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COORDINATE SPACE AND POSITION VARIABLES IN S-MATRIX THEORY

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ABSTRACT

The rule for obtaining coordinate-space probabilities from the momentum-space S matrix is discussed. The distinction between coordinate variables and position variables, raised by Newton and Wigner, is important in this respect.

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The S matrix can be considered to be a set of functions of momentum, spin, and particle type variables. These functions give the transformation relating amplitudes for idealized initial and final experiments performed in the infinite past and infinite future. In particular, one may write[1]

$$a_f(K') = \Sigma \int_K S(K', -\tilde{K})\, a_i(K) \; . \tag{1}$$

We take the integrations to be over the invariant momentum-space elements:

$$\begin{aligned} \Sigma \int_{K''} &\equiv \Sigma \int \prod_i \frac{d^4k_i}{(2\pi)^4} \; 2\pi \; 2m_i \theta(k_i^o)\, \delta(k_i^2 - m_i^2) \\ &= \Sigma \int \prod_i \frac{d^3k_i}{(2\pi)^3} \; \frac{m_i}{\omega_i} \; . \end{aligned} \tag{2}$$

With this convention, the no-scattering part is

$$S_0(K', -\tilde{K}) = \prod_i \frac{(2\pi)^4\, \delta^4(k'_i - k_i)\, \theta(k_i^o)}{(2\pi)\, 2m_i\, \delta(k_i^2 - m_i^2)} \; . \tag{3}$$

One can also consider $S(K', -\tilde{K}'')$ to be the final amplitude for the particular case in which the initial particles have the fixed momenta, spins, and types described by K'', and are represented by the amplitude

$$a_i(K, K'') = \prod_i \left(\frac{(2\pi)^4\, \delta(k_i - k''_i)\, \theta(k_i^o)}{2\pi\; 2m_i \delta(k_i^2 - m_i^2)} \right) \delta_0 \; , \tag{4}$$

where δ_0 represents the unit matrix in the spin and type indices. The

momentum vectors of K, K', and K'' are here restricted to positive energies, and the variables are to be normal-ordered.

The coordinate-space functions are defined by Fourier transformation. Thus one can write

$$S(X, -\tilde{K}'') \equiv \int_{K'} (\exp -i\, X\cdot K')\, S(K', -\tilde{K}''), \tag{5}$$

where the integration is as defined above and

$$X\cdot K' = \prod_i x_i\cdot k_i = \prod_i (x_i^{\,o}\, t_i^{\,o} - \underset{\sim}{x}_i\cdot \underset{\sim}{k}_i) . \tag{6}$$

This function, $S(X,-\tilde{K}'')$, is the final coordinate-space function corresponding to the initial amplitude given in (4).

Let S' be the function obtained from S by the transformation that takes R to M_1. The corresponding coordinate-space functions have the simple invariance property

$$S'(X, -\tilde{K}'') = \Lambda_S^{-1}\, S'(\Lambda X, -\Lambda\tilde{K}''), \tag{7}$$

which follows immediately from the analogous property of $S'(K', -\tilde{K}'')$. One can form linear combinations of the initial states and obtain more general functions

$$\phi_\alpha(X) = \sum \int_{K'K''} (\exp -i\, X\cdot K')\, S'(K', -\tilde{K}'')\, K''\cdot\tilde{\sigma}\ \alpha(K''), \tag{8}$$

which have the property

$$\phi_\alpha(X) = \Lambda_S^{-1}\, \phi_{\Lambda\alpha}(\Lambda X) . \tag{9}$$

Although the coordinate-space functions introduced in this way have nice transformation properties, they are not the usual wave functions of

quantum theory, whose absolute value squared gives the probability density. Indeed, we see that the quantity

$$\int \phi_\alpha^*(X)\, \phi_\alpha(X)\, d^3X \tag{10}$$

is not an invariant. For suppose $\phi_\alpha(X)$ represents a large wave packet that is essentially at rest. Under Lorentz transformation its extent will shrink by the Lorentz-contraction factor, but (for scalar particles, say) the numerical value does not correspondingly increase, due to (9).

It is easy to find a function that has the correct property in this respect. Let us define

$$\psi_\alpha(X) = \Sigma \int_{K'} \left[\prod_i (k'^{\,o}_i/m_i)\right]^{1/2} (\exp -i\, X\cdot K')\, S'(K', \alpha) \quad . \tag{11}$$

Then, for the zero-spin case, one has

$$\int \psi_\alpha^*(X)\, \psi_\alpha(X)\, d^3X$$

$$= \Sigma \int_{K'} \Sigma \int_{K''} \int d^3X \;\; S'^*(K',\alpha)\, S'(K'',\alpha) \left[\prod_i \frac{k'^{\,o}_i\, k''^{\,o}_i}{m_i^2}\right]^{1/2} \exp(-i(X\cdot K' - X\cdot K''))$$

$$= \Sigma \int_{K'} S'^*(K',\alpha)\, S'(K',\alpha) = \text{invariant} \quad . \tag{12}$$

The distinction between $\psi(X)$ and $\phi(X)$ is a fundamental feature of relativistic particle theory. The two functions may be referred to as the wave function and the field, respectively. In field theory, the extra factor $\sqrt{k_i^{\,o}}$ relating the covariant fields ϕ to the wave functions arises in the expansion of the field in terms of the amplitudes $a(k)$, which are the Fourier components of the wave function [see, for instance,

Wentzel's Eq. (6.20)].[2] The same results are obtained by Newton and Wigner[3] directly from invariance arguments. They distinguish between coordinates and position, and construct a position operator in coordinate space. The result is equivalent to going from the coordinate representation function $\phi(X)$ to the position representation frunction $\psi(X)$ --from the field to the wave function. Certain complications due to spin, encountered by Newton and Wigner, are avoided if one uses the two-component formalism developed in reference 1; the same rule for passing to position variables holds for all values of the spin. However, if the particles have spin, one must use for probability density the function

$$P(X) = \psi_\alpha^*(X)\,(K\cdot\tilde{\sigma})\,\psi_\alpha(X) \,. \tag{13}$$

Here the k_i's in K are to be interpreted as differential operators on the position variables x_i of X.

REFERENCES AND FOOTNOTES

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1. H. P. Stapp, Phys. Rev. 125, 2139 (1962).

2. Gregory Wentzel, Quantum Theory of Fields (Interscience Publishers, Inc., New York, 1949).

3. T. D. Newton and E. P. Wigner, Revs. Modern Phys. 21, 400 (1949).

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