UCRL-10384

University of California

Ernest O. Lawrence Radiation Laboratory

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

Berkeley, California

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UCRL-10384

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory

Berkeley, California

Contract No. W-7405-eng-48

FORTRAN PROGRAM TREGGE

Philip G. Burke and Cecil Tate

July 24, 1962

FORTRAN PROGRAM TREGGE

Philip G. Burke and Cecil Tate

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 24, 1962

ABSTRACT

This report describes a Fortran program called TREGGE for an IBM-7090 computer. The program calculates Regge trajectories and S-matrix elements for an arbitrary potential.

FORTRAN PROGRAM TREGGE*

Philip G. Burke and Cecil Tate

Lawrence Radiation Laboratory
University of California
Berkeley, California

July 24, 1962

I. INTRODUCTION

This report describes a Fortran program called TREGGE, which calculates the scattering matrix from a Schroedinger equation in which the angular momentum is complex. In particular, the program will find the positions and residues of Regge poles as functions of energy, or it will compute S matrix and phase shifts at a specified grid of points in the complex-angular-momentum plane for a given set of energies. The program as presented treats a single Yukawa potential, but it is designed so that it could easily be modified to treat any analytically given potential.

The permissible range of variables comprises all complex values of angular momentum except negative half integers and real values of the energy.

The program has been used on an IBM-7090 to calculate Regge trajectories for single Yukawa potentials[1,2] and for a superposition of Yukawas.[3] Program decks are available from the Computer Library of the Lawrence Radiation Laboratory at Berkeley.

II. METHODS AND EQUATIONS

We are interested in calculating the scattering matrix for the Schroedinger equation

$$\left[\frac{d^2}{dr^2} + k^2 - V(r) - \frac{\ell(\ell+1)}{r^2}\right] \cdot u_\ell(r) = 0, \tag{2.1}$$

in which the angular momentum, ℓ, is complex and the energy k^2 can take positive and negative real values. The S-matrix $S(\ell,k)$ is defined by the asymptotic solution

$$u_\ell(r) \underset{r \to \infty}{=} e^{-ikr} - S(\ell,k)e^{-i\pi\ell}\, e^{ikr} , \tag{2.2}$$

and the boundary condition at the origin is

$$u_\ell(r) \underset{r \to 0}{=} r^{\ell+1} . \tag{2.3}$$

Our method for calculating $S(\ell,k)$ is to start with the regular solution for some small value of r and to continue this solution by numerical integration, so that it can be compared with the asymptotic form (2.2). The program is designed for potentials that can be written as a power series

$$V(r) = -\sum_{i=0}^{\infty} C_i r^{i-1} , \tag{2.4}$$

but the main routine is applicable to any analytically known potential. Given the potential (2.4), we then assume a power-series solution of Eq. (2.1):

$$u(r) = r^{\alpha_R+1} \sum_{n=0}^{\infty} [a_n \cos(\alpha_I \ln r) - b_n \sin(\alpha_I \ln r)]r^n \tag{2.5a}$$

$$v(r) = r^{\alpha_R+1} \sum_{n=0}^{\infty} [a_n \sin(\alpha_I \ln r) + b_n \cos(\alpha_I \ln r)]r^n , \tag{2.5b}$$

where we have made the following decompositions into real and imaginary parts:

$$\ell = \alpha_R + i\alpha_I$$

$$u_\ell(r) = u(r) + iv(r) \ .$$

The real coefficients a_n and b_n are obtained by substitution of Eqs. (2.4) and (2.5) into Eq. (2.1). The resulting recurrence relations are

$$a_n = - \frac{\left[(2\alpha_R+n+1)a_{n-2} + 2\alpha_I b_{n-2}\right] k^2 + \sum_{i=0}^{n-1} C_i \left[(2\alpha_R+n+1)a_{n-i-1} + 2\alpha_I b_{n-i-1}\right]}{n\left[(2\alpha_R+n+1)^2 + 4\alpha_I^2\right]} \tag{2.6a}$$

and

$$b_n = - \frac{\left[(2\alpha_R+n+1)b_{n-2} - 2\alpha_I a_{n-2}\right] k^2 + \sum_{i=0}^{n-1} C_i \left[(2\alpha_R+n+1)b_{n-i-1} - 2\alpha_I a_{n-i-1}\right]}{n\left[(2\alpha_R+n+1)^2 + 4\alpha_I^2\right]} \ , \tag{2.6b}$$

with $a_n = b_n = 0$ for $n < 0$.

To calculate the S matrix, we must evaluate two linearly independent solutions of the equation

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{\ell(\ell+1)}{r^2}\right] u_\ell(r) = 0 \ . \tag{2.7}$$

Two such solutions are the modified Bessel functions of complex order,

$$f_\ell(r) = k^{-(\ell+1)} \sqrt{\frac{\pi kr}{2}} \ J_{\ell+1/2}(kr)$$

and

$$g_\ell(r) = k^{\ell} \sqrt{\frac{\pi kr}{2}} \ J_{-(\ell+1/2)}(kr) \ . \tag{2.8}$$

We multiply by the factors $k^{-(\ell+1)}$ and k^{ℓ} to attain greater numerical accuracy for very small k: it will be apparent that the algebraic equations which have to be solved to get the S matrix are better conditioned with this approach.

We can then write the asymptotic solution as

$$u(r) + iv(r) \sim (\lambda_1 + i\lambda_2)\, f_\ell(r) + (\mu_1 + i\mu_2)\, g_\ell(r)\,, \tag{2.9}$$

where λ_1, λ_2, μ_1, and μ_2 are real. Now asymptotically we have

$$\sqrt{\frac{\pi x}{2}}\; J_n(x) \sim \cos\left(x - \frac{\pi}{4} - \frac{1}{2}\, n\pi\right),$$

whence it can be readily shown that if the S matrix is written

$$S = \frac{C + iD}{A + iB}\,, \tag{2.10}$$

then the real numbers A, B, C, and D are given by:

$$A = e^{\pi\alpha_I}\, k^{\alpha_R}[\mu_1 \cos(\pi\alpha_R - \alpha_I \ell nk) + \mu_2 \sin(\pi\alpha_R - \alpha_I \ell nk)]$$

$$+ k^{-(\alpha_R+1)}\, [\lambda_1 \sin(\alpha_I \ell nk) - \lambda_2 \cos(\alpha_I \ell nk)]$$

$$B = e^{\pi\alpha_I}\, k^{\alpha_R}\, [\mu_2 \cos(\pi\alpha_R - \alpha_I \ell nk) - \mu_1 \sin(\pi\alpha_R - \alpha_I \ell nk)]$$

$$+ k^{-(\alpha_R+1)}\, [\lambda_2 \sin(\alpha_I \ell nk) + \lambda_1 \cos(\alpha_I \ell nk)] \tag{2.11}$$

continued

$$C = -e^{-\pi\alpha_I} k^{\alpha_R}[\mu_1 \cos(\pi\alpha_R + \alpha_I \ln k) + \mu_2 \sin(\pi\alpha_R + \alpha_I \ln k)]$$

$$+ k^{-(\alpha_R+1)}[\lambda_1 \sin(\alpha_I \ln k) - \lambda_2 \cos(\alpha_I \ln k)]$$

$$D = -e^{-\pi\alpha_I} k^{\alpha_R}[\mu_2 \cos(\pi\alpha_R + \alpha_I \ln k) + \mu_1 \sin(\pi\alpha_R + \alpha_I \ln k)] \tag{2.11}$$

$$+ k^{-(\alpha_R+1)}[\lambda_2 \sin(\alpha_I \ln k) + \lambda_1 \cos(\alpha_I \ln k)] .$$

As the energy tends to zero, expressions (2.8) tend to the limits

$$f_\ell(r) \underset{k \to 0}{=} \frac{\sqrt{\pi}}{\Gamma(\ell + 3/2)} \left(\frac{r}{2}\right)^{\ell+1}$$

and (2.12)

$$g_\ell(r) \underset{k \to 0}{=} \frac{\sqrt{\pi}}{\Gamma(-\ell + 1/2)} \left(\frac{r}{2}\right)^{-\ell} ,$$

and so we use this pair of asymptotic solutions at zero energy. From Eqs. (2.10) and (2.11) it is clear that the S matrix on the real ℓ axis at zero energy is

$$S(\alpha_R, k=0) = \begin{cases} 1 & \text{for } \alpha_R > -1/2 \\ \exp[i\pi(2\alpha_R+1)] & \text{for } \alpha_R < -1/2 . \end{cases} \tag{2.13}$$

Our procedure for finding poles for $k^2 \geqq 0$ is to look for zeros of $A + iB$. Starting with a value of ℓ that is not too far from a pole, we iterate to the corresponding zero of $A + iB$ by repeated linear extrapolation (Newton's method). If ℓ_0 is the position of a pole, then for a value of ℓ that is sufficiently close to ℓ_0, we have

$$\ell_0 \cong \ell - \frac{A + iB}{\frac{d}{d\ell}(A + iB)} \quad . \tag{2.14}$$

Each iteration thus involves the evaluation of A and B at the two points ℓ and $\ell + d\ell$ in order to get the derivative $d/d\ell(A + iB)$, followed by an extrapolation to a new point in the ℓ plane by using Eq. (2.14). This process is repeated until two successive values for ℓ_0 differ by less than a certain number (HCONV; see glossary). Finally, the residue of the pole is

$$\beta = \frac{C(\ell_0) + iD(\ell_0)}{\frac{d}{d\ell}(A + iB)\big|\ell_0} \quad . \tag{2.15}$$

For negative energies a somewhat different approach is necessary. This is due to the inaccuracy that accumulates when the bound state solution of Eq. (2.1) is integrated outwards. For $k^2 < 0$ we have limited the program to finding the position of a pole without calculating the S matrix. This is easily achieved by integrating inwards starting with the asymptotic bound-state solution. Written in terms of modified Bessel functions of imaginary argument and complex order, this solution is

$$\sqrt{\frac{i\pi kr}{2}} \left[J_{\ell+1/2}(ikr) - e^{i\pi(\ell+1/2)} J_{-(\ell+1/2)}(ikr) \right] \quad . \tag{2.16}$$

The integrated solution is then matched to that given by Eq. (2.5). In this case A and B of Eq. (2.14) are replaced by the real and imaginary parts of the discontinuity of the slope of the wave function at the matching point. If u_1 and u_2 are the two solutions to be matched, and u_1', u_2' are their respective derivations, we put

$$A + iB = u_1 u_2' - u_2 u_1' \tag{2.17}$$

and then perform the same iteration procedure using Eq. (2.14).

III. DESCRIPTION OF THE CODE

A. General Structure of the Code

It was mentioned in the introduction that the program is designed to study any analytically given potential. The unmodified Fortran decks that will be available treat only a single Yukawa potential, but we have tried to organize the coding in such a way that necessary modification for a different potential will be easy. If the potential is expressible in the form of Eq. (2.4) one need only change the two small subroutines POTA and POTB. The latter effectively generates the coefficients C_i in Eq. (2.4), and the former simply evaluates the potential at a given point. For some potentials not expressible in the form of Eq. (2.4), it may be necessary to rewrite subroutine START. An example of such a potential which has been investigated with a modified version of the code is the Yukawa plus repulsive core[2]

$$V(r) = \left(\frac{B^2}{r^4} - \frac{A}{r}\right) e^{-r} .$$

More detailed information on the potential-dependent parts of the program will be found in sections III-C and III-D.

The program is comprised of the main routine, called TREGGE, and the seven subroutines START, POTA, POTB, BESSEL, GAMMA, DEQ, and MLR. The three routines START, POTA, and POTB are described in sections III-C and III-D. Descriptions of the routines BESSEL[4] and GAMMA[5] which are used to evaluate two independent asymptotic solutions of the Schroedinger equation are available under the Share code names C3 EO BESL and C3 EO GAMA at the Computer Library, Lawrence Radiation Laboratory, Berkeley. The two remaining routines, DEQ[6] and MLR,[7] were not written by the present authors; DEQ carries out the step-by-step integration of the Schroedinger equation according to Gill's variation of

the Runge-Kutta methods; MLR is used to obtain λ_1, λ_2, μ_1, and μ_2 from the set of four simultaneous algebraic equations given by the real and imaginary parts of Eq. (2.9) and its derivative.

The Fortran decks to be made available will consist of decimal (symbolic) cards for the routines TREGGE, START, POTA, POTB, BESSEL and GAMMA, and binary cards for DEQ and MLR.

The program has three modes of operation. Two of these involve scanning the three-dimensional space (complex angular momentum x real energy) at a given array of points. In one mode, S matrix and phase shifts are produced over this array, and in a second mode the space is scanned for Regge poles. In the third mode the program follows a Regge trajectory upon being given a sufficiently close approximation to one point on the trajectory.

B. The Main Routine, TREGGE

The main routine is not described in detail here, because it is not expected that users will want to modify it. An outline is given in the accompanying flow diagram (Fig. 1). The numbers attached to same blocks are the corresponding Fortran statement numbers.

C. Subroutine START

The first half of the routine--up to the fifth statement after statement 12--evaluates the starting solution (2.5 and 2.6) and its derivative. It first chooses R1, which is a value of r such that the series is convergent to one part in 10^5 while the number of terms in the series is between 20 and 25. This is done by repeatedly summing the series and changing R1 until the conditions are met. The real and imaginary parts of the solution u(R1) and v(R1) are then stored in Z(3) and Z(4) and the derivatives $du/dr|_{R1}$ and $dv/dr|_{R1}$ are stored in Z(1) and Z(2) in preparation for entrance to the Runge-Kutta integration routine.

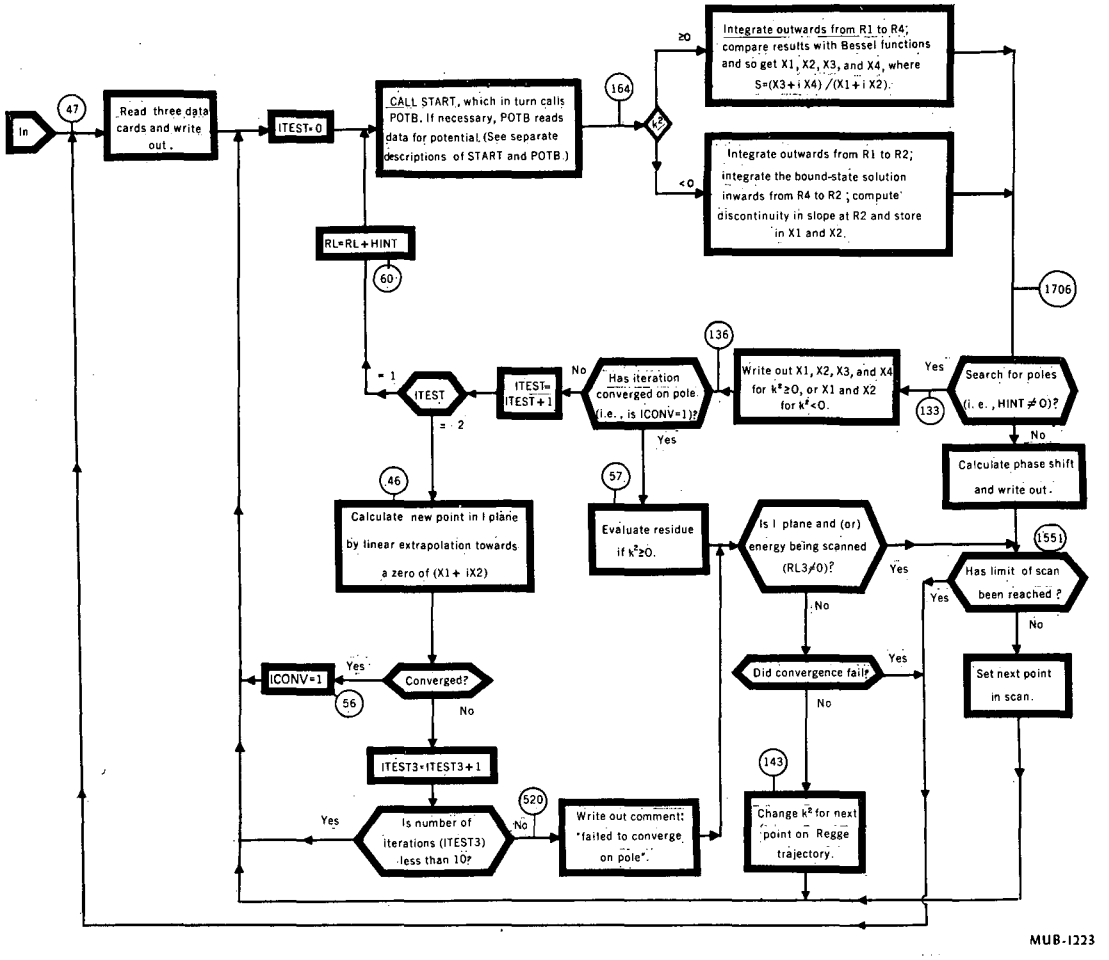

Fig. 1. Flow diagram for the main routine.

START next determines R4, which is the outer limit of the integration. It does so by first setting R4 = R1 + 1 (statement 16) and then increasing R4 in steps of 0.5 until the potential V(R4) at three successive points is negligible. A complication is met at this point if k^2 is negative. For large negative values of k^2 it is found to be numerically impossible to obtain an accurate value for the asymptotic bound-state solution $e^{-|k|R4}$ simply by using the appropriate linear combination of Bessel functions (2.16). We therefore decided to use a special asymptotic expansion for $|k|R4 > 10$, and, if the value for R4 given by the above procedure makes $|k|R4$ lie in the range $2.5 \leqq |k|R4 \leqq 10$, we choose $R4 = 10/|k|$. The coding for the evaluation of the asymptotic solution is contained in the main routine, but these preliminary decisions are made in START so that the integration intervals can be determined.

At statement 22, therefore, the inner and outer limits of integration have been determined. The rest of subroutine START calculates the integration intervals. The distance R1 to R4 is divided into three ranges in the ratio 2:2:3, and each of the three is given a separate integration interval. The interval, H_i , in the ith range is given by the formula

$$H_i = 0.15[k^2 + |V_i| + |\ell(\ell+1)|/R_i^{\,2}]^{-1/2} \tag{3.1}$$

for i = 1, 2, 3, where $|V_i|$ is an estimate of the maximum absolute value of the potential in that range and R_i is the lower bound of the ith range. The estimates $|V_i|$ are obtained by further dividing each of the three ranges into four equal parts and taking $|V_i|$ as the largest of the five numbers thus examined.

D. The Potential Subroutines: POTA and POTB

Subroutine POTA is extremely simple: it evaluates the potential at a given point. Subroutine POTB generates the first 30 terms, C1(I), of the series

$$V(R1) = \sum_{I=1}^{\infty} C1(I) , \tag{3.2}$$

where the C1(I) are related to expression (2.4) by

$$C1(I) = (R1)^{I+1} C_{I-1} . \tag{3.3}$$

This routine also reads the data for the potential. Statement 47 of the main routine sets IRA = 0 as a signal that a new set of data is being read. The signal is transmitted through START to POTB, which then looks for data. After these data are read and are written on the output tape, the index IRA is set to 1 so that the Read Statement in POTB is not executed again until the next time that statement 47 of the main code is reached. In the present code, the potential data consist of just one number: the strength, A, of the single Yukawa Ae^{-r}/r. It would be an easy matter, of course, to rewrite the potential subroutines to accommodate more complicated potentials of the form (2.4).

E. Data Cards

The code requires four data cards, each of which is read with the format statement FORMAT(4E15.7, F10.5). When a calculation is finished, a new set of four cards is read, if available. In the following table each number is represented by the symbol that is used in the Fortran coding (these symbols are defined in the glossary):

Card 1	AK1	AK2	ADL		
Card 2	RLSTRT	SLSTRT	HCONV	HINT	DKIN
Card 3	RL2	SL2	RL3	SL3	DKFIN
Card 4	A				

If a Regge trajectory is to be followed, RL3 must be zero and AK1 must not be zero. The scanning modes are entered by having RL3 $\neq$ 0. If the scan is not to search for poles but simply to calculate S matrix and phse shifts, HINT must be zero.

F. Output

As indicated in Section II and Fig. 1, the methods of calculation are somewhat different for positive and negative energies. For $k^2 \geqq 0$, the residue as well as the position of a pole is calculated; but for $k^2 < 0$, only the position is obtained. Consequently, the output formats for the two cases are different. In the pole-finding modes, each block of output represents a set of iterations to a pole at a particular energy. For $k^2 < 0$, each block contains five columns: the first gives the energy under the heading KSQU; the second and third give the real and imaginary parts of ℓ under the headings RE L and IM L; and the fourth and fifth, headed MATCHING ERROR, give the real and imaginary parts of the discontinuity in the slope of the wave function at R2 [expression (2.17)]. The rate at which these two numbers decrease is a good indication of whether the code has really found a pole. The last line of a block, for $k^2 < 0$, gives the position of a pole (provided, of course, that the iteration is successful) and the preceding lines show the individual iterations.

For $k^2 \geqq 0$, the pair of numbers called MATCHING ERROR is replaced by two pairs under the headings DENOMINATOR OF S MATRIX and NUMERATOR OF S MATRIX.

The other difference is that the next to the last line of a block gives the position of a pole and the last line gives the residue, β, of Eq. (2.15) together with a modified residue which is $\beta' = \beta e^{-i\pi\ell_0}$, where ℓ_0 is the position of the pole.

The output may contain the comment SERIES SOLUTION DIVERGED. It is clear from Eqs. (2.6) that the power-series solution is divergent when ℓ is equal to a negative half-integer. Whenever ℓ is closer than about 0.0003 to a negative half-integer, the calculation fails, and the above comment is printed out.

G. Glossary of Fortran Symbols Used in the Program

1. Symbols in Read Statements

AK1 — In any mode of operation, this is the initial value of the energy, k^2. (The use of the factor DKIN at the start of the trajectory requires that AK1 be nonzero if a Regge trajectory is to be followed.)

AK2 — Final value of k^2. In the scanning modes, AK2 should be algebraically greater than AK1. It can be zero, but in the trajectory mode, AK1 and AK2 should have the same sign.

ADL — This quantity has two functions, depending on the mode of operation. In the scanning modes, it is the interval in k^2 between successive ℓ-plane scans. Alternatively, if a Regge trajectory is to be followed, ADL is the interval in the ℓ plane at which it is desired to follow the trajectory. The energy is usually changed in such a way that successive points on the trajectory are separated by the constant interval ADL in the ℓ plane. It has been observed, however, that when $|k^2|$ becomes very small,

the motion of a pole in the ℓ plane becomes very nonlinear with k^2 (especially when the zero-energy pole is at $\ell = -1/2$). When this happens, the trajectory is followed down to small $|k^2|$ by repeatedly multiplying by the factor DKFIN. This procedure is adopted whenever the linear extrapolation of k^2 would take it beyond AK2 while $|AK2|$ is less than 10^{-3}.

RLSTRT and SLSTRT — Real and imaginary parts of the initial value of ℓ, respectively.

HCONV — A criterion for convergence on a pole. From the point of view of the code, a pole is considered to have been found when two successive iterations differ by less than HCONV in the ℓ plane.

HINT — The increment by which RL is changed in order to get the derivative $d/d\ell(X1 + iX2)$ when iterating towards a zero of $(X1 + iX2)$. For HINT = 0 the code simply calculates S matrix and phase shifts without going on to look for poles.

DKIN — Energy of the second point on a Regge trajectory is given by k^2 = DKIN × AK1. The trajectory is described in the sense of increasing or decreasing $|k^2|$ according as DKIN is greater than or less than one.

RL2 and SL2 — These quantities are used only in the scanning modes. They set the limits of the area of the ℓ plane that is to be scanned: they are the upper limits of Re ℓ and Im ℓ, respectively.

RL3 and SL3 — In an ℓ plane scan, these are the intervals in Re ℓ and Im ℓ respectively. Note that RL3 is the number that is tested to determine whether a Regge trajectory is to be followed; RL3 must be zero in that case.

DKFIN — When $|k^2|$ gets very small, the variation in the position of a pole sometimes becomes very nonlinear (particularly when the zero energy pole is at $\ell = -1/2$). The linear extrapolation of k^2 to step along the trajectory at the interval ADL then becomes inappropriate, and so k^2 is further reduced by simply multiplying by DKFIN. Hence, if a trajectory is to be followed down to very small $|k^2|$, $(< 10^{-3})$, DKFIN should be set at some suitable value in the range $0 < \text{DKFIN} < 1$.

A — Strength of the single Yukawa potential $V(r) = -Ae^{-r}/r$. For an attractive potential, A is greater than zero.

2. Some Important Symbols Which do not Appear in Read Statements

AK — Current value of the energy, k^2.

AKS — $\sqrt{|k^2|}$

C1(I) — Terms of the series $V(R1) = -\sum_{I=1}^{30} C1(I)$ [expression (3.2)].

H — Current value of the integration interval.

H1, H2, and H3 — Integration intervals used in the three regions, respectively [given by Eq. (3.1)].

ICONV — This is set equal to 1 when two successive iterations on the position of a pole differ by less than HCONV in the ℓ plane; otherwise it is zero.

IRA — The flag that indicates to START and POTB that new data is being read (IRA = 0 in that case).

ITEST — Poles are found by looking for zeros of (X1 + iX2) (see the flow diagram). Each iteration towards (X1 + iX2) = 0 requires the evaluation of (X1 + iX2) at the two points (RL, SL) and (RL + HINT, SL). The values ITEST = 1 and 2 are used to label

these two calculations.

ITEST3	An index that counts the number of iterations at a particular value of k^2 in looking for a pole. The iteration is stopped if ITEST3 reaches 10 while ICONV is zero.
R1	Starting point for numerical integration: the point at which the series solution of the Schoedinger equation is evaluated.
R2	Outer limit of the first region of integration. For negative k^2, this is the point at which the outward and inward integrations are matched.
R3 and R4	Inner and outer limits respectively of the third and final region of integration.
RL and SL	Current values of the real and imaginary parts of ℓ respectively.
U	Current value of r during integration.
V	Potential at any point R.

FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U. S. Atomic Energy Commission.

1. A. Ahmadzadeh, P. G. Burke, and C. Tate, Regge Trajectories for a Single Attractive Yukawa Potential, UCRL-10140, March 1962.

2. A. Ahmadzadeh, P. G. Burke, and C. Tate, Regge Trajectories for Yukawa Potentials, UCRL-10216, May 1962.

3. P. G. Burke and C. Tate, Calculations of Regge Trajectories by Using Dispersion Relations, UCRL-10217, May 1962.

4. P. G. Burke and C. Tate, Bessel Functions of Complex Order and Complex Argument: C3 EO BESL, Computer Library, Lawrence Radiation Laboratory, Berkeley, May 1962.

5. P. G. Burke, Gamma Function of a Complex Argument: C3 EO GAMA, Computer Library, Lawrence Radiation Laboratory, May 1962.

6. M. Goldstein, Fortran Differential Equations Solutions (Runge-Kutta): DEQF, AEC Computing Center, New York University, September 1958.

7. R. E. von Holdt, MLR (Minimize Linear Residuals), Computer Library, Lawrence Radiation Laboratory, Livermore, November 1960.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.