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A STUDY OF THE REACTION $\pi^+ + p \rightarrow \Sigma^+ + K^+$ NEAR THRESHOLD

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ABSTRACT

The reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+$ has been studied in the momentum range 1037 through 1065 Mev/c with specific regard to the determination of s- and p-wave amplitudes. If we represent these amplitudes near threshold by $S_{\frac{1}{2}} = a_0 k^{\frac{1}{2}}$, $P_{\frac{1}{2}} + 2P_{\frac{3}{2}} = b_0 k^{3/2} e^{i\chi_b}$, and $P_{\frac{3}{2}} - P_{\frac{1}{2}} = c_0 k^{3/2} e^{i\chi_c}$, the following results have been obtained from an analysis based on 274 events: $a_0^2 = (3.31 \pm 0.87) \times 10^{-5}$ mb/sr-Mev/c, $b_0/a_0 = 0.0075 \pm 0.0026$, $c_0/a_0 = -0.0062 \pm 0.0023$, $\chi_b = 52.1 \pm 8.0$ deg, $\chi_c = 80.8 \pm 27.2$ deg. An alternative set of solutions exists for $\chi_b = -52.1$ deg and $\chi_c = 99.2$ deg.

A STUDY OF THE REACTION $\pi^+ + p \rightarrow \Sigma^+ + K^+$ NEAR THRESHOLD*

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I. INTRODUCTION

The reaction $\pi^+ + p \rightarrow \Sigma^+ + K^+$ has been studied near threshold (1020 Mev/c) in the 72-inch liquid hydrogen bubble chamber. Energy loss in the incident beam by ionization in the hydrogen has permitted an analysis of the reaction in the momentum interval 1037 through 1065 Mev/c. Several other workers have analyzed this reaction at higher momenta in bubble chambers;[1-5] one experiment has been done in this low-energy range.[5] However, since only five events were reported, a detailed analysis of Σ^+K^+ threshold properties was impossible. In this paper, 274 events have been analyzed with specific regard to a partial-wave determination of the reaction near threshold. From the distribution of events as a function of incident pion momentum, c.m. production angle, Σ^+-decay angle with respect to the normal to production plane, and total cross section, the s- and p-wave production amplitudes have been determined from the data. Based on an extrapolation of these results to 1090 Mev/c, an attempt to check the charge-independence hypothesis has been made with currently available data of Σ^- and Σ^0 production by π^-, p interaction.[6,7]

II. EXPERIMENTAL DETAILS

The experimental beam setup has been described previously.[8] In addition to the beam optics shown in Fig. 1 of reference 8, a quadrupole lens and a mass spectrometer were used to produce a separated π^+ beam. The momentum resolution of the beam was $^{+2.2}_{-4.0}$ Mev/c, the same as that given for π^- in Fig. 3 of reference 8. The incident π^+ beam was of such momentum that by

energy degrading in the liquid hydrogen the threshold for Σ^+K^+ was placed in the vicinity of the downstream end of the bubble chamber. A determination of the threshold would then allow a very accurate measure of the incident beam momentum.

In order to estimate the proton contamination in the beam, a sample of two-prong interactions was analyzed for elastic pion-proton and proton-proton scatterings. The cross sections for these reactions were taken to be 10.3 ± 0.9 mb and 25.0 ± 1.0 mb respectively at 1040 Mev/c.[9,10] Proton contamination was measured to be 4.9 ± 2.0%.

Positron and muon contamination was measured from bremmsstrahlung and δ-ray production in the chamber. The principle of this determination was the following. The energies of the δ-rays and the secondary track in bremmsstrahlung events were measured on the scanning table by means of curvature templates, the δ-rays being counted in two momentum intervals: from 53 through 90 Mev/c, and from 90 through 1060 Mev/c. Since the upper limits of the δ-ray spectrum are 53 Mev/c when produced by 1060-Mev/c pions, and 90 Mev/c in the case of 1060-Mev/c muons, the δ-rays selected in the first interval were produced by either positrons or muons. The corresponding absolute cross sections estimated by integration over this momentum interval are equal to 1.85 mb and 0.48 mb respectively.[11] The δ-rays selected in the second energy interval were unambiguously produced by positrons. The integrated cross section is 1.99 mb.[11] The bremmsstrahlung events were selected for secondary track momentum less than 500 Mev/c, to avoid possible confusion with incident pions decaying in the chamber into muons (the lower limit of the muon spectrum in the laboratory system is about 610 Mev/c; in addition, scanning efficiency is optimum for these events). The absolute cross section estimated by integration

from zero momentum up to 500 Mev/c is equal to 8.70 mb for incident positrons of 1060 Mev/c slowed down in the field of hydrogen nuclei.[12] Positron contamination was measured to be 1.3 ± 0.4%, and the muon contamination to be 0.5 ± 0.5%. The combined proton, positron, and muon contamination was 6.7 ± 2.5%.

A total of 105,000 pictures was scanned for Σ^+K^+ events, and approximately 20% of these were rescanned. Events were measured on the Franckenstein measuring machines at the Lawrence Radiation Laboratory (Berkeley). During this experiment the magnetic field in the chamber was 15.7 kgauss. Events were reconstructed and analyzed kinematically by means of the programs PANG and KICK,[13] the latter program providing fitted angles and momenta that were introduced to subsequent detailed analysis programs.

III. PARTIAL-WAVE ANALYSIS OF Σ^+K^+ NEAR THRESHOLD

Preliminary analysis[14] of events indicated large asymmetries in the c.m. production angular distribution and the up-down decay distribution of the Σ^+ in the mode $\Sigma^+ \rightarrow p + \pi^0$. This information implied the presence of at least p-wave in the production mechanism. The experimental data showed no evidence for a significant amount of d-wave.

Assuming only s- and p-waves at the production of Σ^+K^+, the angular distribution $\frac{d\sigma}{d\Omega}$ and polarization P can be written

$$\frac{d\sigma}{d\Omega} = A_0 + A_1 \cos\theta + A_2 \cos^2\theta, \tag{1-A}$$

and

$$P\left(\frac{d\sigma}{d\Omega}\right) = \sin\theta\,(A_3 + A_4 \cos\theta). \tag{1-B}$$

Assume that the Σ^+ is created at an angle θ in the production c.m. system, and decays into a pion making an angle ϕ with respect to the normal

$$n = \frac{\underset{\sim}{P}_\pi \times \underset{\sim}{P}_\Sigma}{|P_\pi|\,|P_\Sigma| \sin\theta_{\pi\Sigma}}$$

to the production plane in the Σ^+ rest frame. Then the probability function for the observation of an event is

$$f = A_0 + A_1 \cos\theta + A_2 \cos^2\theta + \alpha \cos\phi \sin\theta (A_3 + A_4 \cos\theta), \tag{2}$$

where use has been made of the fact that the Σ^+ parity-nonconserving decay distribution of the pion is of the form $(1 + \alpha P \cos\phi)$. The α is the asymmetry parameter for the Σ^+ decay (α^+ for $n\pi^+$, and α^0 for $p\pi^0$). Near threshold, the coefficients of Eq. (2), according to Lee et al.[15], would be

$$\begin{aligned} A_0 &= a_0^2 k + c_0^2 k^3, \\ A_1 &= 2 a_0 b_0 k^2 \cos\chi_b, \\ A_2 &= (b_0^2 - c_0^2) k^3, \\ A_3 &= -2 a_0 c_0 k^2 \sin\chi_c, \\ \text{and} \quad A_4 &= 2 b_0 c_0 k^3 \sin(\chi_b - \chi_c), \end{aligned} \tag{3}$$

where k is the Σ^+ momentum in the production c.m. and the parameters $a_0, b_0, c_0, \chi_b, \chi_c$, are related to the $S_{\frac{1}{2}}$, $P_{\frac{1}{2}}$, and $P_{\frac{3}{2}}$ production amplitudes by

$$\begin{aligned} S_{\frac{1}{2}} &= a_0 k^{\frac{1}{2}}, \\ P_{\frac{1}{2}} + 2 P_{3/2} &= b_0 k^{3/2} e^{i\chi_b}, \\ P_{3/2} - P_{\frac{1}{2}} &= c_0 k^{3/2} e^{i\chi_c}. \end{aligned} \tag{4}$$

The s- and p-wave amplitudes have been determined by the maximum-likelihood method, assuming that the energy dependence of Eq. (3) is still valid in the energy range investigated. The probability function may be written as

$$f = X_0 k + X_1 k^3 + X_2 k^2 \cos\theta + X_3 k^3 \cos^2\theta + X_4 \cos\phi \sin\theta\, k^2 + X_5 \cos\phi \sin\theta \cos\theta\, k^3, \tag{5}$$

where

$$X_0 = a_0^2$$
$$X_1 = c_0^2$$
$$X_2 = 2a_0 b_0 \cos \chi_b,$$
$$X_3 = b_0^2 - c_0^2,$$
$$X_4 = -2a\, a_0 c_0 \sin \chi_c,$$
$$X_5 = -2a\, b_0 c_0 \sin(\chi_c - \chi_b).$$

Near threshold, k is related to the incident pion momentum p, by

$$k(\mathrm{Mev/c}) = r(p - 1020\ \mathrm{Mev/c})^{1/2} = r(\Delta p)^{1/2}, \tag{6}$$

where

$$r = \left\{ \frac{0.991\, m_p}{2} \left[1 - \frac{(m_K^2 - m_\Sigma^2)^2}{(m_K + m_\Sigma)^4} \right] \right\}^{1/2} = 19.63\ (\mathrm{Mev/c})^{1/2}. \tag{7}$$

The probability function, Eq. (5), may now be written in terms of the laboratory-system momentum p,

$$f = Y_0 (\Delta p)^{1/2} + Y_1 (\Delta p)^{3/2} + Y_2 \cos\theta\, (\Delta p) + Y_3 \cos^2\theta\, (\Delta p)^{3/2} + Y_4 \cos\phi \sin\theta\, (\Delta p) + Y_5 \cos\phi \sin\theta \cos\theta\, (\Delta p)^{3/2}, \tag{8}$$

where

$$Y_0 = a_0^2 r,$$
$$Y_1 = c_0^2 r^3,$$
$$Y_2 = 2a_0 b_0 r^2 \cos \chi_b,$$
$$Y_3 = (b_0^2 - c_0^2)\, r^3,$$
$$Y_4 = -2a\, a_0 c_0 r^2 \sin \chi_c,$$

and

$$Y_5 = -2a\, b_0 c_0 r^3 \sin(\chi_c - \chi_b).$$

For each event the values of $\cos\theta$ and $\cos\phi$, and the y coordinate of the production vertex in the chamber are known. The y axis is directed along the 72-in. dimension of the chamber, with the center of the chamber at y = 0 cm.

Δp is related to $\Delta y = y - y_{th}$, where y_{th} is the coordinate of the threshold, by Δp (Mev/c) = 0.243 Δy (cm). The curvature of the tracks over the whole length of the chamber introduces no significant deviation from this formula (< 0.5%). For a given y coordinate there is a corresponding most likely value of the incident momentum $\bar{p}$, around which the real value $p = \bar{p} + \epsilon$ fluctuates, according to the resolution curve $f(\epsilon)$ of Fig. 3 in reference 8. Therefore, the probability function has been folded in with the momentum resolution curve (normalized to unity). The quantities $(\Delta p)^n$ in Eq. (8) have been replaced by

$$\langle \Delta p^n \rangle_{av} = \int_{0.243(y-y_{th})}^{y_{max}} d\epsilon\, f(\epsilon)\,[0.243(y_{th}-y) + \epsilon]^n. \qquad (9)$$

To construct the likelihood function the probability density,

$$\mathcal{P} = \frac{f \exp\left[-\frac{y - y_{min}}{\lambda}\right]}{\int_{-1}^{+1} d(\cos\phi) \int_{a}^{b} d(\cos\theta) \int_{y_{min}}^{y_{max}} f \exp\left[-\frac{y - y_{min}}{\lambda}\right] dy}, \qquad (10)$$

is required. The exponential factor takes into account the beam attenuation by all strong interactions with a total mean free path $\lambda(1.16 \times 10^3\ \text{cm})$.[9] The integrations extend over the whole range of physical quantities in which the experimental results are given. Provision has been made for cutting the production angular distribution in the forward and backward directions. In order to find the best values of the parameters a_0, b_0, c_0, χ_b, χ_c, the likelihood function

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}_i$$

has been maximized. For this purpose, a program has been written for the 709 IBM computer. From the distribution of the events as a function of position y in the chamber, Σ^+ production angle θ, and decay angle, ϕ, the best values of the quantities b_0/a_0, c_0/a_0, and χ_b, χ_c were determined first. A preliminary

analysis of the data indicated that all c.m. production angles could be accepted without any correction for experimental bias. Consequently a and b have been set equal to -1 and +1 respectively in Eq. (10). The parameter α has been assumed known. With the experimental data currently available from Σ-decay analysis taken into account,[16,17] and according to the discussion of Gell-Mann and Rosenfeld,[18] α^0 has been taken equal to +1 for events in which the Σ^+ decayed into $p + \pi^0$, and 0 for the $n + \pi^+$ decay mode. The likelihood function has been maximized for different values of y_{th} distributed in an interval where the threshold was thought to be. The absolute values of the amplitudes have then been computed from the mean cross section defined here by

$$\bar{\sigma}\,(\mathrm{mb}) = \frac{N}{K n \ell} \tag{11}$$

where N = number of events produced by $\pi^+ p$ interactions in the chosen fiducial volume, limited by y_{min} and y_{max},

n = total number of π^+ tracks entering the chamber through the thin window, counted at $y = y_{min}$

ℓ = track length per incident pion in the fiducial volume,

and K = number of target protons per cm^3 of liquid hydrogen (0.35×10^{23}).

After substitution of Eqs. (3) and (6) into Eq. (1), integration over cos θ and cos ϕ gives

$$\sigma = 4\pi\, a_0^{\,2} r\, (\Delta p)^{1/2} + 4\pi r^3 \left(\frac{2c_0^{\,2} + b_0^{\,2}}{3}\right)(\Delta p)^{3/2}. \tag{12}$$

Then the relation

$$\bar{\sigma} = \frac{\int_{y_{min}}^{y_{max}} \sigma \exp\left[-\frac{y - y_{min}}{\lambda}\right] dy}{y_{max} - y_{min}}, \tag{13}$$

with Δp (Mev/c) replaced by $0.243 \times \Delta y$ (cm), allows the determination of the absolute amplitudes.

IV. EXPERIMENTAL RESULTS

In order to analyze experimental biases, the Σ^+ mean lifetime and decay branching ratio were determined.

A. Σ^+ Mean Lifetime

The mean lifetime of the Σ^+, $\bar{\tau}_{\Sigma^+}$, was obtained by the maximum-likelihood method. The likelihood function, maximized for various values of t_{min}, reduces to the following expression when the potential time of flight for all events is large compared to $\bar{\tau}_{\Sigma^+}$:

$$\mathcal{L} = \frac{\prod_{i=1}^{N} \exp\left[-\frac{(t_i - t_{min})}{\tau_{\Sigma^+}}\right]}{\bar{\tau}_{\Sigma^+}^{\,N}}, \tag{14}$$

where t_i is the observed lifetime for the ith decay, N is the total number of decays for which $t_i > t_{min}$, and t_{min} is some lower-limit cutoff below which one believes that the Σ^+ is sufficiently short to cause a bias. The decay modes $\Sigma^+ \rightarrow p + \pi^0$, and $\Sigma^+ \rightarrow n + \pi^+$ have been analyzed separately. The results of the analysis are given in Table I. In $\Sigma^+ \rightarrow p + \pi^0$ mode events in which the proton came off at less than 5 deg with the Σ^+ in the laboratory system, and events in which the Σ^+ - p decay plane made an angle of less than 15 deg with the optic axes of the bubble chamber cameras have been excluded. The latter restriction removed events in which the Σ^+-p decay was nearly edge-on to the line of sight of the cameras. A fiducial volume limited by $-65 \leq y \leq +51$ cm has been taken. The values obtained for both modes are in agreement with several other experiments.[4, 19-21] A strong bias was observed (a) for $t_{min} < 0.50 \times 10^{-10}$ sec, and (b) for $t_{min} < 0.25 \times 10^{-10}$ sec.

B. Σ^+ Decay Branching Ratio

The 104 $\Sigma^+ \rightarrow p + \pi^0$ decays in Table I for which $t_i > 0.50 \times 10^{-10}$ sec have been corrected for $t_i < 0.50 \times 10^{-10}$ sec, using the corresponding calculated lifetime of 0.835×10^{-10} sec. The 5 deg decay cutoff and the 15 deg cutoff with the optic axes have been corrected, assuming that the Σ^+ decays in equal numbers forward and backward in its rest frame, and that in the laboratory system it decays uniformly with respect to its direction of motion. Based on the two scans, the scanning efficiency for $p\pi^0$ within the fiducial volume, and satisfying the above criteria, is 95 ± 5%. These corrections give a total of 257 ± 31 events. The 192 $\Sigma^+ \rightarrow n + \pi^+$ decays need only be corrected for $t_i < 0.25 \times 10^{-10}$ sec and a scanning efficiency of 97 ± 2%. This gives 275 ± 21 events. Thus we have

$$\frac{\Sigma^+ \rightarrow p + \pi^0}{(\Sigma^+ \rightarrow p + \pi^0) + (\Sigma^+ \rightarrow n + \pi^+)} = 0.48 \pm 0.07. \qquad (15)$$

C. Average Cross Section

The average cross section for the energy interval defined by $-65 \leqslant y \leqslant 51$ cm is given in Table II, along with other pertinent scanning information. The numbers of events have been corrected according to decay mode, in similar fashion to that done for the branching ratio calculation of the preceeding section. All useful events were required to have their beam tracks pass through the bubble chamber thin window so that the resolution function mentioned previously would be applicable. No bias against events in which the Σ^+K^+ plane was edge-on with the cameras was observed.

D. s- and p-Wave Analysis

Preliminary analysis indicated that the threshold would be at y = + 130 cm if the incident π^+ had continued in liquid hydrogen outside the chamber. This information was obtained by extrapolating the fitted beam-momentum distribution

as a function of y at the rate 0.243 Mev/c per cm to the known threshold of 1020 Mev/c and, in addition, by extrapolation of the square of the average production angle of the K^+ as a function of y to the threshold by a curve of the form θ^2_{av} = constant $\times (y_{th} - y)$, as given by Wolf et al. for the reaction $\pi^- + p \rightarrow \Sigma^- + K^+$ near threshold.[8] These two methods provided consistent starting values in the search for the maximum-likelihood solution. A fiducial volume defined by $-65 \text{ cm} \leq y \leq +51$ cm was taken; the beam was sufficiently collimated in the x and z directions (two dimensions normal to the beam direction) so that no cutoff was required. The results of the maximum-likelihood search are shown in Fig. 1. The natural logarithm of the likelihood function is maximum in the region y_{th} = 130 to 135 cm. The fit has also been done with $\alpha^0 = +0.80$. Figure 1 indicates that +1.00 is a better value of the asymmetry parameter. The maximum-likelihood solutions to the s- and p-wave parameters are given in Table III. Equation (3) shows that an alternative set of solutions exists for χ_b replaced by $-\chi_b$, and χ_c by $\pi - \chi_c$. The resultant amplitudes do not vary appreciably within their errors for this difference in the asymmetry parameter, nor with position of the threshold within the extreme values of Fig. 1. Because of the asymmetry of the resolution curve of the incident beam momentum, the mean value of the threshold is displaced to $\overline{y}_{th}$ = 122 cm for a most probable value of y_{th} = 130 cm. Here $\overline{y}_{th} = 122 \pm 10$ cm has been taken as the best value of the threshold. The error on $\overline{y}_{th}$ is based primarily on the fitted-beam-momentum and K^+ angle extrapolation to the threshold. By use of the average cross section of Table II and Eq. (13), the square of the absolute amplitude, a_0^2, has been computed. With the chosen values for y_{min} and y_{max} and $\overline{y}_{th}$ = 122 cm, the expression

$$a_0^2 \left(\frac{\text{mb}}{\text{sr-mev/c}}\right) = \frac{\overline{\sigma}\ (\text{mb})}{1.30 \times 10^3 + 5.48 \left(\frac{b_0^2}{a_0^2} + \frac{2c_0^2}{a_0^2}\right) \times 10^6}, \qquad (16)$$

gives $a_0^2 = (3.31 \pm 0.87) \times 10^{-5} \frac{\text{mb}}{\text{sr-Mev/c}}$ for the values of the parameters as given in Table III, with $\alpha^0 = +1.0$, and $y_{th} = 130.0$ cm.

With the information from Table III, the coefficients of Eq. (1) may be evaluated. Figure 2 shows the c.m. production angular distributions for Σ^+ given for the intervals (a) $-65 \text{ cm} \leq y \leq +51$ cm, (b) $-65 \text{ cm} \leq y \leq -7$ cm and (c) $-7 \text{ cm} \leq y \leq +51$ cm. Curves (A) are calculated from Eq. (1-A) with the coefficients (Eq. 3) evaluated for the mean value of k corresponding to each energy interval. Curves (B) are the result of an independent maximum-likelihood fit to a second-order polynomial in cos θ. Curves (A) and (B) are consistent with each other in all cases. In order to compare the results of the fit with the observed polarization, the average polarization

$$\bar{P} = \frac{\int_{-1}^{+1} P \, d\sigma/d\Omega \quad d(\cos\theta)}{\int_{-1}^{+1} d\sigma/d\Omega \quad d(\cos\theta)} = \frac{\pi}{4} \left(\frac{A_3}{A_0 + A_2/3}\right) \tag{17}$$

has been computed, and is (a) + 0.69, (b) + 0.70, and (c) + 0.66, for the three energy intervals given above. The experimental up-down distributions of the decay of the Σ^+ into $p + \pi^0$ are given in Fig. 3. The asymmetry coefficients for each of the energy intervals has been determined by a maximum-likelihood fit to the form $(1 + \alpha^0 \bar{P} \cos\phi)$; the results for $\alpha^0 \bar{P}$ are (a) $= +0.68 \pm 0.14$, (b) $+ 0.76 \pm 0.18$, and (c) $+0.55 \pm 0.23$. The experimental up-down asymmetry of the Σ^+ into $n + \pi^+$ for $-65 \text{ cm} \leq y \leq +51$ cm is given in Fig. 4. The maximum-likelihood fit to $(1 + \alpha^+ \bar{P} \cos\phi)$ gives $\alpha^+ \bar{P} = +0.023 \pm 0.13$. If we use $\bar{P} \geq +0.68 \pm 0.14$ for the corresponding energy interval from the $p\pi^0$ mode, then $\alpha^+ \leq +0.03 \pm 0.21$, which is in close agreement with the results of Cork et al.[16]

Lastly, the prediction of the maximum-likelihood solution with regard to the energy-excitation function has been considered. Upon use of the final solution of Table III, Eq. (12) becomes

$$\sigma(\text{mb}) = 0.00816\,(\Delta p)^{1/2} + 0.000139\,(\Delta p)^{3/2}. \tag{18}$$

The distribution as a function of the production vertex y of the cross section computed from corrected numbers of events, and the path length corrected for beam attenuation, is shown in Fig. 5. The curve represents Eq. (18). Therefore the maximum-likelihood solutions of Table III are completely consistent with the c.m. production angular distribution, polarization, and the energy-excitation function. A summary of experimental results on Σ^+K^+ total cross sections and Σ^+ polarizations is presented in Figs. 6 and 7, along with the prediction of the maximum-likelihood solution to s- and p-wave amplitudes. Deviation of the curves from the data beyond approximately 1150 Mev/c indicates that the energy dependence of the coefficients, represented by Eq. (3), is no longer valid at these energies. Nevertheless, in order to make a check of the charge-independence hypothesis,[22] the results of this analysis were extrapolated to 1090 Mev/c, where data of Σ^- and Σ^0 production by π^-p interactions exists.[6,7] Based on this extrapolation, there is no evidence of a violation in either the total or differential cross sections.

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Table I. Σ^+ lifetimes for various values of t_{min} for (a) $\Sigma^+ \rightarrow p + \pi^0$, and (b) $\Sigma^+ \rightarrow n + \pi^+$

	N	t_{min} ($\times 10^{-10}$ sec)	$\bar{\tau}_{\Sigma^+}$ ($\times 10^{-10}$ sec)
(a)	104	0.50	$0.835 \begin{smallmatrix} +0.088 \\ -0.077 \end{smallmatrix}$
	79	0.75	$0.810 \begin{smallmatrix} +0.106 \\ -0.090 \end{smallmatrix}$
	56	1.00	$0.846 \begin{smallmatrix} +0.124 \\ -0.104 \end{smallmatrix}$
(b)	192	0.25	$0.749 \begin{smallmatrix} +0.056 \\ -0.052 \end{smallmatrix}$
	138	0.50	$0.749 \begin{smallmatrix} +0.068 \\ -0.060 \end{smallmatrix}$
	103	0.75	$0.721 \begin{smallmatrix} +0.076 \\ -0.067 \end{smallmatrix}$

Table II. Scanning information and mean cross section for the energy defined between $y_{min} = -65$ cm and $y_{max} = +51$ cm.

(a) Total number of pictures scanned	105526
(b) Total track length scanned (uncorrected)	$(1741.78 \pm 29.63) \times 10^3$ m
(c) Beam contamination	$6.7 \pm 2.5\%$
(d) Fraction of beam entering through bubble chamber thin window	0.96 ± 0.01
(e) Total π^+ track length, corrected for (c) and (d) above	$(1563.42 \pm 26.60) \times 10^3$ m
(f) Number of events with beam tracks entering bubble chamber through thin window (uncorrected)	274
(g) Scanning efficiency $\begin{Bmatrix} n\,\pi^+ \\ p\,\pi^0 \end{Bmatrix}$	$\begin{Bmatrix} 97 \pm 2\% \\ 95 \pm 5\% \end{Bmatrix}$
(h) Number of events with beam tracks entering through thin window, corrected for (g), lifetime, small-angle proton decay, and loss due to decay plane's being edge-on with cameras	364 ± 26
(i) Mean cross section (σ)	0.067 ± 0.005 mb

Table III. Maximum-likelihood solutions to the s- and p-wave amplitudes for various values of a^0, y_{th}.

Parameters:	$a^0 = +1$, $y_{th} = 130$;	$a^0 = +1$, $y_{th} = 170$;	$a^0 = +0.8$, $y_{th} = 130$
b_0/a_0	0.0075 ± 0.0026	0.0080	0.0075
c_0/a_0	-0.0062 ± 0.0023	-0.0068	-0.0067
χ_b	52.1 ± 8.0 deg	53.3 deg	50.9 deg
χ_c	80.8 ± 27.2 deg	77.1 deg	85.5 deg

Error matrix ($a^0 = +1$, $y_{th} = 130.0$ cm)

b_0/a_0	$+6.97\times10^{-6}$	-5.05×10^{-6}	1.13×10^{-2}	-2.54×10^{-2}
c_0/a_0		$+5.40\times10^{-6}$	-5.37×10^{-3}	$+2.49\times10^{-2}$
χ_b			+63.47	-43.78
χ_c				+740.10

Figure Legends

Fig. 1. Logarithm of the likelihood function versus y_{th} for $\alpha^0 = +1.00$ and $\alpha^0 = +0.80$.

Fig. 2. Angular distributions for the different momentum intervals. The points have been determined from the observed number of events corrected for experimental biases and beam attenuation. The quoted errors are statistical.

Fig. 3. Angular distributions of the decay pion with respect to the normal to the production plane for the $\Sigma^+ \rightarrow p + \pi^0$ mode. The quoted errors are statistical.

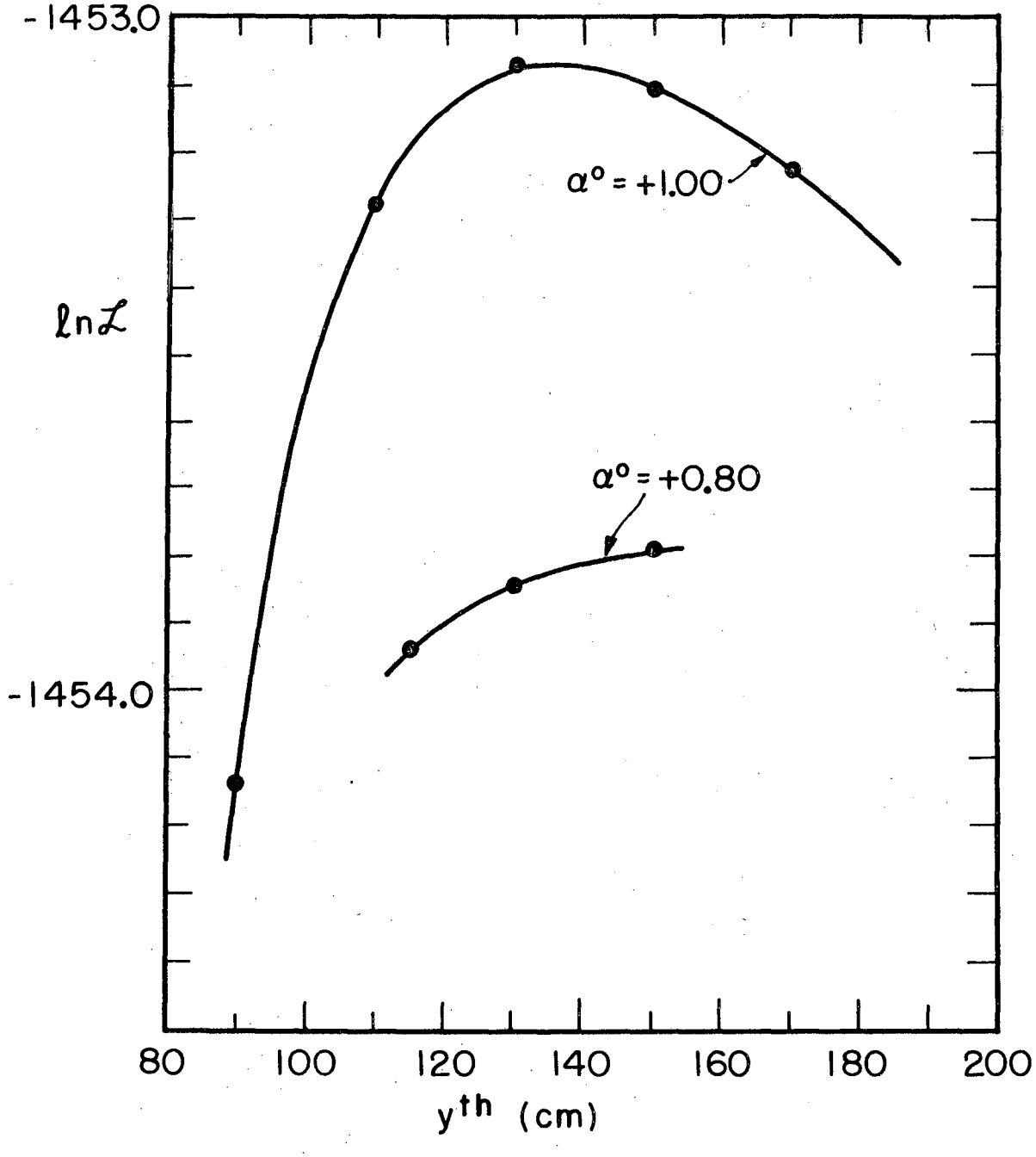
Fig. 4. Angular distribution of the decay pion with respect to the normal to the production plane for the $\Sigma^+ \rightarrow n + \pi^+$ mode. The quoted errors are statistical.

Fig. 5. Production cross section as a function of position in the chamber. The points have been determined from the observed number of events corrected for experimental biases and beam attenuation. The quoted errors are statistical. The solid curve has been computed from Eq. (18).

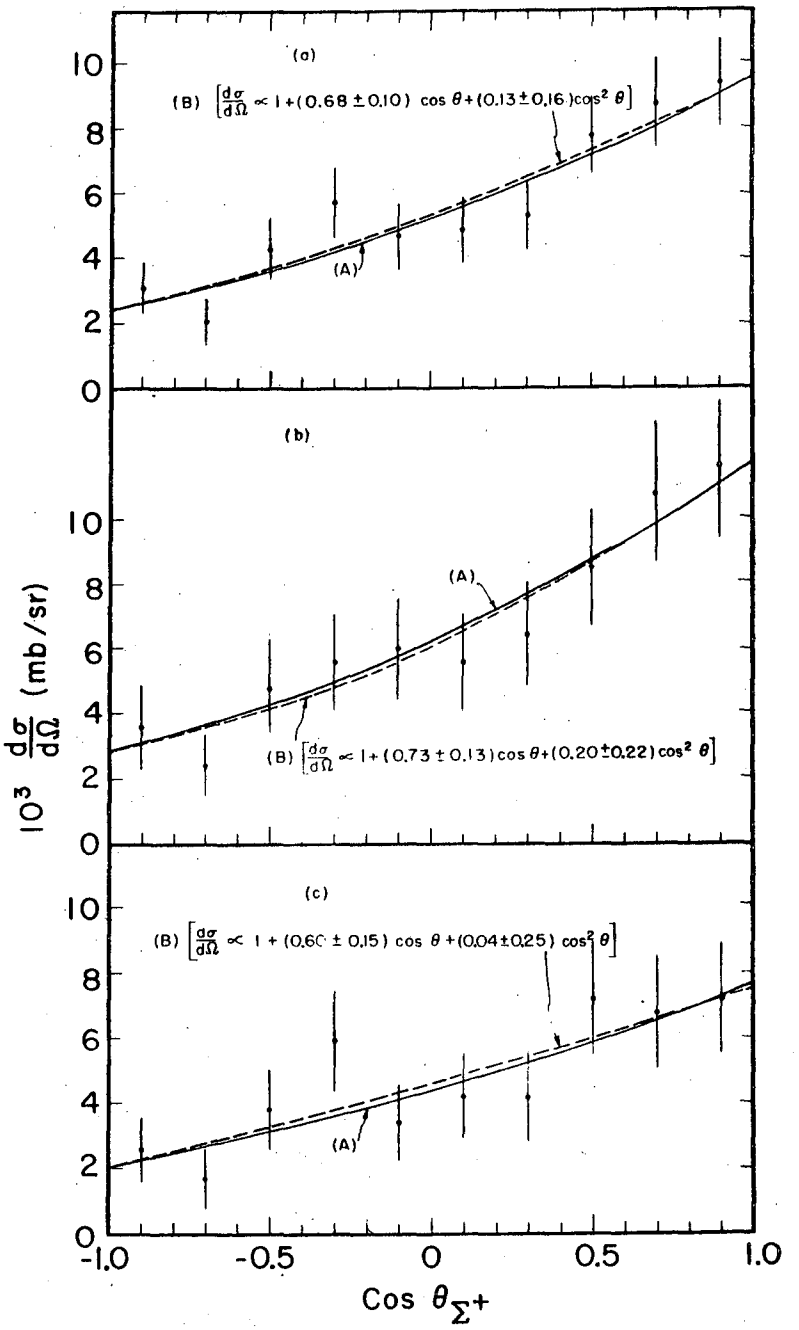
Fig. 6. The total cross section of this experiment compared with other experimental results. The solid curve has been determined from Eq. (18). The dashed curves represent one standard-deviation error, where proper account has been taken of correlations between errors. References to the data, cited from this article, are: (●) present experiment; (0) Baltay, reference 5; (■) Berthelot, reference 4; (▲) Brown, reference 1; (▼) and Erwin, reference 2.

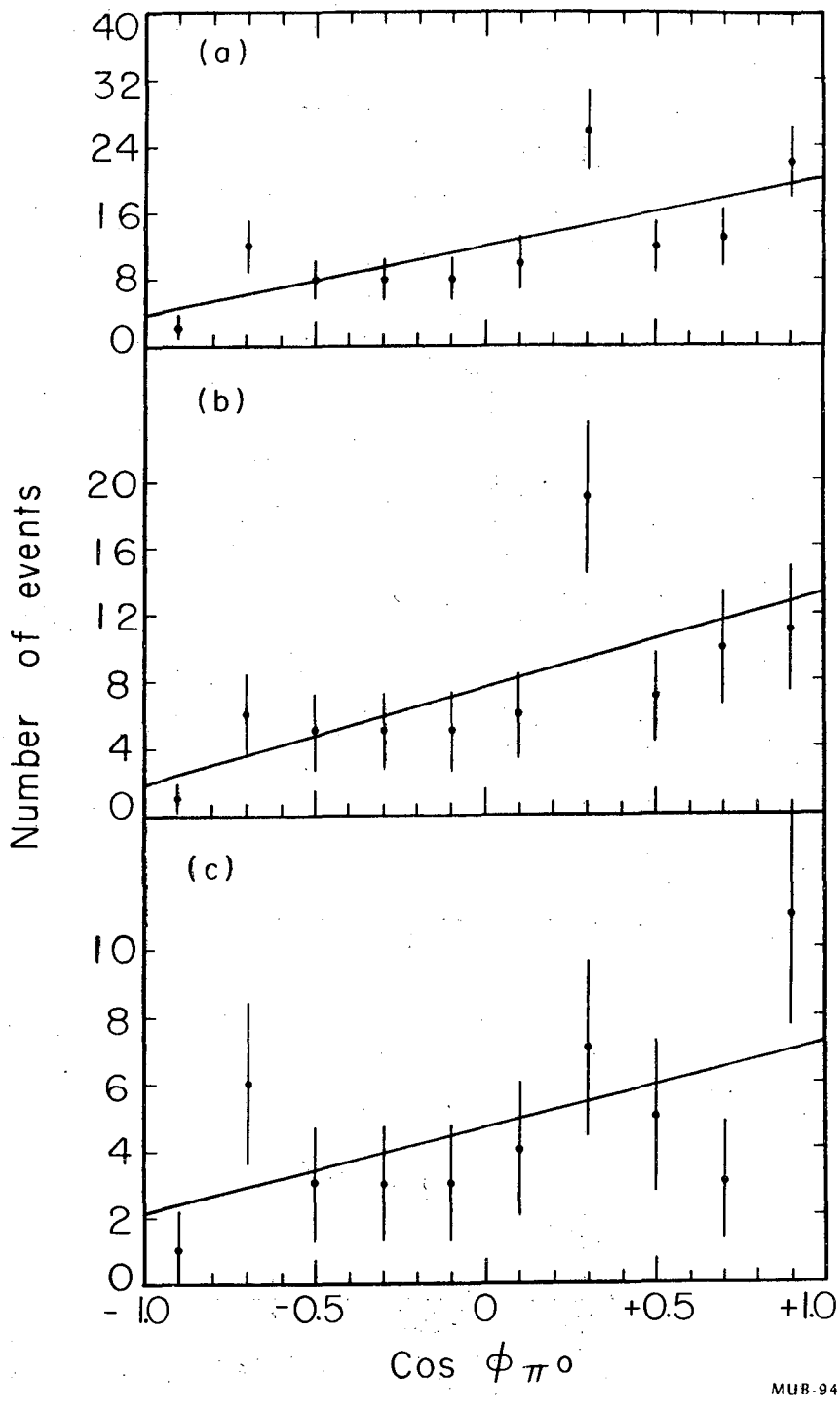
Fig. 7. The up-down asymmetries for the $\Sigma^+ \rightarrow p + \pi^0$ decay mode compared with other experimental results. The solid curve was determined from fitted s and p amplitudes. The result of Cool et al. has been determined by a counter experiment, looking at an angle (c.m.) of 87 ± 15 deg.

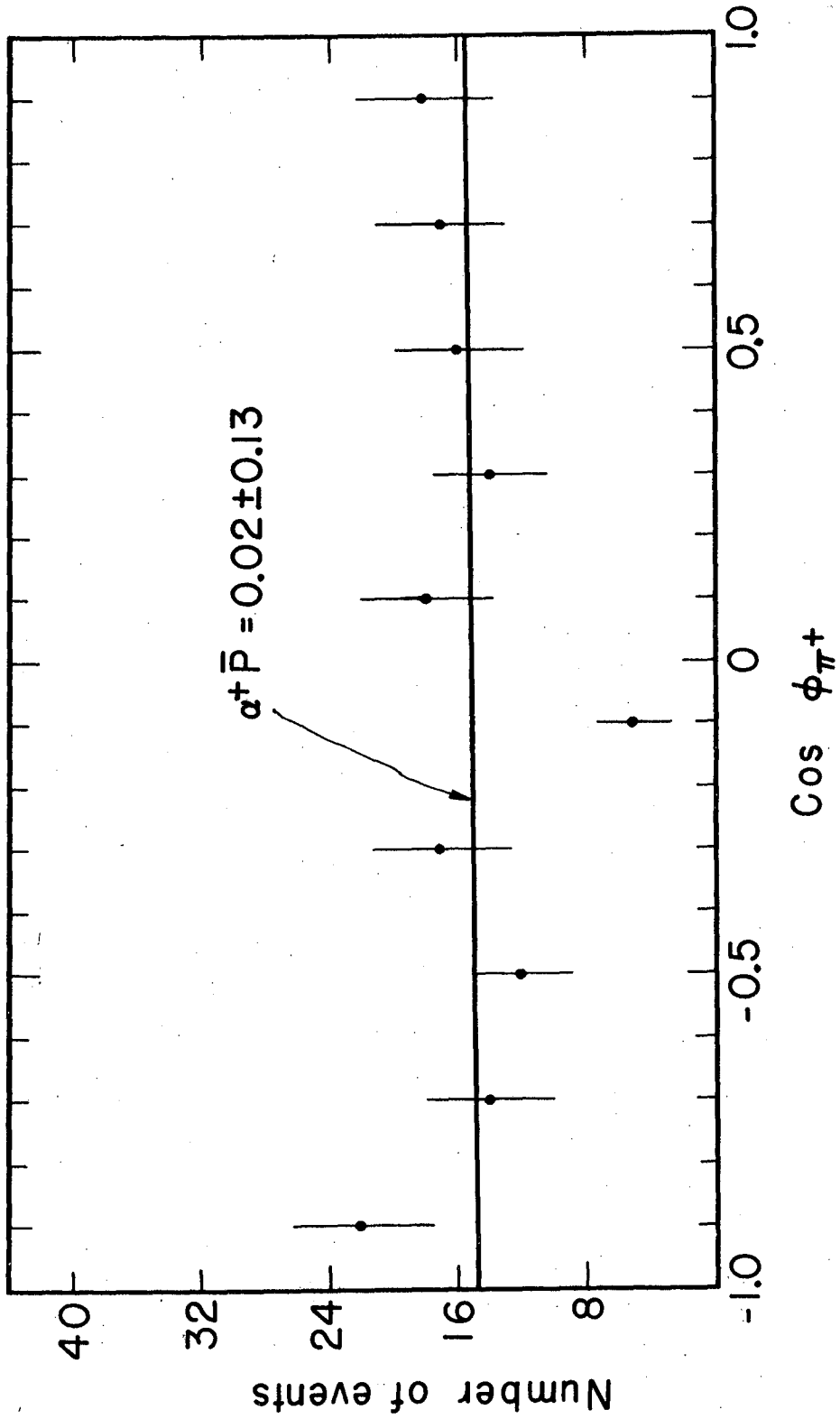
The corresponding value taken from s and p amplitudes is 0.92. Symbols are as follows: (●) from reference 23; (▲) from reference 5; (0) this experiment; (□) from reference 16.


-1453.0
-1454.0
ln$\mathcal{L}$
α^o = +1.00
α^o = +0.80
80
100
120
140
160
180
200
y^{th} (cm)

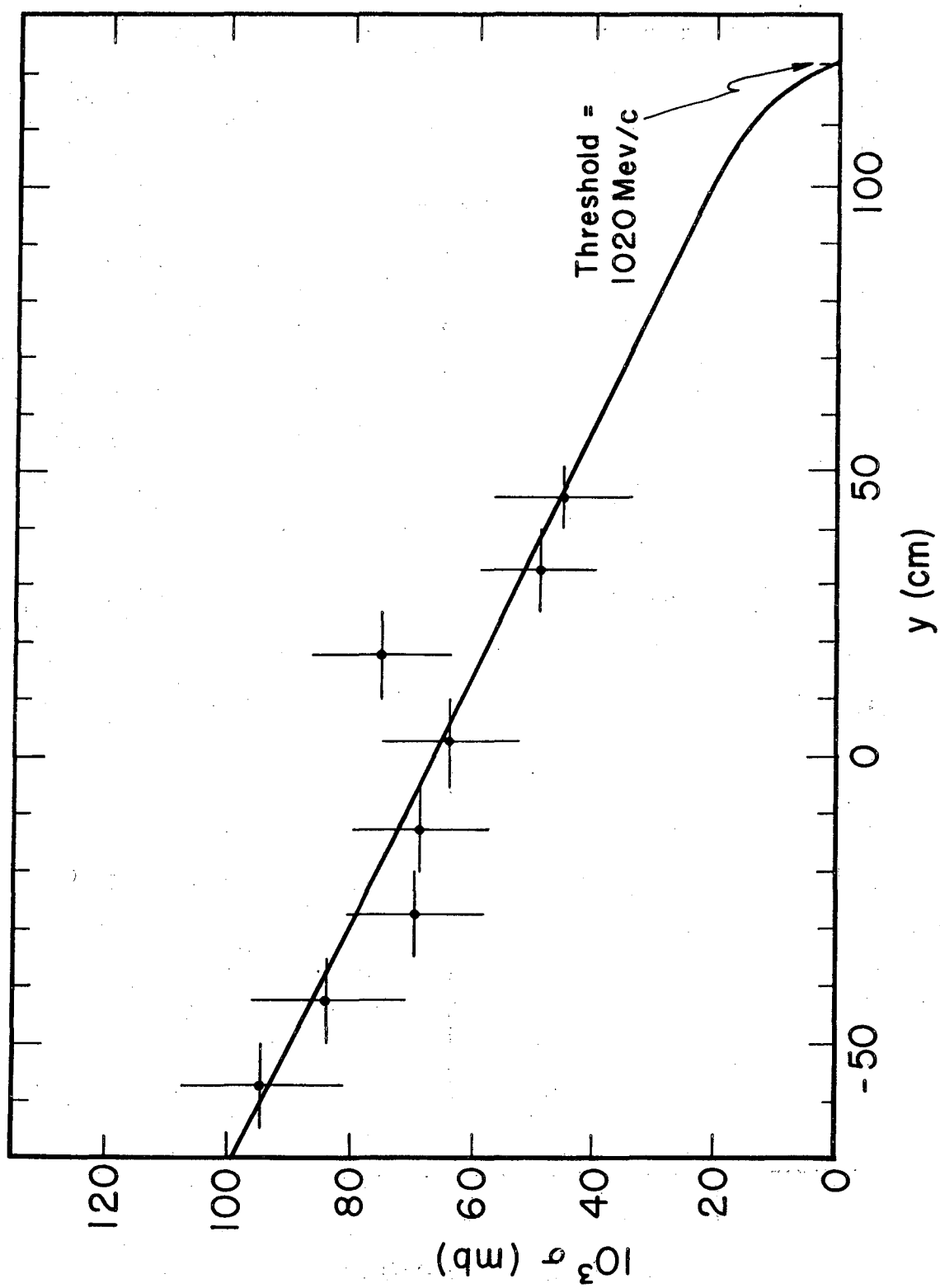
MU-25948


(a)
(B) $\left[\frac{d\sigma}{d\Omega} \propto 1+(0.68\pm0.10)\cos\theta+(0.13\pm0.16)\cos^2\theta\right]$
(A)
(b)
(A)
(B) $\left[\frac{d\sigma}{d\Omega} \propto 1+(0.73\pm0.13)\cos\theta+(0.20\pm0.22)\cos^2\theta\right]$
(c)
(B) $\left[\frac{d\sigma}{d\Omega} \propto 1+(0.60\pm0.15)\cos\theta+(0.04\pm0.25)\cos^2\theta\right]$
(A)
$10^3 \frac{d\sigma}{d\Omega}$ (mb/sr)
Cos θ_{Σ^+}
0
2
4
6
8
10
-1.0
-0.5
0
0.5
1.0

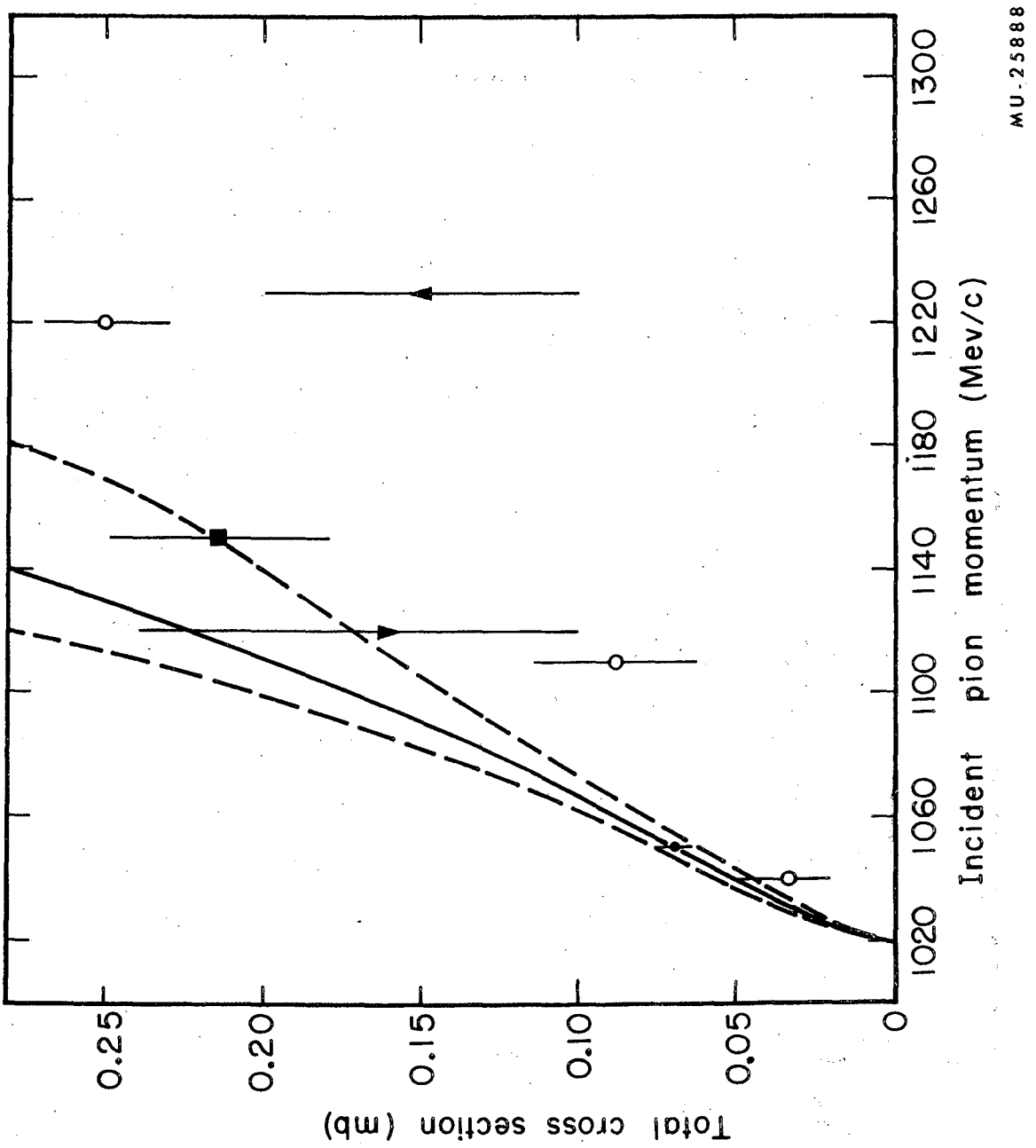

(a)
(b)
(c)
Number of events
Cos ϕ_{π^0}
- 1.0
-0.5
0
+0.5
+1.0
MUB-944

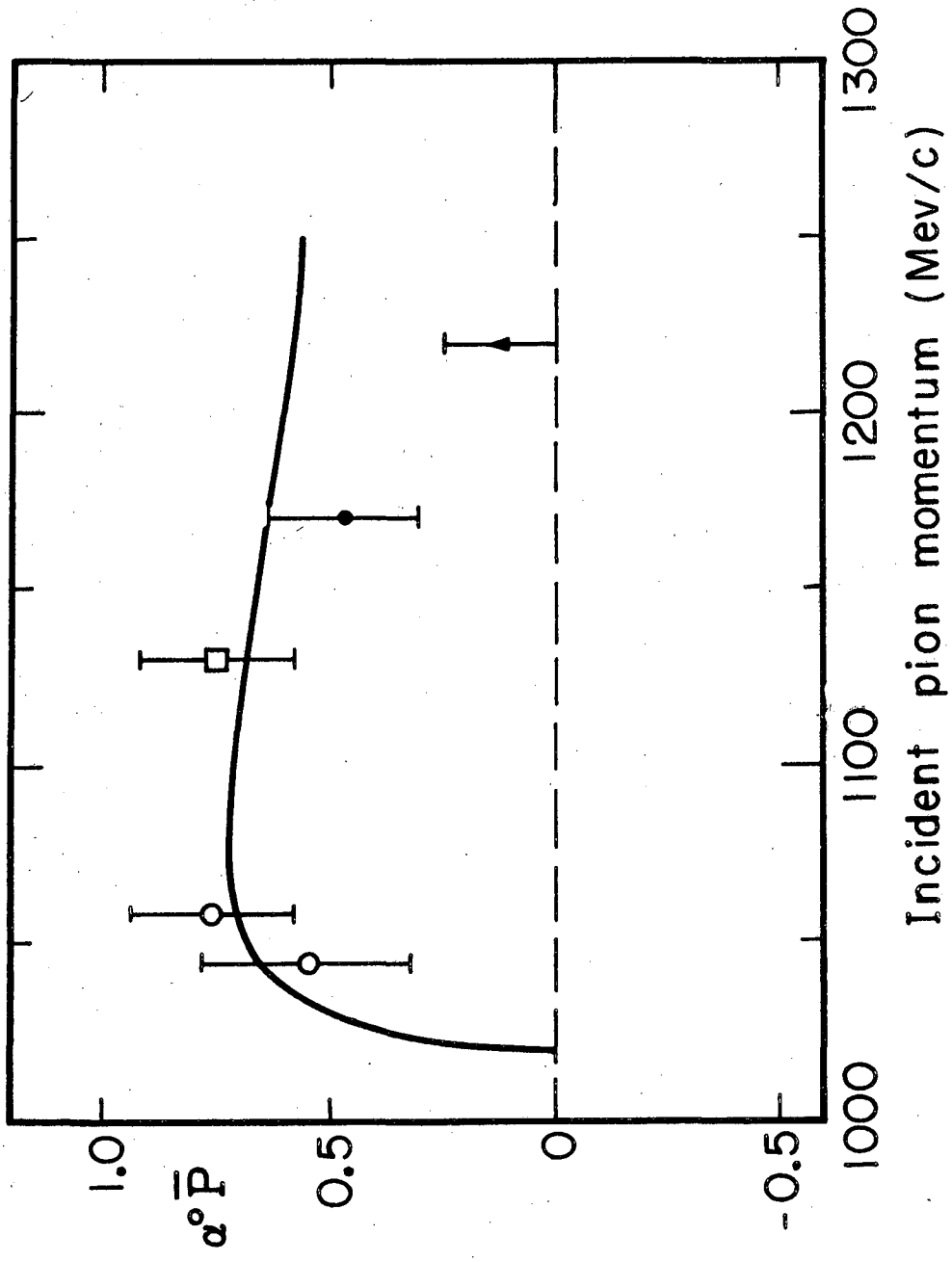

$\alpha^+\bar{P} = 0.02 \pm 0.13$
Number of events
40
32
24
16
8
-1.0
-0.5
0
0.5
1.0
Cos ϕ_{π^+}

MU-25890


Threshold =
1020 Mev/c
120
100
80
60
40
20
0
$10^3\sigma$ (mb)
-50
0
50
100
y (cm)

MU-25891


Total cross section (mb)
0.25
0.20
0.15
0.10
0.05
0
1020
1060
1100
1140
1180
1220
1260
1300
Incident pion momentum (Mev/c)
MU-25888


1.0
0.5
0
-0.5
$\alpha^{\circ}\bar{P}$
1000
1100
1200
1300
Incident pion momentum (Mev/c)

MU-25889

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