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Contract No. W-7405-eng-48

CALCULATION OF REGGE TRAJECTORIES BY USING DISPERSION RELATIONS

Philip G. Burke and Cecil Tate

May 24, 1962

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Blankenbecler et al.[1] have shown that for certain classes of potentials the Schroedinger equation is equivalent to the Mandelstam representation combined with unitarity. Given the first Born approximation for the scattering amplitude the full amplitude can be obtained by a step-by-step procedure.

Explicitly, the equivalent equations are

$$\tilde{D}(t,s) = v(t) + \frac{1}{\pi} \int ds' \frac{\rho(s',t)}{s' - s} , \tag{1}$$

$$\rho(s,t) = \frac{1}{2\pi} \frac{1}{k} \iint dt' \, dt'' \frac{\tilde{D}^*(t',s) \, \tilde{D}(t'',s)}{[t^2+t'^2+t''^2 - 2(tt' + tt'' + t't'') - tt't''/k^2]^{1/2}} . \tag{2}$$

The integration in (2) is carried out over that region of the $t' \; t''$ space where the denominator function is positive. We have

$$(s - 4)/4 = k^2 \quad \text{and} \quad t = -2k^2(1 - \cos\theta) . \tag{3}$$

The function $v(t)$ is related to the potential in configuration space by

$$V(r) = -\frac{1}{\pi} \int_0^\infty dt \, v(t) \exp(-t^{1/2} r)/r . \tag{4}$$

Finally, the full scattering amplitude can be written

$$f(s,t) = \frac{1}{\pi} \int_0^\infty \frac{\tilde{D}(t',s)}{t' - t} \, dt' . \tag{5}$$

Now we know that the scattering amplitude in potential scattering can be represented as a sum of an infinite number of Regge pole terms,[2] and

is dominated for large momentum transfers t by that pole which lies farthest to the right in the complex angular-momentum plane ℓ. We obtain therefore

$$\tilde{D}(t,s) \underset{t \to \infty}{\sim} \beta(s)\, t^{\alpha(s)} , \tag{6}$$

where $\alpha(s)$ is the position of this pole in the ℓ plane for a particular s. It can be shown that the asymptotic form (6) is consistent with Eqs. (1) and (2) provided that $\mathrm{Im}\,\alpha(s) \neq 0$. However, what is not so clear is how accurately it is possible to generate the trajectory $\alpha(s)$ by means of an iterated solution of Eqs. (1) and (2), given the input potential $v(t)$.

We have therefore evaluated the trajectory $\alpha(s)$ of the pole farthest to the right in the ℓ plane by two methods. First, using a potential $v(t)$ ($= 5/9$ for $4 \leqslant t \leqslant 400$ and zero outside this range), we carried out the iteration of (1) and (2) until we reached a value of t (between 2000 and 5000) where the oscillatory behavior implied by (6) had developed. We then evaluated the real and imaginary parts of α at several energies by looking at the frequency and amplitude of the oscillations in the double spectral function. The behavior of the double spectral function as a function of t is exemplified by Fig. 1 where we give the result of an equivalent calculation which uses the relativistic unitarity condition (see below). The second method used was to solve the Schroedinger equation using the potential $V(r)$ related to $v(t)$ through Eq. (4):

$$V(r) = -\frac{10}{9\pi r^3}\left[(1 + 2r)e^{-2r} - (1 + 20r)e^{-20r}\right] . \tag{7}$$

The comparison of the two methods for the Regge trajectory farthest to the right in the ℓ plane is given in Figs. 2 and 3. It is seen that the dispersion-relation approach is accurate to about 0.01 in the ℓ plane. It

proved difficult to obtain the second Regge trajectory from the dispersion-relation method owing to the insufficiency of the numerical accuracy. The calculation was carried out on an IBM 7090 and required about five minutes computing time for the accuracy shown.

Chew and Frautschi[3] have considered the modifications to Eqs. (1) and (2) necessary for the relativistic case. In this case $v(t)$ is replaced by a complex potential $v(t,s)$ obtained from crossing symmetry, and an extra factor $4/\sqrt{s}$ is introduced into the unitarity relation (2). The equations can be solved with the same amount of effort as the original equations, provided that we know $v(t,s)$ for all relevant t and s. It then becomes possible in principle to treat the coupled integral equations arising in pion-pion scattering by this method.[4] The integrals, although formally divergent, can be carried out by assuming a Regge pole behavior for the amplitude. Equations (1) and (2) were solved for a variety of potentials $v(t,s)$ in the relativistic case, and the corresponding Regge trajectories determined. In Fig. 1 we show one example of such a calculation. An additional difficulty, first pointed out by Chew et al.,[4] arises from the presence of Regge pole terms in the crossed channel. This implies that the potential has the behavior

$$v(t,s) \underset{s \to \infty}{\sim} \beta(t) s^{\alpha(t)} , \tag{8}$$

and, if $\mathrm{Re}\,\alpha(t) \geqslant 1$ for some t, the unitarity relation becomes divergent on iterating Eqs. (1) and (2). Chew et al. regard this as evidence that many particle intermediate states, not included in (2), give a strong repulsive contribution at small distances; however, it is difficult to know how to allow for such effects in the theory. The troublesome behavior implied by (8) occurs for those regions of $\rho(s,t)$ with small t and large s, and so

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where a and b are chosen to give continuity in the ρ function and its derivative. In Fig. 4 we show the effect on the first Regge trajectory of applying such a cutoff. The potential used in this run was a function of t only. We see that the regions of the double spectral function omitted do not play a vital role in determining the trajectory, provided α and θ are chosen appropriately. We are thus encouraged to believe that if the conjecture of Chew et al.[4] is correct and the divergence in the unitarity condition is prevented by a hard core in the potential, then the use of the elastic unitarity condition (2) together with this cutoff procedure will be capable of giving reasonably accurate trajectories in the full pion-pion problem.

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REFERENCES

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FIGURE CAPTIONS

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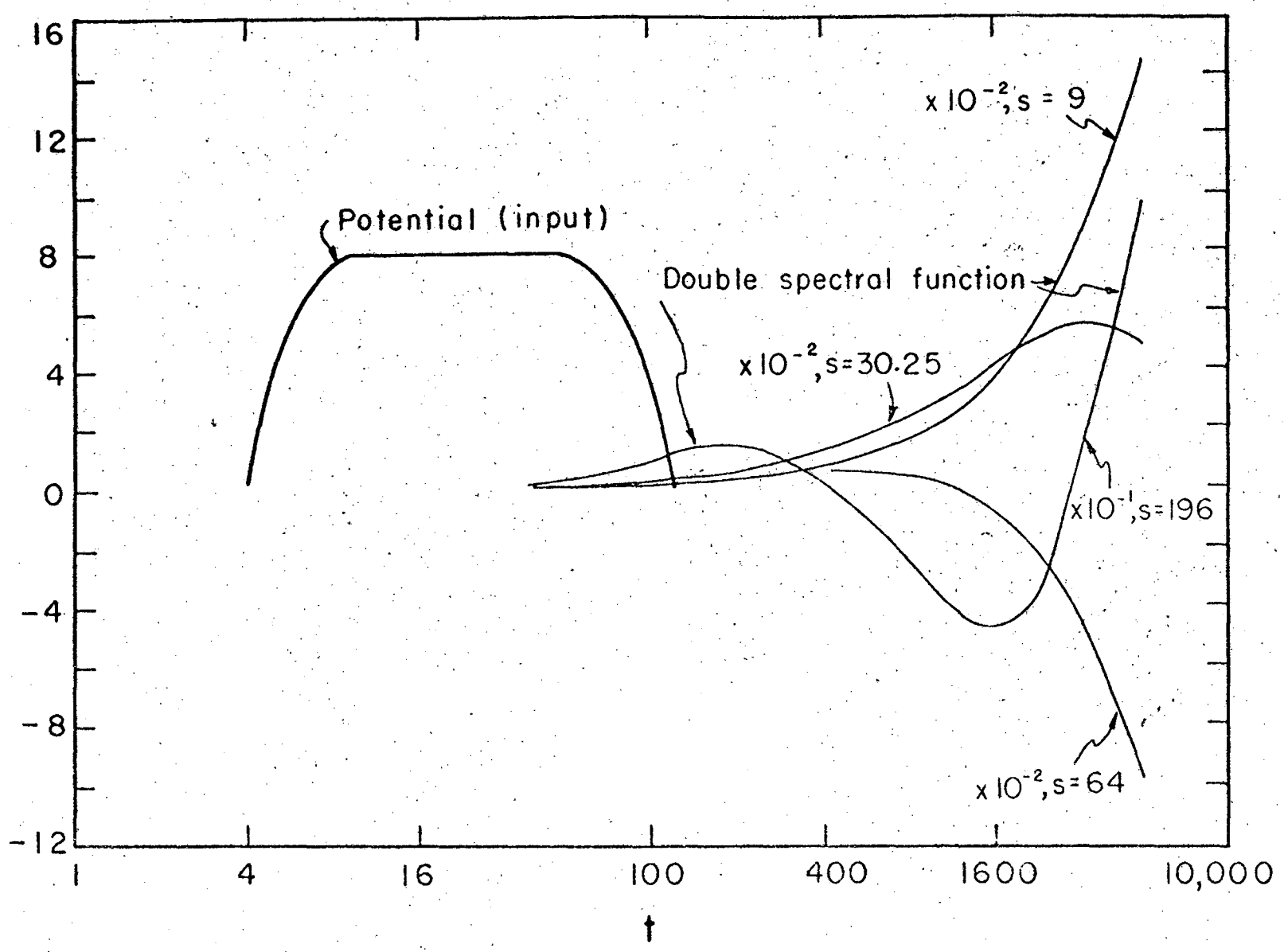

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Fig. 1

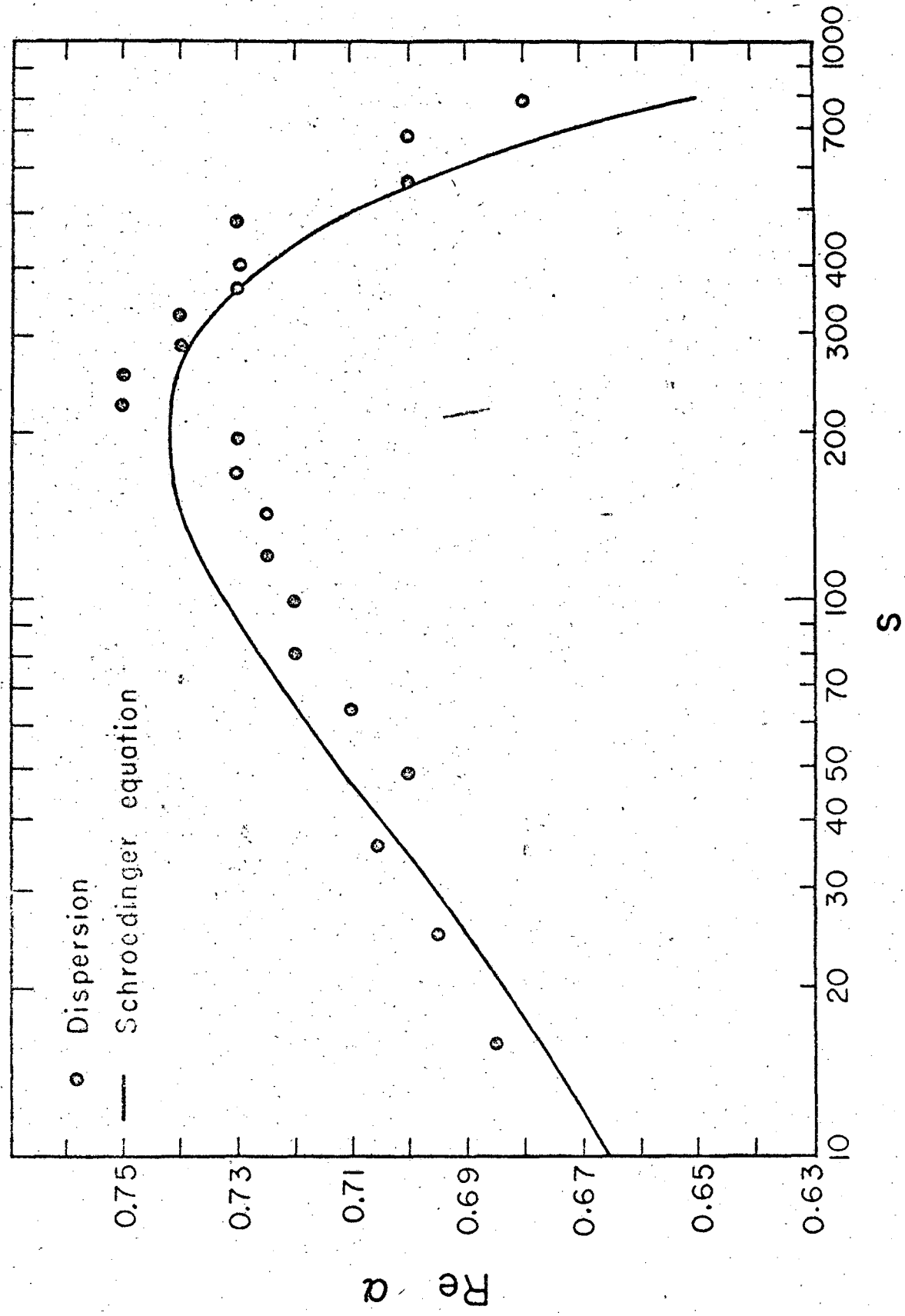

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Fig. 2

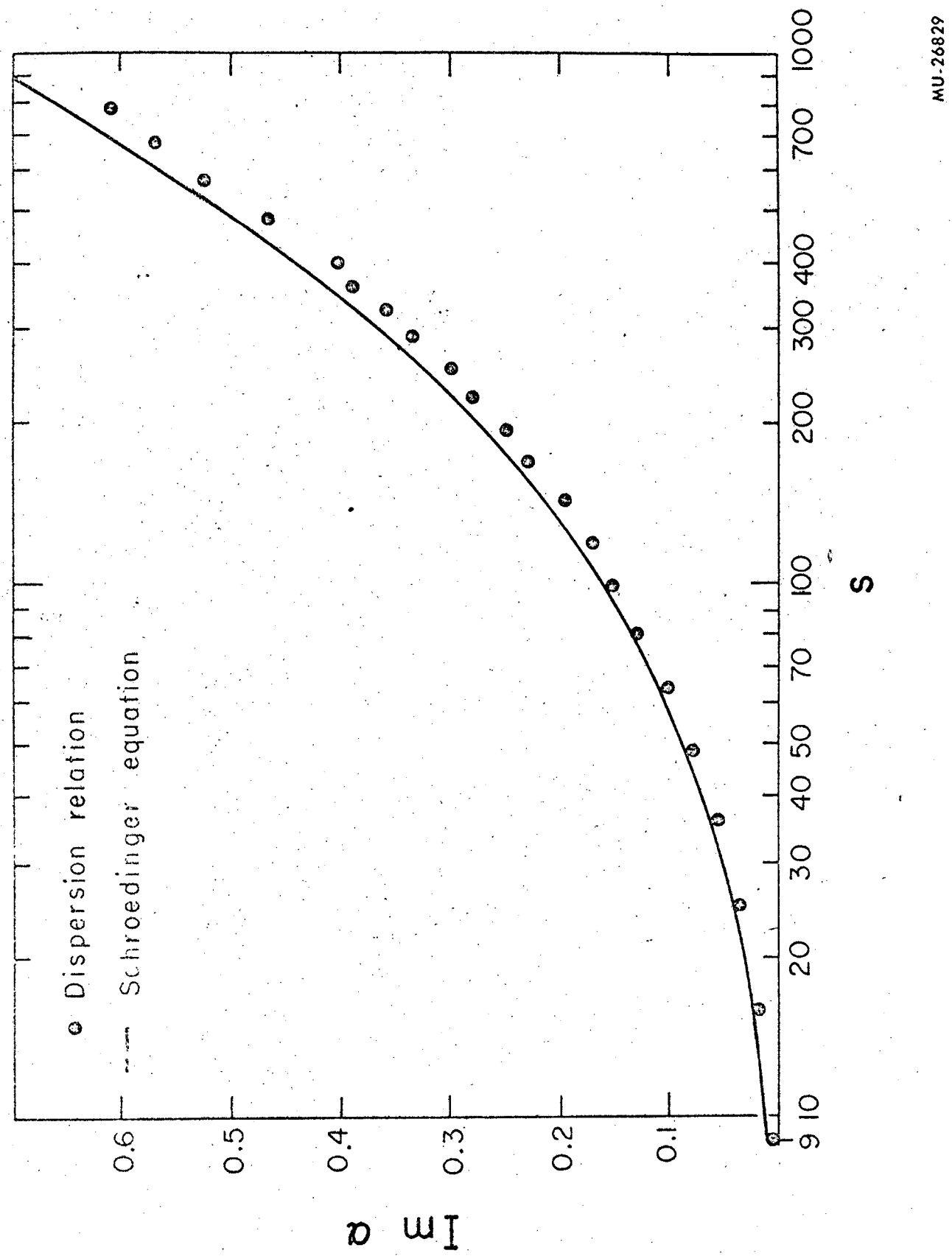
Dispersion relation
Schroedinger equation
Im α
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Fig. 3

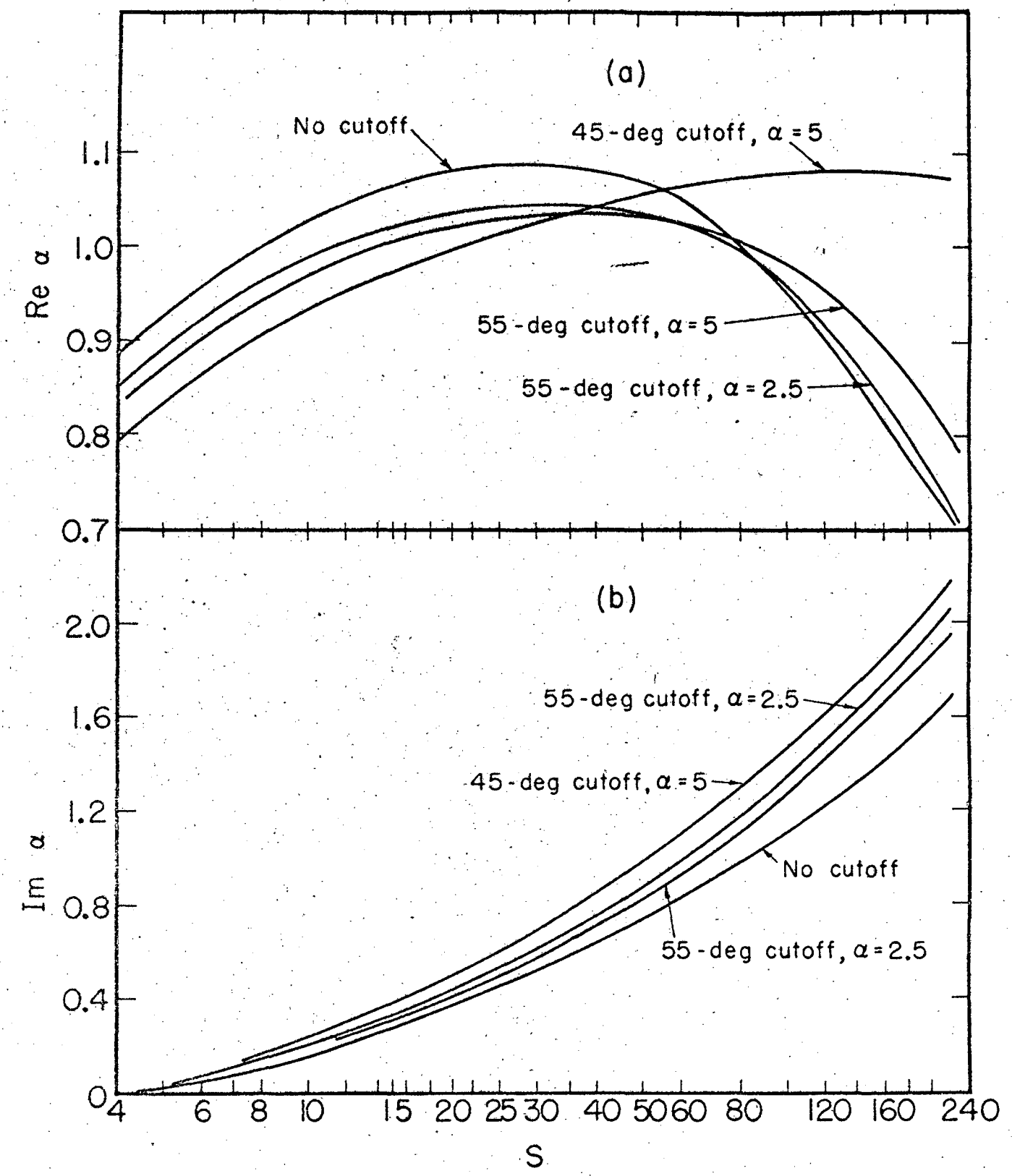

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Fig. 4

Physics + Math

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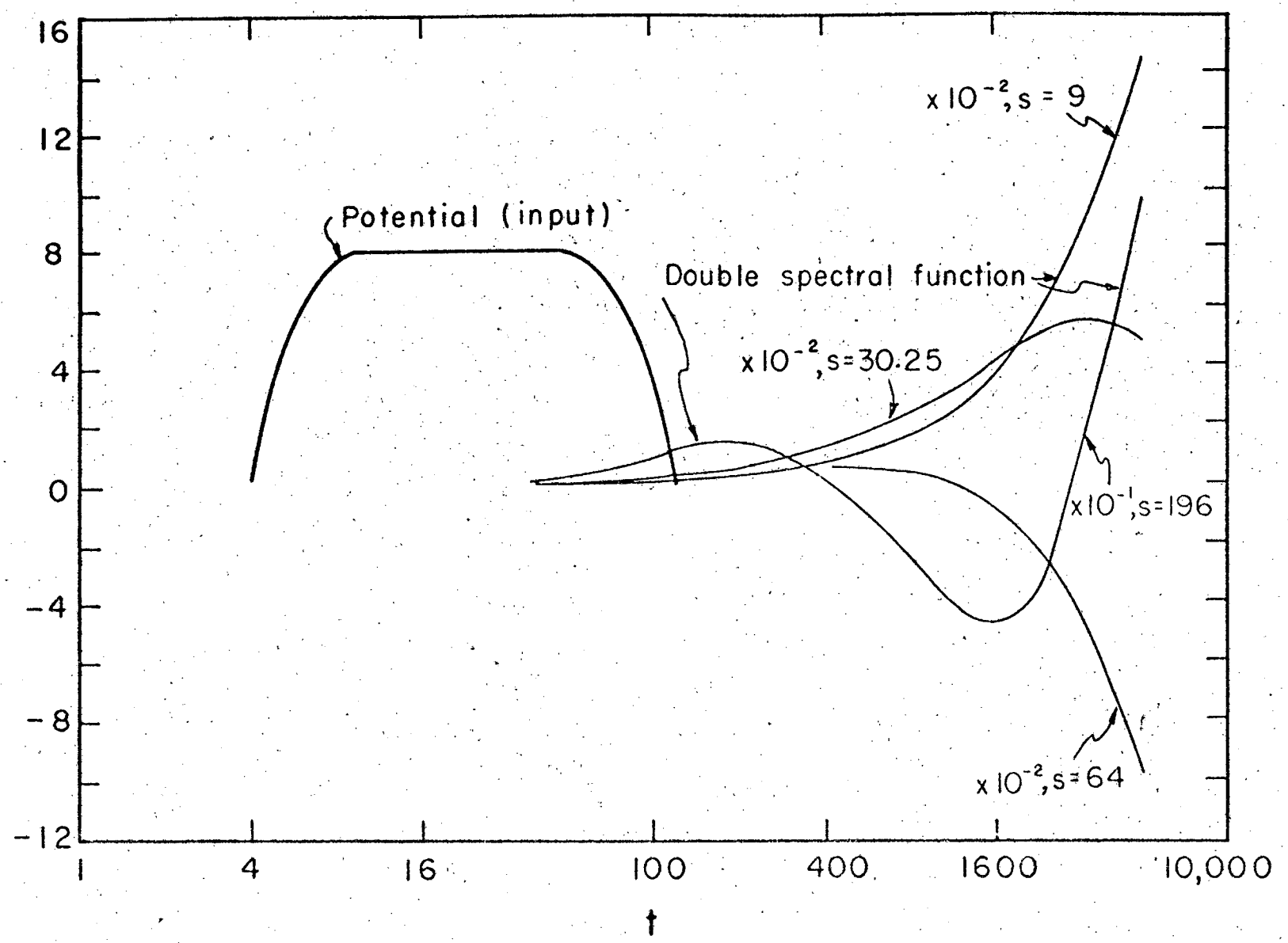

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Fig. 1

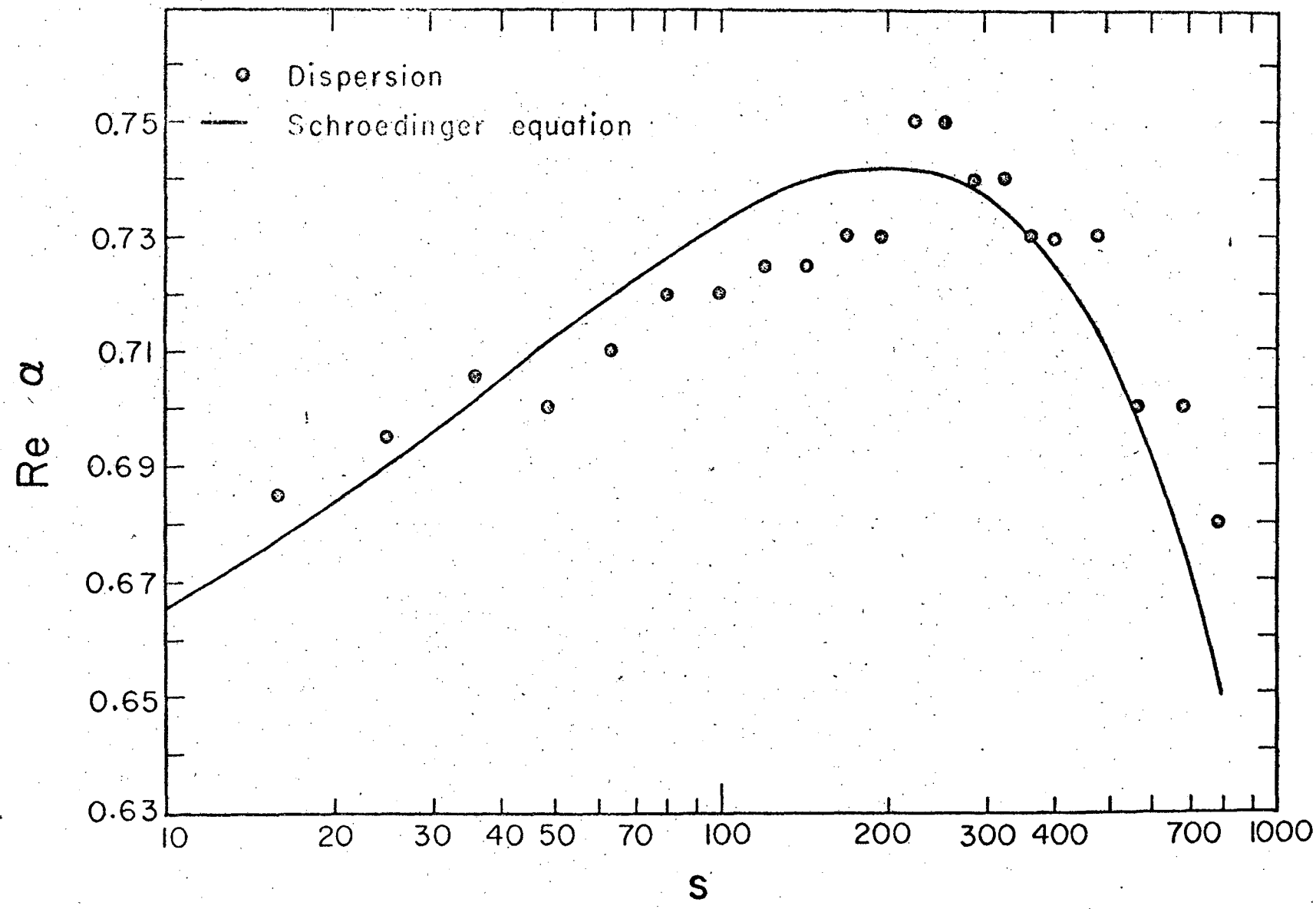

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Fig. 2

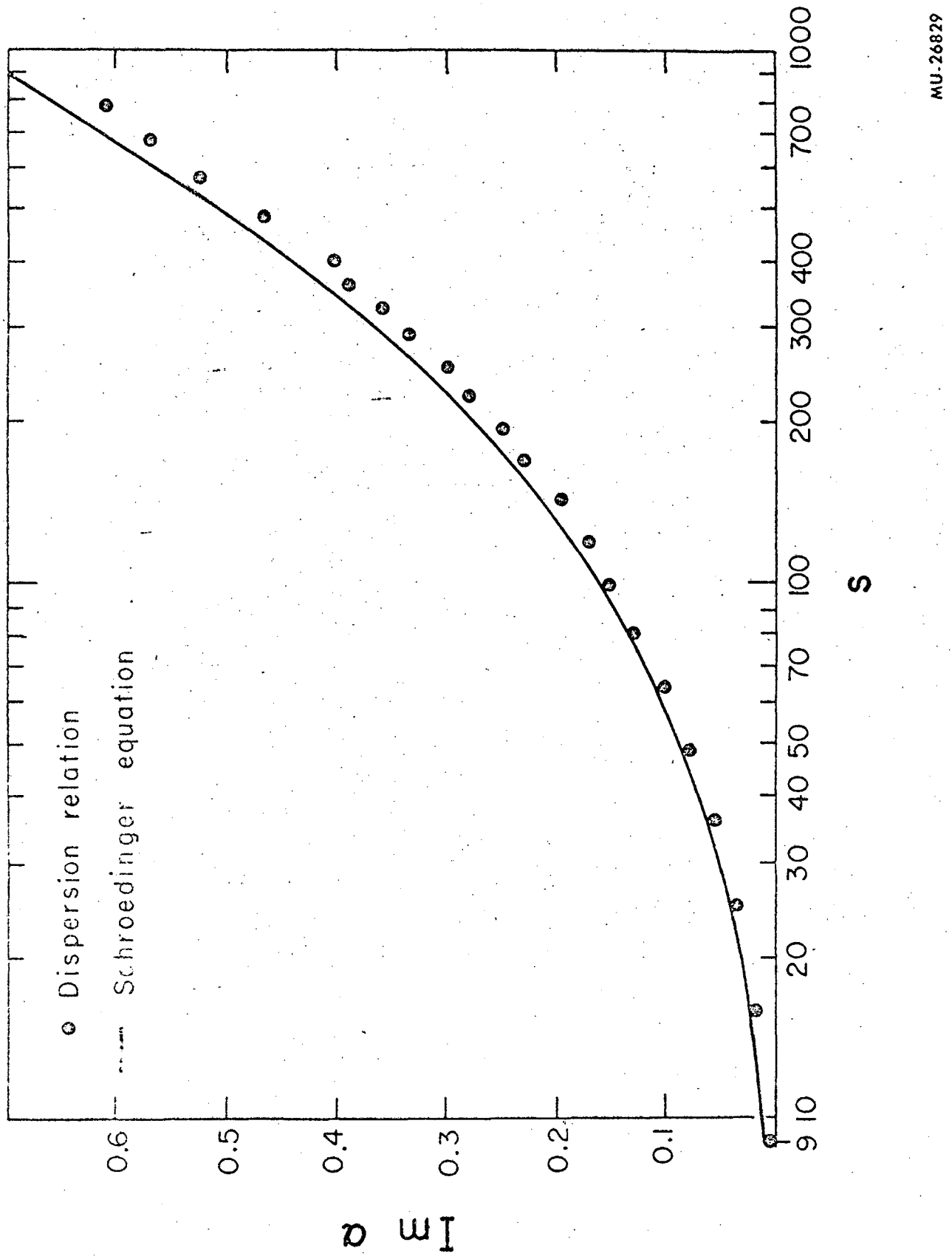
Dispersion relation
Schroedinger equation
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Fig. 3

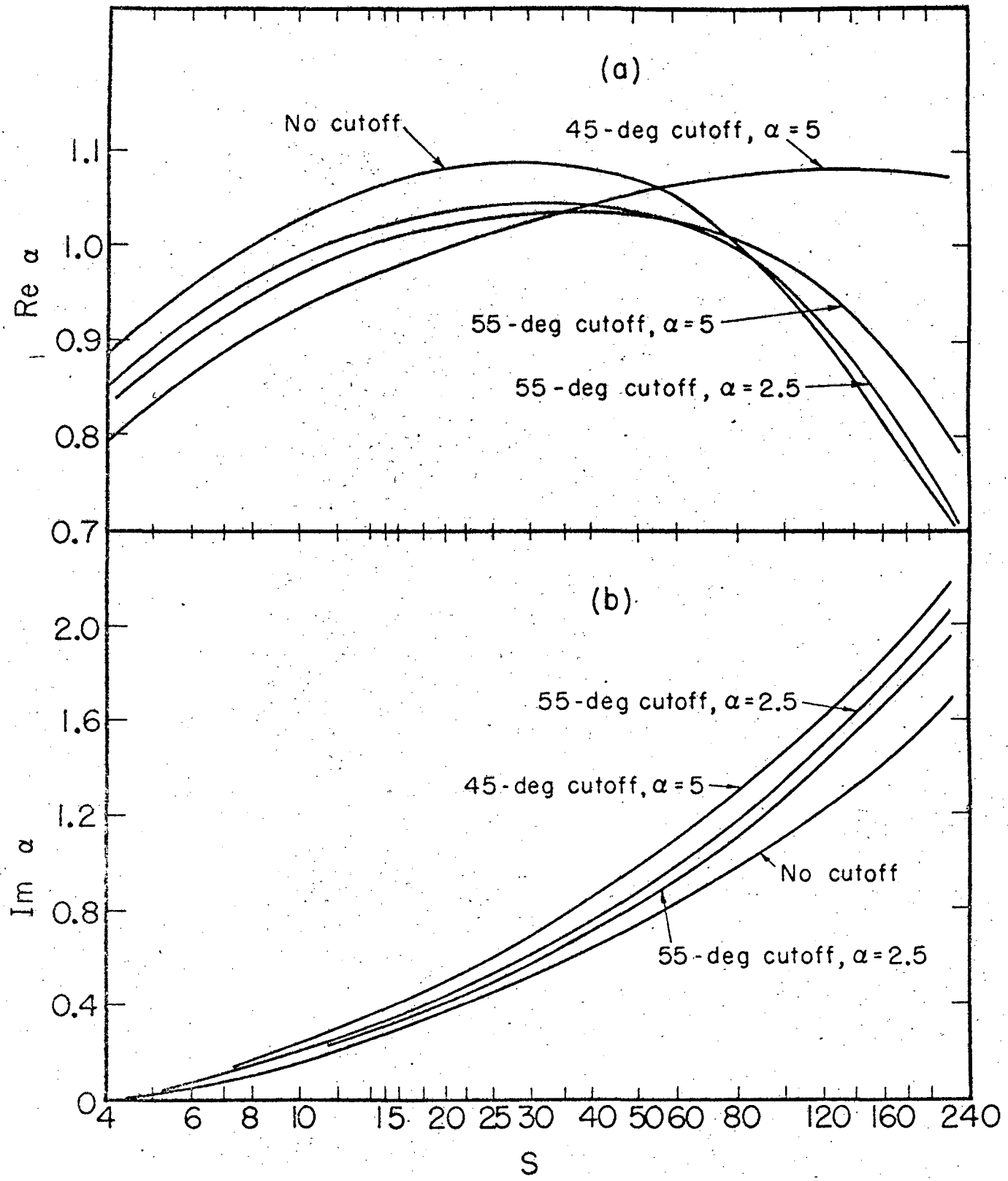

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Fig. 4