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TOWNSEND IONIZATION COEFFICIENT FOR HYDROGEN IN A TRANSVERSE STRONG MAGNETIC FIELD

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ABSTRACT

The Townsend ionization coefficient $\alpha/p(\text{cm-mm Hg at } 20^{\circ}\text{C})^{-1}$ was measured across a strong magnetic field by varying the electrode gap in a cylindrical geometry. Measurements were made for B/p up to 4000 gauss/mm Hg at 20°C and for $E/p(1+\omega_b^2\tau^2)^{1/2}$ less than 150 v/cm—mm Hg, where $\omega_b\tau$ is the ratio of the electron cyclotron to elastic-collision frequencies. A theoretical expression of Blevin and Haydon,

$$\alpha/p = C_1(1+\omega_b^2\tau^2)^{1/2} \exp\left[-C_2(p/E)(1+\omega_b^2\tau^2)^{1/2}\right],$$

is shown to fit the points well over a limited range of values. The effective collision frequency (3.1×10^9 p/sec) that makes the theory fit is about three-fifths of the average momentum-transfer collision frequency. Analysis shows that this descrepancy results from deviations in the electron distribution function because the actual elastic-collision frequency is not the theoretically assumed constant.

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I. INTRODUCTION

The first Townsend ionization coefficient, α, is defined as the average number of ionizing collisions that an electron makes in a gas while drifting 1 cm in the direction of the electric field, E. For no magnetic field, there is an extensive amount of literature on measuring this coefficient which shows that α/p is a function only of E/p, where p is the gas pressure.[1,2] For hydrogen, measurements in a pure system show that α/p can be represented very well by

$$\alpha/p = C_1 \exp(-C_2\, p/E) \tag{1}$$

in the range $20 < E/p < 150$, where C_1 and C_2 are empirical constants.[3] The most dependable method of determining α/p is by measuring the anode current while varying the electrode gap as the other parameters are held constant.[2] This method was used here in a cylindrical geometry even though very cumbrous; a few measurements were also made by varying pressure.[4,5]

A theory of how α/p is altered by a magnetic field B applied perpendicular to E has been derived by Blevin and Haydon.[6] They find that α/p depends upon $\omega_b \tau$, the ratio of the electron cyclotron frequency to the momentum-transfer collision frequency, ν_c, of the electrons with gas molecules. In hydrogen and helium, ν_c is often assumed to be constant.[7] By assuming that ν_c is a constant and using the equivalent pressure concept $p' = p(1+\omega_b^2\tau^2)^{1/2}$, Blevin and Haydon obtain

$$\alpha/p = C_1(1+\omega_b{}^2\tau^2)^{1/2} \exp\left[-C_2(p/E)(1+\omega_b{}^2\tau^2)^{1/2}\right]. \qquad (2)$$

They then suggest that an average value of ν_c could be determined by comparing their theory with measurements made in crossed fields. Results show that the actual average ν_c is not obtained, because ν_c is not constant, which produce significant deviations in the electron distribution function.

2. APPARATUS AND PROCEDURE

The cylindrical electrode geometry, magnet, and vacuum system have been described in the preceding paper. The anode consisted of a stack of three copper rings, insulated from one another and aligned by nylon washers and pins; the top and bottom rings were grounded guard rings for insuring a uniform field in the central gap region. The cathodes were a set of replaceable aluminum cylinders of different diameters so that d could be varied from 0.2 to 1.0 cm as shown in Fig. 1. The electron current, i, to the middle anode ring was measured with a Keithley electrometer. The cathode voltage was variable from 0 to 1000 v with a Helipot potentiometer across a regulated power supply. A small pattern of holes in the anode and a quartz window on the vacuum chamber allowed the cathode to be illuminated with ultraviolet light from a Hanovia mercury lamp which tests showed to have a constant light output.

In order to determine α/p, the anode current, i, was measured as a function of electrode gap, d, while p, E, B, and the cathode emission, i_0, were kept constant. The procedure consisted of taking anode-current data each day at a different d for predetermined values of p, E, and B. The major difficulty in this procedure was that the i_0 corresponding to a given E/p and B/p varied greatly from day to day because of different cathode surfaces. The method for making the measurements taken on different days correspond to the same i_0 is now described. After the cathode photoemission became relatively constant at the beginning of a day's run, the electron emission in vacuum with $B = 0$ was measured at regular intervals during the run. Since the actual electron emission at a given E, p, and B is directly proportional to the vacuum photoemission, these vacuum currents were used to adjust the measurements taken on different days to common values of i_0. Measurements were duplicated for several d to check reproducibility, which was mostly within 20% (it became a little worse at the higher B/p). Also measurements of α/p with $B = 0$ were made using this technique to compare with the values of other experiments. Space-charge effects were negligible, because the range of currents used was 10^{-12} to 10^{-10} amp. Measurement of the current to the guard rings showed that diffusive loss of electrons along B was negligible. Test measurements showed no observable difference when 1 to 3% air was added to the gas in the chamber.[8] The parameters p, E, and B ranged from 1 to 8 mm Hg, 75 to 1000 v/cm, and 0 to 8 kgauss. The random errors on the parameters were less than 1% for p and E and less than 2% for i and B.

A few later measurements were taken by varying the pressure at a constant gap (0.5 cm) while keeping E/p and B/p constant (B/p = 1). This was much easier, because all data could be taken in one day.

3. RESULTS

The coefficient α/p is determined from

$$\alpha/p = (1/pd)\ln(i/i_0), \tag{3}$$

which is obtained from a defining equation for α with the secondary emission considered negligible.[2,4] Thus α/p is obtained by plotting ln i as a function of either d or p at a given E/p and B/p and drawing the best-fit straight line through the points. I assume that any secondary-emission effects will show up in an upward curving of the points at larger d or p. For $E/p > 150$, there is some disagreement as to the analysis needed to correctly determine α/p; secondary emission is supposedly important, even though a plot of ln i vs d shows no upward curving.[9,5] Because my measurements (except for B = 0) are for an effective $E/p' = E/p(1 + \omega_b^2\tau^2)^{1/2}$ less than 150, I have ignored this question. Most data show no upward curving, and typical sets are shown in Figs. 2 and 3

An interesting observation is that the curves for different E/p do not intersect the d = 0 axis at about the same place as is observed for $B = 0 (E/p > 25)$. The effective emission from the cathode is greatly decreased, because many electrons are bent by B and recaptured by the cathode. Examination of the data for different pressures seems to show that i_0 is only a function of E/p and B/p, with a very slight increase in i_0 at higher p. Studies have shown that the effective emission (effect of backscattering) is only a function of E/p.[10] It would be of interest to study whether or not the effective emission in a magnetic field is a function of E/p and B/p, as expected on the basis of simple arguments. Recent attempts to extend the measurements to large E/p at large B/p by varying p with d = 0.3 cm gave results that were too large when compared to the other values. Whether this effect came from diffusive loss out the ends at small p or increased i_0 as p was increased was not discovered.

The error on each value of α/p was obtained by judging the maximum deviation in slopes of the straight lines that could be drawn through the set of points. Also the values of α/p obtained at different p for the same E/p and B/p were averaged together. The values are tabulated in Table I. Any error due to the slight divergence of E in the cylindrical geometry is insignificant. The effect of small amounts of impurities in the gas cannot be completely discounted, but hydrogen does seem to be quite insensitive to contaminants, as observed by other authors.[4,11,12]

As indicated by the theory of Blevin and Haydon, a plot of $\ln(\alpha/p)$ as a function of p/E should give straight lines for various values of B/p. The measured values are plotted thus in Fig. 4 for T = 20°C, with the experimental error shown by the height of each plotted value. My results for B = 0 agree excellently with those of Rose in the range $30 \leq E/p \leq 150$ when corrected to the same temperature.[3] From a straight line drawn through my values for B = 0, I was able to determine the constants in Eq. (1). If we use these constants, Eq. (2) becomes for hydrogen

$$\alpha/p = 5.4(1+\omega_b{}^2\tau^2)^{1/2} \exp\left[-(139\pm 1)(p/E)(1+\omega_b{}^2\tau^2)^{1/2}\right]. \quad (4)$$

Next, a value of the constant $K = \omega_b \tau p/B$ is chosen which gives a family of theoretical straight lines best fitting the experimental points. This fit yields $K = (5.6 \pm 0.2)$ when B is expressed in kgauss and p in mm Hg at 20°C. These theoretical lines, as shown in Fig. 4, fit the data well except at the lowest values of E/p for each B/p. From the value found for K, the effective collision frequency ν_{eff} corresponding to τ is $(3.1 \pm 0.1)\times 10^9$/sec-mm Hg, which is considerably smaller than the actual elastic-collision frequency ν_c determined by other measurements.

The only other results with which these values can be compared are to be found in some unpublished work of Blevin quoted in Blevin and Haydon.[6] He made measurements in a planar electrode geometry for $B/p \leq 0.3$. To compare their theory with experiment, Blevin and Haydon assumed a collision frequency of 3.6×10^9/sec-mm Hg, which was incorrectly obtained from drift-velocity data. Using his empirical constants to compare Eq. (2) with his results, I find values of the effective collision frequency ranging from 2.9 to 3.4×10^9/sec-mm Hg, which agrees with my result.

4. INTERPRETATION OF RESULTS

The momentum-transfer collision frequency ν_c, reasonably well established by cross-section measurements, reaches a maximum value of about 6×10^9 p/sec around 3.5 ev, gradually decreases at higher electron energies, and rapidly decreases below 3 ev.[13] At levels of measurable ionization, the average electron energy is in excess of 4 ev.[7,13] Hence, it is reasonable to conclude that the average ν_c is at least 5×10^9 p/sec, which is considerably larger than the ν_{eff} of 3.1×10^9 p/sec. In effect, the measured values of α/p are smaller than predicted on the basis of the equivalent-pressure concept. I shall analyze the effect of collisions in the cases of B = 0 and the strong-magnetic-field limit ($\omega_b^2 \tau^2 \gg 1$) to explain this discrepancy.

For B = 0, elastic collisions determine the electron distribution by keeping the electrons from running away to high energies, and the transverse drift velocity (along E) is inversely proportional to ν_c.[13] Conversely, in the strong-field limit, the electrons are able to gain energy and form a distribution only by colliding, and the transverse drift velocity is directly proportional to ν_c.[13] Using these ideas, we can see in comparing the electron distribution for B = 0 with the "equivalent-energy" case for the strong-field limit that a larger percentage of electrons have ionizing energies in the former case. Figure 5a illustrates this effect; for elastic collisions only, it was shown that the strong-field distribution is Maxwellian.[13] Thus, just on the basis of elastic collisions, there is slightly more ionization for B = 0 than the equivalent case based on the average ν_c.

When inelastic collisions are considered, the effect of the nonconstant ν_c becomes even more pronounced. Inelastic collisions continuously place significant numbers of electrons in the low-energy region below 3 ev, where ν_c is small. For B = 0, most of these electrons rapidly gain back some energy. But with a magnetic field, many of these displaced electrons become

somewhat

somewhat "trapped" at the low energies. The effect of inelastic collisions on a Maxwellian distribution is shown in Fig. 5b for the B = 0 and strong-field cases.

Calculations show that the collision effects discussed here are easily sufficient to explain the measured α/p being smaller than predicted from the equivalent-pressure concept using the average ν_c. I have estimated the reduction in average energy because of electron build-up at low energies by "magnetic-field trapping." The flux in and out of the low-energy region by drift-velocity and inelastic processes, respectively, have been equated by assuming an average energy loss of 9 to 10 ev per inelastic collision.[14] A much larger percentage of electrons was found to have less than 3 ev for the equivalent strong-field case than for the B = 0 case. Computer calculations, using known values of the elastic and inelastic cross sections in hydrogen, are currently underway to determine the ionization rate in a strong magnetic field.[15] Preliminary results show a very decided build-up of electrons at low energies.

At low E/p, the measured values of α/p are larger than given by the theoretical curves. This is because inelastic collisions are not so dominant at low average energies in determining the energy distribution, and so the distribution approaches that predicted when the average ν_c is used. Similarly, at large E/p, the actual values should fall below those given by the theoretical curves. Again, this is because of the arguments already presented showing that the real values need to fall below those predicted on the basis of the momentum-transfer collision frequency. Measurements of Robertson and Haydon for somewhat larger E/p (B/p < 0.4 kgauss/mm Hg) than reported here seem to follow this trend.[12]

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Table I. Measured values of α/p in ionizations per cm-mm Hg at 20°C as a function of E/p in v/cm-mm Hg and B/p in kgauss/mm Hg. Errors on sets of values and individual values are given in footnotes.

	B/p				
E/p	0.25[a]	0.5[a]	1[a]	2[b]	4[b]
50	0.11				
62.5	0.21	0.04[c]			
75	0.38	0.12[d]			
87.5	0.58	0.17			
100	0.82	0.29	0.04[e]		
125	1.30	0.60	0.12[b]		
150		0.96	0.21[b]		
175		1.45	0.40		
200		1.96	0.64		
225		2.4			
250			1.35	0.20[c]	
300			2.25	0.38	
350				0.80[d]	
400				1.25[d]	
450				1.85	
500				2.6	0.53
600				3.6	0.90

a. 5% error

b. 10% error

c. 25% error

d. 15% error

e. 60% error

FIGURE LEGENDS

Fig. 1. Experimental details for measuring α/p. Aluminum cathodes and a copper anode are centered with an epoxy-resin base having a brass center post.

Fig. 2. Sample data for determining α/p by measuring the anode current as a function of electrode separation. Note that two measurements have been made at d = 2, 4, 5, and 6mm.

Fig. 3. Data for measuring α/p by varying pressure at a gap separation of 0.5 cm. The ratio of magnetic field to pressure is B/p = 1 kgauss/mm Hg at T = 20°C.

Fig. 4. First Townsend coefficient as a function of p/E for T = 20°C. The three additional points (open triangles) for B/p = 1 are those obtained by varying pressure. Heights of points represents experimental error. Family of theoretical lines best fitting points is shown where $\omega_b \tau = 5.7$ B/p.

Fig. 5. Distribution of electrons as a function of the electron velocity. (a) Effect of elastic collisions only. The strong-field case and B = 0 case have the same equivalent energy. (b) Effect of inelastic collisions on a Maxwellian distribution for the strong-field and B = 0 cases.

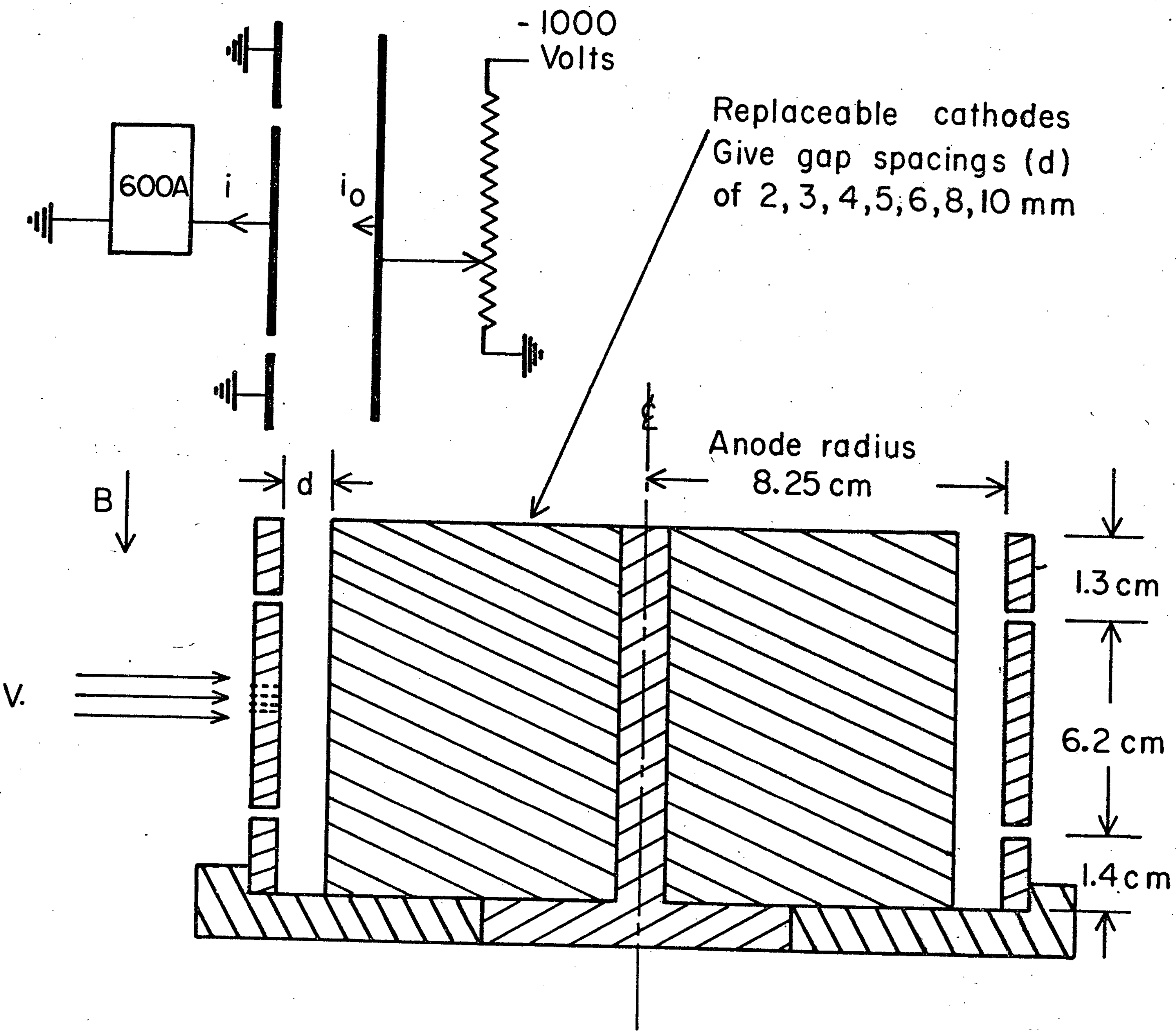

Fig. 1

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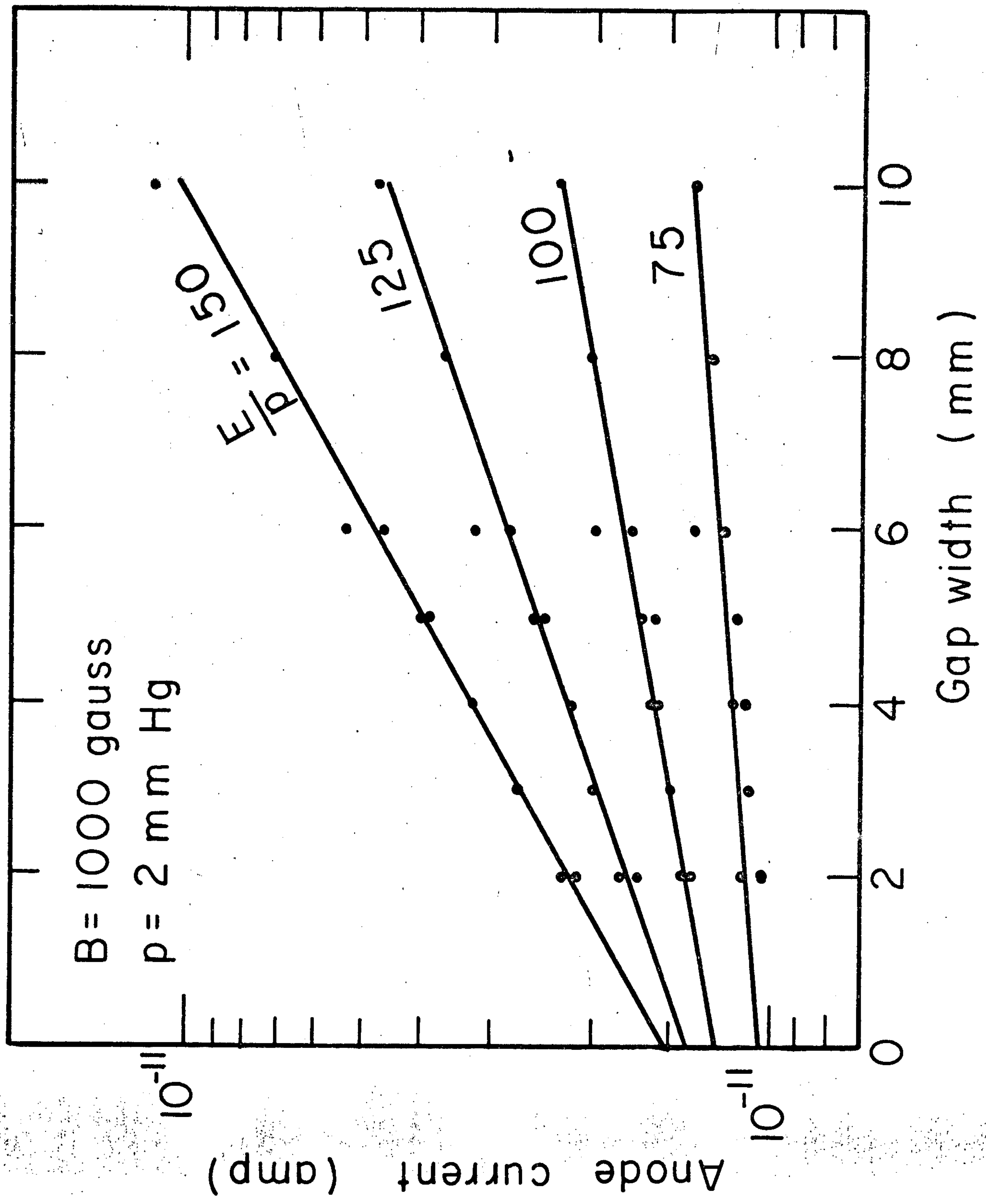

MU-21883

Fig. 2

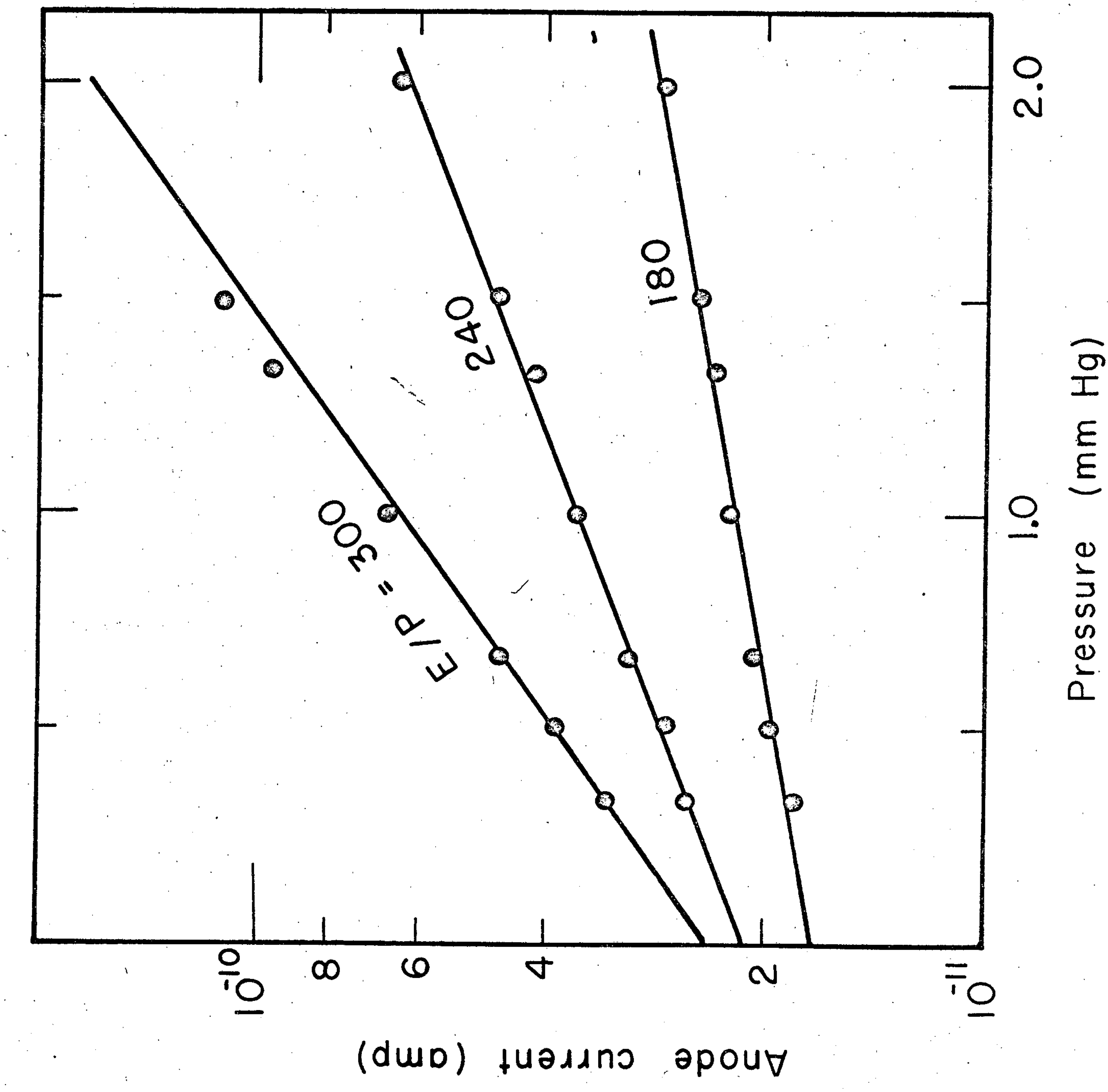

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Fig. 3

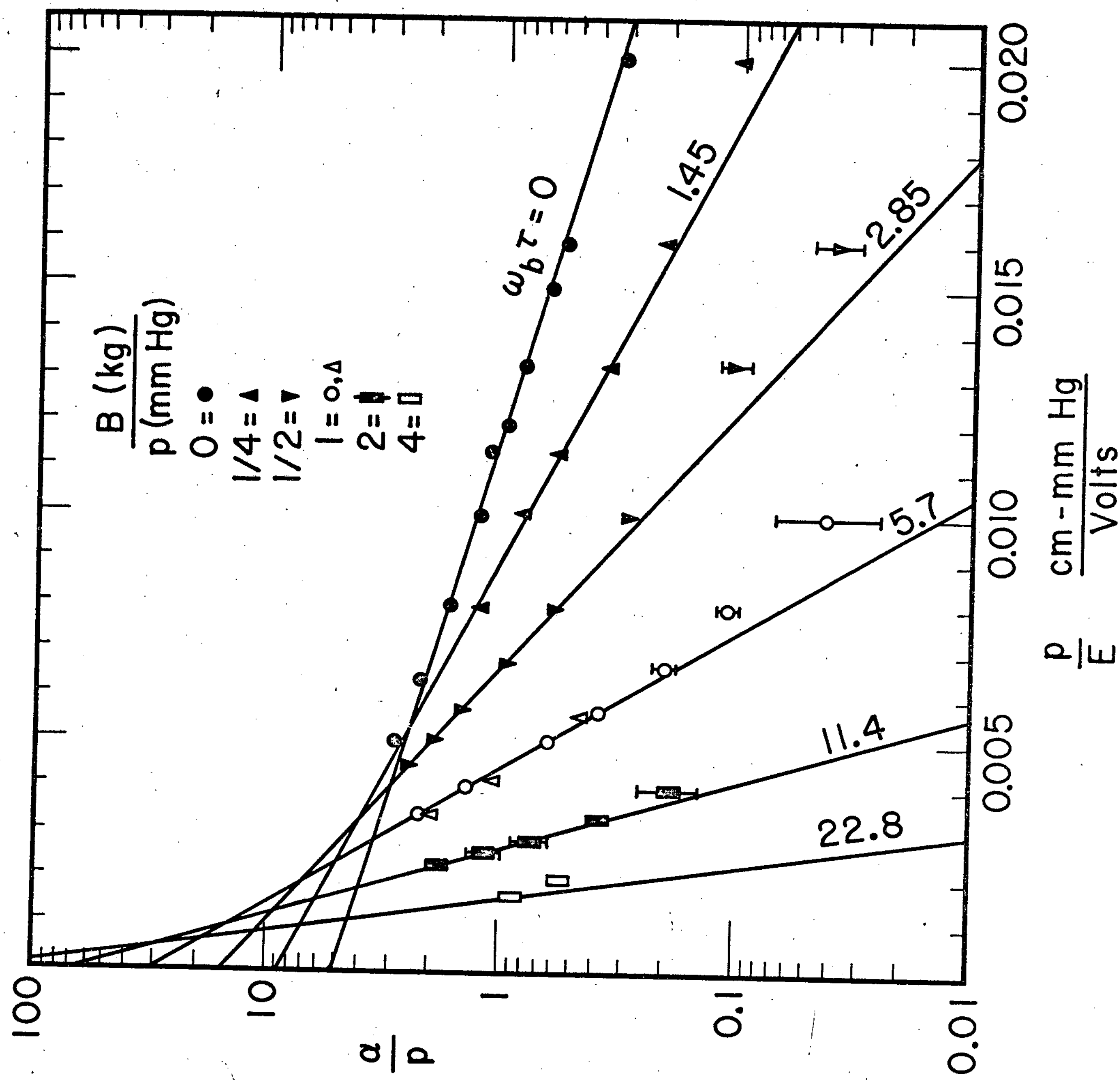

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Fig. 4

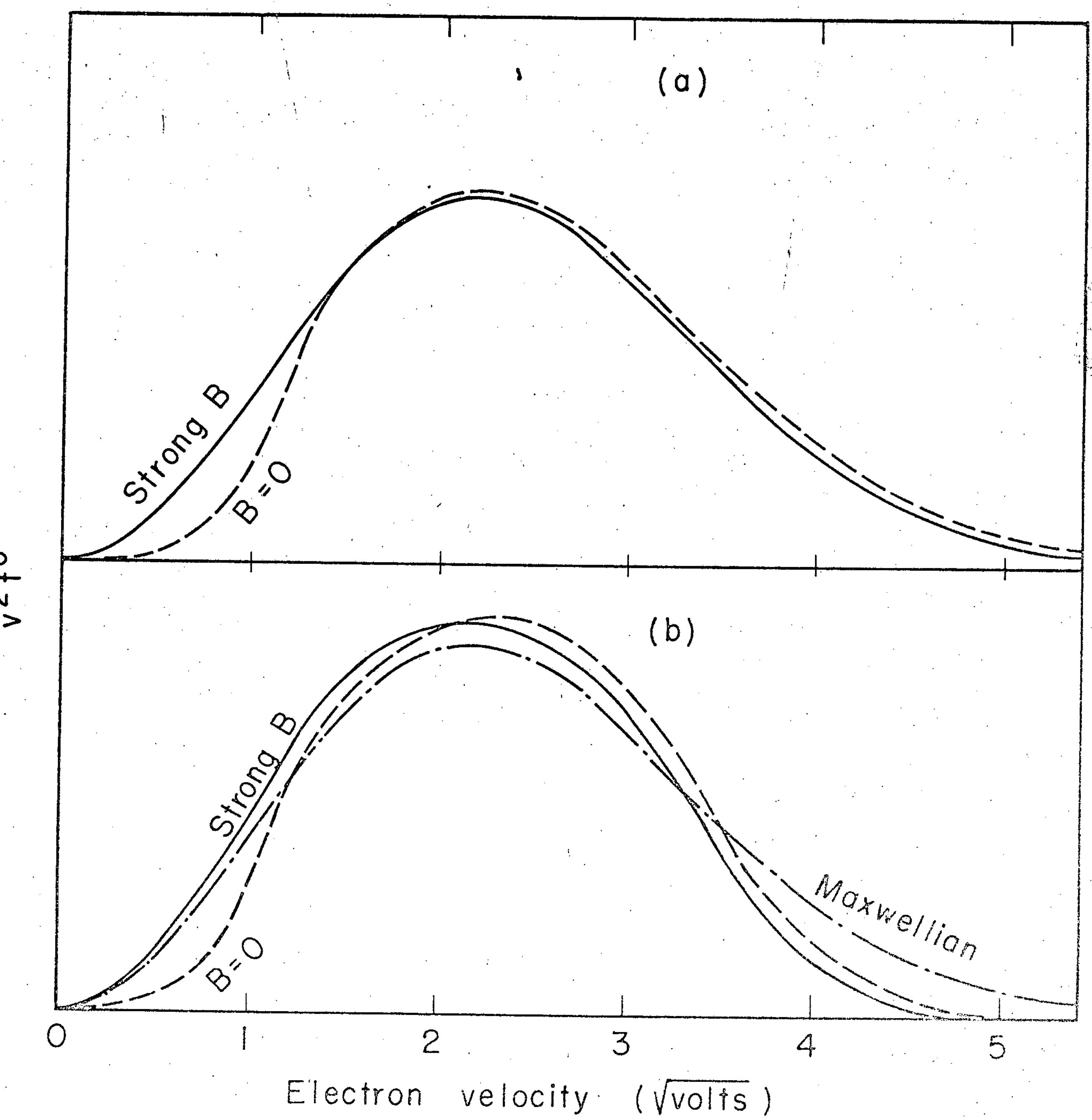
(a)
Strong B
B=O
(b)
Strong B
B=O
Maxwellian
$v^2 f^0$
0
1
2
3
4
5
Electron velocity ($\sqrt{\text{volts}}$)

Fig. 5