

# **Noninvariance of Space/Time-Scale Ranges under a Lorentz Transformation. Implications for the Study of Relativistic Interactions.**

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# Special relativity

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## Lorentz transformation (v along x)

$$\begin{aligned}t' &= \gamma (t - vx/c^2) & \gamma &= (1 - v^2/c^2)^{-1/2} \\x' &= \gamma (x - vt) \\y' &= y \\z' &= z\end{aligned}$$

## Time dilation/space contraction

$$\begin{aligned}\text{at rest: } \Delta t, \Delta x=0 &\rightarrow \text{in motion: } \Delta t' = \gamma \Delta t \\ \Delta x, \Delta t=0 & \Delta x' = \Delta x / \gamma\end{aligned}$$

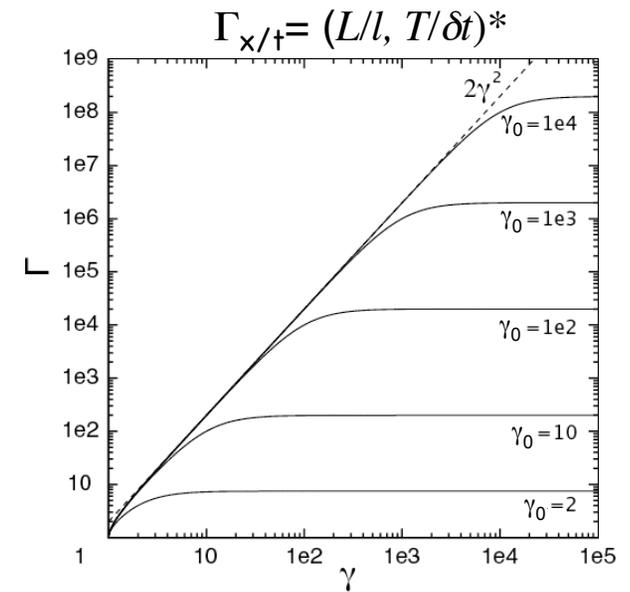
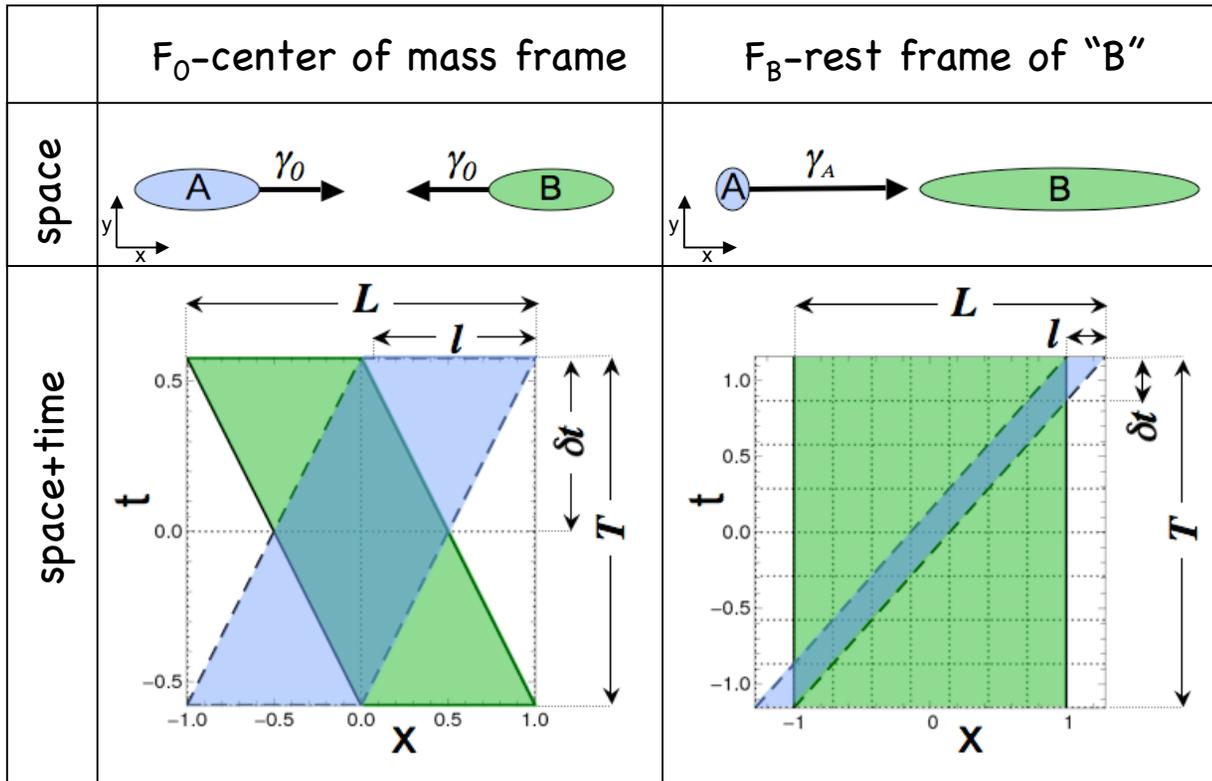
## Lorentz invariant (invariant to change of reference frame)

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2 \Delta t'^2$$

# Range of space and time scale of a simple system

## two identical objects crossing each other

same event as seen in two frames



$$\Gamma_{x/t} \propto \gamma^2$$

$$\Gamma = \Gamma_x \cdot \Gamma_t \propto \gamma^4$$

- $\Gamma$  is **not invariant** under the Lorentz transformation.
- There exists an **optimum** frame which **minimizes** ranges.
- For PIC, Vlasov, fluid methods,  $\text{cost} \propto \Gamma \Rightarrow$  **huge** penalty if calculation **not** performed in optimum frame!

\*J.-L. Vay, PRL 98, 130405 (2007)

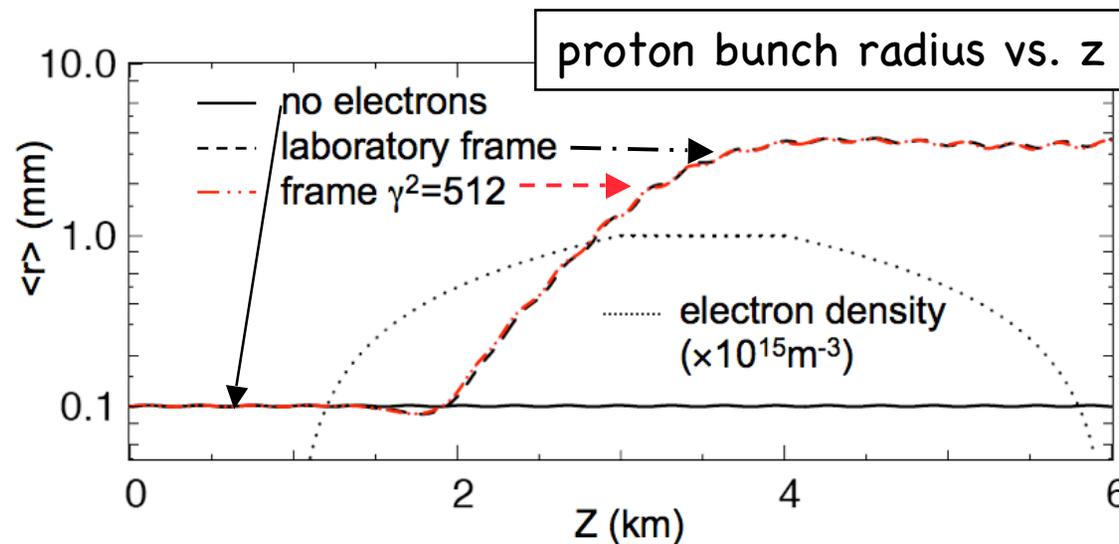
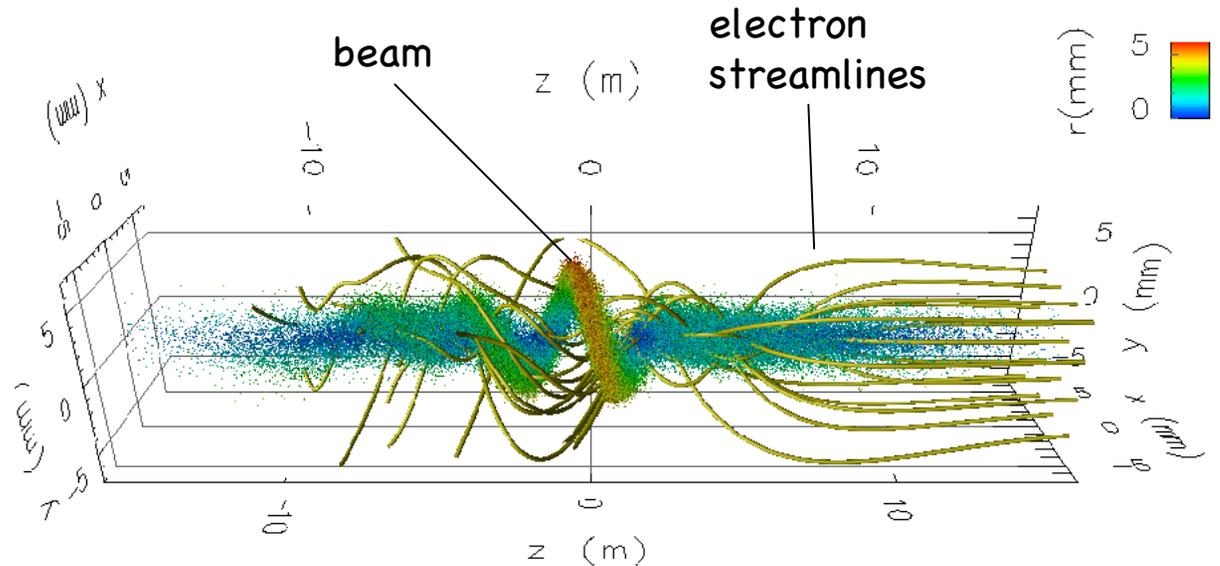
# Boosted frame calculation sample

## proton bunch through a given $e^-$ cloud\*

This is a proof-of-principle computation: hose instability of a proton bunch

Proton energy:  $\gamma=500$  in Lab  
 • L= 5 km, continuous focusing

Code: WARP (Particle-In-Cell)



CPU time:

- lab frame: >2 weeks
- frame with  $\gamma^2=512$ : <30 min

**Speedup x1000**

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# “This sounds like a free lunch”. How is this possible?

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Conventional scientific wisdom: the “complexity” of a system is invariant to a change of reference frame. Is that so?

In order to respond to this question, one needs a definition of complexity which allows quantification, i.e. units of complexity.

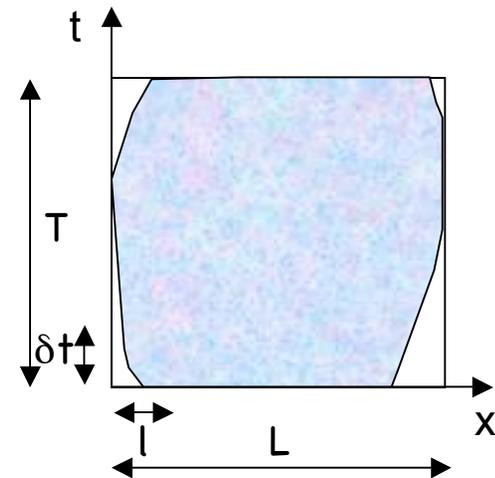


# One possible definition

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Complexity = range of spatial scales x range of time scales

The complexity ( $\Gamma$ ) of a system of total length  $L$ , shorter space scale  $l$ , shorter time scale  $\delta t$ , evolving for a total time  $T$ , is then given by



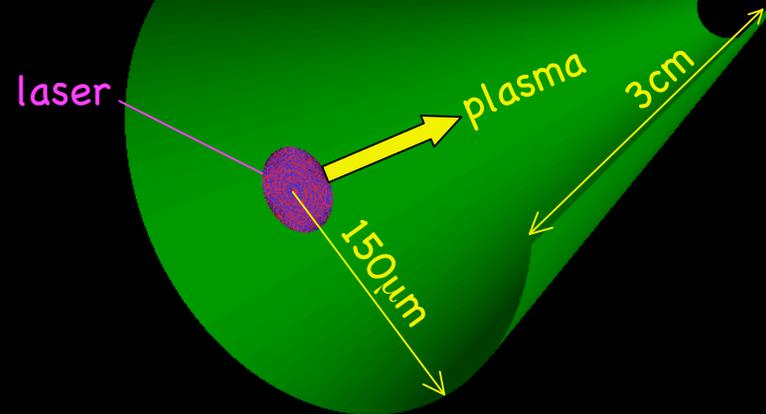
$$\Gamma = L/l \cdot T/\delta t$$

This definition has some practical merit: difficulty in experiments or calculations often scales with ratio of scales to cover.

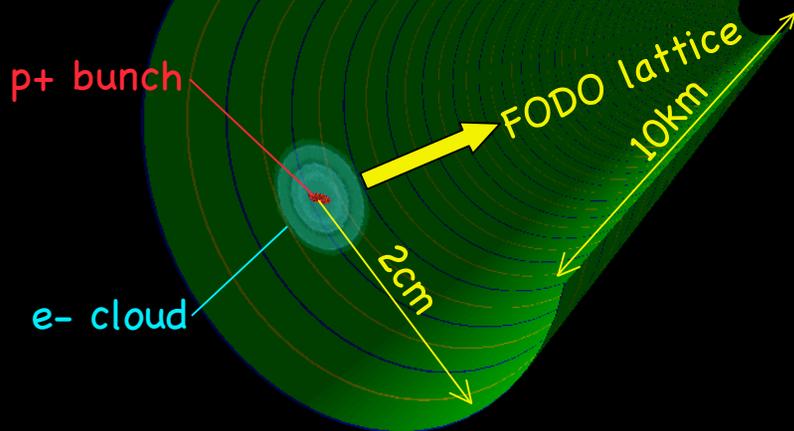
# A few systems of interest

## In the laboratory

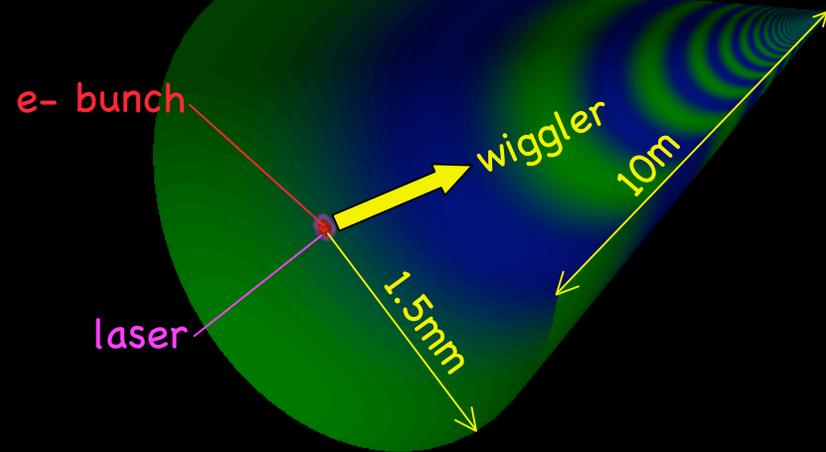
### Laser-plasma acceleration



### HEP accelerators (e-cloud)



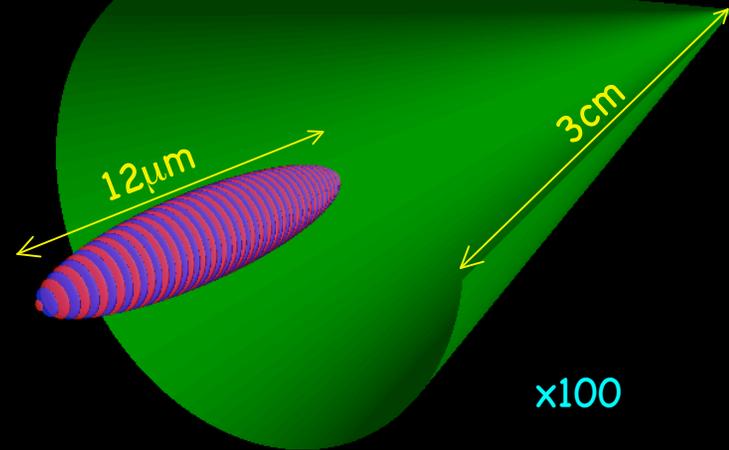
### Free electron lasers



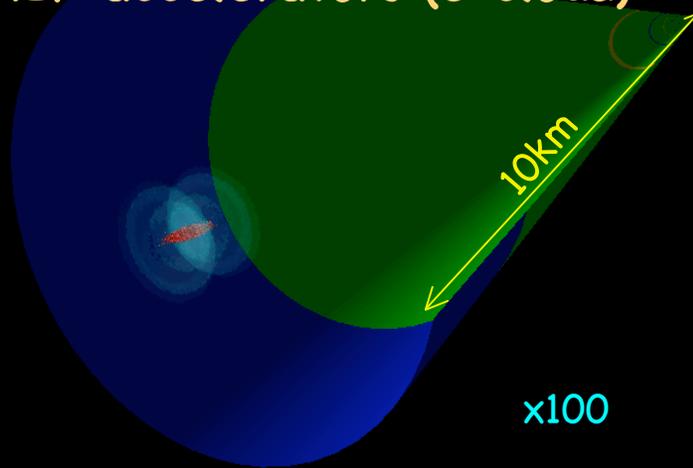
# A few systems of interest

Scaling longitudinally by x100...

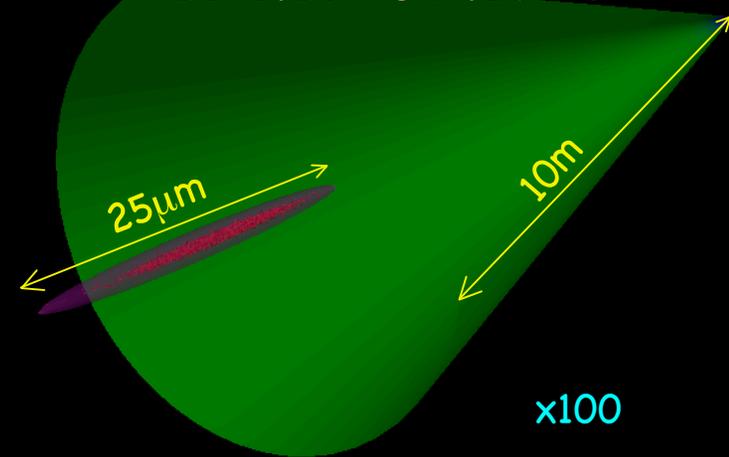
Laser-plasma acceleration



HEP accelerators (e-cloud)



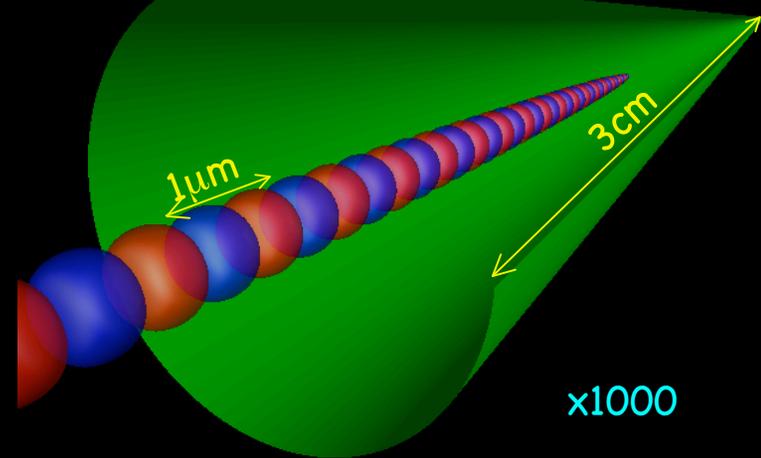
Free electron lasers



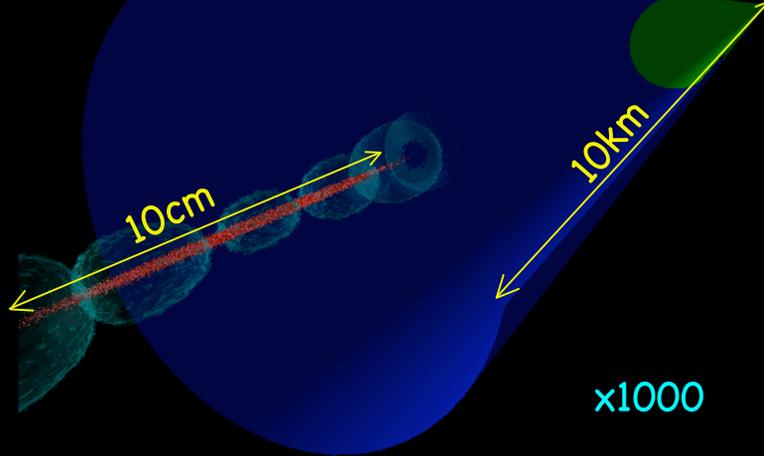
# A few systems of interest

Scaling longitudinally by x1000...

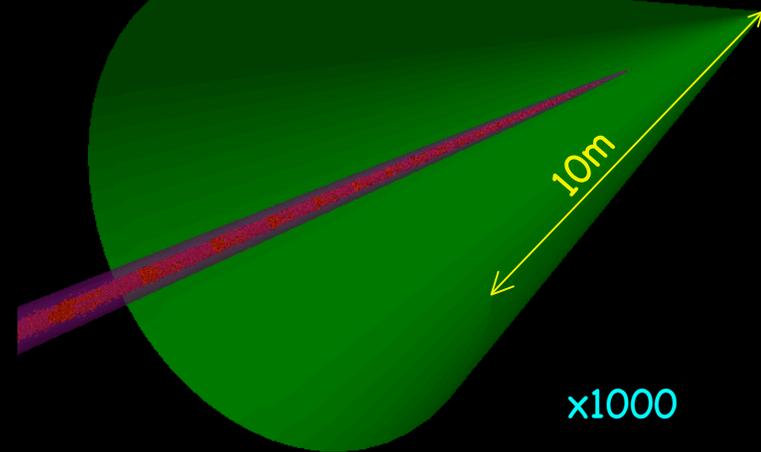
Laser-plasma acceleration



HEP accelerators (e-cloud)



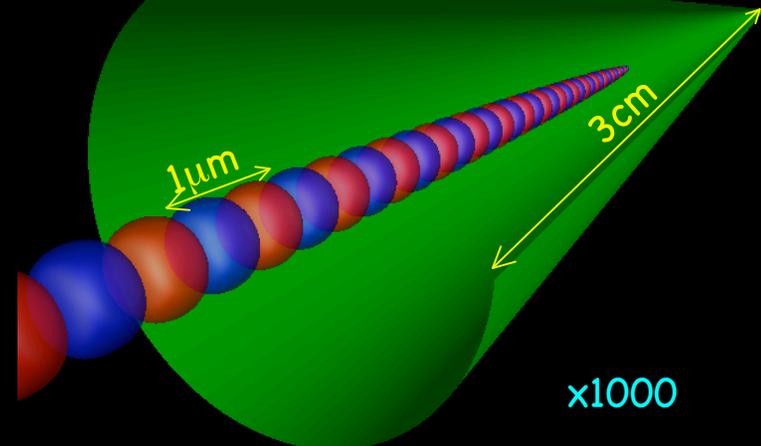
Free electron lasers



# A few systems of interest

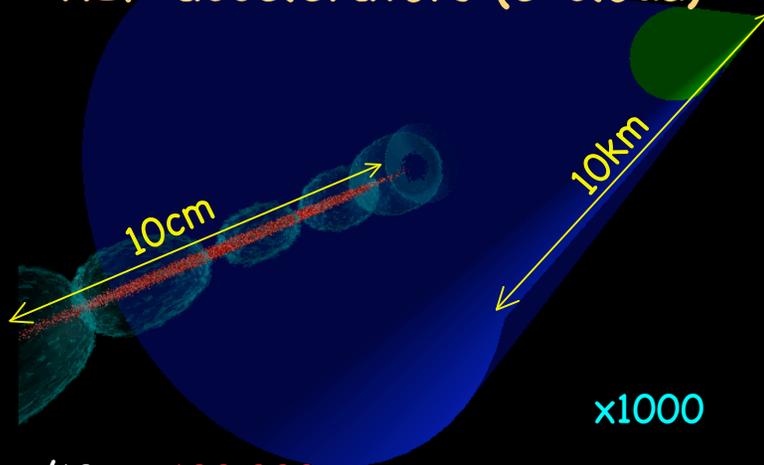
... so-called "multiscale" problems  
= very challenging to model!  
Use of approximations  
(quasi-static, eikonal, ...).

## Laser-plasma acceleration



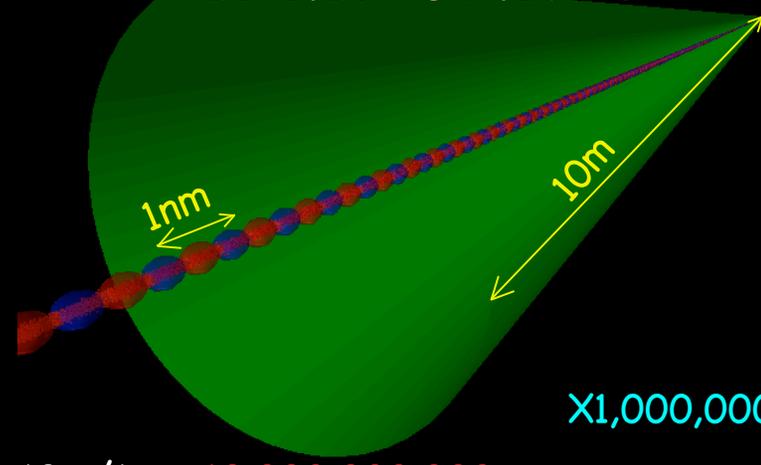
$$3\text{cm}/1\mu\text{m}=30,000.$$

## HEP accelerators (e-cloud)



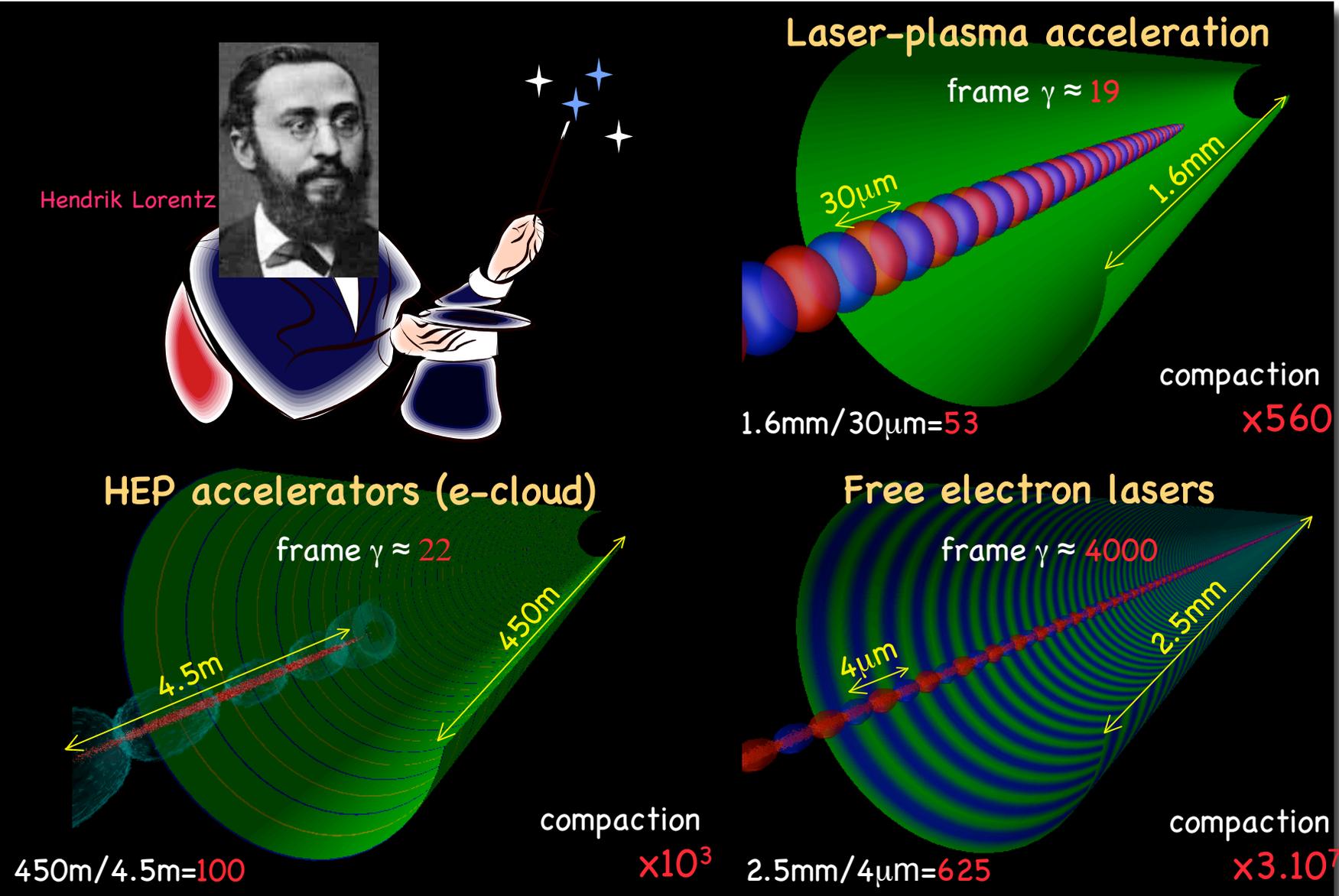
$$10\text{km}/10\text{cm}=100,000.$$

## Free electron lasers



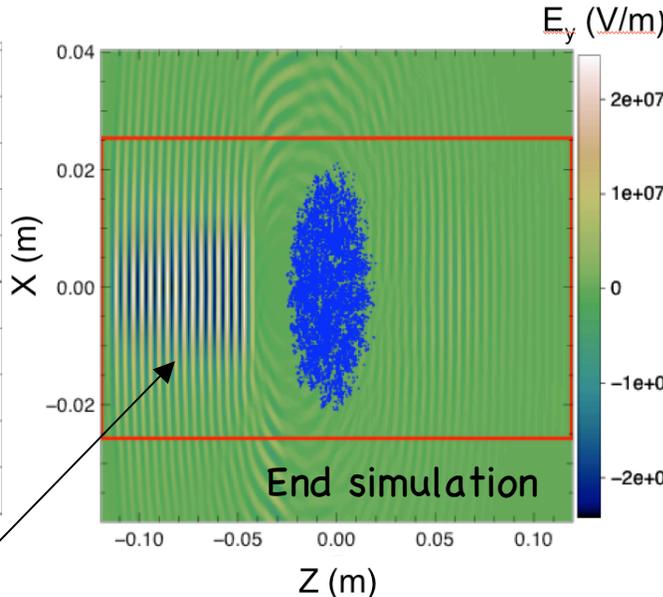
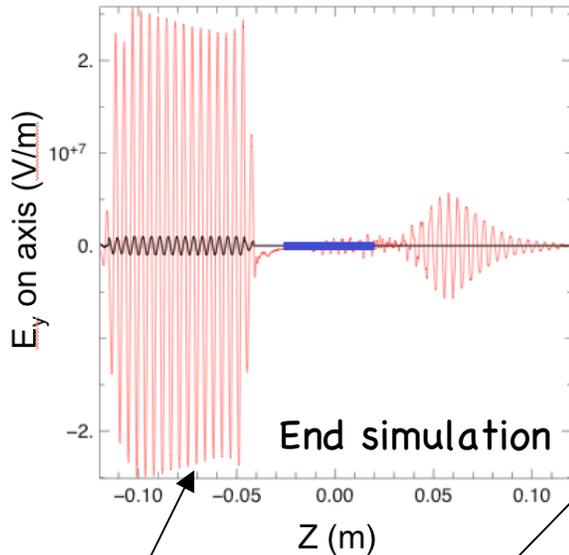
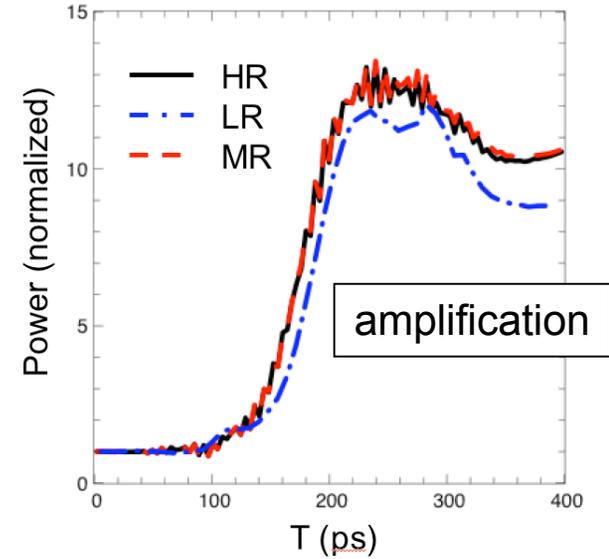
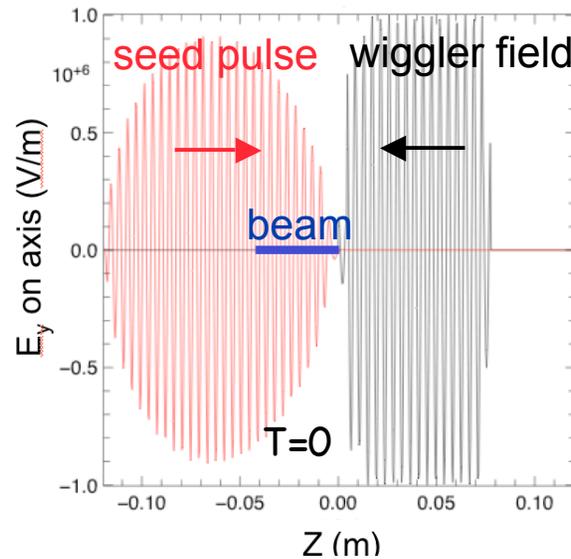
$$10\text{m}/1\text{nm}=10,000,000,000.$$

# Optimum frame => large level of compaction of scale ranges



# Demonstration on the modeling of a toy 2-D FEL problem

- e- and e+ beams
  - o current: 160kA
  - o energy: 76MeV ( $\gamma \approx 150$ )
- wiggler in lab frame:
  - o period: 43.6cm
  - o field: 37.5mT
- seed pulse (in bucket frame):
  - o wavelength: 4.3mm
  - o length: 12.cm
  - o Max electric field: 1.e6V/m



Calculation required **1000** time steps in the "bucket" frame (performed on this laptop). In the **lab frame**, the same calculation would have required  $1000 \times 2\gamma^2 \approx 45,000,000$  time steps.

**Speedup  $\approx 45000$**

Backward emitted wave is neglected in Eikonal approximation: we get more physics!

# Demonstration on the modeling of a toy 1-D LPWA problem

self-modulation instability in capillary discharge channels.

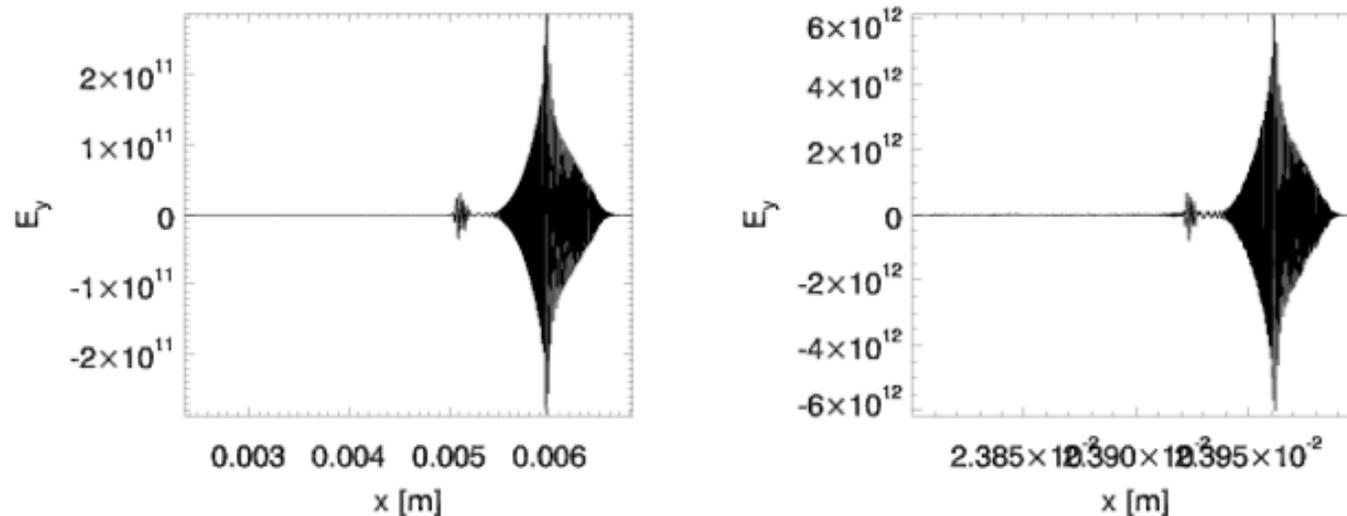


Fig. 1. (left) Self-modulated laser pulse from 1D PIC simulation in boosted frame, with 100x speedup; (right) self-modulated laser pulse from 1D PIC simulation in lab frame.

(courtesy D. Bruhwiler, J. Cary, Tech-X)



The Heavy Ion Fusion Science Virtual National Laboratory



# Summary

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- The range of scales  $\Gamma$  of a system is not a Lorentz invariant and can vary greatly for some systems.
- There exists an optimum frame which minimizes  $\Gamma$ .
- We demonstrated speedup of  $\times 1000$  for PIC simulation of relativistic beam interacting with electron background.
- It is not in contradiction with the conventional scientific wisdom that “complexity” is an invariant.
- We identified three domains of application (laser-plasma acceleration, e-cloud in HEP accelerators, free electron lasers) for which speedup ranging from 2 to 4 orders of magnitude were demonstrated on toy problems.



# Outlook

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- Update codes to accommodate calculations independently of the choice of frame.
- Apply to the modeling of actual experimental facilities for the three identified cases.
- Develop methods which costs do not depend on the range of scales, based on argument that “complexity”, based on a more topological definition, can be made Lorentz invariant.
- Explore other applications: astrophysics,...

