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THE AVERAGE ENERGY AND ANGULAR MOMENTUM
REMOVED FROM D_y COMPOUND NUCLEI BY
NEUTRONS AND PHOTONS

Berkeley, California

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ABSTRACT

Excitation functions are presented for many heavy-ion-induced (HI) reactions that produce Dy^{149} , Dy^{150} and Dy^{151} . Projectiles were C^{12} , N^{14} , N^{15} , O^{16} , O^{18} , F^{19} , and Ne^{20} of 4 to 10.4 MeV per amu. The reactions studied are all of the type (HI,xn), where x ranges from 3 to 9. A large fraction of the total reaction cross section is accounted for by these (HI,xn) reactions— 9/10 at approximately 45 MeV to 1/2 at approximately 105 MeV. An analysis to obtain the energy of the first neutron is presented. Comparison of the results of this analysis to angular-distribution studies suggests that each neutron removes 2 to 4 \hbar units of angular momentum. We obtain the relationship between average total photon energy and average angular momentum removed by photons. Comparison with the average photon energy from other work leads to an average of $1.8 \pm 0.6 \hbar$ for the angular momentum removed by each photon. The excitation energy E_j of the lowest-lying state of spin J has been estimated.

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I. INTRODUCTION

Currently available beams of heavy ions (HI) make it possible to study compound nuclei over a wide range of excitation energy and angular momentum. Radiochemical studies are quite useful because they give information about specific reactions; e.g., the (HI,5n) reaction can be studied without interference from the reactions (HI,6n), (HI,p5n), etc. This specificity is difficult to obtain by physical means because of complex coincidence-detection requirements. The products Tb^{149g}, Dy¹⁵⁰, and Dy¹⁵¹ are particularly useful because they can be easily identified by their characteristic alpha radioactivity.

In previous studies we have presented recoil-range data that give strong evidence that these products are produced by essentially pure compound-nucleus reactions.^{1,2,3} Also reported are angular-distribution measurements from which it has been possible to obtain the average total energies (T_n and T_γ) of neutrons and photons.³

The experimental data reported here consist of excitation functions for 26 reactions of type (HI,xn)Dy¹⁴⁹, (HI,xn)Dy¹⁵⁰, (HI,xn)Dy¹⁵¹. Compound nuclei of masses 154 to 158 have been formed by various projectiles and targets.

The conventional treatment of excitation-function data involves the use of the statistical model with little, if any, allowance for the effect

of angular momentum. This type of treatment may be acceptable for reactions induced by protons and helium ions of several tens of MeV. However, it is clearly unsatisfactory for reactions between complex nuclei that involve angular momenta of several tens of \hbar units.⁴

We analyze the results to obtain the average energy associated with the first emitted neutron. Also, we have estimated the relationship between average total photon energy and average angular momentum removed by the photons. This relationship along with the average individual photon energy⁵ gives the average angular momentum removed by each photon. By an approximate method we have estimated the energy E_j of the lowest level of spin J as a function of J .

II. EXPERIMENTAL PROCEDURES AND RESULTS

We have used the stacked-foil technique to measure cross sections for 4.1-h Tb^{149g} (10% alpha), 7.4-min Dy¹⁵⁰ (17.9% alpha), and 17.9-min Dy¹⁵¹ (6.2% alpha) produced by many reactions between complex nuclei. The experimental conditions (targets, irradiations, counting techniques, etc.) have been described previously.⁶

The product atoms recoiled out of thin target layers (30 to 120 $\mu\text{g}/\text{cm}^2$) and were stopped in Al catcher foils of about 1.8 mg/cm^2 . We measured gross alpha radioactivity with 2π ionization chambers. Activation of impurities in the catcher foils was found to be negligible. Decay curves were graphically analyzed into the three components above. At the lower energies small amounts of 2.5-h Dy¹⁵² activity were observed. The presence of Dy¹⁵² prevented us from measuring the very small cross sections for Tb¹⁴⁹ and Dy¹⁵¹ at lower energies. Separation of the activities of 7.4-min Dy¹⁵⁰ and 17.9-min Dy¹⁵¹ by the decay analysis was usually quite clear. However, for those cases in which the initial activity of either species was dominant (ratio of approx 10:1), the determination of the weaker component was subject to large error.

Various uncertainties have been discussed previously.⁶ In this study the only additional uncertainties are those from analysis of the decay curves, and the decay properties of Dy¹⁵⁰ and Dy¹⁵¹. The half-periods and alpha branching ratios for Dy¹⁵⁰ and Dy¹⁵¹ have been measured by Macfarlane.⁷ The half-periods are uncertain to approx $\pm 3\%$ and lead to negligible error in the cross sections. The absolute uncertainties in the alpha branching ratios are not known but are probably about $\pm 10\%$.⁷

Resolution of the decay curves introduces no additional uncertainties for Dy^{149} cross sections. For those experiments in which the cross sections of Dy^{150} and Dy^{151} are approximately equal, standard errors from decay analysis are about $\pm 20\%$ for Dy^{151} and about $\pm 10\%$ for Dy^{150} . For experiments in which the ratio of these cross sections is approx 8:1, the activity measurement for the species of higher cross section has a standard error of about $\pm 5\%$, and for the other species has a standard error of approx $\pm 50\%$. Isotopically enriched materials were used for targets of Nd^{142} , Nd^{144} , Ce^{140} , Ba^{136} , Ba^{137} , and Ba^{138} . The isotopic composition of these materials is given in Table I. In the table we make a note of those isotopes for which corrections were applied in the calculation of the cross sections. It is important that these corrections be precise for an analysis such as that presented in the next section.

The cross-section results are presented in Table II. In Fig. 1 we show some typical excitation functions (plotted as fractional cross section σ/σ_R against excitation energy E). This figure shows data for two sets of reactions that produce the compound nucleus Dy^{156} . The beam energies were calculated from range-energy curves of Northcliffe⁸ and the initial energy of 10.38 MeV per amu. The cross sections given for Dy^{149} were measured by observation of 4.1-h Tb^{149g} . The values listed for Dy^{149} actually include any 4.1-h Tb^{149g} formed directly by (HI,pxn) reactions. Also, that fraction of the Dy^{149} that decayed to 4-min Tb^{149m} was not observed, and therefore is not included in the listed values. Our estimate is that the direct production of Tb^{149g} is negligible and that about 2/3 of the Dy^{149} decays to Tb^{149m} and is not observed. The former estimate was discussed previously;³ the latter is based on the fact that the fractional cross sections for Dy^{149}

are all about $1/3$ those for Dy^{150} , or Dy^{151} from similar reactions (see Fig. 1). Relative values of the cross sections for Dy^{149} require only that the first estimate be correct; absolute values require a measurement of the branching ratio of Dy^{149} to Tb^{149g} . More detailed studies of the decay properties of each of these nuclides would make the interpretation of these data more definite.

III. AN ANALYSIS TO OBTAIN THE AVERAGE ENERGY OF THE FIRST EMITTED NEUTRON

In this work we have measured excitation functions for a number of different reactions of the type (HI, xn) that lead to Dy^{149} , Dy^{150} , and Dy^{151} . Let us consider the relationship between two of these reactions that lead to the same product, say $(HI, 6n)Dy^{150}$ and $(HI, 5n)Dy^{150}$, where target and projectile in the two reactions are different. It is clear that if atomic and mass numbers, (Z and A) and excitation energy (E) were the only variables, then we could hope to unfold the energy spectrum of the first neutron emitted in the $(HI, 6n)$ reaction by comparing the two excitation functions. This unfolding process would be rather tedious and would require very accurate data; therefore, we attack the more modest goal of extracting the average energy $\langle \epsilon_1 \rangle$ associated with the first emitted neutron (the average kinetic energy $\langle k_1 \rangle$ of the first neutron plus the average total photon energy $\langle \gamma_1 \rangle$ dissipated before emission of the second neutron). It is generally believed that the photon energy $\langle \gamma_1 \rangle$ is very small.⁴

Let us define F_x as the fraction of those reactions in which no charged particle is emitted that lead to a specific product by an (HI, xn) reaction. The fraction of all reactions in which no charged particles are emitted is denoted by f_n . For various excitation energies (E) we have measured the cross section σ for a specific product, and we can calculate the total reaction cross section σ_R .⁹ Therefore we have

$$F_x(E) = \frac{\sigma}{\sigma_R f_n} \quad (1)$$

and

$$\sum_{x=0}^{x_{\max}} F_x(E) = 1. \quad (2)$$

Now let us define the quantity $\langle E \rangle_x$, the average excitation energy associated with the reaction (HI, xn) :

$$\langle E \rangle_x = \frac{\int_0^{\infty} (E) F_x(E) dE}{\int_0^{\infty} F_x(E) dE}. \quad (3)$$

These

$\langle E \rangle_x$ quantities can be obtained from experimental excitation functions provided f_n can be determined.

Let us derive the relationship between $\langle E \rangle_x$ and $\langle E \rangle_{x-1}$. The distribution of energies (ϵ_1) associated with the first emitted neutron is denoted by $P(\epsilon_1)$. Neglecting the effect of angular momentum, we have

$$F_x(E) = \int_0^{\epsilon_{\max}} P(\epsilon_1) F_{x-1}(E - B_1 - \epsilon_1) d\epsilon_1, \quad (4)$$

where $\epsilon_{\max} = E - \sum_{i=1}^x B_i$ and B_i is the separation energy of the i th neutron.

Normalization of $P(\epsilon_1)$ such that $\int_0^{\infty} P(\epsilon_1) d\epsilon_1 = 1$ leads to the result

$$\int_0^{\infty} F_{x-1}(E) dE \approx \int_0^{\infty} F_x(E) dE. \quad (5)$$

Substituting Eq. (4) into Eq. (3), we obtain

$$\langle E \rangle_x = \frac{\int_0^{\infty} (E) \left[\int_0^{\epsilon_{\max}} P(\epsilon_1) F_{x-1}(E - B_1 - \epsilon_1) d\epsilon_1 \right] dE}{\int_0^{\infty} F_x(E) dE}. \quad (6)$$

If ϵ_{\max} is large with respect to $\langle \epsilon_1 \rangle$ then it can be replaced with small error, by ∞ . The order in integration of Eq. (6) can be changed then, provided we assume that $P(\epsilon_1)$ does not vary with E over the region of interest

$$\langle E \rangle_x = \frac{\int_0^{\infty} P(\epsilon_1) \left[\int_0^{\infty} (E) F_{x-1}(E - B_1 - \epsilon_1) dE \right] d\epsilon_1}{\int_0^{\infty} F_x(E) dE}. \quad (7)$$

The quantity in the square bracket is simply

$$(\langle E \rangle_{x-1} + B_1 + \epsilon_1) \int_0^{\infty} F_{x-1}(E) dE.$$

Therefore, we have

$$\langle E \rangle_x = \frac{\int_0^{\infty} P(\epsilon_1) \left\{ \left[\langle E \rangle_{x-1} + B_1 + \epsilon_1 \right] \int_0^{\infty} F_{x-1}(E) dE \right\} d\epsilon_1}{\int_0^{\infty} F_x(E) dE}. \quad (8)$$

Finally, Eq. (8) can be reduced to

$$\langle E \rangle_x = \langle E \rangle_{x-1} + B_1 + \langle \epsilon_1 \rangle. \quad (9)$$

From Eq. (9) one can determine the average energy $\langle \epsilon_1 \rangle$ associated with the first neutron if Z , A , and E are the only variables. Even if the values of $\langle E \rangle_x$ vary with angular momentum, Eq (9) can be useful. It may be possible to calculate or measure the change in angular momentum, ΔJ_1 , associated with the emission of the first neutron. If one knows experimentally the dependence of the $\langle E \rangle_x$ values on angular momentum, then values of $\langle E \rangle_x$ and $\langle E \rangle_{x-1}$ can be chosen corresponding to J values that differ by the ΔJ_1 associated with the first neutron. Alternatively, if one knows $\langle \epsilon_1 \rangle$, he may be able to obtain ΔJ_1 .

In the next section we present values of f_n and $\langle E \rangle_x$ obtained from the excitation functions. We discuss the dependence of $\langle E \rangle_x$ on angular momentum and the significance of the application of Eq. (9).

IV. DISCUSSION

This work and previous studies^{3,4} establish the necessity for including angular-momentum effects in a meaningful analysis of cross-section data. The description of the dependence of nuclear level density on angular momentum requires two parameters:⁴ (a) the nuclear moment of inertia (possibly dependent on E and J) and (b) the excitation energy of E_j of the lowest excited state of spin J . We have not attempted to delimit these quantities by fitting calculated excitation functions to our data. Instead, we use Eq. (9) from the previous section to gain information about the first step in the evaporation cascade, and we use a simple approximation to estimate E_j as a function of J .

We compare these results with average energies of the neutrons and photons obtained from angular distributions³ and try to arrive at an energy and angular-momentum balance. Finally, we obtain a relationship between total photon energy and angular momentum removed by photons.

A. General Relationship of These Results to Other Studies

In a previous study⁶ we have presented cross-section data for reactions of the type $(HI, xn)Tb^{149g}$. The results were compared with the data for $(HI, xn)Dy$ reactions. These two reaction types show large differences in the magnitude of the peak cross sections. We can explain these differences by assuming that only those Tb compound systems of low spin ($<7.5 \pm 1.5$) contribute to the $(HI, xn)Tb^{149g}$ reactions.⁶

Also we have compared angular-distribution measurements for the two reaction types $(HI, xn)Tb^{149g}$ and $(HI, xn)Dy^{149}$, $(HI, xn)Dy^{150}$, $(HI, xn)Dy^{151}$. This comparison leads us to conclude that an increase in angular momentum leads to an

increase in the amount of energy dissipated by photon emission.³ Additional evidence for this conclusion is given by the fact that the excitation functions for (HI,xn)Tb^{149g} reactions peak at 3 to 3.5 MeV per emitted neutron compared with 5 to 6 MeV per emitted neutron for the (HI,xn)Dy reactions (see Fig. 2 of reference 6).

Mollenauer has studied the photons emitted in various nuclear reactions induced by He⁴ and C¹².⁵ His results indicate that total photon energy increases with increasing angular momentum. For all the reactions studied the average individual photon energy was between 1.0 and 1.6 MeV (1.1 MeV for Te + 110-MeV C¹² and 1.2 MeV for Ho + 110-MeV C¹²). His measurements of photon yields at 45 and 90 deg give evidence for quadrupole radiation in several reactions induced by C¹², with the notable exception of Te + 110-MeV C¹². As shown in Sec. IVB the (HI,xn)Dy reactions account for approx 1/2 to 9/10 of the calculated reaction cross sections. Since these cross sections are such a substantial part of all the reactions, it is reasonable to assume that the average photon energy for (HI,xn) reactions is very nearly the same as that measured by Mollenauer.⁵ Therefore from Mollenauer's results it is reasonable to expect for (HI,xn)Dy reactions a value of 1.2 ± 0.3 MeV for the average individual photon energy.

B. The Fraction of the Reactions in Which No Charged Particle Is Emitted

In Table II, cross-section data are given for reactions of the type (HI,xn)Dy. How does the probability for these reactions vary with type and energy of the projectile? We need this information to describe the quantity f_n (the fraction of the reactions in which no charged particle is emitted). We can expect that the probabilities for neutron evaporation from each of the Dy compound nuclei ($A = 154$ to 158) will have very similar dependence

on excitation energy (E). However, we do not know how the probability for compound-nucleus formation depends on type and energy of the projectile.

The simplest assumptions that we can make are as follows: (a) The projectile type (C^{12} , N^{14} , etc.) is not important; (b) The energy dependence of f_n can be described in terms of the initial excitation energy of the compound nucleus.

We show values of f_n plotted against excitation energy in Fig. 2. The values of f_n shown correspond to excitation energies for equal cross sections of Dy^{150} and Dy^{151} . At this energy we approximate f_n as

$$f_n \approx \frac{\sigma_{151} + \sigma_{150} + 8\sigma_{149}}{\sigma_R}, \quad (10)$$

where σ denotes cross section with numerical subscripts for the mass number of the product. The last term ($8\sigma_{149}$) in Eq. (10) is a crude estimate of the sum of the cross sections for Dy^{152} and Dy^{149} . (We estimate that the absolute cross section for Dy^{149} is three times the measured cross section; see Sec. II.) The magnitude of this term is not large as shown by the arrows in Fig. 2. The absolute values shown are uncertain by approx $\pm 20\%$, but the relative values have standard errors of approx $\pm 10\%$ (see reference 6).

We have used a single relationship for $\sigma_R/\pi R^2$ for all reactions. This relationship was obtained from the calculations by Thomas⁹ for reactions of heavy ions with Pr^{141} . The values of $\sigma_R/\pi R^2$ are given in Table III, where they are compared to the classical result

$$\frac{\sigma_R}{\pi R^2} = 1 - \frac{V}{E_{c.m.}} \quad (11)$$

The sum of the radii of target and projectile is denoted by R , Coulomb barrier by V , and center-of-mass energy by $E_{c.m.}$. The energy dependence of σ_R from Eq. (11) and from square-well calculations is very similar for $E_{c.m.}/V \gtrsim 1.10$. We conclude that the relative values of σ_R for $E_{c.m.}/V \gtrsim 1.10$ are quite reliable.

We have drawn a single curve in Fig. 2 for all projectiles and targets, namely

$$f_n = \left(\frac{1}{2}\right)^{(E - 35)/65}, \text{ for } 45 < E < 105 \text{ MeV. (12)}$$

This equation fits all the measurements within the experimental errors.

We conclude that a very substantial fraction of the total cross section leads to (HI,xn)Dy reactions. Also the variations between different projectiles are probably less than approx. 10%. Note that for the calculation of $\langle E \rangle_x$, errors in f_n and σ_R tend to compensate.

C. Values of the Average Excitation Energy $\langle E \rangle_x$ for (HI,xn) Reactions

In Sec. II we have defined the average excitation energy $\langle E \rangle_x$ and discussed the relationship of this quantity to the average energy $\langle \epsilon_1 \rangle$ associated with the first neutron. The value of $\langle E \rangle_x$ is determined by the ratio of two integrals over excitation energy, as given in Sec. III

$$\langle E \rangle_x = \frac{\int_0^{\infty} (E) F_x(E) dE}{\int_0^{\infty} F_x(E) dE}. \quad (13)$$

A graph of a typical pair of these integrands is shown in Fig. 3. The integrations were performed graphically with a planimeter. Values of $\langle E \rangle_x$

have been determined for 26 reactions of type $(\text{HI}, \text{xn})\text{Dy}^{149}$, $(\text{HI}, \text{xn})\text{Dy}^{150}$, or $(\text{HI}, \text{xn})\text{Dy}^{151}$. The results are given in Table IV. Cross-section data in Table II were used with f_n values from Eq. (12) and σ_R values from Table III (column 2). The first column gives the reaction, the second the value of $\langle E \rangle_x$. In the third column is given $\langle E \rangle_x - \sum_{i=1}^x B_i$, where B_i is the separation energy of the i th neutron.

It is important to remember that only relative values of σ , the product $f_n \sigma_R$, and the masses are important for the determination of the relative values of $\langle E \rangle_x$. We are interested in the differences between values of $\langle E \rangle_x$, and therefore relative values are of much more concern than the absolute values. Masses of target and Dy nuclei were taken from Seeger's mass formula.¹⁰ The absolute values of the atomic masses from Seeger's, Cameron's, and Levy's formulae may differ by several MeV¹⁰⁻¹² but the relative values agree to about 0.5 MeV. A major source of error in the relative values of $\langle E \rangle_x - \sum_{i=1}^x B_i$ may be the day-to-day variation in initial energy of the beam from the Hilac. There has been no detailed study of this question, but we estimate a standard error of about ± 1 MeV for the relative values of $\langle E \rangle_x$. In the last column is given the average angular momentum $\langle J \rangle$ that corresponds to each value of $\langle E \rangle_x$. These values have been calculated from the classical approximation,

$$\langle J \rangle = \frac{(8\mu)^{1/2} R(E_{\text{c.m.}} - V)^{1/2}}{3\hbar} \quad (14)$$

where μ is the reduced mass, and R is the sum of the radii of the collision partners (see Eq. 20).

The values of $\langle E \rangle_x - \sum_{i=1}^x B_i$ are plotted against average angular momentum $\langle J \rangle$ in Fig. 4. From the data for reactions with x ranging from 4

to 7, we can establish that increasing $\langle J \rangle$ increases $\langle E \rangle_x - \sum_{i=1}^x B_i$. A linear dependence of $\langle E \rangle_x - \sum_{i=1}^x B_i$ on $\langle J \rangle$ with slope 0.47 ± 0.2 MeV is consistent with all the data. Dashed lines of the same slope are indicated for the other reactions.

In order to use Eq. (9) of Sec. II to extract the average energy associated with the first neutron, we must know the average change ΔJ_1 in angular momentum due to the emission of the first neutron. Pik-Pichak has calculated $\Delta J_1 \approx 1/2$ for a nucleus of mass 50 having the moment of inertia of a rigid sphere, and angular momenta and excitation energy comparable to the Dy nuclei formed in this study.¹³ Thomas has obtained a similar result for a nucleus of mass 209.¹⁴ If no photons accompany the first neutron and if $\Delta J_1 = 1/2$, we have

$$\langle \epsilon_1 \rangle = \langle k_1 \rangle = d_x + 0.23 \text{ MeV}, \quad (15)$$

where d_x is the displacement between the lines for (HI, xn) and $[\text{HI}, (x-1)n]$ reactions. The placement of each line is uncertain by about ± 1 MeV and the extrapolation of several of the lines leads to additional uncertainty. We can expect an overall standard error of about ± 1.5 MeV in the values of d_x .

The value of $\langle k_1 \rangle$ can also be inferred from angular-distribution measurements.³ A comparison of the $\langle k_1 \rangle$ quantities from the two independent studies is interesting. Angular-distribution data have been used to obtain the average total energy T_n of the neutrons. Values of T_n from reactions of $x = 3$ to 7 are approximately proportional to the square root of the excitation energy or to the square root of x . It is therefore reasonable to assume that the average value $\langle k_1 \rangle$ will also be proportional to \sqrt{x} . Using the results

of reference 3 to obtain the proportionality constant, we have

$$\langle k_1 \rangle = 1.7 \sqrt{x} \text{ MeV.} \quad (16)$$

This evaluation refers to the excitation energy $\langle E \rangle_x$ for each reaction. Experimental sources give rise to errors of about $\pm 10\%$ in the proportionality constant 1.7. The assumption of isotropic emission of neutrons if in error makes the values of $\langle k_1 \rangle$ from Eq. (16) too small.³

In Table V we list the values of $\langle k_1 \rangle$ from Eqs. (15) and (16). Also we give their ratio. Even though the uncertainties are rather large, it is interesting that all values derived from excitation functions are smaller than those from angular distributions. It is certainly possible that there is some systematic error of which we are not aware. One possibility is that the lines in Fig. 2 have a slope ≈ 0.2 MeV rather than 0.47 MeV. This would require that the errors in $\langle E \rangle_x - \sum_{i=1}^x B_i$ be somewhat larger than we estimate.

Another possibility is that the ΔJ_1 has been estimated incorrectly. If ΔJ_1 were 3, then Eq. (15) would read

$$\langle k_1 \rangle = d_x + (1.4 \pm 0.6) \text{ MeV.} \quad (17)$$

In this case the degree of consistency would be much greater. Preliminary calculations by Thomas indicate that a moment of inertia (appropriate to the nuclear level density) of about 1/4 that of a rigid sphere is required to give this result.¹⁴ Theoretical arguments have been given to show that the appropriate moment of inertia is not expected to be less than that of a rigid sphere.^{13,15} Additional experimental evidence is certainly required to

determine how much angular momentum is taken away by the neutrons. However, these results seem to suggest that $\Delta J_1 \approx 3$ compared to theoretical estimates of $\Delta J_1 \approx 1/2$.

From the values $\langle E \rangle_x$ it is possible to obtain the relationship between average total photon energy and average total angular momentum removed by photons. Using Eq. (16) as the most reliable estimate of average neutron energy, we can subtract from each value of $\langle E \rangle_x$ the sum of the binding and average kinetic energies of each neutron. The remaining energy $\langle T_\gamma \rangle$ must be dissipated by photons. Similarly, we must subtract from the values of $\langle J \rangle$ the sum of the angular momenta removed by the neutrons. In the preceding paragraph we gave evidence that suggested rather large values of ΔJ_1 . Let us consider the classical approximation for the average orbital angular momentum l_n of the emitted neutrons. If the directions of these angular momenta l_n are parallel to J , then we have

$$l_n = \Delta J_1 \approx \frac{\mu^{1/2} R_c \langle k_1 \rangle^{1/2}}{\hbar} \quad (18)$$

where R_c is the radius of the emitting nucleus. This relationship combined with Eq. (16) leads to ΔJ_1 values of 2 to 4. If we subtract from the values of $\langle J \rangle$ the values of ΔJ from Eq. (18) for each successive neutron, we might expect to arrive at a lower limit for the angular momentum removed by photons. However, there is evidence that the values of $\langle J \rangle$ from Eq. (14) probably overestimate the average angular momenta of the compound nuclei.^{1,16} Noncompound nucleus reactions occur and probably deplete the number of compound nuclei of higher spins. Therefore, the use of Eq. (14) for $\langle J \rangle$ and Eq. (18) for ΔJ probably leads to a reasonable estimate for the total average angular momentum removed by photons.

In Fig. 5 we show the results obtained by the procedure just described. Note that each experimental point in Fig. 5 was obtained from values of $\langle E \rangle_x$ and $\langle J \rangle$ and therefore represents an average over all energies for a given reaction. (Roughly speaking, each point is from the peak of an excitation function.) There are several interesting features of this graph. First, all the different measurements from reactions of $x = 3$ to 9 give a consistent trend - namely, a roughly linear increase of $\langle T_\gamma \rangle$ with average angular momentum. Second, the slope of the line is 0.46 ± 0.15 MeV/ \hbar -essentially the same as that in Fig. 4. Combining Fig. 5 with Mollenauer's measurement of 1.2 ± 0.3 MeV per photon⁵, we obtain an average of $1.8 \pm 0.6\hbar$ for the angular momentum removed by each photon. This result is in accord with the number of photons per reaction that Mollenauer observed for $\text{Te} + \text{C}^{12}$. But it is surprising that Mollenauer's relative photon yields at 45 and 90 deg (for $\text{Te} + \text{C}^{12}$) indicated dipole radiation.⁵

The plot shown in Fig. 5 is, of course, intimately related to the dependence of E_j (the energy of the lowest state of spin J) on J . For each J , the total photon energy $\langle T_\gamma \rangle$ must be greater than the energy E_j by approximately the separation energy of a neutron.⁴ Therefore, the trend in Fig. 5 implies that E_j varies almost linearly with J .

Throughout this discussion we have assumed that in the first step of the evaporation cascade essentially no energy is dissipated by photon emission--that is $\langle \gamma_1 \rangle \approx 0$. There is no direct evidence that this assumption is strictly correct. However, the cross-section and angular-distribution results do indicate that the photon energy $\langle \gamma_1 \rangle$ decreases with increasing number x of emitted neutrons. This conclusion is based on two results:

(a) The values of the quantity $(\langle E \rangle_x - \sum_{i=1}^x B_i)/x$ are all 5.0 to 6.4 MeV per neutron and do not show a trend that increases with x , and (b) The values

T_n/x do increase with x ,³ and thus we infer that $\langle k_1 \rangle$ increases with x (see Eq. 16). The comparison of these two results gives evidence that the ratio $\langle k_1 \rangle / \langle \gamma_1 \rangle$ increases with x .

D. An Estimation of the Dependence of E_j on J

A complete analysis of the results presented here requires a rather difficult calculation. One must consider the distribution in angular momentum of the initial compound nuclei. Then the distributions in energy, angular momentum, and type of emitted particle must be considered for each step of the evaporation cascade. Such a calculation is beyond the scope of this paper. However, with a number of simplifying assumptions and approximations we can arrive at an estimate of the dependence of E_j on J . The essential features of this analysis were suggested to us by Dr. J. Robb Grover of Brookhaven National Laboratory.

The assumptions made are: (a) The distribution function $P(J)$ that describes the initial spectrum of angular momenta is given by the classical sharp-cutoff approximation.

$$P(J)dJ = (2J/J_{\max}^2)dJ \text{ for } J < J_{\max}, \quad (19)$$

$$P(J)dJ = 0 \text{ for } J > J_{\max}, \quad (20)$$

and

$$J_{\max}^2 = 2\mu(E_{c.m.} - V)R^2 \hbar^{-2}; \quad (21)$$

(b) There are only small changes in $P(J)$ as a result of the evaporation of neutrons; (c) The distribution of the total energy T_n of x neutrons is represented as

$$P(T_n)dT_n = \frac{1}{(2x-1)! \tau^{2x} T_n^{2x-1}} \exp(-T_n/\tau) dT_n \quad (22)$$

where τ is a nuclear temperature parameter. This expression originates from the constant-nuclear-temperature approximation developed by Jackson.¹⁷ In this approximation $T_n/x = 2\tau$. Thus we obtain a value of τ for each value of E from the T_n values given in reference 3; (d) The dependence of E_j on J is given by

$$E_j = c_1 J \quad (23)$$

or

$$E_j = c_2 J^2; \quad (24)$$

and (e) Neutron emission takes place if the excitation energy exceeds the sum of E_j and the separation energy of a neutron. The physical consequences of this assumption are described by a very illustrative graphical representation in reference 4.

We will develop an approximate relationship between the constants c_1 or c_2 and the values of fractional cross sections F_x for the reactions (HI, xn) , (see Eq. 1). Let us consider initial excitation energies 10 to 30 MeV less than $\langle E \rangle_{x+1}$ --in other words, the leading edge of the excitation function for the reaction $[HI, (x+1)n]$.

After the emission of x neutrons, we require assumption (e), above, that another neutron will be emitted only if

$$M_x - T_n > E_j + B_{x+1}, \quad (25)$$

where

$$M_x = E - \sum_{i=1}^x B_i \quad (26)$$

and B_i is the separation energy of the i th neutron. Then we have

$$\frac{F_{x+1}}{F_x + F_{x+1}} = \frac{\frac{1}{(2x-1)!} \int_0^{J_c} \int_0^{M_{x+1}/\tau} \left(\frac{T_n}{\tau}\right)^{2x-1} \exp\left(-\frac{T_n}{\tau}\right) d\left(\frac{T_n}{\tau}\right) dJ}{\frac{1}{(2x-1)!} \int_0^{J_x} \int_0^{M_x/\tau} \left(\frac{T_n}{\tau}\right)^{2x-1} \exp\left(-\frac{T_n}{\tau}\right) d\left(\frac{T_n}{\tau}\right) dF} \quad (27)$$

The limit J_c is obtained from Eq. (23) or (24) and Eq. (25). Hence,

$$J_c = (M_{x+1} - T_n)/c_1 \quad (28)$$

or

$$J_c^2 = (M_{x+1} - T_n)/c_2 \quad (29)$$

Expressions similar to Eqs. (28) and (29) can be written for the limit J_x , which, with these assumptions, determines the division between the reactions (HI, xn) and $[HI, (x-1)n]$. The expressions for J_c and J_x depend strongly on assumption (b), namely that $P(J)$ is essentially unchanged by the evaporation of neutrons. We expect that this assumption is reasonable for the smaller J values⁶ (e.g., $J < 25$), but it may be very poor for the higher values of J . (See the discussion in the preceding section.) Therefore we confine this treatment to a portion of the leading edge of the excitation function for the $[HI, (x+1)n]$ reaction. In this region, typical values of F_{x+1} range from about 0.02 to 0.25, F_x is about 0.5, and F_{x-1} ranges from about 0.4 to 0.2. Therefore, values of J_c are not very large ($J_c < 25$), and the values of J_x approach J_{max} . In this energy region we do not lean very heavily on assumption (b) for the higher J values because the value of J_x is not critical.

For simplicity we eliminate J_x from the formulation by the following approximation. In the denominator of Eq. (27) we extend the integration

over J from the limit J_x to J_{\max} , and we extend the integration over T_n from M_x to ∞ . These new limits make a small additional contribution to the integral; this addition depends on initial energy E approximately as does F_{x-1} . With these considerations we change Eq. (27) to read

$$F_{x+1} \approx \frac{F_{x+1}}{F_{x-1} + F_x + F_{x+1}} \approx \frac{\frac{1}{(2x-1)!} \int_0^{J_c} J \int_0^{\frac{M_{x+1}}{\tau}} \left(\frac{T_n}{\tau}\right)^{2x-1} \exp\left(-\frac{T_n}{\tau}\right) d\left(\frac{T_n}{\tau}\right) dJ}{\frac{1}{(2x-1)!} \int_0^{J_{\max}} J \int_0^{\infty} \left(\frac{T_n}{\tau}\right)^{2x-1} \exp\left(-\frac{T_n}{\tau}\right) d\left(\frac{T_n}{\tau}\right) dJ} \quad (30)$$

The denominator of Eq. (30) is simply $J_{\max}^2/2$, and the numerator can be expressed in the terms of the incomplete gamma function

$$\Gamma_y(p+1) = \int_0^y e^{-x} x^p dx \quad (31)$$

Integrating and solving for c_1 or c_2 , we obtain

$$c_1^2 = \frac{1}{F_{x+1} J_{\max}^2} \left[\frac{M_{x+1}^2}{(2x-1)!} \Gamma_{M_{x+1} \tau^{-1}}(2x) - \frac{2M_{x+1} \tau}{(2x-1)!} \Gamma_{M_{x+1} \tau^{-1}}(2x+1) + \frac{\tau^2}{(2x-1)!} \Gamma_{M_{x+1} \tau^{-1}}(2x+2) \right] \quad (32)$$

or

$$c_2 = \frac{1}{F_{x+1} J_{\max}^2} \left[\frac{M_{x+1}}{(2x-1)!} \Gamma_{M_{x+1} \tau^{-1}}(2x) - \frac{\tau}{(2x-1)!} \Gamma_{M_{x+1} \tau^{-1}}(2x+1) \right] \quad (33)$$

One cannot expect this treatment to be very accurate. We can expect only to obtain the trend of the E_j values within about a factor of two.

The application of Eq. (32) yields values of c_1 from 0.10 to 0.27 MeV. Values of c_2 from Eq. (33) range from 0.0025 to 0.015 MeV. The former result is indicated in Fig. 5 by the solid line, the latter result by the dashed line. Both the dashed and the solid lines are consistent with the trend indicated by the values of $\langle T_\gamma \rangle$ shown in Fig. (5). As stated previously, the average total photon energy $\langle T_\gamma \rangle$ is expected to be greater than E_j by about the separation energy of the neutron. The variation in c_1 and c_2 values is large enough that there is a considerable region of overlap of these two representations. The range of values obtained for c_1 is more limited than that of c_2 , which may indicate that E_j is better represented by a linear than a quadratic dependence on J .

We might expect a "cold" spinning nucleus to give a reasonable model of the states of highest angular momentum for a given excitation energy. If the cold nucleus has the moment of inertia of a rigid sphere of radius $1.2A^{1/3}R_F$, then Eq. (24) is appropriate with a c_2 value of 0.0084 MeV. This model is not inconsistent with the approximate analysis presented above.

V. CONCLUSION

A large body of cross-section data has been presented for reactions of type $(HI, xn)Dy^{149}$, $(HI, xn)Dy^{150}$, $(HI, xn)Dy^{151}$. The fraction of the total reaction cross section that leads to these reactions varies with initial excitation energy from about 9/10 at 45 MeV to about 1/2 at 105 MeV. An analysis of the "first moment" of the excitation functions has been presented. This analysis of the cross-section data leads to estimates of the energy of the first emitted neutron. These energies are consistently smaller than estimates obtained from angular-distribution studies. The discrepancy suggests that the first neutron may remove rather large amounts of angular momentum (2 to $4\hbar$). A linear relationship has been obtained between average total photon energy and average total angular momentum removed by the photons. This relationship implies that the average angular momentum removed by each photon is $1.8 \pm 0.6\hbar$. The dependence of E_j on J has been roughly estimated from the cross-section data.

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Footnotes and References

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Table I. Isotopic composition of the targets.

Target nuclide	Mass number and abundance (%) of the isotopes						
	<u>142</u>	<u>143</u>	<u>144</u>	<u>145</u>	<u>146</u>	<u>148</u>	<u>150</u>
Nd ¹⁴²	97.45	1.04	0.89	0.21	0.26	0.08	0.07
Nd ¹⁴⁴	0.56	0.67	97.3	0.8	0.67	<0.05	<0.05
Ce ¹⁴⁰	<u>136</u>		<u>138</u>		<u>140</u>		<u>142</u>
	<0.01		<0.01		99.65		0.35
Ba ¹³⁶	<u>130</u>	<u>132</u>	<u>134</u>	<u>135</u>	<u>136</u>	<u>137</u>	<u>138</u>
	<0.05	<0.05	<0.1	1.08	92.9	1.77 ^a	4.24 ^a
Ba ¹³⁷	<0.03	<0.03	<0.05	<0.1	0.63	81.9	17.4 ^a
Ba ¹³⁸	0.02	0.02	0.05	0.15	0.26	1.45	98.04

^a Corrections for these components were made in calculating the cross sections.

Table II. Cross-section results. (Different experiments separated by dashed lines).

E_b (lab) (MeV)	Cross section (mb)		
	Dy^{149} $\text{Nd}^{142} + \text{C}^{12} \rightarrow \text{Dy}^{154}$	Dy^{150}	Dy^{151}
116.3	109.	24.1	
112.4	172.	34.8	
108.1	230.	50.0	
103.7	331.	92.8	
99.0	408.	178.	
94.3	446.	358.	
89.3	381.	641.	16.9
84.1	234.	856.	100.
78.8	95.1	934.	178.
73.0	11.6	709.	323.
67.2		262.	445.
60.6			327.
53.9			55.7

122.9	52.5		
118.6	90.3		
114.2	146.		
110.2	224.		
85.8	297.		
80.5	141.		

Table II. (Cont'd).

E_b (MeV)	Cross section (mb)		
	Dy^{149} $\text{Pr}^{141} + \text{N}^{14} \rightarrow \text{Dy}^{155}$	Dy^{150} Dy^{155}	Dy^{151}
128.8	85.7	13.6	
122.9	151.	32.6	
116.6	218.	80.5	
110.5	266.	176.	
103.7	280.	376.	
96.7	197.	578.	40.4
89.5	83.0	642.	124.
81.3	12.3	478.	282.
72.8		126.	325.
63.8			138.

142.8	17.4		
137.2	32.7		
132.0	62.0		
126.4	119.		
120.8	179.		
108.6	309.		

137.5	34.8		
131.7	66.1		
125.7	124.5		
119.6	206.8	58.7	
113.0	289.8	153.1	
99.8	245.8	551.4	
92.0	135.7	648.3	

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Pr^{141} + N^{14} \rightarrow Dy^{155}$		
142.8	17.4	2.11	
134.1	45.8	5.68	
125.4	125.	24.0	
115.9	221.	87.2	
109.2	270.	205.	
102.3	235.	375.	
95.1	161.	588.	34.3
87.2	55.2	650.	155.
79.2	3.1	316.	322.
70.3		31.9	282.
60.5			37.0

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Nd^{144} + C^{12} \rightarrow Dy^{156}$		
122.8	280.	274.	
118.8	282.		
114.5	262.	554.	34.2
110.5	214.	705.	67.7
106.1	149.	830.	142.
101.5	82.0	783.	292.
96.8	36.7	656.	437.
92.0	10.9	381.	537.
87.0	5.2	164.	591.
81.7		43.3	472.
76.3		11.8	189.
70.3			33.6

122.8	300.	290.	
95.	27.2	537.	490.

105.6	127.		
100.9	65.6		
96.0	24.4		
91.1	8.7		

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Pr^{141} + N^{15} \rightarrow Dy^{156}$		
132.9	228.	169.	
126.4	243.	344.	
119.7	212.	538.	
113.0	148.	659.	130.
105.9	74.2	618.	327.
98.1	<13.	393.	433.
90.4		117.	489.
153.0	53.5		
147.9	87.0		
142.8	133.		
137.4	190.		
132.0	218.		
126.3	234.		
113.1	147.		
106.8	74.1		
148.5	85.3		
143.0	137.		
137.3	190.		
131.1	204.		
115.2	166.		

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Ce^{140} + O^{16} \rightarrow Dy^{156}$		
163.0	62.5	14.8	
152.2	164.	65.8	
140.8	290.	292.	
132.6	264.	512.	
124.6	180.	645.	126.
115.7	85.5	690.	353.
106.9	12.5	361.	458.
96.6	<0.5	73.6	439.
86.2		<2.8	109.
74.7			<1.5

147.8	181.	100.	
140.2	249.	260.	
128.0	240.	612.	87.1
119.5	141.	745.	220.
110.7	47.9	651.	369.
101.3	<3.5	274.	622.
90.6		<18.3	326.

163.0	71.2	17.2	
151.7	162.	79.3	
131.4	240.	441.	47.5
113.8	51.5	497.	349.
104.2		255.	487.

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Ce^{140} + O^{16} \rightarrow Dy^{156}$		
93.8		17.0	331.
82.7			28.7
163.0	58.4		
155.5	120.		
148.6	222.		
141.3	270.		
133.6	252.		
126.2	211.		
118.4	127.		

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Ba^{136} + Ne^{20} \rightarrow Dy^{156}$		
202.6	1.6	0.3	
190.2	5.6	0.7	
179.5	25.4	3.2	
167.2	92.8	22.1	5.2
154.6	220.	169.	13.3
140.8	233.	508.	63.5
126.7	96.1	607.	277.
111.2	6.1	210.	400.
94.4		3.2	70.8

202.6	1.2		
184.8	10.1	0.2	
172.3	48.4	7.3	
159.8	159.	58.9	11.4
146.4	246.	309.	50.2
131.9	181.	610.	139.
116.6	28.0	429.	364.
99.8		26.0	296.
81.8			2.3

179.6	23.0		
126.6	85.2		
111.4	5.2		

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150} Dy^{157}	Dy^{151}
	Ba ¹³⁷ + Ne ²⁰ → Dy		
202.7	11.3	3.0	
189.1	39.8	12.6	
175.0	117.	66.5	
160.0	147.	252.	33.5
146.7	102.	416.	121.
132.8	23.6	310.	301.
118.0		46.6	241.
101.5			17.1

202.7	10.3	2.3	
191.3	32.2	7.6	
179.4	91.7	38.9	
167.0	170.	147.	36.3
154.5	171.	391.	69.7
141.2	89.1	482.	256.
126.4	9.3	252.	387.
111.4		15.3	150.
95.0		0.5	2.2

191.4	30.9		
167.6	160.		
141.2	85.7		

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Ce^{140} + O^{18} \rightarrow Dy^{158}$		
173.2	127.	168.	<24.
165.8	150.	259.	35.9
154.3	129.	459.	125.2
148.0	81.4	483.	206.8
135.0	34.1	381.	304.8
128.9	10.6	211.	410.7
119.9	<4.7	34.3	291.
110.2			90.2
99.5			5.0

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$La^{139} + F^{19} \rightarrow Dy^{158}$		
192.9	41.9	17.1	
179.6	107.	77.6	
169.9	142.	189.	
159.8	144.	351.	66.1
149.2	95.9	452.	168.
138.1	34.8	351.	353.
126.0	6.9	136.	341.
114.8		8.3	141.
101.1			4.9
192.9	40.0		
182.4	87.5		
170.8	137.		
160.7	142.		
149.7	97.6		
192.9	43.0		
183.2	86.5		
173.1	135.		
162.3	154.		
139.8	46.5		
128.8	12.1		

Table II. (Cont'd)

E_b (MeV)	Cross section (mb)		
	Dy^{149}	Dy^{150}	Dy^{151}
	$Ba^{138} + Ne^{20} \rightarrow Dy^{158}$		
187.2	114.	94.6	
171.2	152.	349.	50.3
153.8	72.4	466.	255.
140.4	13.5	243.	400.
125.6	<5.5	26.5	211.
110.6		1.9	8.6
202.6	36.7		
188.8	95.4		
175.6	152.		
162.0	122.		
148.0	44.		
202.8	42.2		
154.8	67.1		

Table III. Calculated total reaction cross sections.

$\frac{E}{V}$ c.m.	$\frac{\sigma_R}{\pi R^2}$	
	Square well ^a	Classical ^b
0.98	0.022	0.000
1.00	0.030	0.000
1.05	0.054	0.048
1.10	0.084	0.091
1.15	0.116	0.130
1.20	0.146	0.167
1.25	0.175	0.200
1.30	0.201	0.231
1.40	0.249	0.286
1.50	0.292	0.333
1.60	0.333	0.375
1.70	0.375	0.412
1.80	0.412	0.445
2.00	0.455	0.500
2.20	0.495	0.545
2.40	0.532	0.583

^aSee reference 9.

^bEq. 11 in text.

Table IV. Average energies and angular momenta.

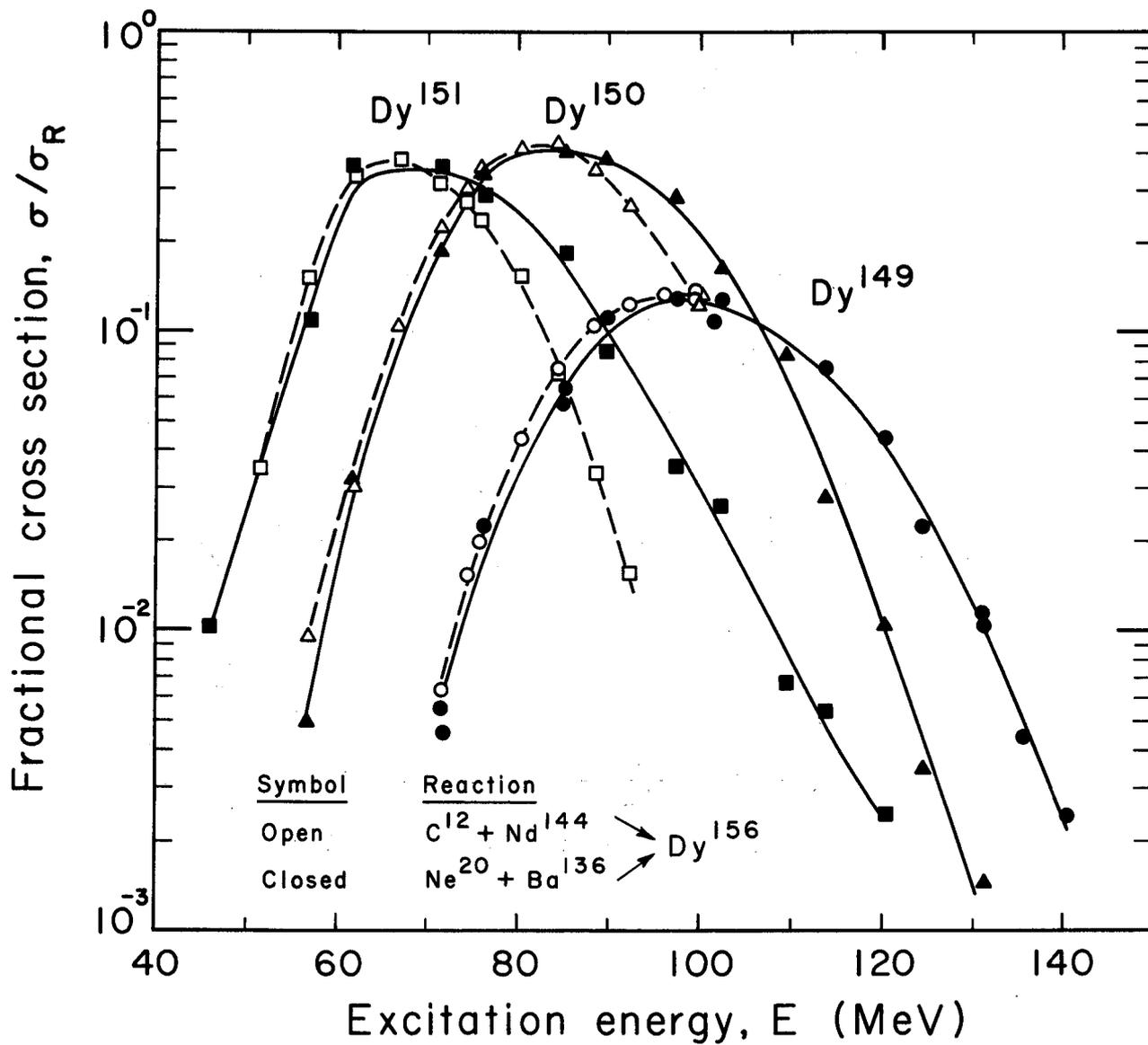
Reaction	$\langle E \rangle_x$ (MeV)	$\langle E \rangle_x - \sum_{i=1}^n B_i$ (MeV)	$\langle J \rangle$
Nd ¹⁴² (C ¹² ,3n)Dy ¹⁵¹	45.4	18.7	21.1
Nd ¹⁴² (C ¹² ,4n)Dy ¹⁵⁰	59.8	25.2	29.5
Nd ¹⁴² (C ¹² ,5n)Dy ¹⁴⁹	75.6	30.8	36.7
Pr ¹⁴¹ (N ¹⁴ ,4n)Dy ¹⁵¹	54.0	20.2	23.6
Pr ¹⁴¹ (N ¹⁴ ,5n)Dy ¹⁵⁰	70.2	28.5	33.7
Pr ¹⁴¹ (N ¹⁴ ,6n)Dy ¹⁴⁹	86.1	34.2	41.3
Nd ¹⁴⁴ (C ¹² ,5n)Dy ¹⁵¹	68.6	25.8	33.1
Nd ¹⁴⁴ (C ¹² ,6n)Dy ¹⁵⁰	85.2	34.5	40.0
Pr ¹⁴¹ (N ¹⁵ ,6n)Dy ¹⁵⁰	85.5	34.8	43.6
Pr ¹⁴¹ (N ¹⁵ ,7n)Dy ¹⁴⁹	100.8	39.9	49.9
Ce ¹⁴⁰ (O ¹⁶ ,5n)Dy ¹⁵¹	71.4	28.6	38.5
Ce ¹⁴⁰ (O ¹⁶ ,6n)Dy ¹⁵⁰	87.2	36.5	46.2
Ce ¹⁴⁰ (O ¹⁶ ,7n)Dy ¹⁴⁹	102.3	41.4	52.5
Ba ¹³⁶ (Ne ²⁰ ,5n)Dy ¹⁵¹	73.0	30.2	39.5
Ba ¹³⁶ (Ne ²⁰ ,6n)Dy ¹⁵⁰	88.0	37.3	48.4
Ba ¹³⁶ (Ne ²⁰ ,7n)Dy ¹⁴⁹	102.5	41.6	55.6
Ba ¹³⁷ (Ne ²⁰ ,6n)Dy ¹⁵¹	88.2	38.6	48.5
Ba ¹³⁷ (Ne ²⁰ ,7n)Dy ¹⁵⁰	101.9	44.4	55.4
Ba ¹³⁷ (Ne ²⁰ ,8n)Dy ¹⁴⁹	116.9	49.2	62.0
Ce ¹⁴⁰ (O ¹⁸ ,7n)Dy ¹⁵¹	100.4	42.1	54.2
La ¹³⁹ (F ¹⁹ ,7n)Dy ¹⁵¹	99.8	41.5	52.9
La ¹³⁹ (F ¹⁹ ,8n)Dy ¹⁵⁰	114.5	48.3	59.4
La ¹³⁹ (F ¹⁹ ,9n)Dy ¹⁴⁹	129.0	52.6	65.2
Ba ¹³⁸ (Ne ²⁰ ,7n)Dy ¹⁵¹	100.1	41.8	54.6
Ba ¹³⁸ (Ne ²⁰ ,8n)Dy ¹⁵⁰	115.5	49.3	61.5
Ba ¹³⁸ (Ne ²⁰ ,9n)Dy ¹⁴⁹	129.4	53.0	67.2

Table V. Average energy of the first emitted neutron $\langle k_1 \rangle$ in (HI,xn)Dy reactions.

$\langle k_1 \rangle$ (MeV)			
x	angular distribution ^a	cross sections ^b	ratio
4	3.4 ± 0.3	1.8 ± 1.5	0.53 ± 0.47
5	3.8 ± 0.4	1.6 ± 1.5	0.42 ± 0.42
6	4.2 ± 0.4	3.4 ± 1.5	0.81 ± 0.38
7	4.5 ± 0.5	1.6 ± 1.5	0.36 ± 0.36
8	4.8 ± 0.5	3.9 ± 1.5	0.81 ± 0.33
9	5.1 ± 0.5	1.7 ± 1.5	0.33 ± 0.31
Av.	4.3	2.3	0.53

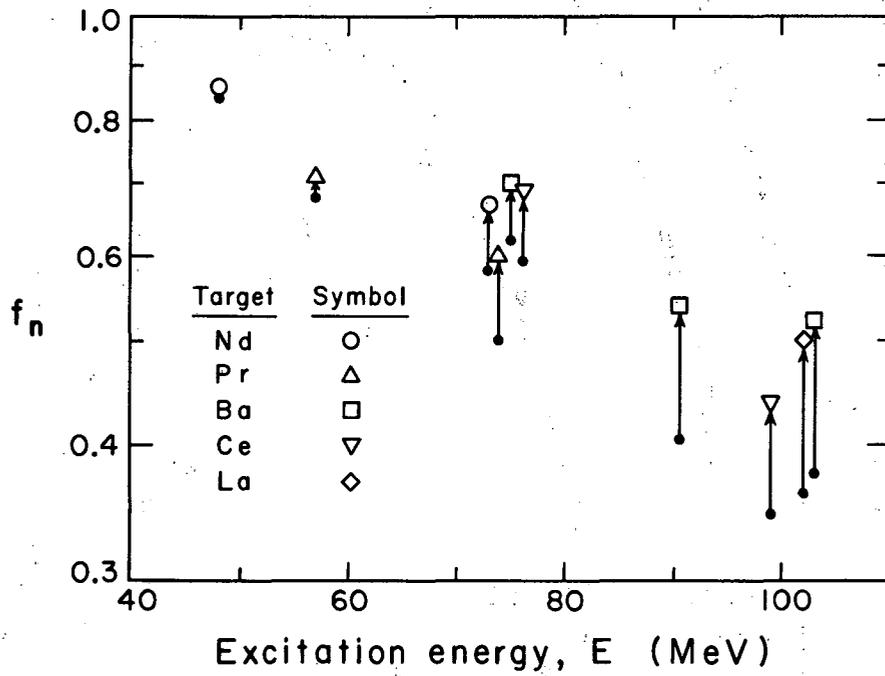
^a Eq. 16.

^b Eq. 15.



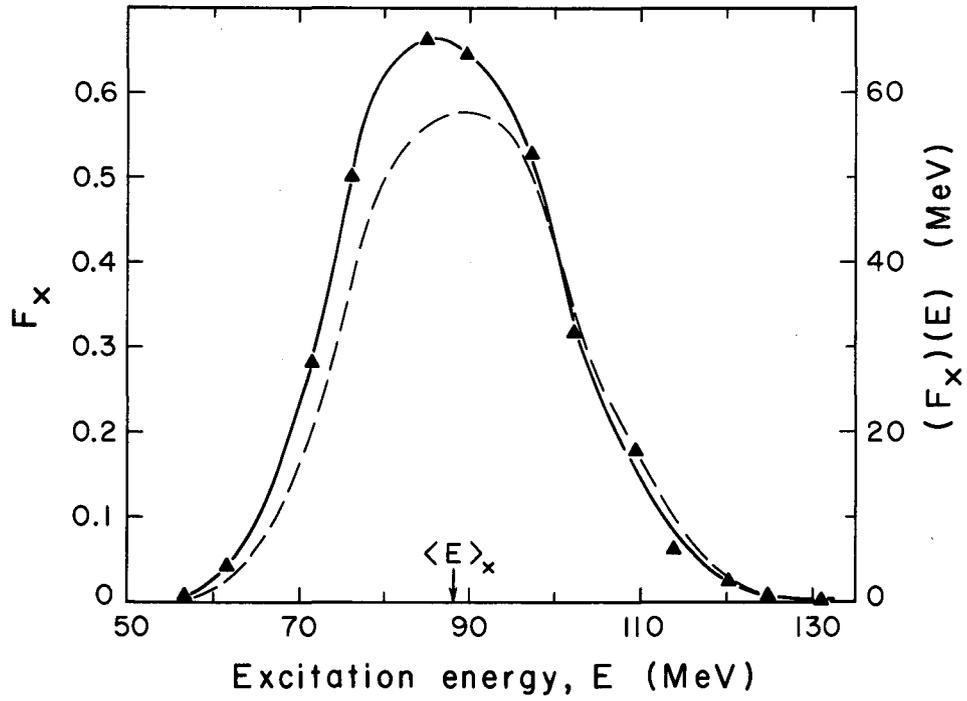
MUB-1575

Fig. 1. Measured cross section σ divided by calculated total reaction cross section σ_R as a function of excitation energy E.



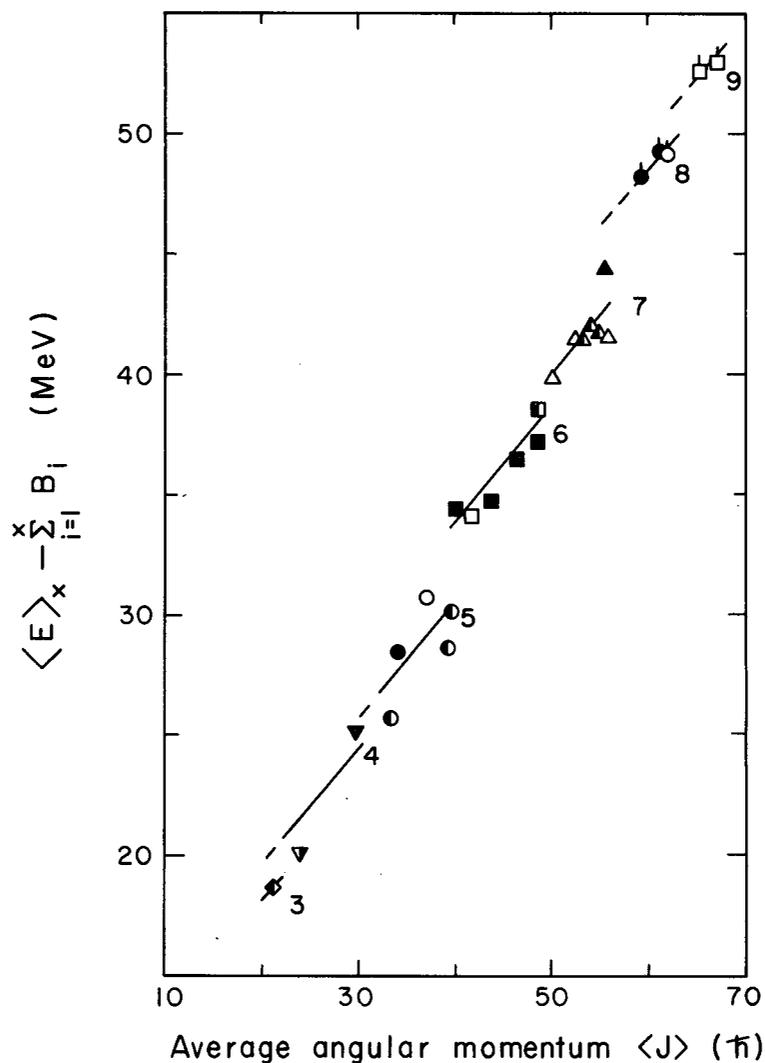
MU-29077

Fig. 2. The fraction f_n of the calculated total reaction cross section that leads to (HI, xn) reactions as a function of excitation energy E . The different symbols are for different target materials as shown. The arrows indicate the estimated magnitude of the contribution from reactions producing Dy^{149} and Dy^{152} . The major products are Dy^{150} and Dy^{151} (see text).



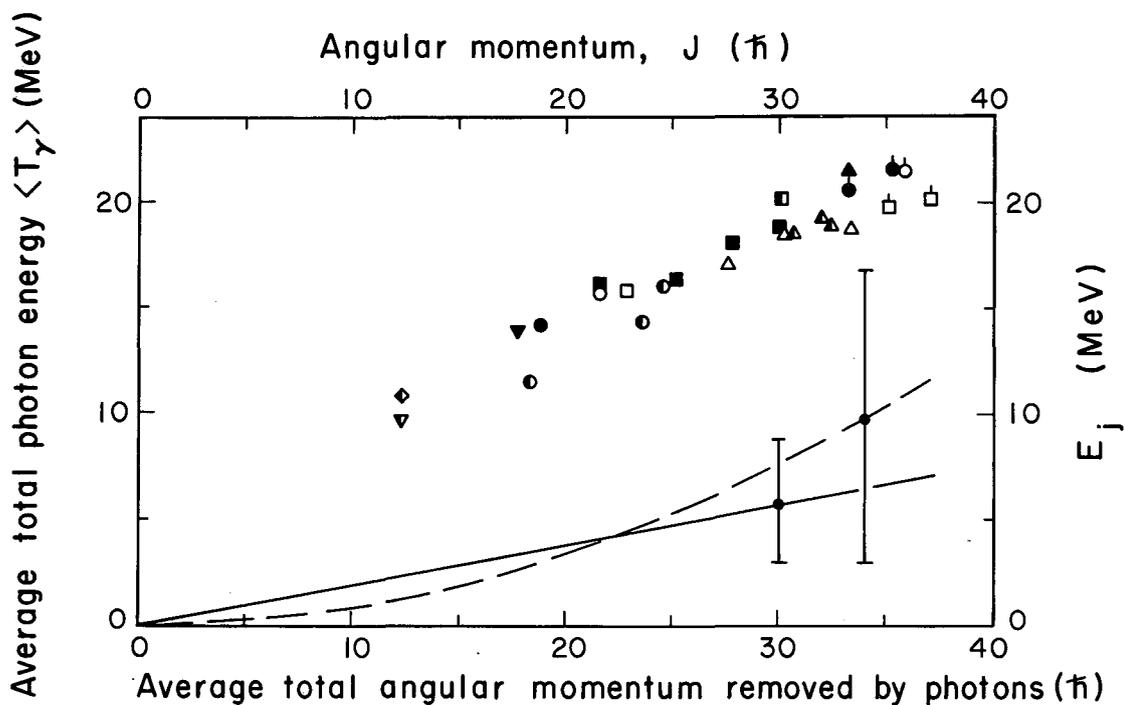
MU-29078

Fig. 3. F_x (solid curve) and $(F_x)(E)$ (dashed curve) vs excitation energy for the reaction $Ba^{136}(Ne^{20},6n)Dy^{150}$. The value of $\langle E \rangle_x$ is indicated.



MU-29076

Fig. 4. The average excitation energy $\langle E \rangle_x$ minus the sum of the binding energies B_i of the neutrons as a function of the average angular momentum $\langle J \rangle$. Different symbols are used for the different (HI, xn) reactions \blacklozenge , $x = 3$; \blacktriangledown , $x = 4$; \bullet , $x = 5$; \blacksquare , $x = 6$; \blacktriangle , $x = 7$; \bullet , $x = 8$; \blacksquare , $x = 9$. Open points are for Dy^{149} ; closed for Dy^{150} ; and half open for Dy^{151} .



MU-29079

Fig. 5. Average total photon energy $\langle T_\gamma \rangle$ versus average total angular momentum removed by the photons. Symbols are as in Fig. 4. Also indicated is the relationship between E_j and J . The solid line was obtained with the assumption $E_j = c_1 J$; the dashed line with $E_j = c_2 J^2$.

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