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REGGE TRAJECTORIES FOR A SINGLE ATTRACTIVE YUKAWA POTENTIAL

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ABSTRACT

The Regge trajectories of the poles in the S matrix for nonrelativistic scattering by a single attractive Yukawa potential have been investigated. They are found to be closed loops in the complex angular-momentum plane starting from, and returning to, the negative integers as the energy goes from $-\infty$ to $+\infty$. Detailed results are graphically presented.

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Recent conclusions by Regge and coworkers[1] concerning poles of the scattering amplitude in the complex angular-momentum plane for nonrelativistic scattering have been shown by Chew et al.[2] to have an important bearing on the interpretation of relativistic dispersion relations. Many features of low-energy resonances and high-energy cross sections can also be understood in terms of these poles.[2] It is of interest to investigate in detail the behavior of these pole "trajectories" in the nonrelativistic case to gain further insight into this and also the more general relativistic problem.

The Schrödinger equation for a single Yukawa potential has been solved numerically on IBM 709/7090 computers, and the poles of the S matrix found. Both positive and negative energies were investigated, and the Regge trajectories of the poles as a function of energy were traced in the complex angular-momentum plane. Several attractive potential strengths were investigated.

The Schrödinger equation can be written

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell+1)}{r^2} + \frac{A\exp(-r)}{r}\right] u(r) = 0 . \qquad (1)$$

The boundary condition imposed at the origin for all ℓ was

$$u(r) \sim r^{\ell+1} , \qquad (2)$$

and Eq. (1) was then integrated numerically, by using standard techniques, to to a region where the asymptotic form

$$u(r) \sim f_+(\ell,k)e^{ikr} - f_-(\ell,k)e^{-ikr} \tag{3}$$

is applicable.

The S matrix is defined by

$$S(\ell,k) = \frac{f_+(\ell,k)}{f_-(\ell,k)} e^{i\pi\ell} . \tag{4}$$

For a fixed energy, k^2, we vary ℓ to search for the zeros of $[S(\ell,k)]^{-1}$ corresponding to the poles of $S(\ell,k)$. A standard Newton's method is used in this search. Once a pole is located, k^2 is varied slightly and the search repeated starting in the neighborhood of the original zero. By varying k^2 from large negative to large positive values, we trace a complete trajectory of the Regge poles, which is found to be a closed loop in the complex ℓ plane.

It should be noted that, at least as far as the first Regge trajectory (that trajectory which lies furthest to the right in the complex ℓ plane) is concerned, no matter how small the potential strength, Re ℓ is greater than -0.5 near $k^2 = 0$. It is an interesting hypothesis that this may be true for all Regge curves (see Fig. 1). However, we have not yet been able to obtain conclusive evidence on this point.

For small k^2, the standard threshold behavior for the curves is

$$\text{Im } \ell \sim k^{2\ell_0+1} \tag{5a}$$

and

$$\text{Re } \ell \sim A k^2 + B k^{2\ell_0+1} + \ell_0 , \tag{5b}$$

where ℓ_0 is the threshold value of the position of the pole. This gives confidence in the correctness of the numerical procedure. Another check is provided in the large-energy behavior $(k^2 \to \pm\infty)$ of Eq. (1). In these limits,

only that region of r near the origin, where the potential is comparable to k^2, is important. The potential can be replaced in Eq. (1) by A/r in this region, and Singh[3] has shown analytically that for this Coulomb potential, the position of the poles is given by

$$\ell = -n + \frac{i\,A}{2\sqrt{k^2}} \tag{6}$$

where $n = 1, 2, 3, \cdots$ gives the first, second, third, etc. poles respectively. It is found that the Regge curves for large $|k^2|$ go over smoothly to the analytic form in Eq. (6), and, in particular, the curves go to $-n$ as $|k^2| \to \infty$. It is only necessary to carry out the numerical investigation in a region where Eq. (6) is not applicable in order to trace the full curve. The final results are presented in Figs. 1 and 2.

We are greatly indebted to Professor G. F. Chew and other members of the Physics Department of the Lawrence Radiation Laboratory for advice and encouragement in this work.

FOOTNOTES AND REFERENCES

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

1. T. Regge, Nuovo Cimento 14, 951 (1959); 18, 947 (1960). See also A. Bottino, A. M. Longini, and T. Regge, Potential Scattering for Complex Energy and Angular Momentum (University of Turin preprint, 1961).

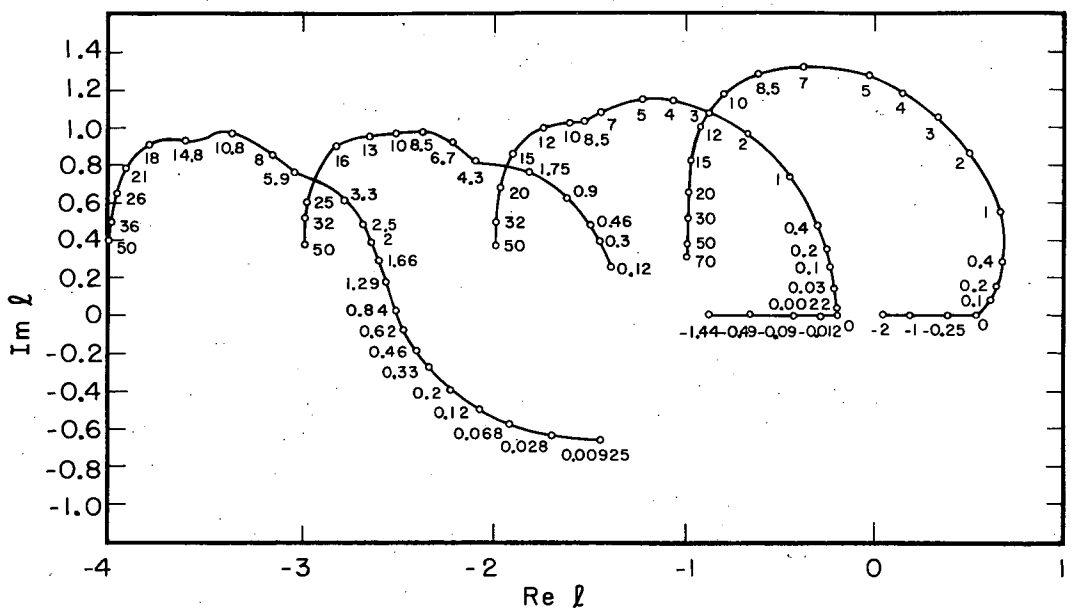
2. G. F. Chew, S. C. Frautschi, and S. Mandelstam, Regge Poles in $\pi\pi$ Scattering, UCRL-9925, November 9, 1961 and Phys. Rev. (to be published). See also Geoffrey F. Chew, S-Matrix Theory of Strong Interactions without Elementary Particles, UCRL-10058, February 1962.

3. Virendra Singh, Analyticity in the Complex Angular-Momentum Plane of the Coulomb-Scattering Amplitude, UCRL-9972, December 1961.

FIGURE LEGENDS

Fig. 1. The first few Regge trajectories for the potential strength $A = 5$. The k^2 values are written adjacent to the curves. The small positive k^2 values for the third and fourth curves have not yet been obtained owing to numerical difficulties.

Fig. 2. The first Regge trajectory for a number of potential strengths A. The k^2 values are written adjacent to the curves (When k^2 is negative, some of these values are shown displaced from the real axis for the sake of clarity.)



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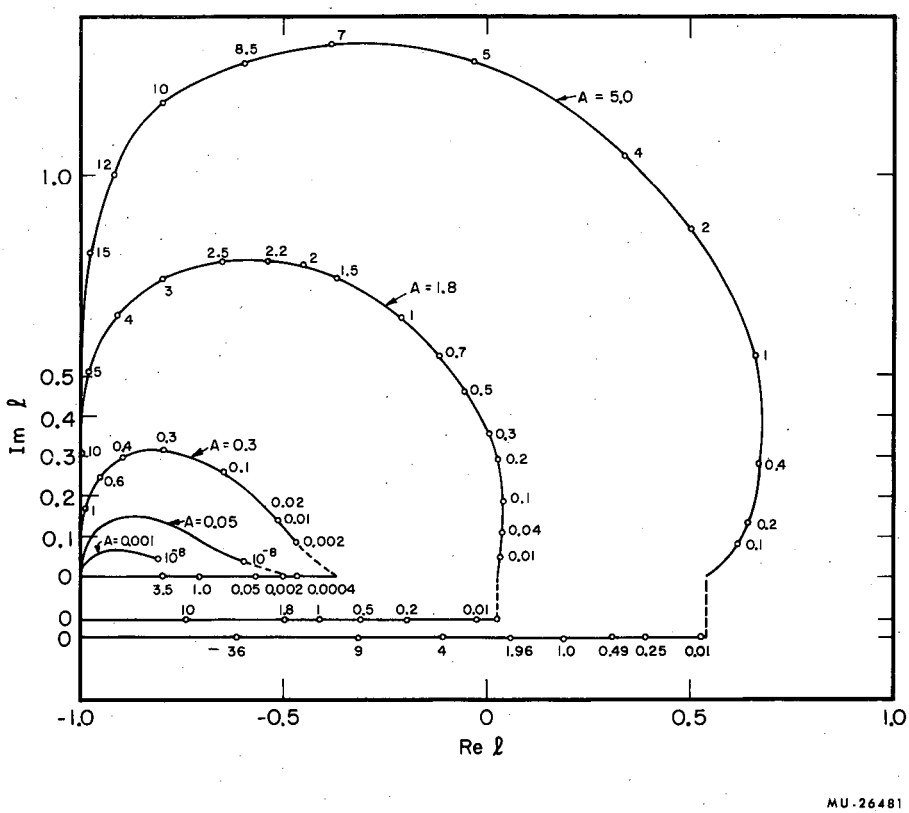

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