmetry is not broken by the large gaugino (and squark) masses because the masslessness of the axion leaves the phase of the gaugino masses undetermined. We argue that this implies that the symmetries of the effective theory at the supersymmetry-breaking scale must be reflected in the effective quark-gluon theory just above the QCD condensation scale, implying

a correction to the axion-gluon coupling. This result is confirmed by an explicit calculation of the heavy gaugino loop contribution in Appendix A, and we recover the result of Section 2 for the axion mass.

For QCD with $N_c = 3$, the point of enhanced symmetry (1.1) has

$$b_c = \frac{3}{8\pi^2} = .038, \tag{1.2}$$

which is in the preferred range $.3 \leq b_c \leq .4$ found in studies of electroweak symmetry-breaking [5] and of dark matter candidates [6] in the context of the models considered here. As a consequence the axion mass is suppressed and higher dimension operators [1, 7] might lead to strong CP violation. We address this question in Section 4.

Our results are summarized in Section 5. Here we use the linear supermultiplet formulation for the dilaton superfield, but we expect that our results can be reproduced in the chiral multiplet formulation.

2 Supersymmetric models

The supersymmetry breaking models of [2, 3] are based on strongly coupled hidden sector gauge groups of the form $\prod_a \mathcal{G}_a$. The generalization to supergravity [8, 9] of the VYT effective action [10] is obtained by introducing composite field operators U_a and Π_a^α that are $\mathcal{G}_a$-charged gauge and matter condensate chiral superfields, respectively:

$$U_a \simeq \mathcal{W}_a^\alpha \mathcal{W}_\alpha^a, \qquad \Pi_a^\alpha \simeq \prod_A \left(\Phi_a^A\right)^{n_{\alpha,a}^A}, \tag{2.1}$$

and by matching the anomalies of the effective theory to those of the underlying theory. The Lagrangian[1]

$$\mathcal{L}_{VYT} = \frac{1}{8}\int \mathrm{d}^4\theta\, \frac{E}{R} \sum_a U_a \left[b'_a \ln(e^{-K/2} U_a) + \sum_\alpha b_a^\alpha \ln \Pi^\alpha \right] + \text{h.c.}, \tag{2.2}$$

has the correct anomaly structure under Kähler $U(1)$ R-symmetry:

$$\begin{aligned} \lambda_\alpha^a &= \mathcal{W}_\alpha^a| \to e^{i\alpha/2}\lambda_\alpha^a, \qquad \chi_\alpha^A = \frac{1}{\sqrt{2}}\, \mathcal{D}_\alpha \Phi^A\Big| \to e^{-i\alpha/2}\chi_\alpha^A, \\ U_a &\to e^{i\alpha} U_a, \qquad \Pi_a^\alpha \to \Pi_a^\alpha, \end{aligned} \tag{2.3}$$

[1] We work in the Kähler $U(1)$ superspace formalism of [11].

conformal transformations:

$$\begin{aligned} \lambda_a &\to e^{3\sigma/2}\lambda_a, \qquad \Phi^A \to e^{\sigma}\Phi^A, \qquad etc., \\ U_a &\to e^{3\sigma}U_a, \qquad \Pi_a^\alpha \to e^{d_a^\alpha \sigma}\Pi_a^\alpha, \qquad d_a^\alpha = \sum_A n_{\alpha,a}^A, \end{aligned} \tag{2.4}$$

and modular (T-duality) transformations that involve functions of the moduli chiral supermultiplets T^I:

$$\begin{aligned} \lambda_a &\to e^{-i\mathrm{Im}F/2}\lambda_a, \qquad \chi^A \to e^{-F^A + i\mathrm{Im}F/2}\chi^A, \qquad F^A = \sum_I q_I^A F^I(T^I), \qquad F = \sum_I F^I, \\ U_a &\to e^{-i\mathrm{Im}F}U_a, \qquad \Pi_a^\alpha \to e^{-F_a^\alpha}\Pi_a^\alpha, \qquad F_a^\alpha = \sum_A n_{\alpha,a}^A F^A, \end{aligned} \tag{2.5}$$

provided the conditions

$$b_a' = \frac{1}{8\pi^2}\left(C_a - \sum_A C_a^A\right), \qquad b_a^\alpha = \sum_{A\in\alpha} \frac{C_a^A}{4\pi^2 d_a^\alpha}, \tag{2.6}$$

are satisfied, where $C_a(C_a^A)$ is the quadratic Casimir in the adjoint (Φ^A) representation of $\mathcal{G}_a$. In the heterotic superstring theory, the anomaly under (2.5) is canceled by moduli dependent string loop threshold corrections [12] and a four-dimensional version [13] of the Green-Schwarz [14] term. In effective supergravity theories with an anomalous gauge group $U(1)_X$, there are additional anomaly matching conditions [3] and an additional Green-Schwarz term [15] to restore the $U(1)_X$ invariance of the underlying string theory. The composite chiral superfields Π_a^α are invariant under the nonanomalous symmetries, and may be used to construct an invariant superpotential [2, 3]. Provided there are no invariant chiral fields of dimension two, and no additional global symmetries (such as chiral flavor symmetries), the dynamical degrees of freedom associated with the composite fields (2.1) acquire masses [16] larger than the condensation scale Λ_a, and may be integrated out, resulting in an effective theory constructed as described above with the composite fields taken to be nonpropagating; that is, they do not appear in the Kähler potential. The dynamical axion is massless if there is a single condensate, or if the condensing gauge groups all have the same β-function coefficient b_a, defined by

$$\frac{\partial g_a(\mu)}{\partial \ln \mu} = -\frac{3b_a}{2} g_a^3(\mu). \tag{2.7}$$

In this case there is a nonanomalous R-symmetry: the axion shift compensates the anomaly arising from the transformation (2.3). The symmetry is broken if there are condensing gauge groups with different coefficients b_a. For example in the case of two condensates with

coefficients $b_1 > b_2$ and condensation scales $\Lambda_1 \gg \Lambda_2$, the axion mass is approximately given by [2]

$$m_a \approx \frac{3\langle\ell\rangle\sqrt{b_1}}{b_2}\left[(b_1-b_2)\frac{\Lambda_2}{\Lambda_1}\right]^{\frac{3}{2}} m_{\frac{3}{2}}, \tag{2.8}$$

where ℓ is the dilaton field in the linear multiplet formulation. In the classical approximation $\langle\ell\rangle = g_s^2/2 \approx .25$, with g_s the string coupling constant. Generally we expect $\langle\ell\rangle \sim 1$ when string nonperturbative [17] and/or field theoretic quantum corrections [1, 18, 19] to the dilaton Kähler potential are invoked to assure dilaton stabilization [20].

In the case that there is just one hidden sector condensing gauge group $\mathcal{G}_c$, the axion remains massless above the scale Λ_{QCD} of quark and gluon condensation in the standard model, and is a candidate for the Peccei-Quinn axion. However in this case there is an enlarged symmetry in the limit of one or more massless quarks, leading to light condensates $m_\pi < \Lambda_{QCD}$, and the result (2.8) is modified.

In order to more closely model QCD, in this section we consider supergravity models with a strongly coupled gauge group $\mathcal{G}_c \otimes SU(N_c)$ with $N \equiv N_f < N_c$ vector-like flavors. We first extend the construction of the effective VYT action to this case, and show that in the flat SUSY limit it reduces to the results [4] based on the holomorphy of the superpotential.

2.1 The VYT action for $SU(N_c)$ with chiral flavor symmetry

We have N "quark" and N "anti-quark" chiral supermultiplets Q^A and Q^c_A, respectively. We take the quark condensates to be the matrix-valued "meson" superfield $\mathbf{\Pi}^A_B = Q^A Q^c_B$. We do not assume *a priori* that these are static fields. For $\mathcal{G}_a = SU(N_c) \equiv \mathcal{G}_Q$ we take

$$\mathcal{L}^Q_{VYT} = \frac{1}{8}\int \mathrm{d}^4\theta\, \frac{E}{R} U_Q \left[b'_Q \ln(e^{-K/2}U_Q) + b^\alpha_Q \ln(\det \mathbf{\Pi})\right] + \text{h.c.} \tag{2.9}$$

For the elementary fields we have $C_Q = N_c$, $C^A_Q = \frac{1}{2}$, $\sum_A C^A_Q = N$. Under Kähler $U(1)$ R-symmetry (2.3) anomaly matching requires

$$b'_Q = \frac{1}{8\pi^2}\left(N_c - N\right), \tag{2.10}$$

and under the conformal transformation (2.4) with $\mathbf{\Pi} \to e^{2\sigma}\mathbf{\Pi}$, we require

$$3b'_Q + 2Nb^\alpha_Q = \frac{1}{8\pi^2}\left(3N_c - N\right) = 3b_Q, \tag{2.11}$$

where b_Q is the β-function coefficient as defined in (2.7). Putting these together gives (2.10) and

$$b^\alpha_Q = \frac{1}{8\pi^2}, \qquad b_Q = b'_Q + \frac{2N}{3} b^\alpha_Q, \tag{2.12}$$

in agreement with the general result (2.6) with $d_Q^\alpha = 2N$ for $\pi_Q^\alpha = \det \mathbf{\Pi}$. If Q, Q^c have modular weights $\mathbf{q}_I, \mathbf{q}_I^c$, under T-duality (2.5)

$$Q \to e^{-\mathbf{F}^Q} Q, \quad Q^c \to e^{-\mathbf{F}^{Q^c}} Q^c, \qquad \mathbf{F}^Q = \sum_I \mathbf{q}_I F^I(T^I), \quad \mathbf{q}_I = \mathrm{diag}(q_I^1, \dots q_I^N), \tag{2.13}$$

and similarly for $\mathbf{F}^{Q^c}$. The modular anomaly matching condition

$$b'_Q + b_Q^\alpha q_I^\alpha = \frac{1}{8\pi^2}\left\{C_Q + \sum_A C_Q^A \left[2q_I^A + 2(q^c)_I^A - 1\right]\right\}, \qquad q_I^\alpha = \sum_A \left(q_I^A + (q^c)_I^A\right), \tag{2.14}$$

is also satisfied by (2.10) and (2.12). Finally, like the underlying theory, (2.9) is invariant under flavor $SU(N)_L \otimes SU(N)_R$, while under chiral $U(1)$ transformations

$$Q \to e^{i\beta} Q, \quad Q^c \to e^{i\beta} Q^c \tag{2.15}$$

the anomaly matching condition

$$2N b_Q^\alpha = \sum_A \frac{C_Q^A}{4\pi^2} \tag{2.16}$$

is also satisfied.

For the superpotential we take

$$W(\mathbf{\Pi}) = \eta^{-2} \mathrm{Tr}\left[\prod_I \eta_I^{2\mathbf{q}_I} \mathbf{\Pi} \prod_J \eta_J^{2\mathbf{q}_J^c} \mathbf{M}\right], \tag{2.17}$$

where $\mathbf{M}$ is the mass matrix and $\eta_I = \eta(T^I)$, $\eta = \prod_I \eta_I$. The component Lagrangian for the effective theory with just this $SU(N_c)$ condensate can easily be inferred from the results of [2]. Solving the equation of motion for $\mathrm{Re} F^Q$ gives

$$\begin{aligned} u_Q &= e^{i\omega_Q} \lambda_Q [\det(\mathbf{\Pi}\mathbf{\Pi}^\dagger)]^{-1/2(N_c - N)}, \\ \lambda_Q &= e^{-1} e^{[k + K(\mathbf{\Pi})]/2} \prod_I \left[2\mathrm{Re} t^I |\eta(t^I)|^4\right]^{(b - b'_Q)/2b'_Q} |\eta(t^I)|^{-2b_Q^\alpha q_I^\alpha / b'_Q} \Lambda_Q^{(3N_c - N)/(N_c - N)}, \end{aligned} \tag{2.18}$$

where

$$\Lambda_a = \exp(-s(\ell)/3b_a), \qquad s(\langle \ell \rangle) = g_s^{-2}, \tag{2.19}$$

is defined as the scale at which the one loop running coupling $g_a(\mu)$ blows up:

$$g_a^{-2}(\Lambda_a) = g_s^{-2} + 3b_a \ln(\Lambda_a / m_P), \tag{2.20}$$

in reduced Planck mass units, $m_P = 1$, that we use throughout.[2] To compare with previous results [4] we take the rigid SUSY limit, and neglect the moduli and the dilaton; $s(\ell) \to g_0^{-2}$.

[2] A correction that accounts for the fact that the string scale differs from the Planck scale by $\mu_s = g_s m_P$ is encoded in the e^k factor in $|\bar{u}_a|^2$.

Then the superpotential reduces to the standard VY one:

$$W(U_Q) = \frac{1}{4} U_Q \left[g_0^{-2} + b'_Q \ln(U_Q) + b^\alpha_Q \ln(\det \mathbf{\Pi}) \right]. \tag{2.21}$$

Keeping U_Q static and imposing the equation of motion for the auxiliary field F^Q gives the potential

$$-V = \mathrm{Tr}\left[\bar{\mathbf{F}}^\pi \mathbf{K}'' \mathbf{F}^\pi + \left\{ \mathbf{F}^\pi \left(\mathbf{M} + \frac{1}{4} b^\alpha_Q u_Q \mathbf{\Pi}^{-1} \right) + \text{h.c.} \right\} \right], \qquad u_Q = e^{-1} \left(\frac{\Lambda_Q^{3N_c - N}}{\det \mathbf{\Pi}} \right)^{1/(N_c - N)} \tag{2.22}$$

where $\mathbf{K}''$ is the (tensor-valued) Kähler metric for $\mathbf{\Pi}$. Since $\partial(\det \mathbf{\Pi})^p / \partial \mathbf{\Pi} = p \mathbf{\Pi}^{-1} \det \mathbf{\Pi}^p$, the potential (2.22) is derivable from the following superpotential for the dynamical superfield $\mathbf{\Pi}$:

$$W_{\mathbf{\Pi}} = \mathrm{Tr}(\mathbf{M}\mathbf{\Pi}) - \frac{(N_c - N)}{32\pi^2 e} \left(\frac{\Lambda_Q^{3N_c - N}}{(\det \mathbf{\Pi})} \right)^{1/(N_c - N)}, \tag{2.23}$$

which, up to a factor[3] $-2/e$, is the superpotential found in [4].

We may also consider the case – more closely resembling QCD – where only $n < N$ chiral supermultiplets have masses below the condensation scale $u_Q^{1/3} \sim \Lambda_Q$, while $m = N - n$ chiral supermultiplets have masses M_A above that scale. The latter decouple at scales below their masses, which explicitly break the nonanomalous $U(N)_L \otimes U(N)_R$ symmetry to a $U(n)_L \otimes U(n)_R$ symmetry if $m = N - n$ quarks are massive. They do not contribute to the chiral anomaly at the $SU(N_c)$ condensation scale. To account for these effects we replace (2.9) by

$$\mathcal{L}^Q_{VYT} = \frac{1}{8} \int \mathrm{d}^4\theta \, \frac{E}{R} U_Q \left\{ b'_n \ln(e^{-K/2} U_Q) + b^\alpha_Q \left[\ln(\det \mathbf{\Pi}_n) - \sum_{A=1}^{m} \ln M_A \right] \right\} + \text{h.c.}, \tag{2.24}$$

where $b'_n = (N_c - n)/8\pi^2$ and $\mathbf{\Pi}_n$ is an $n \times n$ matrix-valued composite operator constructed only from light quarks. (2.24) can be formally obtained from (2.9) by integrating out the heavy quark condensates as follows. As the threshold M_A is crossed, set $\det \mathbf{\Pi}_{n+A} \to \pi^A \det \mathbf{\Pi}_{n+A-1}$ and take the condensate $\pi^A \sim Q^A Q^c_A$ to be static: $K(\mathbf{\Pi}_{n+A}) \to K(\mathbf{\Pi}_{n+A-1})$. Then including the superpotential term $W(\pi^A) = -M_A \pi^A$, the equation of motion for F^A gives $\pi^A = e^{-K/2} u_Q / 32\pi^2 M_A$, giving (2.24) up to some constant threshold corrections. The flat SUSY analogue of (2.21) is now

$$W = \frac{1}{4} U_Q \left\{ g_0^{-2} + b'_n \ln(U_Q) + b^\alpha_Q \left[\ln(\det \mathbf{\Pi}) - \sum_{A=1}^{m} \ln M_A \right] \right\}, \tag{2.25}$$

[3] The factor e comes from the fact that we take the derivative of $\int U \ln U$, while the authors of [4] start with $\int < \lambda\lambda > \ln \Lambda$ and determine $< \lambda\lambda >$ from threshold matching. The minus sign comes from the convention of [11]: $u \sim \mathcal{W}^\alpha \mathcal{W}_\alpha| = -\lambda\lambda$.

and we recover (2.22)–(2.23) with now

$$\Lambda_Q = e^{-1/3b_n g^2} \prod_{A=1}^{m} M_A^{b_3/3b_n}, \qquad 3b_n = \frac{3N_c - n}{8\pi^2} = 3b'_n + 2nb_3, \tag{2.26}$$

which corresponds to running $g^{-2}(\mu)$ from $g^{-2}(1) = g_0^{-2}$ to $g^{-2}(\Lambda_Q) = 0$ using the β-function coefficient $(3N_c - n - A)/8\pi^2$ for $m_A \leq \mu \leq m_{A+1}$, again in agreement with the results of nonperturbative flat SUSY analyses [4].

2.2 Supergravity with strongly coupled $\mathcal{G}_c \otimes SU(N_c)$

We consider the supergravity action defined by

$$\mathcal{L} = \mathcal{L}_K + \mathcal{L}_P + \mathcal{L}_{VYT} + \mathcal{L}_{GS} + \mathcal{L}_{Th}, \tag{2.27}$$

where $\mathcal{L}_{GS}$ and $\mathcal{L}_{Th}$ contain, respectively, the Green-Schwarz terms and threshold corrections discussed above,

$$\mathcal{L}_P = \frac{1}{2} \int \mathrm{d}^4\theta \, \frac{E}{R} e^{K/2} W, \tag{2.28}$$

is the superpotential term, and

$$\mathcal{L}_K = \int \mathrm{d}^4\theta \, E \left[-3 + 2Ls(L) \right] \tag{2.29}$$

contains the locally supersymmetric extension of the Einstein and Yang-Mills terms as well as the kinetic terms for matter through the Kähler potential

$$K = k(L) + K(\mathbf{\Pi}) - \sum_I \ln \left(T^I + \bar{T}^{\bar{I}} \right), \qquad k'(L) + 2Ls'(L) = 0. \tag{2.30}$$

The Yang-Mills term arises from the modified linearity conditions [11] defined by the chiral projections of the real field L:

$$\left(\bar{\mathcal{D}}^2 - 8R \right) L = -\sum_a \left(\mathcal{W}^\alpha \mathcal{W}_\alpha \right)_a, \qquad \left(\mathcal{D}^2 - 8\bar{R} \right) L = -\sum_a \left(\overline{\mathcal{W}}^{\dot\beta} \overline{\mathcal{W}}_{\dot\beta} \right)_a. \tag{2.31}$$

Below the condensation scale Λ_a we make the replacement $\left(\mathcal{W}^\alpha \mathcal{W}_\alpha \right)_a \to U_a$ in (2.31). The VYT term is given by (2.9)–(2.12) for $a = Q$. For $\mathcal{G}_c$ we follow [2] and take dimension-three operators for the Π_c^α in (2.2), giving the anomaly matching conditions:

$$\begin{aligned} b_c &= b'_c + \sum_\alpha b_c^\alpha = \frac{1}{8\pi^2} \left(C_c - \tfrac{1}{3} \textstyle\sum_A C_c^A \right), \\ b'_c &= \frac{1}{8\pi^2} \left(C_c - \sum_A C_c^A \right), \qquad b_c^\alpha = \frac{1}{12\pi^2} \sum_A C_c^A. \end{aligned} \tag{2.32}$$

For the superpotential we now take

$$W = \sum_\alpha c_\alpha \prod_I \eta_I^{2(q_I^\alpha - 1)} \Pi_c^\alpha + \eta^{-2} \mathrm{Tr} \left[\prod_I \eta_I^{2\mathbf{q}_I} \mathbf{\Pi} \prod_J \eta_J^{2\mathbf{q}_J^c} \mathbf{M} \right], \tag{2.33}$$

and we approximate the Kähler potential for $\mathbf{\Pi}$ by[4]

$$K(\mathbf{\Pi}) = \mu^{-2} \mathrm{Tr} \left[\prod_I (T^I + \bar{T}^{\bar{I}})^{-\mathbf{q}_I} \mathbf{\Pi}\mathbf{\Pi}^\dagger \prod_J (T^J + \bar{T}^{\bar{J}})^{-\mathbf{q}_J^c} \right]. \tag{2.34}$$

We can make a holomorphic field redefinition such that[5] it becomes obvious that the moduli are still stabilized at self dual points with vanishing F-terms, namely

$$\mathbf{\Pi} = \prod_I \eta_I^{-2q_I} \mathbf{\Pi}' \prod_J \eta_J^{-2q_J^c}, \tag{2.35}$$

and then drop the prime. Then the last line in (2.18) becomes

$$\lambda_Q = e^{-1} e^{[k+K(\mathbf{\Pi})]/2} \prod_I \left[2\mathrm{Re} t^I |\eta(t^I)|^4 \right]^{(b - b'_Q - b_Q^\alpha q_I^\alpha)/2b'_Q} \Lambda_Q^{(3N_c - N)/(N - N_c)}, \tag{2.36}$$

and the Kähler potential and superpotential are now

$$\begin{aligned} K(\mathbf{\Pi}) &= \mu^{-2} \mathrm{Tr} \left\{ \prod_I \left[(T^I + \bar{T}^{\bar{I}}) |\eta_I|^4 \right]^{-\mathbf{q}_I} \mathbf{\Pi}\mathbf{\Pi}^\dagger \prod_J \left[(T^J + \bar{T}^{\bar{J}}) |\eta_J|^4 \right]^{-\mathbf{q}_J^c} \right\}, \\ W &= \sum_\alpha c_\alpha \prod_I \eta_I^{2(q_I^\alpha - 1)} \Pi_c^\alpha + \eta^{-2} \mathrm{Tr} \left(\mathbf{\Pi}\mathbf{M} \right). \end{aligned} \tag{2.37}$$

Then the moduli derivatives $K_I(\mathbf{\Pi})$ and $\partial_I[e^K W(\mathbf{\Pi})]$ with the new $\mathbf{\Pi}$ variables fixed vanish at the self dual points. To study the potential for the other fields we may set the moduli at their ground state values. We could neglect the moduli altogether in this toy model since modular invariance and Kähler R-symmetry give the same anomaly matching conditions. However it is also interesting to check that mixing of the axion with $\mathrm{Im} t^I = \mathrm{Im}\, T^I \Big|$ makes no

[4] We could take a more general form of the Kähler potential, *e.g.,* $K \sim \mathrm{Tr}\mathbf{\Pi}\mathbf{\Pi}^\dagger / \mathrm{Tr}\mathbf{\Pi}$, $\mathrm{Tr}\mathbf{\Pi}\mathbf{\Pi}^\dagger / (\det \mathbf{\Pi})^{1/N}$ or $(\mathrm{Tr}\mathbf{\Pi}\mathbf{\Pi}^\dagger)^{\frac{1}{2}}$; these would give essentially the same result with $\mu^2 \sim v$.

[5] We took the superpotential to be modular invariant; in the context of string theory, this implies an implicit assumption that the *vev* that induces the mass term does not break modular invariance. This need not be the case for the more realistic QCD model discussed in Section 3 where the masses are generated by the MSSM Higgs *vev* $v_H \ll \Lambda_c$; as a result there can be small corrections to the effective theory of order v_H/Λ_c. These do not affect our conclusions which depend only on the residual R-symmetry above the QCD confinement scale and, for CP violation discussed in Section 4, on unbroken T-duality above Λ_c.

difference. Since we are only concerned with the phase $N\delta$ of $\det \mathbf{M}$, we make the simplifying assumption that $\mathbf{M} = me^{i\delta}\mathbf{1}$. Then $\langle \mathbf{\Pi} \rangle = ve^{i\phi_\pi}\mathbf{1}$, and a convenient parameterization is

$$\begin{aligned}
\mathbf{\Pi} &= \mathbf{S}e^{i\mathbf{P}}, \qquad \mathbf{S} = v\left[1 + \frac{\mu}{v\sqrt{2}}\left(\boldsymbol{\sigma} + \frac{\sigma_0}{\sqrt{N}}\right)\right], \qquad \mathbf{P} = \phi_\pi + \frac{\mu}{v\sqrt{2}}\left(\mathbf{a} + \frac{a_0}{\sqrt{N}}\right), \\
\det \mathbf{\Pi} &= \det \mathbf{S} e^{i(N\phi_\pi + \mu\sqrt{N}a_0/v\sqrt{2})}, \qquad \mathrm{Tr}|\mathbf{\Pi}|^2 = \mathrm{Tr}\mathbf{S}^2, \qquad \mathrm{Tr}|\mathbf{\Pi}|^{-2} = \mathrm{Tr}\mathbf{S}^{-2}, \\
\langle \boldsymbol{\sigma} \rangle &= \langle \sigma_0 \rangle = \langle \mathbf{a} \rangle = \langle a_0 \rangle = 0, \qquad \boldsymbol{\sigma} = \sqrt{2}\sum_i T_i \sigma^i, \qquad \mathbf{a} = \sqrt{2}\sum_i T_i a^i,
\end{aligned} \tag{2.38}$$

where T_i is a generator of flavor $SU(N)$ in the fundamental representation:

$$\mathrm{Tr}T_i = 0, \qquad \mathrm{Tr}T_i^2 = \frac{1}{2}. \tag{2.39}$$

The component Lagrangian for bosons then takes the form

$$\mathcal{L} = \mathcal{L}_{KE} - V, \tag{2.40}$$

The potential is

$$\begin{aligned}
V &= \frac{k'}{16\ell}\left|\rho_c e^{i\omega_c}(1 + b_c\ell) + \lambda_Q |\det \mathbf{\Pi}|^{-1/(N_c - N)} e^{i\omega_Q}(1 + \ell b'_Q) - 4e^{K/2}\ell\eta^{-2} m e^{i\delta}\mathrm{Tr}\mathbf{\Pi}\right|^2 \\
&\quad - \frac{3}{16}\left|b_c\rho_c e^{i\omega_c} + b'_Q\lambda_Q |\det \mathbf{\Pi}|^{-1/(N_c - N)} e^{i\omega_Q} - 4e^{K/2}\eta^{-2} m e^{i\delta}\mathrm{Tr}\mathbf{\Pi}\right|^2 \\
&\quad + \sum_I K_{I\bar{I}} F^I \bar{F}^{\bar{I}} + \mathrm{Tr}\bar{\mathbf{F}}^\pi \mathbf{K}'' \mathbf{F}^\pi, \\
K_{I\bar{I}}\bar{F}^{\bar{I}} &= \frac{4\mathrm{Re}t^I \zeta(t^I) + 1}{8\mathrm{Re}t^I}\Big[(b - b_c)\rho_c e^{i\omega_c} + (b - b'_Q)\lambda_Q |\det \mathbf{\Pi}|^{-1/(N_c - N)} e^{i\omega_Q} \\
&\qquad + 4e^{K/2}\eta^{-2} m e^{i\delta}\mathrm{Tr}\mathbf{\Pi}\Big] \\
(\bar{\mathbf{F}}^\pi \mathbf{K}'')^A_B &= \frac{1}{4}\Big\{\mu^{-2}\overline{\mathbf{\Pi}}^A_B\left[b_c\rho_c e^{i\omega_c} + b'_Q\lambda_Q |\det \mathbf{\Pi}|^{-1/(N_c - N)} e^{i\omega_Q} - 4e^{K/2}\eta^{-2} m e^{i\delta}\mathrm{Tr}\mathbf{\Pi}\right] \\
&\qquad - b^\alpha_Q \lambda_Q \left(\mathbf{\Pi}^{-1}\right)^A_B |\det \mathbf{\Pi}|^{-1/(N_c - N)} e^{i\omega_Q} - 4e^{K/2}\eta^{-2} m \delta^A_B e^{i\delta}\Big\},
\end{aligned} \tag{2.41}$$

which can be written in the form

$$\begin{aligned}
V &= A_1(\mathbf{S}) + A_2(\mathbf{S})\cos(\omega'_c - \omega'_Q) + m_0 \mathrm{Tr}\left\{e^{-i\mu\mathbf{a}/v\sqrt{2}}\left[A_3(\mathbf{S})e^{i\omega'_c} + A_4(\mathbf{S})e^{i\omega'_Q}\right] + \mathrm{h.c.}\right\} \\
&\quad + m_0^2 A_5(\mathbf{S}, \mathbf{a}^2), \qquad \omega'_a = \omega_a - \delta - \phi_\pi - \nu a_0 - i\ln(\eta/\bar{\eta}), \qquad \nu = \frac{\mu}{v\sqrt{2N}},
\end{aligned} \tag{2.42}$$

where the A_i are real and independent of the condensate phases, and

$$m_0 = e^{K/2}|\eta|^{-2} m \tag{2.43}$$

is the quark mass. Using the parameterization (2.38), the kinetic energy term is, dropping a total derivative,

$$\begin{aligned}
\mathcal{L}_{KE} &= \frac{k'}{4\ell} B^m B_m - b_c \tilde{\omega}_c \nabla^m B^c_m - b'_Q \tilde{\omega}_Q \nabla^m B^Q_m + i\frac{b}{2} B^m \sum_I \left(\ell'^I \partial_m t^I - \text{h.c.}\right) \\
&\quad -\frac{1}{2}\left(\partial_m \boldsymbol{\sigma} \partial^m \boldsymbol{\sigma} + \partial_m \sigma_0 \partial^m \sigma_0 + \partial_m \mathbf{a} \partial^m \mathbf{a} + \partial_m a_0 \partial^m a_0\right) - K_{I\bar{J}} \partial t^I \partial \bar{t}^{\bar{J}}, \\
\tilde{\omega}_c &= \omega_c - i\ln(\eta/\bar{\eta}), \quad \tilde{\omega}_Q = \omega_Q - i\ln(\eta/\bar{\eta}) + \frac{N}{N_c - N}\left(\phi_\pi + \nu a_0\right), \\
\ell'^I &= \frac{\partial \ell^I}{\partial t^I} = \frac{\partial}{\partial t^I} \ln\left[|\eta(t^I)|^4 (t^I + \bar{t}^I)\right].
\end{aligned} \tag{2.44}$$

Defining $\tilde{B}^m = B^m_c - B^m_Q$, the equations of motion for ω_a and a_0 give

$$\begin{aligned}
\nabla^m B_m &= -\frac{1}{b_c}\frac{\partial V}{\partial \omega_c}\bigg|_{a_0,\omega_Q} - \frac{1}{b'_Q}\frac{\partial V}{\partial \omega_Q}\bigg|_{a_0,\omega_c} = -\frac{1}{b_c}\frac{\partial V}{\partial \omega'_c}\bigg|_{\omega'_Q} - \frac{1}{b'_Q}\frac{\partial V}{\partial \omega'_Q}\bigg|_{\omega'_c}, \\
\nabla^m \tilde{B}_m &= -\frac{1}{b_c}\frac{\partial V}{\partial \omega_c}\bigg|_{a_0,\omega_Q} + \frac{1}{b'_Q}\frac{\partial V}{\partial \omega_Q}\bigg|_{a_0,\omega_c} = -\frac{1}{b_c}\frac{\partial V}{\partial \omega'_c}\bigg|_{\omega'_Q} + \frac{1}{b'_Q}\frac{\partial V}{\partial \omega'_Q}\bigg|_{\omega'_c}, \\
\Box a_0 &= \frac{b'_Q N \nu}{2(N_c - N)}\left(\nabla^m B_m - \nabla^m \tilde{B}_m\right) + \frac{\partial V}{\partial a_0}\bigg|_{\omega_c,\omega_Q} \\
&= -\nu\left(\frac{\partial V}{\partial \omega'_c}\bigg|_{\omega'_Q} + \frac{N_c}{N - N_c}\frac{\partial V}{\partial \omega'_Q}\bigg|_{\omega'_c}\right),
\end{aligned} \tag{2.45}$$

where subscripted fields are held fixed. The equations of motion for the three-form potentials $\Gamma = {}^*B$, $\tilde{\Gamma} = {}^*\tilde{B}$, that are dual to the one-forms $B_m, \tilde{B}_m$, give

$$\begin{aligned}
\frac{k'}{\ell} B_m &= -\nabla_m\left(b_c \tilde{\omega}_c + b'_Q \tilde{\omega}_Q\right) - i\sum_I \frac{b}{2}\left(\ell'^I \partial_m t^I - \text{h.c.}\right), \\
0 &= -\nabla_m\left(b_c \tilde{\omega}_c - b'_Q \tilde{\omega}_Q\right) \quad \Rightarrow \quad \tilde{\omega}_Q = \frac{b_c}{b'_Q}\tilde{\omega}_c + \phi_0,
\end{aligned} \tag{2.46}$$

where ϕ_0 is a constant phase. There are therefore two independent neutral axions (besides $\text{Im}t^I$) that we can take to be a_0 and

$$\omega = b_c\left(\omega'_c + \nu a_0 + \phi_c\right) = b_c \tilde{\omega}_c = b'_Q\left(\omega'_Q + \frac{N_c \nu a_0}{N_c - N} + \phi_Q\right), \tag{2.47}$$

where ϕ_c and ϕ_Q are constant phases. Using

$$\frac{\partial V}{\partial \omega}\bigg|_{a_0} = \frac{1}{b_c}\frac{\partial V}{\partial \omega'_c}\bigg|_{\omega'_Q} + \frac{1}{b_Q}\frac{\partial V}{\partial \omega'_Q}\bigg|_{\omega'_c} \qquad \frac{\partial V}{\partial a_0}\bigg|_{\omega} = -\nu\left(\frac{\partial V}{\partial \omega'_c}\bigg|_{\omega'_Q} + \frac{N_c}{N - N_c}\frac{\partial V}{\partial \omega'_Q}\bigg|_{\omega'_c}\right), \tag{2.48}$$

and combining (2.45) and (2.46), we obtain the equations of motion of the dual scalar Lagrangian[6]

$$\begin{aligned}\mathcal{L} &= -\frac{2\ell}{k'}\left(\partial_m\omega - \sum_I \frac{b}{2}\mathrm{Im}\ell'^I\partial_m t^I\right)\left(\partial^m\omega - \sum_I \frac{b}{2}\mathrm{Im}\ell'^I\partial_m t^I\right) - V \\ &\quad -\frac{1}{2}\left(\partial_m\boldsymbol{\sigma}\partial^m\boldsymbol{\sigma} + \partial_m\sigma_0\partial^m\sigma_0 + \partial_m\mathbf{a}\partial^m\mathbf{a} + \partial_m a_0\partial^m a_0\right) - K_{I\bar{J}}\partial t^I\partial\bar{t}^{\bar{J}},\end{aligned} \tag{2.49}$$

with V given by (2.42) and (2.47). There are two dynamical degrees of freedom associated with the phases relevant for the strong CP problem, the axion ω and the phase ϕ_π. Setting the other fields at their vacuum values, (2.42) takes the form

$$V(\omega,\phi_\pi) = \tilde{A}_1 + \tilde{A}_2\cos\omega_c' + \tilde{A}_3\cos\omega_Q' + \tilde{A}_4\cos(\omega_c' - \omega_Q'), \tag{2.50}$$

where $\tilde{A}_1 = A_1(v) + m_0^2 A_5(v,0)$, *etc.* The potential (2.50) is minimized for

$$0 = \frac{\partial V}{\partial\omega_a} = -\tilde{A}_2\sin\omega_c' - \tilde{A}_4\sin(\omega_c' - \omega_Q') = -\tilde{A}_3\sin\omega_Q' + \tilde{A}_4\sin(\omega_c' - \omega_Q'). \tag{2.51}$$

This has CP conserving solutions $\omega_a' = 0, \pi$. There might also be a CP violating solution $\omega_a' \neq 0$ provided

$$\begin{aligned}-1 < \cos\omega_c' &= -\frac{1}{2}\left(\frac{\tilde{A}_4}{\tilde{A}_3} + \frac{\tilde{A}_3}{\tilde{A}_4} - \frac{\tilde{A}_3\tilde{A}_4}{\tilde{A}_2^2}\right) < 1, \\ -1 < \cos\omega_Q' &= -\frac{1}{2}\left(\frac{\tilde{A}_4}{\tilde{A}_2} + \frac{\tilde{A}_2}{\tilde{A}_4} - \frac{\tilde{A}_2\tilde{A}_4}{\tilde{A}_3^2}\right) < 1,\end{aligned} \tag{2.52}$$

However the global minimum will occur for the CP conserving vacuum with $\omega_a' = 0, \pi$ that maximizes the (negative) coefficients of the two largest of $|\tilde{A}_{i\neq 1}|$. For example if $\tilde{A}_2 > \tilde{A}_3 > \tilde{A}_4 > 0$, the global minimum occurs[7] for $\langle V\rangle = \tilde{A}_1 - \tilde{A}_2 - \tilde{A}_3 + \tilde{A}_4$.

To study the effective theory for the axion and the mesons we can set the moduli and the static condensates at their ground state values. The canonically normalized mesons $\sigma_0, \boldsymbol{\sigma}, a_0, \mathbf{a}$ are defined as in (2.38) and

$$a = -\sqrt{\frac{2\ell}{k'}}\omega \tag{2.53}$$

is the canonically normalized axion.[8] In the limit of vanishing meson masses, $m_0 \to 0$, the potential depends only on the scalars σ and one linear combination, mostly a_0, of neutral

[6] It is straightforward to check that all the equations of motion are the same as for the original Lagrangian; see, for example [2].

[7] As usual we fine-tune the dilaton Kähler potential to make the potential positive semi-definite.

[8] The sign is chosen to give the standard coupling, (2.71) below; see (32) of [7].

pseudoscalars:

$$\begin{aligned}\sqrt{1+c_a^2}\eta &= \frac{N_c-N}{\nu N}(\omega_c'-\omega_Q'+\phi_c-\phi_Q)=a_0+c_a a\approx a_o,\\ c_a &= \frac{v}{\mu}8\pi^2\sqrt{\frac{k'}{N\ell}}\left(1-\frac{b_Q'}{b_c}\right)\ll 1.\end{aligned} \tag{2.54}$$

The other (mostly axion) neutral pseudoscalar and the charged pseudoscalars are massless Goldstone bosons of the nonanomalous symmetry $SU(N)_L\otimes SU(N)_R\otimes U(1)$, where the nonanomalous $U(1)$ is defined by (2.3) and (2.15) with

$$\alpha b_c=\frac{1}{8\pi^2}\left[\alpha\left(N_c-N\right)+2\beta N\right]. \tag{2.55}$$

If $m_0\neq 0$, flavor-chiral $U(1)$ symmetry is broken, and there is no longer the freedom to choose the R-parity of Q; in this case the classical R-symmetry has $\beta=\alpha/2$, and it is anomalous at the quantum level unless

$$b_c=\frac{N_c}{8\pi^2}=\frac{b_Q'N_c}{N_c-N}. \tag{2.56}$$

Writing

$$\begin{aligned}\omega_c' &= -\nu\sqrt{1+c_a^2}\eta-\frac{\omega}{Nb_c}\left(8\pi^2b_c-N_c\right)-\phi_c,\\ \omega_Q' &= -\frac{\nu N_c\sqrt{1+c_a^2}}{N_c-N}\eta-\frac{\omega}{Nb_c}\left(8\pi^2b_c-N_c\right)-\phi_Q,\end{aligned} \tag{2.57}$$

we see explicitly that the potential (2.42) depends only on one neutral pseudoscalar η at the point of enhanced symmetry (2.56). With the *vev*'s $\langle\omega_a'\rangle$ determined as described above, the potential for the light pseudoscalars takes the form

$$\begin{aligned}V(\mathbf{a},a) &= -c\mathrm{Tr}e^{i\mu(\mathbf{a}-c_a'a/\sqrt{N})/v\sqrt{2}}+\text{h.c.},\qquad c=m_0\left|A_3(v)-\frac{A_2(v)}{|A_2(v)|}A_4(v)\right|=\frac{v^2}{\mu^2}m_{\mathbf{a}}^2,\\ c_a' &= \frac{m_a}{m_{\mathbf{a}}}=\sqrt{\frac{k'}{N\ell}}\frac{v(8\pi^2b_c-N_c)}{\mu b_c}.\end{aligned} \tag{2.58}$$

For example, if we assume $\rho_c\gg|u_c|,m_0|\mathbf{\Pi}|$, and use the condition of (approximately) vanishing vacuum energy [2]:

$$k'/\ell\approx 3b_c^2/(1+b_c\ell)^2, \tag{2.59}$$

the minimum of the potential when $m_0\to 0$ is given by

$$\langle\cos(\omega_c'-\omega_Q')\rangle=\frac{\alpha}{|\alpha|},\quad \mu_v=v^{\frac{1}{2}}\approx\left(\frac{\gamma\mu^2\Lambda^{3b_Q/b_1}}{m_{\frac{3}{2}}}\right)^{1/2(2+3b_QN/b_3}\quad,\quad \Lambda=\left(\frac{b_Q'\lambda_Q}{4}\right)^{b_1/3b_Q}\sim\Lambda_Q, \tag{2.60}$$

where

$$\alpha = \frac{b_Q^\alpha}{b'_Q} - \frac{3(b_c - b'_Q)}{2b'_Q(1+b_c\ell)}, \qquad \gamma = \sqrt{\alpha^2+3} - |\alpha| > 0, \tag{2.61}$$

and the η, σ masses are

$$m_\eta \approx 2m_{\frac{3}{2}}\sqrt{|\alpha|/\gamma}, \qquad m_{\sigma_0} \approx 2m_{\frac{3}{2}}\sqrt{6-2|\alpha|\gamma}/\gamma, \qquad m_{\boldsymbol{\sigma}} \approx 2m_{\frac{3}{2}}\sqrt{2-|\alpha|\gamma}/\gamma, \tag{2.62}$$

where

$$m_{\frac{3}{2}} \approx \frac{b_c}{4}\rho_c \tag{2.63}$$

is the gravitino mass. When m_0 is turned on there are small shifts in the vacuum value (2.60) and the masses (2.62), and the pseudoscalars $\mathbf{a}, a$ acquire masses as in (2.58) with

$$c = 4m_0^2\Lambda^{\frac{7}{4}}(m_{\frac{3}{2}})^{\frac{3}{4}}\mu^{\frac{1}{2}}\left(3\langle\cos\omega'_Q\rangle - \beta\gamma\langle\cos\omega'_c\rangle\right), \qquad \beta = 1 - \frac{3}{1+b_c\ell}, \tag{2.64}$$

and the minimum is CP conserving with $\langle\cos\omega'_{Q,c}\rangle = \pm 1$ so as to make c positive.

The above supersymmetric model is not a realistic model for QCD for several reasons. The composite operators u_Q and $\mathbf{\Pi}|_{\theta=\bar\theta=0}$ are composed of gauginos and squarks that get large masses proportional to $m_{\frac{3}{2}}$, while the true light degrees of freedom are the quarks and gauge bosons. The corresponding composite operators are the axillary fields $F_Q, \mathbf{F}$ that have been eliminated by their equations of motion. We would like to trade the former for the latter. More precisely, the composite gauge fields are

$$S \sim (F\cdot F)_{QCD} = -F_Q + u_Q\bar M + \text{h.c.}, \qquad P \sim (F\cdot\tilde F)_{QCD} = 4\nabla^m B^Q_m. \tag{2.65}$$

The equation of motion for F_a forces the coefficient of S_a to vanish, consistent with the definition $g^{-2}(\Lambda_a) = 0$ of the $\mathcal{G}_a$ scale Λ_a, and (2.2) correctly reproduces [2] the running of g^{-2} from the string scale to the condensation scale provided supersymmetry is unbroken above that scale. This is not the case for QCD, and the effective "QCD" Lagrangian (2.9) or (2.24) is not valid below the scale Λ_c of supersymmetry breaking. The arguments of the logs are effective infra-red cut-offs. For gauginos and squarks, they should be replaced by the actual masses, as was done in (2.24) for quark supermultiplets with masses above the QCD condensation scale.

The gaugino and squark mass terms

$$\mathcal{L}_{\text{mass}} = -\frac{k'\rho_c}{16\ell}\left(e^{i\omega_c}\bar\lambda_R\lambda_L + \text{h.c.}\right) - m^2_{\frac{3}{2}}|\tilde q|^2 \tag{2.66}$$

are invariant under (2.3) which is spontaneously broken by the vacuum value $u_c \neq 0$, but remains an exact (nonlinearly realized) symmetry of the Lagrangian, since the anomaly can

be canceled by an axion shift as long as QCD nonperturbative effects can be neglected. Since U_c transforms the same way as U_Q an effective theory with the correct anomaly structure under (2.3) and (2.15) is obtained by replacing (2.24) by

$$\mathcal{L}^Q_{VYT} = \frac{1}{8}\int \mathrm{d}^4\theta\, \frac{E}{R} U_Q \left\{ b_1 \ln(e^{-K/2}U_Q) + b_2 \ln(e^{-K/2}U_c) + b_3 \left[\ln(\det \mathbf{\Pi}_n) - \sum_{A=1} \ln M_A \right] \right\} + \text{h.c.}, \tag{2.67}$$

provided

$$b_3 = \frac{1}{8\pi^2} = b^\alpha_Q, \qquad b_1 + b_2 = \frac{N_c - n}{8\pi^2} \equiv b'_n, \tag{2.68}$$

and we can choose b_1 and b_2 to better reflect the correct infrared cut-offs for squarks and gauginos. The potential is still of the form (2.42), except that the functions A_i depend on the parameter b_2/b_1, which modifies the masses (2.62), but the axion mass is unchanged since it depends only on $b_1 + b_2 = b'_n$. We may write the effective Lagrangian below the QCD scale in terms of the quark condensate $\mathbf{F}_n$ by using its equation of motion. Putting everything except the light pseudoscalars at their vacuum values, to leading order in $1/m_P$ we obtain

$$\begin{aligned} \bar{\mathbf{F}}_n &\approx \left(\sigma e^{-i\mathbf{P}'} - c_0\right) e^{i\delta}, \\ \mathbf{P}' &= \frac{\mu}{v\sqrt{2}}\left(\mathbf{a} - \frac{c'_a a}{\sqrt{n}}\right) = \frac{\mu}{v\sqrt{2}}\mathbf{a} - \frac{8\pi^2 b_c - N_c}{b_c n}\sqrt{\frac{k'}{2\ell}}a, \qquad c_0 = \mu^2 m_0, \end{aligned} \tag{2.69}$$

and the effective potential for the light pseudoscalars takes the form

$$c_0 \sigma' e^{i\mathbf{P}'} + \text{h.c.} + O(m_0^2) = c\,\mathrm{Tr}\mathbf{F}_n + \text{h.c.} + O(m_0^2), \qquad c = c_0 \frac{\sigma'}{\sigma}, \tag{2.70}$$

which is the standard result in QCD if Tr$\mathbf{F}$ is identified with the quark condensate. To check that this identification is correct, we note that above the condensation scales the Lagrangian contains the coupling

$$\mathcal{L} \ni -\frac{a}{4}\sqrt{\frac{k'}{2\ell}} \sum_a (F \cdot \tilde{F})_a \equiv -\frac{a}{4F} \sum_a (F \cdot \tilde{F})_a. \tag{2.71}$$

Under the Kähler $U(1)$ transformation (2.3) and the transformation (2.15) on the n light quark supermultiplets, the anomalies induce a shift

$$\delta\mathcal{L} \ni -\frac{1}{4}\left[\alpha b_c (F \cdot \tilde{F})_c + (\alpha b'_n + 2n\beta_3)(F \cdot \tilde{F})_Q\right], \tag{2.72}$$

which is canceled in the nonanomalous case (2.55) by the axion shift

$$a \to a - \alpha b_c \sqrt{\frac{2\ell}{k'}}, \tag{2.73}$$

This gives

$$\mathbf{F}_n \to e^{i\alpha b_{\mathbf{F}}}\mathbf{F}_n, \qquad b_{\mathbf{F}} = \frac{8\pi^2 b_c - N_c}{n}, \tag{2.74}$$

which matches the phase transformation of the quark condensate:

$$\chi_L \chi_L^c \to e^{i\alpha b_\chi}\chi_L\chi_L^c, \qquad b_\chi = 2\frac{\beta}{\alpha} - 1 = \frac{8\pi^2 b_c - N_c + n}{n} - 1 = b_{\mathbf{F}}. \tag{2.75}$$

For $n = 2$ we identify the factor $\exp(i\mu\mathbf{a}/v\sqrt{2})$ in the parametrization (2.38) with the operator $\Sigma = e^{2i\pi^i T_i/F_\pi}$ of standard chiral Lagrangians, where π^i are the canonically normalized pions, and T_i is a generator of $SU(2)$ normalized as in (2.39). That is, we identify a_i with π_i and

$$\mu/v = 2/F_\pi, \qquad F_\pi \approx 93\text{MeV} \tag{2.76}$$

giving

$$m_a = \frac{|8\pi^2 b_c - N_c|}{b_c n}\frac{\sqrt{n}F_\pi}{F\sqrt{2}}m_\pi = \frac{\sqrt{3}|8\pi^2 b_c - N_c|F_\pi}{2\sqrt{n}(1 + b_c\ell)}m_\pi, \tag{2.77}$$

where F is the axion coupling defined in (2.71), and we used (2.59). If we assume $b_c\ell \ll 1$, $b_c = .036$, which is the preferred [6] value for LSP dark matter in the BGW model [2], we are very close to the symmetric point for $N_c = 3$: $8\pi^2 b_c = 2.84$, so we get an (accidental) suppression of the axion mass; if $b_c\ell \ll 1$ and $n = 2$:

$$m_a \approx 5 \times 10^{-13}\text{eV}, \qquad F \approx 5.5 \times 10^{19}\text{GeV}. \tag{2.78}$$

The value of F is larger than the classical value with $k' = 1/\ell$, $\ell = g^2/2 \approx .25$, giving $F^{\text{class}} \approx 1/2\sqrt{2} \approx 8.6 \times 10^{17}$GeV, due to the corrections to the dilaton Kähler potential needed for dilaton stabilization. One also finds in the literature a different normalization for the axion couping

$$\mathcal{L} \ni -\frac{na}{32\pi^2 f_a}\sum_b (F \cdot \tilde{F})_b, \qquad f_a = \frac{nF}{8\pi^2}, \tag{2.79}$$

giving $f_a = 1.4 \times 10^{18}$GeV, and $f_a^{\text{class}} = 2.2 \times 10^{16}$, in agreement with the calculation of [21]. In models with an anomalous $U(1)$, there are other factors that determine the spectrum, and b_c can be quite different. In general these factors tend to raise the scale of supersymmetry breaking unless b_c is smaller and/or ℓ is considerably larger than its classical perturbative value $g_s^2/2 \approx 0.25$. Either of these would increase f_a and probably increase m_a by moving b_c away from the symmetric point.

The result (2.77) appears to differ from the standard result by a factor $1 - N_c/8\pi^2 b_c$. However, F is the axion coupling to Yang Mills fields *above* the scale of supersymmetry breaking. We will see in the next section that when we integrate out the gluinos of the supersymmetric extension of the Standard Model, we generate a correction that modifies the axion coupling strength F^{-1} to $(F\tilde{F})_Q$ by precisely that factor.

3 QCD

In the real world squarks and gauginos are unconfined at the scale of $\mathcal{G}_c$ condensation; they get masses proportional to $m_{\frac{3}{2}}$. Therefore we integrate them out, as well as the dilaton and moduli, to get an effective theory for quarks and gauge bosons. For present purposes, we can ignore the fact that quark masses come from Higgs couplings and just take the quark superpotential and Kähler potential to be

$$\begin{aligned} W_q(\hat{q},\hat{Q}) &= \eta^{-2}\left(me^{i\delta}\hat{q}^T\hat{q}^c+\hat{Q}^TM\hat{Q}^c\right), \qquad K_q=\hat{q}^\dagger\hat{q}+(\hat{q}^c)^\dagger\hat{q}^c+\hat{Q}^\dagger\hat{Q}+(\hat{Q}^c)^\dagger\hat{Q}^c, \\ \hat{q}^T &= (\hat{q}_1,\ldots,\hat{q}_n), \qquad \hat{Q}^T=(\hat{Q}_1,\ldots,\hat{Q}_m), \qquad m+n=N. \end{aligned} \tag{3.1}$$

The chiral superfields $\hat{Q}_A=(\tilde{Q}_A,Q_A,F_A)$ have masses $M_A\gg\Lambda_{QCD}$, and we have used a nonanomalous $SU(N)$ transformation to make their mass matrix real and diagonal and to diagonalize the mass matrix of the light quarks $\hat{q}_i=(\tilde{q}_i,q_i,f_i)$ which we take to have degenerate eigenvalues: $|m_i|=m\ll\Lambda_{QCD}$. The relevant part of the Lagrangian at the SUSY-breaking scale is (dropping kinetic terms for heavy fields and setting the moduli at self-dual points)

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm kin}-V-\mathcal{L}_Y, \qquad \chi=(q,q^c), \qquad \phi=(\tilde{q},\tilde{q}^c), \qquad X=(Q,Q^c), \qquad \Phi=(\tilde{Q},\tilde{Q}^c), \\ \mathcal{L}_{\rm kin} &= \frac{k'}{4\ell}B^mB_m-b_c\omega_c\nabla^mB^c_m-\frac{i}{2}\left(\bar{\chi}_L\gamma^mD_m\chi_L-\text{h.c.}\right)-\frac{1}{4g^2(m_{\frac{3}{2}})}F\cdot F, \\ V &= \frac{k'}{16\ell}\left|\rho_ce^{i\omega_c}(1+b_c\ell)+\bar{\lambda}_R\lambda_L-4\ell e^{K/2}W_q(\tilde{q},\tilde{Q})\right|^2 \\ &\quad -\frac{3}{16}\left|b_c\rho_ce^{i\omega_c}-4e^{K/2}W_q(\tilde{q},\tilde{Q})\right|^2+F^T\bar{F}+f^T\bar{f}, \\ \bar{F}_A &= \frac{1}{4}\left[b_c\rho_ce^{i\omega_c}\Phi^\dagger_A-4e^{K/2}W_q(\tilde{q},\tilde{Q})\Phi^\dagger_A-4(M_0)_A\Phi_A\right], \\ \bar{f}_i &= \frac{1}{4}\left[b_c\rho_ce^{i\omega_c}\phi^\dagger_i-4e^{K/2}W_q(\tilde{q},\tilde{Q})\phi^\dagger_i-4m_0e^{i\delta}\phi_i\right], \\ \mathcal{L}_Y &= Q^TM_0Q^c+e^{i\delta}m_0q^Tq^c+\left(\Phi^\dagger X+\phi^\dagger\chi\right)^2W_q(\tilde{q},\tilde{Q}) \\ &\quad -i\sqrt{2}\left[\chi^T(\lambda\cdot T)\bar{\phi}+X^T(\lambda\cdot T)\bar{\Phi}\right]+\text{h.c.}, \\ B_m &= {}^*(db+\Gamma)_m+\omega_m, \qquad D_m\chi=\mathcal{D}_m\chi+\frac{ik'}{4}B_m\chi, \end{aligned} \tag{3.2}$$

where $\mathcal{D}_m$ is a gauge covariant derivative, M_0 is defined analogously to m_0, and b_{mn}, Γ_{mnp} and ω_m are, respectively, a two-form potential, a three-from potential, and the Chern-Simons one-form for unconfined gauge fields: $\nabla^m\omega_m=\frac{1}{4}F\cdot\tilde{F}$.

If we neglect the small mass m_0, the Lagrangian (3.2) is invariant under the nonanomalous

transformation

$$\begin{aligned}
\lambda_a &\to e^{i\alpha/2}\lambda_a, \quad \Phi_A \to e^{i\alpha/2}\Phi_A, \quad X_A \to X_A, \quad \phi_i \to e^{i\beta\alpha}\phi_i, \quad \chi_i \to e^{i\gamma\alpha}\chi_i, \\
\omega_c &\to \omega_c + \alpha, \qquad \beta = \frac{b_c - b'_n}{2nb_3}, \qquad \gamma = \beta - \frac{1}{2} = \frac{8\pi^2 b_c - N_c}{2n},
\end{aligned} \tag{3.3}$$

where b'_n and b_3 are defined as in (2.68). In order to keep this approximate symmetry manifest in the low energy effective theory, we redefine the squark and quark fields so as to remove the ω_c-dependence from all terms in the Lagrangian for the heavy fields that do not involve the mass m_0:[9]

$$\lambda_a = e^{i\omega_c/2}\lambda'_a, \quad \Phi_A = e^{i\omega_c/2}\Phi'_A, \quad X_A = X'_A, \quad \phi_i = e^{i\beta\omega_c}\phi'_i, \quad \chi_i = e^{i\gamma\omega_c}\chi'_i. \tag{3.4}$$

The primed fields are invariant under (3.3), and when expressed in terms of them, V and $\mathcal{L}_Y$ have no dependence on ω_c when $m_0 \to 0$; this assures that any effects of integrating out the heavy fields will be suppressed by powers of m_0/M_A, $m_0/m_{\frac{3}{2}}$ relative to the terms retained. However, these transformations induce new terms in the effective Lagrangian. First, because the transformation (3.4) with ω_c held fixed is anomalous, it induces a term

$$\mathcal{L}' \ni \Delta\mathcal{L} = -\frac{\omega_c b_c}{4}(F \cdot \tilde{F})_Q. \tag{3.5}$$

Secondly there are shifts in the kinetic terms; the ones that concern us here are the fermion derivatives:

$$\partial_m \lambda_L = e^{i\omega_c/2}\left(\partial_m \lambda'_L + \frac{i}{2}\partial_m \omega_c \lambda'_L\right), \qquad \partial_m \chi_L = e^{i\gamma\omega_c}\left(\partial_m \chi'_L + i\gamma\partial_m \omega_c \chi'_L\right), \tag{3.6}$$

which corresponds to a shift in the axial connections A_m in the fermion connections:

$$\Delta A^\lambda_m = -\frac{1}{2}\partial_m \omega_c, \qquad \Delta A^\chi_m = -\gamma\partial_m \omega_c. \tag{3.7}$$

Quantum corrections induce a nonlocal operator coupling the axial connection to $F\tilde{F}$; at scales $\mu^2 \sim \Box \gg m_\lambda^2$ through the anomalous triangle diagram:

$$\mathcal{L}_{\rm qu} \ni -\frac{1}{4}(F \cdot \tilde{F})_Q \frac{1}{\Box}\left(\frac{N_c}{4\pi^2}\partial^m A^\lambda_m + \frac{n}{2\pi^2}\partial^m A^\chi_m\right). \tag{3.8}$$

The contribution to (3.8) from the shift (3.7) exactly cancels the shift (3.5) in the tree level Lagrangian, leaving the $\omega F\tilde{F}$ S-matrix element unchanged by the redefinition (3.4). However

[9] We are implicitly making invariant other heavy fields, such as the dilatino ($\chi'_\ell = e^{i\omega_c}\chi_\ell$) and gravitino ($\psi'_\mu = e^{-i\omega_c}\psi_\mu$), that also transform under (3.3) in order to insure invariance of the full classical Lagrangian.

at scales $\mu^2 \ll m_\lambda^2$, we replace $\Box \to m_\lambda^2$ in the first term of (3.8) because the contribution decouples, but the analogous contribution (3.5) to the tree Lagrangian $\mathcal{L}'$ remains in the effective low energy Lagrangian. This is a reflection of the fact that the classical symmetry (3.3) of the unprimed variables is anomalous. The gluino contribution to that anomaly is not canceled by the gluino mass term, because the gluino mass does not break the symmetry; its phase ω_c is undetermined above Λ_{QCD} and transforms so as to make the mass term invariant. To see that the gluino contribution to the anomaly does not decouple, we write the (unprimed) gaugino contribution to the one-loop action as

$$S_1 = -\frac{i}{2}\mathrm{Tr}\ln(i\not{D} + m_\lambda) = S_A + S_N, \tag{3.9}$$

where

$$S_A = -\frac{i}{2}\mathrm{Tr}\ln(i\not{D}) \tag{3.10}$$

is mass-independent and contains the gaugino contribution to the anomaly:

$$\delta\mathcal{L} \ni \delta S_A = -\frac{\alpha N_c}{32\pi^2}(F\cdot\tilde{F})_Q. \tag{3.11}$$

The mass-dependent piece

$$S_N = -\frac{i}{2}\mathrm{Tr}\ln(-i\not{D} + m_\lambda) + \frac{i}{2}\mathrm{Tr}\ln(-i\not{D}) \tag{3.12}$$

is finite and therefore nonanomalous. A constant mass term would break the symmetry and the contribution from S_N would exactly cancel that from S_A in the limit $\mu/m_\lambda \to 0$. However it clear that S_N is invariant under (2.3) because the gaugino mass is covariant. In Appendix A we explicitly show by direct calculation that gaugino loops give the contribution (3.11) under (3.3) in the limit $m_\lambda \gg \mu$, which in this limit arises only from the phase of the mass matrix. This implies that the effective low energy theory must contain a coupling

$$\mathcal{L}_{\mathrm{eff}} \ni \mathcal{L}_{\mathrm{anom}} = -\frac{\omega_c N_c}{32\pi^2}(F\cdot\tilde{F})_Q, \tag{3.13}$$

which is precisely the term that is generated by the redefinitions in (3.4). The difference $(S_1)_\lambda - (S_1)_{\lambda'}$ in the one-loop actions calculated with primed and unprimed gaugino fields is just given by the first expression for δS_1 in (A.4), in the limit of small $\omega \to \alpha$, which can trivially be integrated to include arbitrary ω. In order to respect the full classical invariance of the Lagrangian, we have to include the transformation on the squarks and quarks, giving the effective coupling in (3.5).

Setting the squarks and gauginos, as well as the heavy quarks, to zero gives the effective light field tree Lagrangian at a scale $\Lambda_{QCD} < \mu < M_A$:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\rm kin} - V - \mathcal{L}_Y + \Delta\mathcal{L}, \qquad V = \frac{1}{16\ell}\left[k'(1+b_c\ell)^2 - 3b_c^2\ell\right]\rho_c^2, \\
\mathcal{L}_{\rm kin} &= \frac{k'}{4\ell}B^m B_m - b_c\omega_c\nabla^m B_m^c - \frac{i}{2}\left(\bar{\chi}'\gamma^m D_m\chi' - \text{h.c.}\right) - \sum_a \frac{1}{4g_a^2(\mu)}(F\cdot F)_a, \\
D_m\chi'_L &= \mathcal{D}_m\chi'_L + \frac{ik'}{4}B_m\chi'_L + i\gamma\partial_m\omega_c\chi'_L, \qquad \mathcal{L}_Y = e^{i(\delta+2\gamma\omega_c)}m_0\bar{q}'_R q'^c_L + \text{h.c.}, \\
\nabla^m B_m &= \nabla^m B_m^c + \frac{1}{4}F\cdot\tilde{F} = \nabla^m({}^*\Gamma)_m + \frac{1}{4}F\cdot\tilde{F}.
\end{aligned} \tag{3.14}
$$

The equations of motion for Γ and ω_c give (the equation for b_{mn} is redundant; this field can be absorbed into Γ by a gauge transformation $\Gamma \to \Gamma - db$)

$$
\begin{aligned}
0 &= \frac{k'}{2\ell}B_m + b_c\nabla_m\omega_c + \frac{k'}{2}j_m \qquad j_m = \bar{\chi}'_L\sigma_m\gamma_5\chi'_L, \\
0 &= b_c\nabla^m B_m^c + \frac{\partial}{\partial\omega_c}\mathcal{L}_Y - 2\gamma\nabla^m j_m + \frac{b_c}{4}(F\cdot\tilde{F})_Q \\
&= b_c\left(\nabla^m B_m - \frac{1}{4}F\cdot\tilde{F}\right) + \frac{\partial}{\partial\omega_c}\mathcal{L}_Y - 2\gamma\nabla^m j_m + \frac{b_c}{4}(F\cdot\tilde{F})_Q \\
&= -b_c\left[\nabla^m\left(\frac{2\ell}{k'}b_c\nabla_m\omega_c + \ell j_m\right) + \frac{1}{4}F\cdot\tilde{F}\right] \\
&\quad + \frac{\partial}{\partial\omega_c}\mathcal{L}_Y + 2\gamma\nabla^m j_m + \frac{b_c}{4}(F\cdot\tilde{F})_Q,
\end{aligned} \tag{3.15}
$$

which is the scalar equation of motion for the equivalent Lagrangian

$$
\begin{aligned}
\mathcal{L} &= -\frac{\ell}{k'}\partial^m\omega\partial_m\omega - \frac{i}{2}\left(\bar{\chi}'_L\gamma^m D_m\chi'_L - \text{h.c.}\right) - \left[e^{i(\delta+2\gamma\omega/b_c)}m_0 q'^T q'^c + \text{h.c.}\right] \\
&\quad - \sum_a \frac{1}{4g_a^2(\mu)}(F\cdot F)_a + \frac{\omega}{4}[F\cdot\tilde{F} - (F\cdot\tilde{F})_Q], \\
D_m\chi'_L &= \mathcal{D}_m\chi'_L + \frac{i}{2b_c}\left(2\gamma - b_c\ell\right)\partial_m\omega\chi'_L, \qquad \omega = b_c\omega_c.
\end{aligned} \tag{3.16}
$$

If we ignore m_0 the QCD part of (3.16) is invariant under a shift in ω by a constant, which is the same as the nonanomalous symmetry (3.3), after the redefinitions (3.4).

From now on we drop the primes on the quark fields. We define the canonically normalized axion as in (2.53) and include explicitly the QCD instanton-induced term, since it cannot be treated as a total derivative when we approach the QCD scale where we obtain an effective Lagrangian for pseudoscalars. Then, in terms of four-component Dirac spinors,

the Lagrangian (3.16) reads,

$$\begin{aligned}
\mathcal{L} &= -i\bar{q}\,\not{D}q - m_0\left(e^{i(\delta-a/f)}\bar{q}_R q_L + \text{h.c.}\right) - \frac{1}{2}\partial_m a\partial^m a \\
&\quad -\frac{1}{4}\left(g_\gamma^{-2}(\mu)F^2 + \sqrt{\frac{k'}{2\ell}}aF\cdot\tilde{F}\right)_\gamma - \frac{1}{4}\left(g_Q^{-2}(\mu)F^2 + \frac{\theta}{8\pi^2}F\cdot\tilde{F}\right)_Q, \\
\not{D}q &= \not{\partial}q - \frac{i}{2}\sqrt{\frac{k'\ell}{2}}\left(1-\frac{2\gamma}{b_c\ell}\right)\not{\partial}a\gamma_5 q, \qquad f^{-1} = \sqrt{\frac{k'}{2\ell}}\frac{8\pi^2 b_c - N_c}{nb_c},
\end{aligned} \tag{3.17}$$

where the subscript γ stands for[10] QED. The first term in the quark connection is the standard one; in the classical limit $k' = \ell^{-1}$ it reduces to $-i\gamma_5 a/2\sqrt{2} = -i\gamma_5 \text{Im}s/4\text{Re}s$. The second term is a result of the quark field redefinition in (3.4). The Lagrangian (3.17) has a classical symmetry

$$a \to a + \alpha, \qquad q \to e^{-i\gamma_5\alpha/f}q \tag{3.18}$$

that is anomalous unless (neglecting $\delta\mathcal{L}_{QED}$) $f^{-1} = 0$, which is just the condition (2.56) found previously. We can now make an anomalous chiral transformation on the quarks to remove the θ term from (3.17):[11]

$$q \to e^{i\theta\gamma_5/n}q. \tag{3.19}$$

Below the QCD confinement scale the physical degrees of freedom are the pions; with the usual parameterization

$$v^3 e^{i\phi}\Sigma^a_b = v^3(e^{i(\phi+2\boldsymbol{\pi}/F_\pi)})^a_b \sim q^a_L(\bar{q}_R)_b, \qquad \boldsymbol{\pi} = \frac{1}{2}\sum_i \pi^i\lambda_i, \qquad \langle\boldsymbol{\pi}\rangle = 0. \tag{3.20}$$

Using standard chiral symmetry arguments we get the effective Lagrangian

$$\begin{aligned}
\mathcal{L}_{eff} &= \frac{1}{4}F_\pi^2\text{Tr}\left(\partial_\mu\Sigma\partial^\mu\Sigma^\dagger\right) + \frac{1}{2}\partial_\mu a\partial^\mu a + iF_\pi^2\sqrt{\frac{k'\ell}{2}}\left(1-2\gamma/b_c\ell\right)\partial_\mu a\text{Tr}\{\Sigma,\partial^\mu\Sigma^\dagger\} \\
&\quad +\frac{1}{2}\lambda v^3\left[\text{Tr}(e^{-ia'/f}\Sigma m_0) + \text{h.c.}\right], \qquad a' = a - f(\delta+\phi-2\theta/n).
\end{aligned} \tag{3.21}$$

The third term on the RHS's of (3.21) is the coupling of the universal axion to the chiral $U(1)$ Noether current as implied by (3.17). Since $\text{Tr}\boldsymbol{\pi} = 0$ it does not introduce any mixing of the axions with the pions. The potential for the light pseudoscalars $a, \boldsymbol{\pi}$ is identical to that in (2.70). Since since $m_0\langle\Sigma\rangle$ is real the potential is an even function of a' and has a

[10] We have ignored a shift in the axion coupling to $(F\tilde{F})_\gamma$ analogous to (3.5) which is canceled in the S-matrix element by the loop contribution analogous to (3.8).

[11] See for example the discussion in Ch. 23.6 of [22].

minimum at $\langle a' \rangle = 0$, so CP is conserved. If we take $n = 2$ and allow for $m_u \neq m_d$, the mass terms are

$$\begin{aligned} V_m &= \frac{1}{2}\lambda v^2 \left[(m_u + m_d) \left(\frac{\vec{\pi}^2}{F_\pi^2} + \frac{a'^2}{f^2} \right) - 2\,(m_u - m_d)\,\frac{a'\pi^0}{f F_\pi} \right] \\ &\approx \frac{1}{2} m_\pi^2 \left[2\pi^+\pi^- + \left(\pi^0 + \frac{F_\pi(1-z)}{f(1+z)} a' \right)^2 \right] + \frac{1}{2} m_a^2 a_0^2, \end{aligned} \tag{3.22}$$

where

$$a_0 \approx a' - \frac{F_\pi(1-z)}{f(1+z)}\pi^0, \qquad m_a \approx 2m_\pi \frac{F_\pi\sqrt{z}}{f(1+z)}, \qquad z = \frac{m_u}{m_d}. \tag{3.23}$$

To see that this[12] is the standard result [23] for $n = 2$, we note that if we undo the redefinition of the quarks in (3.4) by a transformation $q \to q'' = e^{-ia\gamma_5/2f} q$, we put back a term

$$\mathcal{L} \ni -\frac{na}{32\pi^2 f}(F \cdot \tilde{F})_Q = -\sqrt{\frac{k'}{2\ell}} \left(1 - \frac{b_0}{b_c} \right) \frac{a}{4} (F \cdot \tilde{F})_Q, \tag{3.24}$$

which differs from the universal axion coupling (2.71) by the gaugino contribution (3.13) that is generated when the gauginos are integrated out.

4 CP violation

At the point of enhanced symmetry $b_c = 8\pi^2 N_c$, the nonanomalous symmetry (3.3) does not include a chiral transformation on the quarks, and one loses the solution to the CP problem. The axion decouples from the quarks in the effective Lagrangian (3.17), and its *vev* cannot be adjusted to make the quark mass matrix real in the $\theta = 0$ basis. There is no reason to expect that nature sits at this point, but if the axion mass is very small one should worry about other sources of an axion potential, such as higher dimension operators [1]. These were studied in [7] in the context of the modular invariant gaugino condensation models considered here. Modular invariance severely restricts the allowed couplings; the leading contribution to the axion mass takes the form

$$m_a'^2 \approx \frac{p^3 |u|^2 k' \lambda |\eta^2 e^{-K/2} u|^p}{4 b_c^2 \ell} \left[3b_c - (1 + b_c \ell) k' \right] \tag{4.1}$$

where λ is a dimensionless coupling constant, and p is the smallest integer allowed by T-duality. An orbifold compactification model with three complex moduli and an $[SL(2, \mathbf{Z})]^3$

[12] The coupling constant f_a used in [23] is a factor two larger than the one defined in (2.79) and used in [21]. Taking this into account we agree with [23], but differ by a factor two with [21] and [22]. The latter uses $F_\pi = 186$MeV, and factors of two are missing in the arguments of substitutions $\bar{q}q \to f(\pi^0/F_\pi)$.

symmetry has $p = 12$, and, with the values of the various parameters used above and $\lambda \approx 1$, one finds $m'_a \approx 10^{-63}$eV, which is completely negligible. However if the symmetry is restricted, for example, to just $SL(2, \mathbf{Z})$ one has $p = 4$ and the contribution from (4.1) is of the order of (2.78). The axion potential is now

$$V(a) = -f^2 m_a^2 \cos(a/f + \phi_0) - f'^2 m_a'^2 \cos(a/f'), \qquad f'^{-1} = \frac{p}{b_c} F^{-1} = \frac{pn}{8\pi^2 b_c - N_c} f^{-1}, \quad (4.2)$$

where we have absorbed constant phases in a and/or in $\phi_0 = -(\delta + \phi + 2\theta/n)$ so as to make the coefficients negative. The strong CP problem is avoided if for some value of b_c the vacuum has $\langle \bar{\theta} \rangle = \langle n(a/f + \phi_0)/2 \rangle < 10^{-9}$ for any value of ϕ_0. For values of b_c in the preferred [5, 6] range $.3 \leq b_c \leq .4$ this does not occur. For example for $b_c = .036$ with $p = 4$ and $f'/f \approx 1/50$, this requires $f'^2 m_a'^2 / f^2 m_a^2 < 10^{-10}$, whereas evaluating (4.2) gives $f'^2 m_a'^2 / f^2 m_a^2 \approx 4 \times 10^{-4}$ in this case. A numerical analysis shows that the CP problem is avoided provided $p \geq 5$, that is, provided the T-duality group is not the minimal one, which is the case for most compactifications of the weakly coupled heterotic string.

5 Conclusions

We have shown that there is an enhanced symmetry point where the universal string axion mass vanishes even in the presence of quark masses in string-derived models where supersymmetry is broken by condensation of a simple gauge group in a hidden sector. As a consequence, the axion mass can be suppressed relative to conventional estimates [21] for the mass of the string axion. The conditions under which the universal axion can serve as the Peccei-Quinn axion were examined and it was found that the strong CP problem is avoided for all but the minimal $SL(2, \mathbf{Z})$ version of the T-duality group of the weakly coupled heterotic string. Most compactifications have a larger T-duality group, and from a phenomenological point of view, a larger group is desirable for generating [25] the R-parity of the MSSM. Although our results were obtained using the linear supermultiplet formulation for the dilaton superfield, we expect that they can be reproduced in the chiral multiplet formulation. The implications of our results for cosmological observation will be presented elsewhere.

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Appendix

A Anomaly

Consider the tree Lagrangian for a Majorana fermion λ

$$\begin{aligned}
\mathcal{L} &= -\frac{i}{2}\bar{\lambda}\,\not{D}\lambda - \frac{1}{2}\bar{\lambda}_L\bar{m}\lambda_R + \text{h.c.}, \qquad D_m = \mathcal{D}_m + iA_m\gamma_5, \qquad \mathcal{D}_m = \partial_m + iT\cdot a_m, \\
A_m &= \frac{1}{2}\partial_m\omega, \qquad \bar{m} = m^\dagger = e^{i\omega}\mu,
\end{aligned} \tag{A.1}$$

where a_m is a gauge field. The classical Lagrangian is invariant under

$$\lambda_L \to e^{i\alpha/2}\lambda_L, \qquad \omega \to \omega + \alpha. \tag{A.2}$$

This symmetry is broken at the quantum level by the anomaly:

$$\delta\mathcal{L} \ni -\frac{\alpha}{32\pi^2}F\cdot\tilde{F}. \tag{A.3}$$

For $\mu \to 0$ this is just determined by the standard triangle diagram for the a^2A three-point function. If the mass of λ were constant it would explicitly break the symmetry (A.2) and the explicit breaking would exactly cancel the anomalous breaking, given no contribution to the 3-point function at momentum scales $|p^2| \ll \mu^2$. However since the mass term in (A.1) respects the symmetry, the contribution to (A.3) is independent of the mass parameter μ. Here we show this explicitly in the limit of very large mass, using the methods of [24]. Under (A.2) the effective action (3.9) changes by

$$\begin{aligned}
\delta S_1 &= -\frac{i}{2}\text{Tr}\ln(i\,\not{D} - \gamma_5\delta\,\not{A} + m_\lambda + \delta m) + \frac{i}{2}\text{Tr}\ln(i\,\not{D} + m_\lambda) \\
&= -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\sum_{n=0}^{\infty}\text{Tr}(-\mathcal{R})^n\delta\mathcal{R}, \\
\mathcal{R} &= -\frac{1}{p^2-\mu^2}\left(\{p^m, G_m\} + G^mG_m - \frac{i}{2}\sigma\cdot\hat{G} + i[\widehat{\not{D}, m_\lambda}]\right), \qquad \sigma_{mn} = \frac{i}{2}[\gamma_m,\gamma_n], \\
\delta\mathcal{R} &= -\frac{1}{p^2-\mu^2}\left(\not{p} - m_\lambda\right)\left(\delta\,\not{A} - \delta m_\lambda\right), \\
m_\lambda &= \mu e^{i\omega\gamma_5}, \qquad \hat{f} = e^{-iD\cdot\partial/\partial p}f(x)e^{iD\cdot\partial/\partial p}, \qquad G_{mn} = [D_m, D_n],
\end{aligned} \tag{A.4}$$

and G_m is also an expansion in the operator $D \cdot \partial/\partial p$ acting on G_{mn}. For $\mu \to 0$, each term in the sum is infrared divergent, and the derivative expansion must be resummed to give (A.3). For large μ the only contribution to (A.3) that is not proportional to an inverse power of μ^2 involves $\delta m_\lambda = i\gamma_5 \alpha m_\lambda$ and $G_{mn} \ni iF_{mn}$:

$$\begin{aligned} \delta S_1 &\ni -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\mathcal{R}^2 \delta\mathcal{R} \ni \frac{i}{8}\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\frac{(\sigma\cdot G)^2 m_\lambda \delta m_\lambda}{(p^2+\mu^2)^3} \\ &= -\int \frac{ip^2 dp^2}{16\pi^2}\frac{\mu^2 i\alpha}{(p^2+\mu^2)^3} G\cdot\tilde{G} = -\frac{\alpha}{32\pi^2} F\cdot\tilde{F}. \end{aligned} \tag{A.5}$$

In the models considered the symmetry is only global, $\alpha =$ constant in (A.2), but this has no bearing on the calculation or the anomaly. When we sum over all the gaugino contributions we get a factor N_c.

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