UCRL-10557

University of California

Ernest O. Lawrence Radiation Laboratory

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

COULOMB EXCITATION OF Tb^{159}, Ho^{165}, AND Tm^{169} WITH O^{16} IONS

Berkeley, California

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Nuclear Physics

UCRL-10557

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

COULOMB EXCITATION OF Tb^{159}, Ho^{165}, AND Tm^{169} WITH O^{16} IONS

Richard M. Diamond, Bent Elbek, and F. S. Stephens

November 28, 1962

COULOMB EXCITATION OF Tb^{159}, Ho^{165}, AND Tm^{169} WITH O^{16} IONS

Richard M. Diamond, Bent Elbek, and F. S. Stephens

Lawrence Radiation Laboratory
University of California
Berkeley, California

November 28, 1962

ABSTRACT

The Coulomb excitation of Tb^{159}, Ho^{165}, and Tm^{169} has been studied using principally 60-MeV O^{16} ions as projectiles. Both gamma-ray and conversion-electron spectra were taken. Rotational bands observed at 514 and 687 keV in Ho^{165}, at 580 (and possibly ~1280) keV in Tb^{159}, and at 570 (and possibly ~1170) keV in Tm^{169} have been assigned as collective (gamma-vibrational) bands. The bands at 361 keV in Ho^{165}, at 348 and 971 keV in Tb^{159}, and at ~900 keV in Tm^{169} are ascribed to intrinsic (Nilsson) configurations. The properties of these bands are discussed in some detail.

By multiple excitation six or eight members of each ground-state rotational band were excited. The analysis of the energies of these levels indicates the presence of higher-order Coriolis terms in the rotational-energy formula in the cases of Tb^{159} and Tm^{169}.

COULOMB EXCITATION OF Tb^{159}, Ho^{165}, AND Tm^{169} WITH O^{16} IONS*

Richard M. Diamond, Bent Elbek,† and F. S. Stephens

Lawrence Radiation Laboratory
University of California
Berkeley, California

November 28, 1962

1. INTRODUCTION

Coulomb-excitation measurements have provided a considerable amount of information on the energies, spins, and transition moments of the low-lying excited states of nuclei. For the most part, either protons or alpha particles have been the projectile ions, and by now most of the readily available isotopes have been used as targets. Most of the excited states observed are collective states, that is, rotational members of the ground-state band in deformed nuclei, and vibrational states in spherical nuclei. The reason for this is that the transition moments connecting these collective states with the ground state are greatly enhanced, and correspondingly the probability for excitation is large. On the other hand, transition moments between most other states and the ground state are much smaller, and the probability for Coulomb excitation is thus so small that the states are usually not observed.

The use of heavier ions such as O^{16}, Ne^{20} or A^{40} rather than protons and alpha particles for the excitation is proving to be advantageous. The increase in the excitation probability with the heavier ions allows observation of higher-lying levels and those connected with the ground state by smaller transition moments, and especially promotes the occurrence of

---

*Work performed under the auspices of the U.S. Atomic Energy Commission.

†Present address: Institute for Theoretical Physics, Blegdamsvej 17, Copenhagen, Denmark.

multiple excitation. Several levels in a rotational band, whether based on the ground state or on an excited state, can usually be observed by excitation with heavy ions, and the spacing of these levels, their excitation probabilities, and the branching ratios of their decay radiation are important clues as to the nature of the base state.

The Coulomb-excitation process can be observed by studying the inelastically scattered particles or the radiations de-exciting the target nucleus, that is, either gamma rays or internal conversion electrons. When heavy ions are employed, analysis of the inelastically scattered particles is generally more difficult, because even thin targets produce large energy spreads. In addition, stringent conditions must be imposed on the energy definition of the heavy-ion beam to perform this type of spectroscopy. Thus, a study of the de-exciting radiation appears to be more convenient in this case. Information about excited states and the mechanism for their excitation can be obtained in principle by analyzing either the gamma rays or conversion electrons. In practice, it is often more convenient to look at one or the other, and by measuring both, additional information such as conversion coefficients can be obtained.

The present study is the beginning of an investigation of medium and heavy odd-mass deformed nuclei using O^{16} ions and observing both the de-excitation gamma-rays and internal conversion electrons. In previous studies of such nuclei (cf. review articles, references 1 and 2) the first two rotational levels of the ground-state band have been excited, and the transitions between them and to the ground state have been observed. The spins and energies of these excited levels and their E2 and M1 transition moments to the ground state have been determined. Because of the enhanced probability for excitation with O^{16} ions, we hoped, in this work, to excite and study

still higher-lying states, both collective and intrinsic, and to gain some insight into their nature. Nathan and Popov,[3] using 14 to 20-MeV alpha particles, have excited some higher levels in a number of these odd-mass deformed nuclei, but because of their limited resolution and poor statistics they could not say very much about the nature of the excited levels.

For odd-mass nuclei in the region of the deformed rare earth nuclei, three even parity bands of collective levels based on the ground state might be expected; a beta vibrational band having $K = K_0$, and two gamma vibrational bands having $K = K_0 \pm 2$. On the Davidov-Filippov model[4] these gamma vibrational states are interpreted differently, and although we will call them vibrational states, this does not imply a preference for one model over another. In fact, we will use the term, vibrational, as simply a convenient label for states of this type. Determination of the energies of these states and of the spacing in their rotational bands is certainly the first step in observing the expected systematic behavior of such types of collective motion in going from one nucleus to the next in this region. The collective levels would be expected to de-excite to the ground state by E2 transitions, and the corresponding E2 transition moments should be comparable to or greater than a single particle unit in strength.

The single-particle or intrinsic states expected from shell-model-type calculations in these deformed nuclei, have been computed by Nilsson,[5] and agreement with experimental data has been shown to be excellent by Mottelson and Nilsson.[6] Occasionally we would expect to Coulomb-excite this type of level, but in general the E2 transition moments to these levels would be expected to be considerably less than to the collective states.

The present paper reports on the levels observed by Coulomb excitation with O^{16} ions in the odd-proton deformed nuclei, Tb^{159}, Ho^{165}, and Tm^{169}.

2. EXPERIMENTAL TECHNIQUES

The Lawrence Radiation Laboratory's heavy-ion linear accelerator (HILAC) was the source for the beam of oxygen ions used as projectiles in the present study. The beam energy was usually 60 MeV, but it could be varied up to the maximum energy of 167 MeV by changing the tilt of the gradient in the post-stripper tank and adjusting the tank tuners to cause the beam to fall out of phase at the desired energy and coast the rest of the way down the tank. After leaving the accelerator, the beam passes through a deflecting magnet which bends it 22 deg through a quadrupole focusing magnet into a steel and concrete cave containing the gamma-ray and conversion-electron spectrometers. Beams of +6-charged O^{16} ions of about 1-μA average current can be obtained through a 2-by-2-mm collimator at the target position. Actually, the instantaneous beam intensity is higher by a factor of about 20, as the HILAC is a pulsed machine with a pulse duration of 2-1/2 to 3 msec, and a variable repetition rate, normally set at 15 pulses per second during our runs. Thin (~1 mg/cm^2) gold foils can be moved into the beam at the deflecting magnet and also near our target position in order to scatter the beam into silicon detectors in order to determine the beam energy and energy distribution. These detectors are calibrated against the energy of the full energy beam (167 MeV).

The targets were disks of the rare-earth metals 20 to 25-mm in diameter by 1 to 2-mm-thick, obtained from Research Chemical Corp., Phoenix, Arizona.

The Coulomb excitation of each target was first surveyed through its gamma-ray spectrum, which could be recorded more quickly than the conversion electron spectrum. Both singles spectra and spectra in coincidence with back-scattered O^{16} particles were taken with a 7.5 by 7.5-cm NaI crystal integrally mounted on a Dumont 6363 photomultiplier tube. The spectra were recorded on

a 100-channel pulse-height analyzer which was gated "on" during the beam pulse only. The back-scattered particles were detected in an argon-flow scintillation counter; a diagram of the arrangement of the counters is shown in Fig. 1. The targets were mounted on 6.3-mm-thick aluminum plugs which were at the end of the gas counter, and the NaI crystal was placed 1 cm from the front of the targets and along the beam axis.

The singles gamma-ray spectra suffered from a large background produced partly from external sources, but mainly from the target. Surface impurities such as an oxide coating and possibly pump oil are the likely origins. By means of the coincidence technique developed earlier in which only the gamma rays in coincidence with back-scattered O^{16} particles are recorded,[7] this background was reduced, and far superior spectra were obtained. The O^{16} ions that back-scattered from the target through angles of 120 to 160 deg passed through the wall of the argon scintillation chamber, which consisted of a 1/4-mil-thick ($\sim$ 700 $\mu g/cm^2$) aluminized mylar foil supported on a cylindrical steel cage. Ordinary tank argon at just above atmospheric pressure was passed through the counter at a rate of a few ml/min. The dimensions of the counter were large enough to accomodate particles with ranges up to 5 cm, except for some corner effects. (For O^{16} ions a range of 5 cm corresponds to an energy of about 45 MeV.) Higher particle energies were poorly defined because of the limited size of the counter, but this was not of primary importance. The principal source of back-ground coincidences came from protons, alpha particles, and other low-energy reaction products, and these could be eliminated by setting the discriminator bias at 10 to 15 MeV. Some scattered heavy ions were thus lost because the thick targets used gave rise to a continuous distribution of energies of back-scattered projectiles; accounting for these was one of the several corrections needed to obtain excitation yields.

The inside of the gas scintillation counter was coated with white Tygon paint over which a layer of diphenylstilbene was evaporated to function as a wave-length shifter. The light pulses from the counter were detected by two matched RCA 6655A photomultiplier tubes whose faces were also covered with the wavelength shifter. The amplifications of the two tubes were equalized and the output pulses mixed and sent to a conventional fast-slow coincidence circuit. Fast coincidences (10^{-8} sec) with the fast output of the gamma-ray counter were then used to gate the 100-channel analyzer for recording the gamma-ray spectrum.

Because of the low duty cycle of the HILAC (~ 4%) the instantaneous counting rates in the gamma-ray counter were quite high and usually set the limit on the maximum usable beam. The photomultiplier tube showed considerable fatigue and drift at the counting rates used, and it was necessary therefore to regulate the photomultiplier gain by means of a stabilizing circuit operating on a well-defined photopeak. For this purpose a weak Na^{22} or Cs^{137} source was introduced near the NaI crystal, and the stabilizer was gated "on" during the period between beam pulses. By so gating the stabilizer it was possible to use a 25-fold weaker source than would otherwise be necessary, and the Coulomb-excitation spectrum was thus less disturbed by random coincidences from this source.

For a more detailed investigation of the de-excitation transitions following Coulomb excitation, and one employing higher resolution than is possible through gamma-ray spectroscopy, we studied the internal conversion electron lines using a single wedge-gap spectrometer of the type developed by Kofoed-Hansen, Lindhard, and Nielsen.[8] A short description of our spectrometer has been given elsewhere.[9] The detector on the spectrometer was a 5- or 6-mm-thick anthracene crystal cemented to an RCA 6655A photomultiplier

tube. Since the pulse-height spectrum from the anthracene crystal showed a well-resolved full energy peak, this peak was passed through a single-channel analyzer to discriminate further against the continuum of background pulses. The detector was usually collimated to a 2 mm width, although sometimes a 1- or a 4-mm-wide collimator was used. A collimator of the same width was placed before the target. The thick metal targets were usually inclined at 10 deg, sometimes 5 or 15 deg, to the beam to reduce the projectile penetration and hence the electron path in the target. Energy degradation in the target was nevertheless the limiting factor on the resolution for conversion electron lines of less than ~ 300 keV when the usual range of spectrometer operating conditions were employed. For higher-energy electrons, the spectrometer yielded a momentum resolution which was varied between 0.5 and 1.0%. However, still another factor enters into line-width considerations. When the half-life of the excited state emitting the electron is shorter than the time necessary to stop the recoiling target nucleus, Doppler broadening becomes discernible. This effect is, of course, more prominent when heavy ions are employed than with protons and alpha particles.

The Doppler broadening could take on one of two forms. If the transitions were very fast, $\lesssim 5 \times 10^{-13}$ sec, the conversion electrons would be emitted before the recoiling nucleus stopped, whether in the target (away from the spectrometer) or in the vacuum (toward the spectrometer). Consequently, the conversion line would be broadened symmetrically, that is, toward both higher and lower energies. A good example of this type of broadening is shown by the 580-keV M1 transition in Tb^{159}. In general, M1 transitions might be expected to be fast enough to behave this way, and in fact all the M1 transitions observed in this work appeared to be broadened to some extent. Thus a symmetrical Doppler shift might be considered as an empirical indicator

of (fast) M1 transitions in spectra taken under our conditions. On the other hand we know from the Coulomb-excitation yields that the E2 transitions we are dealing with are not sufficiently fast to be broadened in this way. For these, a small shoulder appeared on the high-energy side only, corresponding to recoils into the vacuum and toward the spectrometer. Only target nuclei recoiling into a relatively small and discrete solid angle could leave the target, therefore this type of Doppler shift appeared not as a smooth broadening of the high-energy side of the line, but as a discrete shoulder, somewhat like a separate transition of about 1% higher energy. Examples of this type of broadening are shown by the Au^{197} 547-keV E2 transition and the relatively fast E2 cross-over transitions of the ground-state rotational bands observed in all three nuclei studied here. Some rough calculations made for this type of shift show that the effect should be a function of the angle that the target makes with the beam and of the mass and velocity of the projectile. These dependencies were verified experimentally.

The spectrometer magnet current was obtained from an electronic supply regulated to one part in 5000. The magnet was always put through a standard cycling procedure before use to minimize hysteresis irregularities. The momentum of the internal conversion electrons was determined as a magnet current reading from a potentiometer in a bridge circuit. The reproducibility of the current value for the peak position of a conversion electron line was usually better than 0.5%.

The operation of the spectrometer was semi-automatic. After a predetermined amount of beam current was collected on the target, the number of counts that had passed through the single-channel pulse-height analyzer was recorded in a 100-channel analyzer modified for multi-scaler operation. The

analyzer was then advanced one channel, the magnet current was increased by a predetermined amount, the single-channel analyzer was reset, and a new count for the same amount of beam current on the target was started at the slightly higher magnet current setting. After 98 such points were taken, the magnet had to be recycled, and a new series of 98 points could then be run off.

3. DATA ANALYSIS

Examples of the gamma-ray spectra coincident with scattered O^{16} ions and of the electron spectra are shown in figs. 2-7. From such primary data, one derives the energy and yield of each gamma-ray and conversion electron transition. The yields can be used to determine the excitation function of each transition, and thus the multipolarity of the excitation, and further to give the excitation transition moment--B(E2) in our cases. In this section we outline the methods used to obtain the above information from the data.

The energies of the transitions were determined on the electron spectrometer by interpolation from the magnet-current readings for standards inserted during the same magnet sweep as the conversion lines of interest. The energies of the first two excited levels in Tb^{159}, Ho^{165} (but with a correction), Tm^{169}, and Ta^{181}, as determined in the very accurate Coulomb-excitation experiments of Chupp et al.[10] were used as standards, as well as the well-known 279-keV transition of Hg^{203}, the 569-and 1060-keV transitions of Bi^{207}, and the 662-keV transition of Cs^{137}. From the consistency of the energies as calculated in various ways from different runs, we estimate that the energies are probably good to about 1 keV. Some transitions used as secondary standards and quoted to the nearest 0.1 keV in the tables are probably somewhat better than this, and one group, around 950 keV in Tb^{159}, is probably not this good.

The gamma-ray yields were all determined from the coincidence spectra taken in the manner already described. The absolute photon yields, for a given number of O^{16} projectiles, were obtained by integrating the number of counts in the photopeak, subtracting out background, and then correcting for geometry, photopeak efficiencies, and absorption in the target and target holder. Additional corrections for the particle-counter geometry and efficiency

are necessary. The geometrical solid angle of the particle counter was calculated to be ~ 19%. However, a variety of factors combine to reduce the effective transmission of the counter significantly. Particle losses in the target, the energy cutoff of the discriminator, losses in the coincidence circuit at the high instantaneous rates employed, small effects from the angular anistropy between scattered particles and the gamma rays, and corrections to the calculated solid angle due to the motion of the center of mass of the particle—target nucleus system--all contribute. Since the calculation of all these effects is rather involved, we have determined the yields relative to the yield of the 547-keV transition in Au^{197} (known from earlier Coulomb-excitation studies) by running a gold target under the same conditions employed for each rare-earth target. In this way, the corrections for the particle counter geometry and efficiency and the NaI-counter geometry approximately cancelled. The gamma-ray absorption in the target and target holder was determined experimentally by using radioactive gamma-ray sources and the same target assemblies. Angular-anistropy effects were estimated to be negligible in all cases, mainly because of the large solid angle subtended by the NaI crystal.

The absolute yields of the internal conversion electron lines were determined similarly by comparison with the yield of the K-shell conversion electron line of the 547-keV transition Coulomb excited in Au^{197} under the same conditions. The yields were obtained from the conversion electron spectra, after subtraction of background, by summing over the conversion peak and then correcting for the fact that the counts were taken for equal intervals of magnet current and not equal intervals of $H\rho$. Although the electron path to the spectrometer aperature makes a 90 deg angle to the beam axis, the estimated corrections due to electron anisotropy were less than 5% for the cases studied

and were neglected. The corrections for detector width, effective solid angle, and spectrometer dispersion cancel when the transition is compared to the standard gold transition under the same conditions. Even the small loss of counts in the detector and single-channel analyzer is essentially compenstated for.

The partial reduced transition probability, $\epsilon B(E\lambda)$, for excitation leading to a particular gamma ray or internal conversion electron was calculated by equating the experimentally determined yield with the theoretical yield obtained by integrating the differential cross section over the range of the exciting projectile. For particles scattered through an angle θ, we have

$$Y_{th}(\theta) = N\, I \int_0^{E_{max}} d\sigma_{E\lambda}(\theta)\, dE/(dE/ds), \qquad (1)$$

where N is the number of target nuclei per cm^3, I is the number of particles striking the target, and

$$d\sigma_{E\lambda}(\theta) = C_{E\lambda}\, E^{2\lambda-3}\, B(E\lambda)\, df_{E\lambda}(\theta, \eta_i, \xi) \qquad (2)$$

$$C_{E\lambda} = \frac{Z_1^{\,2} A_1}{40.03} \left[0.07199 \left(1 + \frac{A_1}{A_2}\right) Z_1 Z_2\right]^{-2\lambda + 2} \qquad (3)$$

$$\xi = \frac{Z_1 Z_2 A_1^{\,1/2}\, \Delta E'}{12.65\, E^{3/2}} \qquad (4)$$

$$\Delta E' = (1 + A_1/A_2)\,\Delta E. \qquad (5)$$

$$\eta_i = \frac{Z_1 Z_2}{2}\left(\frac{A_1}{10.008E}\right)^{1/2}. \qquad (6)$$

Here we employ the notation of Alder et al.[1]

Then we have

$$Y_{th}(\theta) = N\,I\,C_{E\lambda}\,B(E\lambda)\int_0^{E_{max}} E^{2\lambda-3}\,df_{E\lambda}(\theta,\,\eta_i,\,\xi)\,dE/(dE/ds), \qquad (7)$$

and setting

$$dF_{E\lambda}(E_{max},\,\theta) = \int_0^{E_{max}} E^{2\lambda-3}\,df_{E\lambda}(\theta,\,\eta_i,\,\xi)\,dE/(dE/ds) \qquad (8)$$

gives

$$Y_{th}(\theta) = N\,I\,C_{E\lambda}\,B(E\lambda)\,dF_{E\lambda}(E_{max},\theta). \qquad (9)$$

It can be seen from eq. (9) that the shape of the excitation function is given directly by $dF_{E\lambda}(E_{max},\theta)$; all other factors are independent of the projectile energy.

In principle, one would have to evaluate the integral, $dF_{E\lambda}(E_{max},\theta)$, for each combination of projectile, target, and excitation energy. However, in practice certain simplifications are possible.

Firstly it turns out that one can to some extent simplify the differential energy loss (dE/ds) used in the evaluation of eq. (8). Figure 8 shows the ranges of O^{16} ions plotted against the atomic number Z_2 of the target material. The data are taken from a compilation of semiempirical ranges by Hubbard.[11] It is seen that in the energy interval of interest for these experiments

(20 through 100 MeV), the relation between range and Z_2 is remarkably linear and to a good approximation, the lines for different energies cut the Z_2 axis at about -37. The range of O^{16} ions in any element can thus be related simply to the range in a standard material; we have chosen gold as the standard material. For the range in an element with atomic number Z, one then gets

$$R_Z = R_{Au} \frac{Z + 37}{116} , \tag{10}$$

and for the differential energy loss,

$$\left(\frac{dE}{ds}\right)_Z = \left(\frac{dE}{ds}\right)_{Au} \frac{116}{Z + 37} . \tag{11}$$

From the integral (8) evaluated for gold, one then obtains the value for any other target by multiplication by $(Z + 37)/116$.

Secondly, one notes that the term ξ can be split into two factors, one containing the bombarding energy only, the second depending on the other experimental conditions only. That is, we can write

$$\xi = \frac{Z_1 Z_2 A_1^{1/2} \Delta E'}{12.65} \frac{1}{E^{3/2}} = \Lambda \frac{1}{E^{3/2}} . \tag{12}$$

One can then evaluate eq. (8) for a set of values of Λ and E_{max} covering the range of practical interest, and since the integral (8) is a fairly smooth function of Λ, it is easy to interpolate for other values.

Similarly, when the scattered projectiles are not observed, the yield now depends on the total rather than the differential cross section. The previous method of analysis carries over with substitution of $f_{E\lambda}(\eta_i, \xi)$ for $df_{E\lambda}(\theta, \eta_i, \xi)$ and consequently $F_{E\lambda}(E_{max})$ for $dF_{E\lambda}(E_{max}, \theta)$.

4. RESULTS

The energies and intensities of the transitions observed in the present work on the Coulomb excitation of Tb^{159}, Ho^{165}, and Tm^{169} are listed in tables 1 through 7. From these data the partial transition moments for excitation have been calculated and are included in the tables. These B(E2) values have been calculated relative to the values assumed for the Au^{197} 547-keV transition, namely, $\epsilon_K B(E2) = 0.00675$ and $\epsilon_\gamma \; B(E2) = 0.43$ in units of $e^2 \times 10^{-48}$ cm^4 (reference 1). These data will be analyzed in the discussion section, but some general remarks can be made here.

A comparison of our B(E2) values for the excitation of a particular excited band with the values obtained by Nathan and Popov[3] shows that our values are 2 to 3 times larger than those of the latter authors. Also, the greater detail available to us with heavy-ion excitation and a study of the conversion electron spectrum under moderately good resolution permitted the determination of (somewhat different) partial level schemes in considerably greater detail.

The errors assigned to the B(E2) values determined from the conversion electron lines are obtained by adding twice the standard deviation of the determination of the actual intensity of the line to a flat 10% to account for systematic errors. Most such errors should cancel when the comparison with the Au^{197} standard is used, as described earlier. The errors so determined are larger than the observed variation in yields obtained from the different spectrometer runs (usually three) made on different days. The errors assigned to the B(E2) values determined from the gamma-ray spectra are estimates that were made large enough to include the range of values of B(E2) calculated from three or four different spectra taken at different O^{16} beam energies (44 through 60 MeV).

Holmium-165.

The a priori expectation of transitions resulting from Coulomb excitation is that those most readily seen will involve members of the ground-state rotational band and vibrational states based upon the ground-state configuration. It has been possible to interpret almost all of the transitions observed for Ho^{165} in just such a scheme. This is shown in fig. 9, in which the ground-state rotational band with $K_0 = 7/2-$ is seen together with its two γ-vibrational bands having $K = K_0 \pm 2$. Another level thought to have $K = 3/2+$ is also excited. This interpretation is discussed in section 5; for the present we shall review the experimental data.

Of the transitions seen in the spectrum of fig. 5 and listed in tables 1 and 2, all but those at ~245 and 346 keV are included in the partial-level scheme shown in fig. 9. Those at 327, 346, and 154 keV are actually so weak as to be questionable, but they do appear in at least two spectra each. The intensities of the L conversion electron lines of the 566- and 593-keV transititons are too large and the lines are somewhat broad to be single lines; contributions from K-shell conversion of the 610- and 638-keV transitions are thus suggested, although it was not possible to resolve such weak lines.

With the two exceptions just noted, all the transitions observed below 361 keV can be fitted into the ground-state rotational band (table 1), and so should have lifetimes of order 10^{-10} to 10^{-11} sec. The more intense of these transitions do indeed show the asymmetric Doppler effect described earlier (a small shoulder on the high-energy side). In contrast, the 361-keV M2 decay (see later discussion) is too slow to show any Doppler effect ($\tau_{1/2} = 1.5$ μsec), and the electron peak shows a very steep decrease down to background on the high-energy side.

The many transitions present in the gamma-ray spectrum could not be resolved, but two groups are apparent, clustered about 530 and 685 keV. To

get some idea of their multipolarities we calculated average conversion coefficients for each group. The results are shown in table 3 from which it can be seen that each group must be predominantly E2 in character. In particular, it can be deduced that none of the main transitions can have strong M1 admixture.

Information on the nature of excited states can be obtained from the mode of Coulomb excitation, which in turn can be deduced from excitation functions. For example, it has been shown in U^{238} that certain E1 de-excitation radiations arise from states for which the primary Coulomb excitation from the ground state is E3 in character.[13] Often, it is sufficient to use just two different particle energies to establish the mode. Table 8 gives a comparison of yields at O^{16} ion energies of 60 and 44 MeV for some of the prominent electron lines in the three nuclei under investigation. Gold targets were used as internal standards to calibrate the beam energies in order to conform with the known E2 excitation of the gold 547-keV level.

Comparison with theory for the 514-keV transition in Ho^{165} clearly establishes the E2 character for excitation of the state or states giving rise to this radiation. The 361-keV transition presents a more difficult problem. It might be expected that the 361-keV state would be populated from the K = 3/2 band which had been excited in the primary (E2) process (see discussion in section 5). However, the comparison given in table 8 shows that the observed excitation function is too steep even for that of the highest member of this band at 638 keV. A calculation was made for the direct E3 excitation of the 361-keV level, and this is seen to give a much steeper slope to the excitation function. Another mechanism which would provide qualitatively the same effect consists of multiple excitation of the upper levels of the 514-keV band accompanied by decay to the levels of the 361-keV

band. At present we can say only that all of these mechanisms could contribute to the population of the 361-keV level.

Terbium-159.

The electron spectrum for Tb^{159} is shown in fig. 6 and the analyzed information is presented in tables 1 and 4. The interpretation in terms of a level scheme is given in fig. 11 and will be discussed fully in section 5. All observed transitions were placed in the proposed level scheme except those at 331 and 362 keV. Except for the 289-keV transition, all of the lower-energy peaks (below the 331-keV transition) were fitted into the ground-state rotational band. The others provided evidence for the excitation of two different intrinsic bands and a vibrational band. Possibly, a second vibrational band was also observed.

Many of the conclusions drawn are based upon detailed examination of electron line intensities and widths. Weak, and perhaps questionable, transitions such as those at 429 and 522 keV do not appear at all in fig. 6 but seemed to be present when these regions were examined separately with better statistics than the run shown here. Careful comparison of line widths against standard line shapes revealed that the lines at 536, 540, 580, and 920, 949, and 978 keV exhibited symmetrical Doppler broadening. As mentioned earlier, this would require lifetimes of the order of 5×10^{-13} sec or less, and such lifetimes suggest M1 transitions.

The position of the 522-keV transition in the decay scheme is uncertain. As drawn in fig. 11 it is an E2 transition which must compete with a 580-keV M1 transition. Because of the short half-life for the latter, this is somewhat improbable. Alternatively, the 522-keV transition might be an M1 decay from the 7/2+, 1/2 level of the 580 keV band to the 9/2+, 3/2 level of the

ground-state band; the other transitions from this level would be weak and (or) obscured by stronger lines. It was not possible to determine the conversion coefficient for the 522-keV transition, so the uncertainty remains.

A comparison of the average conversion coefficients for the gamma rays centered at 580 and ~950 keV with the theoretical values (table 5) shows that the 580-keV group is indeed predominantly M1, but that the higher-energy group is indeterminate from the present data. (The 950-keV gamma-ray group is very broad, including transitions with higher-energy than were observed with the electron spectrometer. Therefore that portion of the ϵ_γ B(E2)↑ corresponding to the observed electron lines is also listed in table 5, and is used in calculating the experimental α_K.)

The ratio of thick-target yields at two different O^{16} energies for the K lines of the 348-and 580-keV transitions shows clearly that the primary Coulomb-excitation processes are E2 (see table 8).

Thulium-169.

The analysis of the Tm^{169} spectra follows the same pattern as that already presented for Ho^{165} and Tb^{159}. The level scheme suggested by the data contained in tables 1, 6, and 7 is shown in fig. 13. The 348-and the 294- (or 245-) keV transitions do not fit into this scheme. All other transitions less than 348 keV in energy appear to fit into the ground-state rotational band. The conversion electron lines of the 323- and 387-keV transitions listed in tables 1 and 6 are very weak, but each appears on at least two spectra. The K conversion-electron lines of the 452-, 515-, 562-, and 570-keV transitions are somewhat symmetrically broadened, although not as much as that of the 580-keV transition in Tb^{159}. Such Doppler broadening again suggests that M1 de-excitation radiation is involved, and the average con-

version coefficient for the 562- to 570-keV group does yield a value equal to the theoretical one for an M1 transition (table 7).

The ratio of the thick-target yields at the two different O^{16} beam energies for the K conversion electron lines of the 562- and 570-keV transitions, table 8, shows the excitation to be E2.

5. DISCUSSION

5.1. Holmium-165

The gamma-ray transitions observed in Ho^{165} and listed in tables 1, 2, and 3 have been used to construct the level scheme shown in fig. 9. In addition to multiple excitation within the ground-state band, two bands are excited at energies of 514 and 687 keV. In the following discussion we consider each of the excited bands in some detail, and then analyze the energy spacings of the ground-state band.

The level at 514 keV is almost surely the one populated in the decay of 1.25-min Dy^{165m} and previously observed by Cranston et al.[14] and Tornau.[15] These authors report that the level de-excites both directly to ground by a 516-keV transition, and through a level at 361 keV. They further found that the 361-keV transition was M2 and that its parent level has a lifetime of 1.51 μsec. Finally these authors showed that the 516-keV level very likely has spin and parity 3/2-.[†]

In addition to the level at 514 keV we see levels at 566 keV and 638 keV which seem to be related to the 514-keV level as members of a rotational band. The spacings are in excellent agreement with the assignment to a K = 3/2 band having a rotational constant $(\hbar^2/2\mathfrak{J})$ of 10.4 keV. This assignment agrees with that of previous workers.[14,15] With the assignment K = 7/2- for the ground state, the parity of the K = 3/2 state is established as <u>odd</u> because the Coulomb excitation follows an E2 dependence (table 8) and the 514-keV transition itself is E2 (table 3).

[†]When a plus or minus sign precedes a K value, it indicates the direction along the symmetry axis; when it follows the K value, it refers to the parity. Where spin, parity, and K value are given, they are indicated: $I\pi$, K. Nilsson states are given in the conventional way: $K\pi[Nn_z\Lambda]$.

Bunker et al. have pointed out that there is no 3/2- Nilsson level in the vicinity of the 7/2-[523] Ho^{165} ground state.[14] They offer two possible explanations of the 3/2-level: first, that it is a vibrational level of the ground-state configuration; and second, that it is a level dropping down from the next higher shell of levels. We strongly prefer the former assignment. The principal reason for this is the large B(E2) between this level and the ground state. Summing all transitions depopulating the K = 3/2 band† gives $B(E2)_T(7/2-, 7/2 \rightarrow K = 3/2) = 5.2\ e^2 \times 10^{-50} cm^4$. This summation is made for two reasons. First the calculations of Alder and Winther[16] indicate that whereas multiple Coulomb excitation may affect the population of any particular member of an excited band, it does not affect the total E2 strength to the band. The second reason is that this summation gives an effective vector addition coefficient of unity, which is just what we want to compare with the $0 \rightarrow 2$ transition in even-even nuclei. Using the expression $B(E2)_{sp} = 3 \times 10^{-5}\ A^{4/3}\ e^2 \times 10^{-48}\ cm^4$ (reference 1), we have $B(E2)_T = 1.9\ B(E2)_{sp}$. In an odd-mass nucleus one expects two γ-vibrational bands having $K = K_0 \pm 2$, where K_0 is the ground-state K value. In the case of Ho^{165}, one then expects γ-vibrational bands of K = 3/2- and K = 11/2-. In even-even nuclei the γ-vibrational bands are generally connected with the ground state by E2 strengths of about five single particle units, and so in an odd-mass nuclei where the γ-vibrational band has been split into two bands, it is reasonable to assume that each might have an E2 strength of about two single-particle units. Such a strength is rather large where different intrinsic states are involved unless they are rather heavily mixed with each other, and such mixing (10 to 15% in the amplitude) is not likely for K values differing by two units. Thus we take the E2 strength between these two bands to indicate

---

†We include here the 361-keV transition, although it is not clear that all the population of this state is via the 514-keV band.

that the 3/2- band is the K_0-2 γ-vibrational band based on the ground state 7/2-[523].

An additional, though weaker, argument for the vibrational nature of the 514-keV band is the value of the rotational constant for this band (10.4 keV). This constant is very nearly the same as that for the ground-state band (10.5 keV), and both are appreciably smaller than those for other odd proton nuclei in this region. The reason for the small value of the rotational constant for the ground-state band in Ho^{165} is understood,[17] and we would expect the vibrational state to have nearly the same value. On the other hand, other states in general should have larger rotational constants.

Although there are not many de-excitation branching ratios known for levels of the 3/2 band, it is of interest to examine those for which there is information. For the 566-keV level, we compute, correcting the $\epsilon_\gamma B(E2)\uparrow$ values of table 2 for energy dependence: B(E2)566|B(E2)471 = 0.73±0.16|1.00 The value given by the vector addition coefficients for this ratio is 0.80|1.00, which is in good agreement with the experimental number. From the 638-keV level, we get: B(E2)638|B(E2)543|B(E2)428 = 0.26±0.16|0.64±0.33|1.0. The theoretical ratios are 0.35|1.27|1.00.

O. B. Nielsen[18] has pointed out that simple application of vector addition coefficients may not be adequate to explain such branching ratios where a vibrational band and the ground-state band can be admixed. The present case is a poor one to test these arguments because of the broad limits of error in the experimental information; however, such a mixing correction will be presented here and will be used more fruitfully in the discussion of the 687-keV level. The corrections can be expressed in terms of a parameter, Z_I, which is defined by

$$Z_I = [(I-K_L - 1)(I - K_L)(I + K_L + 1)(I + K_L + 2)]^{1/2}\epsilon \frac{Q_{grd}}{Q_{\gamma\text{-}grd}} .$$

In this equation $[(I - K_L - 1)(I - K_L)(I + K_L + 1)(I + K_L + 2)]^{1/2}\epsilon$ is the admixed amplitude in the state of spin I apart from a sign. The sign of ϵ is different in the two admixed bands, and in our definition of Z_I we pick the sign of ϵ corresponding to the band where the transitions terminate. The symbols Q_{grd} and $Q_{\gamma\text{-}grd}$ represent the E2 transition amplitudes within the ground band and between the ground and gamma vibrational bands, respectively. The lesser of the two K values involved is designated K_L. Nielsen has thus far dealt only with even-even nuclei, for which K_L is zero. In this case he found it convenient to use an I-independent Z defined for I = 2; so that the quantity under the radical becomes $\sqrt{24}$. We will continue to use Nielsen's Z in these odd-mass cases; however, to obtain the admixed amplitude from Z, it is necessary to convert first to Z_I as defined above. The corrections to the B(E2) values can then be expressed as†

$$B(E2;\ I_i\ K\pm 2\ \ I_f\ K) = Q^2_{\gamma\text{-}grd}\langle I_i\ K\pm 2\ 2\ \mp 2|I_f K\rangle^2$$

$$\times\left[1 + Z + \sqrt{(I_i \mp K - 1)(I_i \pm K + 2)}\ \frac{\langle I_i\ K\pm 1\ 2\ \mp 1|I_f K\rangle}{\langle I_i\ K\pm 2\ 2\ \mp 2|I_f K\rangle}\ Z\right]^2$$

It is well to keep in mind when using this equation that the sign of Z is different for excitation and de-excitation in any particular case in accordance with our sign convention for ϵ. The sign of Z also depends on the relative signs of Q_{grd}, and $Q_{\gamma\text{-}grd}$; however, for de-excitation Nielsen has found that in all cases where experimental data are available, Z is positive,

† An expression closely related to this one has recently been given elsewhere.[19]

and for even-even nuclei in the region of Ho^{165} it varies between 3 and 6%. If we use a Z value of 3%, and make no correction for the Q values or the energy separation of the bands (contained in ϵ) which are not well known for the adjacent even-even nuclei, then: B(E2)566|B(E2)471 = 0.61:1.00 and B(E2)638|B(E2)543|B(E2)428 = 0.19|0.93|1.00. All ratios have changed in the direction of the experimental data, and are now within the experimental limits of error; however, the ratio from the 5/2 level has overshot and is now below the experimental number. We do not feel the imporvement is of great significance for this band, but at least the correction is in the right direction, and, as will be seen, the improvement is more definite for the 687-keV level.

There are difficulties in the analysis of the relative Coulomb-excitation probabilities to the three levels of the 3/2 band. In order to calculate these ratios from the experimental data, the branching of each level to the 361-keV band is needed. These are not known, but if we make the simple assumption of the same branching of each level to the 361-keV band, then B(E2)7/2 → 3/2| B(E2)7/2 → 5/2| B(E2)7/2 → 7/2 = 1.0|0.62±0.12|0.20±0.08. The simple vector addition coefficients give 1.0|0.67|0.27 for the above ratios, but these should be modified to take account of mixing of the two bands and the effects of multiple Coulomb excitation. Also, in this case the intraband rotational transitions probably compete appreciably with the interband transitions. We cannot make a quantitative estimate of all these effects, but we will try to show that the observed result is reasonable. It is easy to calculate the effect of band mixing and, using the same admixed amplitudes as in the previous paragraph, we find for the above ratios: 1.0|0.57|0.18. For the effects of multiple excitation we can make only a perturbation-theory estimate, which will almost certainly not be adequate

quantitatively. Therefore, we will use it only to indicate the direction of change, which in this case favors the 5/2 and especially the 7/2 members of the band over the 3/2 member. In fact, multiple excitation tends to go in just the opposite way as the intraband transitions, and we would guess that these two effects largely cancel each other, giving the reasonable agreement of the data with the ratios quoted above.

The branching of the 3/2- band to the 3/2+[411] band is interesting. Because of the small conversion coefficients of such E1 transitions, we do not observe them, but the one from the 514-keV state to the 361-keV state was seen by Bunker et al.[14] From their work, the ratio of the 154-keV E1 transition to the 514-keV E2 is 0.16. (If there is no direct population of the 361-keV band, a considerably larger E1 branching for the upper levels of the 514-keV band than for the 514 level itself is implied, but this is not unreasonable in view of the higher energy of these E1 transitions. Also, the excitation function of the 361-keV transition is steeper than that of the 514-keV transition, which again presumably indicates that much of the population of the 361-keV band is coming from the higher members of the 514-keV band. An alternative explanation is appreciable direct E3 excitation to the 361-keV band.) Using the above E1/E2 ratio and our $B(E2)\uparrow$ for the 514-keV band, the 154-keV E1 half-life of the 514-keV level becomes 1.9×10^{-10} sec, which makes it hindered by 3.5×10^{3} over the single-particle estimate.

Electric-dipole transitions in these deformed nuclei are generally highly hindered over the single-particle estimate. For odd-proton nuclei in the region of holmium, about half a dozen such transitions are known, having hindrances varying between 10^{4} and 5×10^{6}. The above result then shows that the 154-keV E1 transition in Ho^{165} is one of the very fastest E1 transitions in this region. On a simple vibrational picture this transition is forbidden,

and thus we would expect it to be considerably slower than corresponding transitions where neither of the levels involved have vibrational character. We can only conclude that, while it seems quite likely that the 514-keV band has considerable collective character, its detailed description is not clear.

There is less information about the 687-keV band. We strongly suspect that the K value of this band is 11/2, but this assignment is not so conclusive as is the 3/2- assignment for the 514-keV band. There are several reasons for suspecting 11/2 to be the K value. First, there are the 725- and 610-keV transitions which, if placed as shown in fig. 9, define a state at 820 keV. The spacing between this state and the one at 687 keV gives a reasonable rotational constant (10.2 keV) for K = 11/2. The only other plausible K value on this basis would be 9/2 for which the rotational constant becomes 12.1 keV.

We consider the 830-keV level to be tentative as indicated by the dashed line designating it in fig. 9, because it is established only by the 725-keV transition and the one of questionable existence at 610 keV. (This latter transition will be largely ignored in the following discussion.) However, certain problems are created if this level is eliminated. Most acute of these is that the 687-keV level is heavily populated, almost certainly by direct excitation, and one should see at least one other member of its rotational band. If the K value were 7/2 or 9/2, for example, one would expect comparable population of the second band member and the base level. If the 725-(and 610-) keV transition is removed, there is no evidence for this second band member, the limits being 3 to 4% of that going to the 687-keV level. (This limit should be raised, perhaps by as much as a factor of two because of the competition of the intraband rotational transition with the interband decays.) Even if we accept the 725-keV transition as coming from this second member, its low

intensity relative to the 687-keV level (~5%) becomes an added argument for the K = 11/2 assignment, because then it arises from a spin-13/2 state which can only be excited by multiple excitation. Also placing the 725-keV transition elsewhere in the level scheme would be difficult. Either it would have to define an entirely new band, or possibly it could come from the otherwise unobserved 9/2 member of the K = 3/2 band. For this latter assignment the intensity of the 725-keV transition would appear to be too high, and the energy differs by about 6 keV from that expected.

Another argument for the K = 11/2 assignment for this band is that it has a ready explanation as the K_0+2 vibrational configuration of the ground band, whereas any other K assignment is difficult to explain. The parity of this band is negative, as shown by the E2 nature of the 687-keV transition and also by a rough photon excitation function of this transition, which indicated E2 excitation. The transition moment for excitation (including the 820-keV level) is 1.7 $B(E2)_{sp}$, which is nearly the same as for the K = 3/2- band assigned as the K_0-2 vibrational state. About the only plausible odd-parity Nilsson state in this region is 9/2- [514], which is not particularly close. Also we would expect this state to de-excite to the ground-state band by strong M1 transitions. Finally, it might be conceivable that this state is the 7/2- β-vibrational state, but in even-even nuclei in this region such states lie considerably higher than the γ-vibrational states.

The branching ratios from the 687-keV level are reasonably accurately determined. Using the ϵ_γ $B(E2)\uparrow\uparrow$'s from table 2, and correcting for energy dependence, we have B(E2)687|B(E2)593|B(E2)478 = 1.00|0.60 ± 0.12|0.26 ± 0.10. The vector addition coefficients with K = 11/2 for the upper band give 1.00|0.38|0.10. Here we feel our data are sufficiently accurate to indicate a deviation from the simple theory. Considering a mixing between the ground

and 687-keV bands of the type described for the 514-keV band, we can compute correction factors to the vector addition coefficients as follows: $B(E2)687|B(E2)593|B(E2)478 = 1.0[1-Z]^2|0.38[1+3.5Z]^2|0.10[1+9Z]^2$. The parameter Z has been defined previously. Using a Z value of +5% we get $B(E2)687|B(E2)593|B(E2)478 = 1.00|0.59|0.23$. The agreement here with the experimental numbers is probably fortuitously good, but it nevertheless shows that these transitions are entirely consistent with the interpretation of the band as a K_0+2 gamma vibrational band.

We shall now turn to the energy-level spacings in the ground-state rotational band. From the rotational-energy formula, $E_I = E_0+AI(I + 1) + BI^2(I + 1)^2$, one can easily derive the equation $(E_{I+1}-E_I)/2(I + 1) = A + (B/2)[2(I + 1)]^2$. Thus a plot of $(E_{I+1}-E_I)/2(I + 1)$ vs $[2(I + 1)]^2$ should give a straight line for which the intercept is A, and the slope is B/2. As shown in fig. 10, the data are very well fitted by a straight line and yield values of 10.65 ± 0.04 keV for A and $-(3.2 \pm 0.7) \times 10^{-3}$ keV for B. These values are both slightly smaller than those for neighboring nuclei, but this is expected for this particular Nilsson state.[17] In addition to showing graphically the fit, the plot, enables us to show the error limits directly, since the points are just the experimental cascade gamma-ray energies divided by a small whole number. The first point is the very precise energy measured on a bent-crystal spectrometer by Chupp et al.,[10] and here the error limits are within the point itself. The second energy reported by Chupp et al., 109.93 keV, appears to be in error. The line they observed apparently belongs to F^{19}, and the second cascade transition of Ho^{165}, measured by us to be 115.1 ± 0.4 keV, was seen by these workers but attributed by them to a line excited in Zn^{65} by the (p, n) reaction on the copper target backing.[20] The precise energy of the

Zn^{65} line is 115.13 ± 0.05 keV, and apparently the Ho^{165} transition accidently lies within these narrow limits of error. There appears to be nothing unexpected about the ground-state band of Ho^{165}, which will prove not to be the case for the remaining two nuclei to be discussed.

5.2. Terbium 159

The level scheme of Tb^{159} constructed from the data is shown in fig. 11. In addition to considerable multiple excitation of the ground-state band, rotational bands at 580 and 348 keV are well-established, and two others at 971 and 1280 keV are indicated, although detailed information on these last is lacking. As in the case of Ho^{165}, we will discuss these bands individually, concluding with the analysis of the ground-state band.

The evidence for the K = 1/2 assignment for the band of 580-keV is rather convincing. The spacing between the first two members is only 37 keV which gives unreasonable values for the rotational constant for all values of the base-state spin except 1/2. Furthermore, the ratio of the spacings between the three observed levels does not agree in simple fashion with any spin assignment, and such perturbations are most commonly encountered in K = 1/2 bands. The constant in the decoupling term of the level-spacing formula which accounts for the anomalous spacing in this case is $a = +0.054 \pm 0.012$. The parity of the K = 1/2 band must be <u>even</u> since its excitation function follows an E2 dependence (table 8) and the prominent de-excitation gamma rays are M1 (table 5).

That the transitions between the 580-keV band and the ground-state band are predominantly M1 was shown both by the composite conversion coefficient in table 5 and by the symmetrical Doppler broadening of the electron lines as discussed previously. In fig. 11 we see that the spin assignments

are such that all transitions with the exception of the very weak and questionable one at 522 keV can indeed be M1. In the following discussion we will assume that they are. Since the 617-keV transition is probably not a single line in the experimental spectrum, only one branching ratio can be directly compared with theory. This ratio is B(M1)674|B(M1)536, for which the data give 0.13±0.06|1.00. The vector addition coefficients for K = 1/2→ K = 3/2 give 0.14 for this ratio. The agreement is probably accidentally good in view of the large limit of error. As a further check on the branching ratios, we can calculate that the total expected intensity of the 617-keV line based on the 536-and 560-keV lines is some 35% larger than the experimental intensity of this line. This deviation is probably slightly larger than we would estimate our limits of error to be, but, summarizing, we can say that these transitions do not seem to deviate greatly, if at all, from the predictions for M1 transitions given by the vector addition coefficients.

The nature of the K = 1/2 band is subject to some conflicting evidence; however, it seems best explained as the K_0-2 gamma vibrational band of the ground state. An intrinsic Nilsson state, 1/2+[411], is expected around this energy in Tb^{159}, but the decoupling parameter of the band does not seem appropriate to that state (as seen in fig. 11 a tentative indentification of the 1/2 +[411] orbital has been made for a state at 971 keV). The measured $B(E2)_T$ is 1.5 $B(E2)_{sp}$. Although this is somewhat smaller than that for the vibrational bands in Ho^{165}, it is large for the excitation of an intrinsic state. The 1/2+[411] band is known in a number of other nuclei in this region, and for these the decoupling parameter is ~-0.8. For a simple vibrational state, the decoupling parameter would be zero. The experimentally determined value, +0.054, is very nearly zero, and it seems entirely plausible that the small finite value is introduced by band mixing.

Information on the nature of this band can be derived in principle from the relative B(E2) values for excitation of the three members. If we arbitrarily say that the ground state is a K = +3/2 state, the vector addition coefficients will be different if the excited band has K = +1/2 or K = -1/2, both of which can be reached by E2 excitation. For the vibrational assignment (K_0-2) the K = -1/2 configuration is expected. If the band is unrelated to the ground state then both K = +1/2 and K = -1/2 are possible, and the E2 transition will populate these in a ratio that depends on the details of the wave function. However, it seems unlikely that an entirely unrelated band would have an E2 transition probability to the ground state as large as 1.5 $B(E2)_{sp}$. In this case the large moment would most likely be explained by coriolis mixing with the ground band, and then the relative excitation probabilities turn out to be the same as for the K = +1/2 choice above. The experimental information is difficult to resolve because there is a 617-keV transition from both the 617- and 674-keV levels. If we divide the measured intensity equally between the two transitions and set the limits of error large enough to encompass the uncertainty so introduced, the following comparison results:

	$B(E2)(3/2 \rightarrow 1/2)$ /	$B(E2)(3/2 \rightarrow 3/2)$ /	$B(E2)(3/2 \rightarrow 5/2)$
Experimental	1.0	0.50 ± 0.21	0.66 ± 0.21
K = +3/2→K = +1/2 (band mixing)	1.0	4.0	3.9
K = +3/2→K = -1/2 (vibrational)	1.0	1.0	0.43

The vibrational case is much closer to the experimental data than is the band-mixing case. Furthermore, our estimates of multiple excitation coupled with the Nielsen-type corrections discussed previously indicate changes in the right direction to bring the vibrational case into agreement with the data.

Similar calculations indicate that for the band-mixing case multiple excitation makes the situation even worse. Intraband rotational transitions probably do not compete appreciably with the fast M1 interband transitions in this case.

Perhaps the greatest difficulty with the assignment of the K = 1/2 band as a vibrational band lies with the mode of gamma ray de-excitation. The observed M1 transitions would clearly be forbidden if the band were of pure vibrational character. On the other hand, when K_0 is 1/2 or 3/2 the curious situation arises where $|K_0-2| = |K_0| \pm 1$. In these cases, because of symmetrization of the wave function, the above M1 transitions are not forbidden by the K selection rule (although, of course, on a pure vibrational model there are other strong selection rules that would forbid them). In fact, the vector-addition coefficients appropriate in the above case are just those for $|K_i| \rightarrow |K_f|$, which are found to give reasonable agreement with the data. An explanation of the M1 transitions on the basis of lack of K purity in either or both bands would not necessarily give these same vector-addition coefficients.

The 348-keV band (fig. 11) was somewhat difficult to analyze because some of the gamma rays lie within the energy range of transitions in the ground-state rotational band. The 348- and 289-keV transitions can be explained well by a state at 348 keV, while the questionable 429-keV transition and the transitions of 371 and 289 keV can be fitted to a state at 429 keV. We have chosen to assign these two states to the 5/2+[413] orbital, which is the ground-state configuration for Eu^{153}. The arguments follow.

The rotational constant derived from these two states for K = 5/2 is 11.6 keV, which is the same as for the ground-state band. If the value of K were 3/2 or 7/2, the rotational constant would be 16.2 or 9.0 keV, respectively, which are much less reasonable values. The parity of the band must be <u>even</u> because the excitation function follows an E2 dependence (table 8) and the

average conversion coefficient of the transitions around 350 keV indicates an M1-E2 mixture. The 348-keV level is probably the same level reported by Ketelle and Brosi[21] in the decay of Dy^{159}. These authors found gamma rays leading to the 3/2, 5/2, and 7/2 members of the ground-state band in the intensity ratios 1.0|0.2|0.2, respectively. In the present study, the transition to the spin-7/2 member is obscured by the intense lines of the ground-state rotational band, but the other two transitions seem quite consistent with the above ratios. We cannot say more than this since M1-E2 ratios are unknown for all these transitions.

As already mentioned the state assigned here as 5/2+[413] is the ground state for Eu^{153}, while the ground-state configuration of Tb^{159} (3/2+[411]) is found in Eu^{153} at 103 keV.[6] The half life of the 103-keV state in Eu^{153} is known, as is the E2 component. From these data we can calculate B(E2; 3/2→5/2) as $7.3 \times 10^{-3}\ e^2 \times 10^{-48}\ cm^4$. In Tb^{159}, the Coulomb-excitation yield permits us to estimate for the comparable transition a B(E2; 3/2→5/2) of $8.6 \times 10^{-3}\ e^2 \times 10^{-48}\ cm^4$. This agreement would seem to give added evidence that the states are assigned properly.

We would have guessed that the relatively large B(E2) between these two bands – $\sim 0.46\ B(E2)_{sp}$ – was introduced by an appreciable Coriolis mixing of the bands. The magnitude of the mixing is quite reasonable in Tb^{159}. The value of the Coriolis matrix element, $\langle \psi_{3/2} | j_- | \psi_{5/2} \rangle$, indicated by the E2 strength (neglecting any intrinsic E2 moment) is 0.66; whereas the value calculated from the Nilsson wave functions is 0.65. On the other hand, R. L. Graham[22] has analyzed the mixing in Eu^{153} and has found that the rather large body of data there can be explained by using this same Coriolis matrix element, but requiring a sizeable intrinsic E2 moment. It does not seem possible to understand our Tb^{159} results using such parameters. Clearly more detailed information on this band is needed.

The information available on the band at 971 keV is considerably poorer than for the other two bands. This is because we cannot resolve the complex gamma-ray peak into components, and in the electron spectrum the conversion lines are only about 5% of background. Nevertheless, from the electron spectrum, we can say that transitions of 920, 949, and 978 keV are essentially certain, and a peak at 965 keV is probable.

The interpretation of this rather sparse information in terms of a K = 1/2 rotational band as shown in fig. 11 is based upon a supposition with which all of the available data are consistent. It is assumed that these transitions stem from the 1/2+[411] orbital which is the ground state configuration for Tm^{167}, Tm^{169}, and Tm^{171} and appears at 200 to 300 keV in the holmium isotopes. The two lines of 978 and 920 keV are assumed to arise from the I = 3/2 member and lead to the ground state and first excited state, respectively. Then if the other two transitions define states at 1087 and 1103 keV as shown, the rotational constant and decoupling parameter can be calculated. These become $\hbar^2/2\mathfrak{J}$ = 12.0 keV and a = -0.81. This rather large rotational constant is much like that in Tm^{169} and Tm^{171} (12.3 keV) and the decoupling parameter also agrees with that found in these two nuclei (-0.77 and -0.87).

The base state of this band would lie at 971 keV. No transitions from it were observed, but the 971-keV transition would have been obscured by the 965- and 978-keV transitions even though it had been present in its expected (lower) intensity. Similarly other possible transitions from this band which were not observed present no serious challenge to the interpretation.

The widths of the electron lines are such as to indicate Doppler broadening as a result of lifetimes of $\lesssim 5 \times 10^{-13}$ sec. An average conversion coefficient was determined (table 8) and is consistent with an M1-E2 mixture

with the limits of uncertainty permitting either component to predominate. The total B(E2) for exciting this band is $(2.5 \pm 1.0)e^2 \times 10^{-50}$ cm^4, which is just 1.0 $B(E2)_{sp}$. The E2 de-excitation lifetime expected would then be $\sim 3 \times 10^{-12}$ sec which, in view of the Doppler broadening of the electron lines, suggests that the transitions are largely M1. The rather large B(E2) for Coulomb excitation is worthy of further comment. The expected Coriolis mixing between this band and the ground-state band would produce such a result and furthermore would favor excitation to the I = 3/2 and I = 5/2 members, a requirement consistent with our observations.

Very little is known about the 1270-keV band. It was not possible to observe the electron lines from this band. From the gamma-ray spectrum it is apparent that this peak is complex, but a reliable resolution cannot be made. The intensity of the line indicates: $B(E2)_T = (5.3 \pm 1.6)\ e^2 \times 10^{-50} cm^4 = 2.0$ $B(E2)_{sp}$. It is very tempting to suggest that this is the K_0+2 gamma vibrational band; but aside from the fact that this B(E2) value is almost the same as that for the K_0-2 gamma band (as was found for the two bands in Ho^{165}), there is no evidence for this.

The analysis of the energies of the ground-state rotational band members is shown in fig. 12. As was the case for Ho^{165} we would expect the plot of $(E_{I+1} - E_I)/2(I + 1)$ versus $[2(I + 1)]^2$ to give a single straight line if the data are represented by the equation $E_I = E_0 + AI(I + 1) + BI^2(I + 1)^2$. In fact, the data are not at all consistent with a single line, but rather convincingly suggest two straight lines having very nearly a common intercept. This behavior is not completely unexpected. It has been predicted[19,23] that in a K = 3/2 band there will be a term in the above equation for E_I of the form $+C(-)^{I+1/2}(I - 1/2)(I + 1/2)(I + 3/2)$. This term comes from the Coriolis coupling of the K = 3/2 band to K = 1/2 bands, and is the direct result of

the anomalous energy spacing of the K = 1/2 bands. This term will cause just the behavior noted in fig. 12; namely, two lines having a common intercept. From the lines in fig. 12, we can evaluate the constants as follows: A = 11.61 ± 0.04 keV, B = -(5.8 ± 1.0) × 10^{-3} keV, and C = -(8.0 ± 2.0) × 10^{-3} keV. The values obtained for A and B are quite reasonable for this region of the periodic table, and require no further comment. In terms of the mixing of a K = 1/2 band into the K = 3/2 band, C can be written

$$C = \frac{a\hbar^2}{2\mathfrak{J}} \left[\frac{\hbar^2/2\mathfrak{J}\langle\psi_{3/2} \mid j_+ \mid \psi_{1/2}\rangle}{W} \right]^2 ,$$

where a is the decoupling parameter of the 1/2 band, $\hbar^2/2\mathfrak{J}$ is the rotational constant (A in the above equation for E_I), W is the energy separation of the 1/2 and 3/2 bands, and $\langle\psi_{3/2} \mid j_+ \mid \psi_{1/2}\rangle$ is the coriolis matrix element between the two bands. It is clear that the experimental C is the sum of a number of terms of the type written above, representing the coupling to this state of all the K = 1/2 bands in the vicinity of the 3/2+[411] state. However, it also seems clear that the contributions of the various K = 1/2 states will vary widely, with the largest effects coming from nearby (small W) bands having a large decoupling parameter (a) and also a large Coriolis matrix element coupling them to this K = 3/2 band. In table 9 the values of the decoupling parameters and the above-mentioned Coriolis matrix elements are listed for all the 1/2+ states in the N = 4 shell. Of these, the only one which has been experimentally seen in the region of Tb^{159}, and almost certainly the lowest lying in this nucleus is 1/2+[411]. In fact, we have tentatively identified this band as the group of levels around 970 keV. On the other hand, the 1/2+[420] state has by far the largest calculated matrix element connecting it with the 3/2+[411] state, and it is probably the next lowest-

lying intrinsic 1/2+ state in Tb^{159}. This state has never been seen in any nucleus, since it is already filled before the deformation sets in. These two 1/2+ states have decoupling parameters calculated to be about equal in magnitude and of opposite sign. The sign of C is determined only by the sign of the decoupling parameter of the admixed band, and this is known experimentally to be negative. Thus we conclude that either the C term is of a more complicated nature than we are considering, or the term is due principally to admixture of the 1/2+[411] band. The former possibility we cannot analyze; however, the latter we can test in two ways.

Firstly, the collective B(E2) between the two bands as a result of the mixing can be calculated to be:

$$B(E2)_T(3/2+,\ 3/2\ \ K = 1/2) = \left[\frac{\hbar^2/2\mathfrak{J}\langle\psi_{3/2}\,|j_+|\psi_{1/2}\rangle}{W}\right]^2 [6B(E2)_R]\ ,$$

where $B(E2)_R$ is the collective E2 transition strength of the ground-state rotational band (and assumed to be equal for the 1/2+[411] band) of Tb^{159}. The factor in the brackets can be evaluated directly from the expression for C by putting in the observed product of $(a\hbar^2/2\mathfrak{J})$ for the 971-keV band. Using $B(E2)_R = 5.6\ e^2 \times 10^{-48}\ cm^4$, we get $B(E2)_T(3/2+,3/2\rightarrow K = 1/2) = (2.6 \pm 0.6) \times 10^{-2}\ e^2 \times 10^{-48}\ cm^4$, compared with the experimental value of $(2.5 \pm 1.0) \times 10^{-2}\ e^2 \times 10^{-48}\ cm^4$. These numbers are in good agreement, lending considerable support to the argument that essentially only the 1/2+[411] band is causing the C term observed. The agreement need not be exact in any case, since a small intrinsic E2 transition moment between these two bands can interfere with the admixed collective moment calculated above.

Secondly, the matrix element, $\langle\psi_{3/2}\,|j_+|\psi_{1/2}\rangle$, can also be evaluated from the expression for C on the assumption that the 1/2+[411] band at 971 keV

is causing most of the effect; it turns out to be 1.9. The theoretical value from table 9 is 0.56, and the discrepancy here is rather large. However, that the C term is negative is a reasonably direct indication that either it is caused by an effect more subtle than direct Coriolis mixing or that the relative matrix elements for the 1/2+[420] and 1/2+[411] bands must be different than the calculated ones given in table 9. Otherwise the mixing of the 1/2+[420] band would almost surely predominate giving a positive C term. Unfortunately, few Coriolis matrix elements have been experimentally measured, so that there is little indication of how good those calculated from Nilsson's wave functions can be expected to be. Summarizing, we feel that on the basis of the present data it is most likely that the C term arises predominantly from Coriolis mixing with the state, 1/2+[411]. This point is by no means proved, but will receive additional support in the following discussion of Tm^{169}, where 1/2+[411] is the ground state.

5.3. Thulium-169

The level scheme constructed for Tm^{169} is shown in fig. 13. In this case there is detailed information only on the ground and 570-keV bands. Those of 900 and 1170 keV are seen only in the gamma-ray spectra, and the $B(E2)\uparrow\uparrow$ value is essentially the only information available.

The band at 570 keV is assigned a K value of 3/2 principally on the basis of the indicated value for the rotational constant. This value is 9.1 or 12.4 keV for K assignments of 5/2 and 3/2, respectively. Since the rotational constant for the ground-state band is 12.5 keV, the K value of 3/2 seems clearly indicated. The relative energy spacings do not help much in this case, since the limits of error on the transition energies are rather large. We have: $(E_{718} - E_{633})/(E_{633} - E_{570}) = 1.35 \pm 0.08$, whereas

the theoretical values for K = 5/2 and K = 3/2 are 1.29 and 1.40, respectively. The parity of this band is even, since the excitation is shown to be E2 from the excitation function of the electron lines of the 562- and 570-keV transitions in table 8. Thus we assign K and parity values of 3/2+ to the 570-keV band.

The transitions de-exciting this band are principally M1. This is shown by the average conversion coefficient of the 565-keV group in table 7, and also by the fact that the lines are Doppler-broadened symmetrically. The transitions from the 718-keV level are quite weak, and although we have observed these lines on more than one spectrum to establish their existence, it does not seem feasible to analyze their intensities. From the 633-keV level, we get B(M1)625|B(M1)515|B(M1)494 = 1.0|1.0 ±0.4|0.5 ± 0.3. The vector addition coefficients for K = 3/2→K = 1/2 give 1.0|1.14|0.36 for these transitions, which is in satisfactory agreement with the data. The 633-keV transition itself is puzzling. From its position in the decay scheme, this transition would have to be E2, and it is quite surprising that it competes with the M1 transitions de-exciting this level. However, the transition has been seen on several spectra, and there is no other obvious place for it in the level scheme. Since the theoretical E2 branching ratios from this level (and also the 570-keV level) are similar to those for M1 radiations, appreciable E2 admixtures would not cause disagreement with the data. The intensities of the transitions from the 570-keV level give B(M1)570|B(M1)562|B(M1)452 = 1.0|0.81 ±0.13|0.23 ± 0.04. The vector addition coefficients give 1.0|0.80|0.20 for these ratios, which is in good agreement with the experimental data.

The nature of this 3/2+ band presents an interesting problem. There are two very good possibilities which are not easy to distinguish from each

other. These are (1) the K_0-2 gamma vibrational band and (2) the intrinsic state, 3/2+[411], which, as in the case of Tb^{159}, may be mixed with the 1/2+[411] state. The E2 strength between these two bands is found to be $B(E2)_T(1/2+, 1/2 \rightarrow K = 3/2) = (2.9 \pm 0.5)e^2 \times 10^{-50}$ cm^4. This corresponds to 1.0 single particle unit. This E2 strength is somewhat less than was found for the K_0-2 vibrational state in Tb^{159}, but probably represents a reasonable value for such a vibrational state. On the other hand, if the mixing between the 1/2+[411] and 3/2+[411] bands, which we evaluated for the Tb^{159} case is corrected for this lower excitation energy, one single particle unit is a very plausible E2 strength to be introduced. Thus we cannot distinguish between these two interpretations of the 570-keV band on this basis.

A more fruitful approach is to examine the relative population of the 633- and 570-keV levels. Assuming the M1 transitions to the ground band are sufficiently fast so that no rotational transitions occur within the 570-keV band, we have $B(E2)633|B(E2)570 = 0.21 \pm 0.07|1.00$. If the E2 transitions were connecting two intrinsic states, not by virtue of their mixing, then there would be no unique prediction from the vector addition coefficients, since we have $L \geqq K_i + K_f$. This, however, we feel is not likely to be the case because of the magnitude of the B(E2) between the states. If the E2 strength is by virtue of the admixing of two intrinsic states, then we expect the above ratio to be 4.0. This is a factor of 20 different from the experimental data, and it certainly does not seem likely to us that the effects of multiple excitation could explain this difference. Thus, this interpretation seems unlikely. If this is the K_0-2 gamma vibrational band, the vector addition coefficients for $K_f = -3/2$ give 0.25 for the above ratio. A reasonable correction for band mixing, as described earlier for these vibrational states, will reduce this calculated number to 0.19, which is still

in good agreement with the experimental data. Our rough estimate shows that multiple excitation largely cancels in this case, but probably further lowers this number slightly.

Another weak argument favoring the vibrational interpretation has to do with the rotational constant for this band. The experimental value, 12.4 keV, is extremely close to that of the ground-state band, 12.5 keV. The rotational constants of the vibrational bands in both Tb^{159} and Ho^{165} are almost exactly the same as their respective ground states, so that this behavior is apparently characteristic for these gamma vibrational bands in this region of the periodic table. On the other hand the 3/2+[411] state in Tb^{159} had a rotational constant of 11.6 keV. Although it might well be different in Tm^{169}, it would nevertheless be somewhat accidental if it came very close to that of the ground-state band of Tm^{169}. The M1 transitions de-exciting this band are not expected in the pure vibrational interpretation; however, they did occur in Tb^{159}, where the vibrational (collective) character of the state was considerably more strongly indicated. Since in this case, as in Tb^{159}, they are not K-forbidden, we do not feel that their existence argues against a vibrational assignment. Again we find that these M1 transitions follow the $K = 3/2 \rightarrow K = 1/2$ vector addition coefficients. This suggests that they are not going simply by virtue of K impurity in one or both bands. Summarizing, we feel that this band is most likely vibrational, but this assignment is by no means conclusive.

There is very little information available about the 900- and 1170-keV bands in Tm^{169}. The transitions from both these bands are seen only in photon spectra, and both appear as broad unresolved peaks. The E2 strengths are $B(E2)_T(1/2+,\ 1/2 \rightarrow 900\ \text{keV}) = (0.8 \pm 0.4)e^2 \times 10^{-50}\ \text{cm}^4$ and $B(E2)_T(1/2+,\ 1/2 \rightarrow 1170\ \text{keV}) = (4.1 \pm 1.2)\ e^2 \times 10^{-50}\ \text{cm}^4$. In terms of the

single-particle unit, this becomes 0.3 and 1.5 units for the 900- and 1170-keV bands respectively. With no more data available we can only guess as to the nature of these levels. On the basis of the E2 strength, it seems most likely that the 1170-keV band would be the K_0+2 gamma vibrational band. The 900-keV band might be based on the 3/2+[411] state, where its position relative to the 1/2+[411] state is almost exactly reversed from Tb^{159}. We emphasize again that these are merely guesses.

The analysis of the energies of the members of the ground-state rotational band is shown in fig. 14, where we have plotted $(E_{I+1}-E_I)/2(I+1)$ versus $[2(I+1)]^2$. This is the same type of plot made for Ho^{165} and Tb^{159}. We would naively expect the data to be represented by

$$E_I = E_0 + A[I(I+1)+a(-)^{I+1/2}](I+1/2] + B[I(I+1)+a'(-)^{I+1/2}(I+1/2)]^2,$$

where a' and a are identical. The plot should then show two nearly straight lines with different slopes and intercepts. This is what fig. 14 shows, and the constants may be evaluated as

$A = 12.48 \pm 0.02$ keV, $a = -(0.777 \pm 0.002)$, $B = -(4.7 \pm 1.0) \times 10^{-3}$ keV, and $a' = -(2.6 \pm 0.3)$. Clearly a' is not identical with a, and cannot be made so even by adding another term in the power-series expansion. The principal effect of a' is in the cross term, $2Ba'(-)^{I+1/2}I(I+1/2)(I+1)$. This is a term very similar to the one in the Tb^{159} ground-state band; and, in fact, the ground-state band in Tm^{169}, 1/2+[411], is just the band we proposed to lie at 971 keV in Tb^{159} and to be causing the oscillating term in the Tb^{159} ground-state band. If that analysis was correct, we would certainly expect to find a similar term here in the analysis of the K = 1/2 band, and the fact that a is not equal to a' indicates just such a term. There is one important difference, however. When we are dealing with a K=3/2 band, each

admixed K = 1/2 band will introduce a term of the form, $C(-)^{I+1/2}(I - 1/2)(I + 1/2)(I + 3/2)$, where the sign of C depends on the sign of the decoupling parameter in the K = 1/2 band. The observed C will be the sum of contributions from (in general) many K = 1/2 bands, and there will be some cancellation since bands with decoupling parameters of opposite sign will be admixed. If we consider the K = 1/2 band, however, all admixed K = 3/2 bands will give C values of the same sign, and only other admixed K = 1/2 bands with decoupling parameters larger in magnitude and opposite in sign will introduce C terms of opposite sign. Thus the effect is largely coherent in the case of the K = 1/2 band, and the observed magnitude of C should be larger than for a K = 3/2 band. If we take the rotational equation to be of the form:

$$E_I = E_0 + A[I(I + 1)+a(-)^{I+1/2}(I + 1/2)] + B[I(I + 1) +a(-)^{I+1/2}(I + 1/2)]^2$$
$$+C(-)^{I+1/2}(I - 1/2)(I + 1/2)(I + 3/2),$$

then the constants are A = 12.48 ± 0.02 keV, a = -(0.777 ± 0.002), B = -(4.7 ± 1.0) × 10^{-3} keV, and C = +(1.76 ± 0.25) × 10^{-2} keV. The positive sign of C is expected in a K = 1/2 band having a negative decoupling parameter, and a negative C should occur in the admixed K = 3/2 bands. The C value found for Tb^{159} was -8.0 × 10^{-3} keV; so that the magnitude of the effect is larger in the K = 1/2 band as expected. From these data, we can estimate that about one-half of the C term in Tm^{169} is contributed by the 3/2+[411] state, provided that as we have guessed, it lies around 900 keV and has the same matrix element connecting it with the 1/2+[411] state as was indicated in Tb^{159}. As a check on the plausibility of the C term being due to mixing, we can estimate that the difference in moment of inertia between even-even

nuclei and the Tb^{169} ground band is related to C by the expression

$$\Delta\hbar^2/2\mathfrak{J} = \frac{WC}{-a\,\hbar^2/2\mathfrak{J}} = 1.8 \times 10^{-3}\,W,$$

where W is the average energy in keV of the admixed bands. Taking W to be 1 MeV, we find $\Delta\hbar^2/2 = 1.8$ keV, or $(\hbar^2/2\mathfrak{J})_{e-e} = 14.3$ keV. Although this is slightly larger than might be expected (Yb^{170}, 14.0 keV; Er^{168}, 13.3 keV), it suggests to us that this explanation is reasonable. We assume here that admixed K = 1/2 bands have decoupling parameters that average to be zero. We assume also that the only difference between the even-even and odd-A nuclei with regard to the moment of inertia is the admixture of single-particle states in the latter case. It will be quite interesting to examine other K = 1/2 and K = 3/2 bands in this respect. Occasionally these effects should be large enough to show up in K = 5/2 bands as well.

6. CONCLUSION

In the present study of Tb^{159}, Ho^{165}, and Tm^{169}, we have Coulomb-excited three types of levels. First, in the cases of Tb^{159} and Tm^{169}, E2 excitation has been seen to two (and possibly three) intrinsic (or single-particle) states. In each case, assignment could be made to an expected Nilsson orbit. The only further comment we will make is that the E2 strengths to these levels (0.3 to 1.0 single-particle unit) are rather large and most likely due to Coriolis admixtures of these states into their respective ground states. Such mixing, even though small, can introduce large E2-transition probabilities because of the very large collective quadrupole moments of nuclei in this region. The other two types of level seen are both collective in nature. As expected, the most prominent excitations were those in the ground-state rotational band. Through the multiple-excitation process, the large E2 strengths made it possible to excite as many as eight members of the band. The other type of collective excitation produced bands which seem to be related to the ground state by $K = K_0 \pm 2$. We call these "gamma-vibrational states" following the terminology of Bohr and Mottelson, although it must be recognized that the results of our measurements do not give any obvious means of distinguishing between different models which provide collective excitations with change of K by two units. Some of the systematic properties observed in the two types of collective levels will be summarized in the following paragraphs.

The K_0-2 gamma vibrational band is almost certainly seen in Ho^{165}, very likely in Tb^{159}, and probably in Tm^{169} as well. The K_0+2 gamma band is probably seen in Ho^{165} and possibly in the other two nuclei. In all three cases the K_0-2 band lies lower in energy; by 170 keV in Ho^{165}, and probably

by 600 or 700 keV in the other cases. It is interesting to note that in Re^{187}, the only other odd-mass nuclei where gamma bands have been established reasonably well,[24] it is the K_0-2 bands alone that are seen. A reasonable implication is that these bands lie lower in energy than the K_0+2 bands. As far as we know there has been no theoretical treatment of the relative energies expected for such bands. While it seems quite reasonable to us that the K_0-2 band should lie lower, the magnitude of the splitting in Tb^{159} and Tm^{169} is surprising. Even if our tentative K_0+2 assignments are not correct in these cases, the failure of this band to show up at lower energies still implies a large splitting. The difference between these cases and that of Ho^{165}, which has a considerably smaller splitting, is not clear.

The E2-transition probabilities (for a vector-addition coefficient of unity) between these vibrational states and their respective ground states is in all cases between one and two single-particle units. Furthermore in a given nucleus the E2 strengths to the K_0+2 and K_0-2 bands from the ground state are apparently nearly equal. The relative E2-transition probabilities for de-excitation of the bands in Ho^{165} (in Tb^{159} and Tm^{169} they de-excite principally by M1 radiation) followed the predictions of the vector-addition coefficients with a slight modification for mixing of the ground and vibrational states, as has been observed by O. B. Nielsen in even-even nuclei. The evidence is fairly strong that this mixing correction is necessary. The relative E2-transition probabilities for Coulomb-exciting the members of these bands in all three nuclei seemed also to follow the modified vector-addition coefficients. However, these probabilities are subject to additional corrections which are not well known and may be large because of the effects of multiple Coulomb excitation and intraband rotational de-excitations.

A surprising result was that when $|K_{vib}| = |K_{grd}| \pm 1$, as is the case for the K_0-2 bands of both Tb^{159} and Tm^{169}, the de-exciting transitions were predominantly M1 rather than E2. For these particular cases, M1 transitions are not K-forbidden. In fact, the appropriate vector-addition coefficients should be those for $|K_{vib}| \rightarrow |K_{grd}|$, and the data followed these within our limits of error. However, such transitions are not expected in a pure vibrational model. On this model we also would not have expected the relatively fast E1 transition(s) observed to occur in Ho^{165} between the K_0-2 gamma band and a nearby intrinsic (Nilsson) state.

The occurence of such transitions does not argue against the collective nature of these states, which seems to us to be rather strongly suggested. Even a lack of K purity in these states is not indicated. The disagreement is with the detailed vibrational description, and particularly in the case of the E1 transitions, it is not clear to us that reasonable admixtures of nearby states can account for the observed transition moments. In this regard it would be interesting to see calculations based on the Davydov-Filippov model.[4] In this model such transitions are not forbidden in general. On the other hand Davydov's calculation for $j = \Omega = 1/2$ is in very poor quantitative agreement with our data on Tm^{169} where $\Omega = 1/2$. It remains to be seen if relaxing the requirement $j = 1/2$, can bring this calculation into agreement with the experimental data.

The feature most interesting to us in the ground-state rotational bands was the term in the energy formula of the form; $+C(-)^{I+1/2}(I - 1/2)(I + 1/2)(I + 3/2)$. This term occurred in both Tb^{159}, 3/2+[411], and Tm^{169}, 1/2+[411]. Such a term was predicted, and is the result of mixing between K = 3/2 and anomalously spaced ($a \neq 0$) K = 1/2

bands. We have tentatively ascribed almost all the effect in Tb^{159}, and about one-half of the effect in Tm^{169}, to Coriolis mixing between these two particular levels. As far as we can tell, all the data are consistent with this analysis; the only unexpected result is a Coriolis matrix element between these two states which is about three times that calculated from the Nilsson wave functions.

ACKNOWLEDGMENTS

The authors would like to thank Prof. I. Perlman for his encouragement and interest in this work, and his critical reading of the manuscript. They would also like to acknowledge their gratitude to Dr. A. Ghiorso and all the personnel of the HILAC for the support and collaboration that made these experiments possible.

REFERENCES

1. K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Mod. Phys. 28, 432 (1956).
2. T. Huus, Nucl. Instr. and Meth. 11, 19 (1961).
3. O. Nathan and V. I. Popov, Nuclear Physics 21, 631 (1960).
4. A. S. Davydov and G. F. Filippov, J. Exptl. Theoret. Phys. 35, 440 (1958); A. S. Davydov, J. Exptl. Theoret. Phys. 36, 1555 (1959); A. S. Davydov, Nuclear Phys. 16, 597 (1960); and others.
5. S. G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selek 29, No. 16 (1955).
6. B. R. Mottelson and S. G. Nilsson, Mat. Fys. Skr. Dan. Vid. Selsk. 1, No. 8 (1959).
7. F. S. Stephens, R. M. Diamond, and I. Perlman, Phys. Rev. Letters, 3, 435 (1959).
8. O. Kofoed-Hansen, J. Lindhard, and O. B. Nielsen, Mat. Fys. Medd. Dan. Vid. Selsk. 25, No. 16 (1950).
9. B. Elbek and M. Nakamura, Nucl. Instr. and Meth. 10, 164 (1961).
10. E. L. Chupp, J. W. M. DuMond, F. S. Gordon, R. C. Jopson, and H. Mark, Phys. Rev. 112, 518 (1958).
11. E. L. Hubbard, Lawrence Radiation Laboratory Report UCRL-9053, January 1960.
12. L. A. Sliv and I. M. Band, Coefficients of the Internal Conversion of Gamma Radiation, Part I, K-Shell, Academy of Sciences of the USSR, Moscow, Leningrad, 1956, as given in Repart 57 ICC K1, University of Illinois, Urbana, Illinois, 1957.

13. R. Diamond, B. Elbek, G. Igo, and F. Stephens, Proceedings of the International Conference on Nuclear Structure, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto 1960), p. 563.

14. F. P. Cranston, Jr., M. E. Bunker, and J. W. Starner, Bull. Am. Phys. Soc. 5, 255 (1960); M. E. Bunker, F. P. Cranston, Jr., and J. W. Starner, private communication, September 1959.

15. R. Tornau, Z. Physik 159, 101 (1960).

16. K. Alder and A. Winther, Mat. Fys. Medd. Dan. Vid. Selsk. 32, No. 8 (1960).

17. O. Prior, Ark. Fysik 14, 451 (1959).

18. O. B. Nielsen, Proceedings of the Rutherford Jubilee International Conference (Heywood, London 1961), p. 317; P. Gregers Hansen, O. B. Nielsen, and R. K. Sheline, Nucl. Phys. 12, 413 (1959).

19. A. Bohr and B. R. Mottelson, to be published in At. Energ. (USSR).

20. H. Mark, private communication (October 1961).

21. B. H. Ketelle and A. R. Brosi, Phys. Rev. 116, 98 (1959).

22. R. L. Graham, private communication (November 1962).

23. B. R. Mottelson, private communication (May 1961).

24. C. J. Gallagher, W. F. Edwards, and G. Manning, Nucl. Phys. 19, 18 (1960).

Table 1. Rotational transitions of ground-state bands

Initial state	Final state	Ho^{165} $I_0 = 7/2-$ (keV)	Tb^{159} $I_0 = 3/2+$ (keV)	Tm^{169} $I_0 = 1/2+$ (keV)
$I_0 + 1$	I_0	94.7	(58.0)[a]	(8.4)
$I_0 + 2$	$I_0 + 1$	115.1	(79.5)	109.8
$I_0 + 2$	I_0	209.8	137.5	118.2
$I_0 + 3$	$I_0 + 2$	135.1	103.7	(20.7)
$I_0 + 3$	$I_0 + 1$	250	183	130.5
$I_0 + 4$	$I_0 + 3$	154(?)	121(?)	193.3
$I_0 + 4$	$I_0 + 2$	289	~225	214
$I_0 + 5$	$I_0 + 4$			
$I_0 + 5$	$I_0 + 3$	327(?)	~269	229
$I_0 + 6$	$I + 5$			269
$I_0 + 6$	$I_0 + 4$		306	305
$I_0 + 7$	$I_0 + 6$			
$I_0 + 7$	$I_0 + 5$			323
$I_0 + 8$	$I_0 + 7$			336(?)

[a]Values in parentheses have been seen in previous work, but not observed here because of low energy.

Table 2. Electron Lines of Ho^{165}

Transition (keV)	Relative intensity Ke-	Parent level	$\epsilon_K B(E2)\uparrow\uparrow$ ($e^2 \times 10^{-53} cm^4$)	Error (%)	Assumed Multipolarity	$\alpha_K \times 10^2$ (Ref. 12)	Calculated Relative intensity[b] γ	Calculated $\epsilon_\gamma B(E2)\uparrow\uparrow$ ($e^2 \times 10^{-51} cm^4$)
~245								
346(?)								
361.2	1765	{514, 566}	{222, 242}	± 10	M2	22.5	78.5	{9.8, 10.8}
428	11	638	1.7	± 41	E2	1.90	6	0.9
471	55	566	7.5	± 14	E2	1.45	38	5.2
478	11	687	1.9	± 35	E2	1.40	8	1.4
514.4	222	514	27.9	± 12	E2	1.18	188	23.6
543	13	638	2.0	± 31	E2	1.02	12	2.0
566	63	566	8.7	± 16	E2	0.92	69	9.5
593	45	687	7.7	± 19	E2	0.82	55	9.4
~610(?)		820						
~638(?)	~ 8	638	1.3	± 47	E2	0.70	11	1.8
686.7	112	687	19.2	± 11	E2	0.59	190	32.5
725	~ 5	820			E2	0.525	~10	
Au 547	2970	547	(675)		E2	1.57	1890	(430)[c]

[a]These values are all calculated neglecting any intra-band (rotational) de-excitation. See text.

[b]Values should be multiplied by 100 to compare with column 2.

[c]Taken as standard from Alder et al.[1]

Table 3. Gamma Rays of Ho^{165}

Energy	Average parent level	$\epsilon_\gamma B(E2)\pi$ ($e^2 \times 10^{-51} cm^4$)	Included Transitions (Table 2)	$\Sigma \epsilon_K B(E2)\pi$ ($e^2 \times 10^{-53} cm^4$) (Table 2)	$\bar{\alpha}_K \times 10^2$	Theoretical $\alpha_K \times 10^2$ (Ref. 12) M1	E2	Assignment
530	562	50±15	471, 478 514, 543 566, 593	56±8	1.1±0.4	2.3	1.1	E2
685	685	39±8	638 687 724	22±3	0.56±0.14	1.2	0.59	E2

Table 4. Electron Lines of Tb^{159}

Transition (keV)	Relative intensity Ke^-	Parent level	$\epsilon_K B(E2)\uparrow\uparrow$ ($e^2 \times 10^{-53} cm^4$)	Error (%)	Assumed Multipolarity	$\alpha_K \times 10^2$ (Ref. 12)	Calculated: Relative intensity[a] γ	Calculated: $\epsilon_\gamma B(E2)\uparrow\uparrow$ ($e^2 \times 10^{-51} cm^4$)
289 (double)	66	429 348	6.3 5.7	± 21	M1 E2	9.7 5.5		
331	34							
347.7	222	348	19.0	± 11	M1 E2	5.9 3.1		
362	22							
370.6	137	429	13.1	± 12	M1 E2	4.9 2.6		
429(?)		429			E2			
522(?)	~ 3	580	0.4	± 85	E2	1.05	3	0.4
536	77	674	11.2	± 12	M1	1.9	41	5.9
560	55	617	7.3	± 13	M1	1.7	33	4.3
580.2	228	580	27.2	± 10	M1	1.58	144	17.2
617 (double)	86	674 617	12.4 11.3	± 13	M1	1.35	64	9.2 8.4
674	11	674	1.6	± 43	M1	1.08	11	1.5
920, 949, 965, 978	~ 34	~1028	6	± 30	M1	0.47	71	13
Au 547	3090	547	(675)		E2	1.57	1970	(430)[b]

[a] Values should be multiplied by 100 to compare with column 2.

[b] Taken as standard from Alder et al.[1]

Table 5. Gamma Rays of Tb^{159}

Energy	Average parent level	$\epsilon_\gamma B(E2)\uparrow$ ($e^2 \times 10^{-51} cm^4$)	Included Transitions (Table 4)	$\Sigma\epsilon_K B(E2)\uparrow$ ($e^2 \times 10^{-53} cm^4$) (Table 4)	$\bar{\alpha}_K \times 10^2$	Theoretical $\alpha_K \times 10^2$ (Ref. 12) M1	Theoretical $\alpha_K \times 10^2$ (Ref. 12) E2	Assignment
580	612	32±6	536, 560 580, 617	57.5±7	1.8±0.4	1.6	0.8	M1
950	1028	25±10 (18±7)[a]	920, 949 965, 978	6±2	0.33±0.16	0.47	0.27	M1 E2, or M1-E2
1280	1280	53±16						

[a] That portion corresponding to the observed electron lines.

Table 6. Electron Lines of Tm^{169}

Transition (keV)	Relative intensity Ke^-	Parent level	$\epsilon_K B(E2)\Uparrow$ ($e^2 \times 10^{-53} cm^4$)	Error (%)	Assumed Multipolarity	$\alpha_K \times 10^2$ (Ref. 12)	Calculated: Relative intensity[a] γ	Calculated: $\epsilon_\gamma B(E2)\Uparrow$ ($e^2 \times 10^{-51} cm^4$)
294(or 245)[b]								
348								
387(?)								
452	43	570	6.2	± 14	M1	4.15	10	1.5
494	11	633	1.8	± 56	M1	3.35	3	0.5
515	22	633	3.6	± 28	M1	3.00	7	1.2
562.4	165	570	23.7	± 12	M1	2.37	70	10.0
570.4	205	570	29.3	± 11	M1	2.29	90	12.8
579								
600								
625	23	633	3.8	± 24	M1	1.80	13	2.1
633	6	633	1.0	± 70	E2	0.77	8	1.3
Au 547	3140	547	(675)		E2	1.57	2000	(430)[c]

[a] Values should be multiplied by 100 to compare with column 2.

[b] Depending on whether this is a K or L line.

[c] Taken as standard from Alder et al.[1]

Table 7. Gamma Rays of Tm^{169}

Energy	Average parent level	$\epsilon_\gamma B(E2)\Uparrow$ ($e^2 \times 10^{-51} cm^4$)	Included Transitions (Table 6)	$\Sigma\epsilon_K B(E2)\Uparrow$ ($e^2 \times 10^{-53} cm^4$) (Table 6)	$\bar{\alpha}_K \times 10^2$	Theoretical $\alpha_K \times 10^2$ (Ref. 12) M1	E2	Assignment
565	580	28 ± 6	452, 494 515, 562 570, 579 600, 625 633	70 ± 10	2.5 ± 0.6	2.3	1.0	M1
~900	900	8 ± 4						
~1170	1170	41 ± 12						

Table 8. Electron Line Excitation Functions

Isotope	Transition Energy (keV)	Experimental $\frac{Y_{60\ MeV}}{Y_{44\ MeV}}$	Theoretical $E_{excit.}$	Theoretical Multipolarity	Theoretical $\frac{Y_{60\ MeV}}{Y_{44\ MeV}}$
Au^{197}	547	4.02	547	E2	(4.0)[a]
Ho^{165}	514	3.40	514	E2	3.4
Ho^{165}	361	4.23	514 638 361	E2 E2 E3	3.4 4.0 4.7
Tb^{159}	580	3.62	580	E2	3.7
Tb^{159}	348	2.85	348	E2	2.8
Tm^{169}	562-570	3.67	566	E2	3.7

[a] Lower beam energy adjusted by 2 MeV to give agreement here.

Table 9. Calculated Properties of K = 1/2 States[a]

Nilsson state	1/2+[440]	1/2+[431]	1/2+[420]	1/2+[411]	1/2+[400]
a (η = 6)	+ 4.14	- 1.60	+ 0.81	- 0.79	+ 0.44
$\langle\psi_{3/2+[411]}\|j_+^-\|\psi_{1/2}\rangle$ (η = 6)	- 0.13	- 0.11	+ 2.87	+ 0.56	- 1.31

[a]From calculations of S. G. Nilsson.

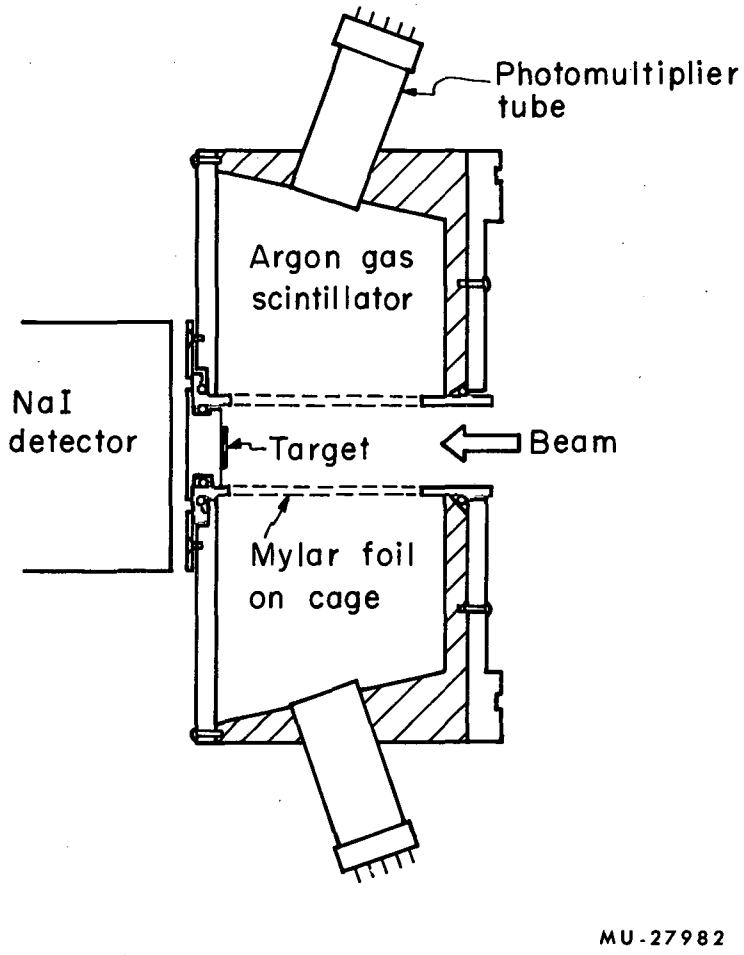


Fig. 1. Diagram of the argon-gas scintillation counter (not to scale).

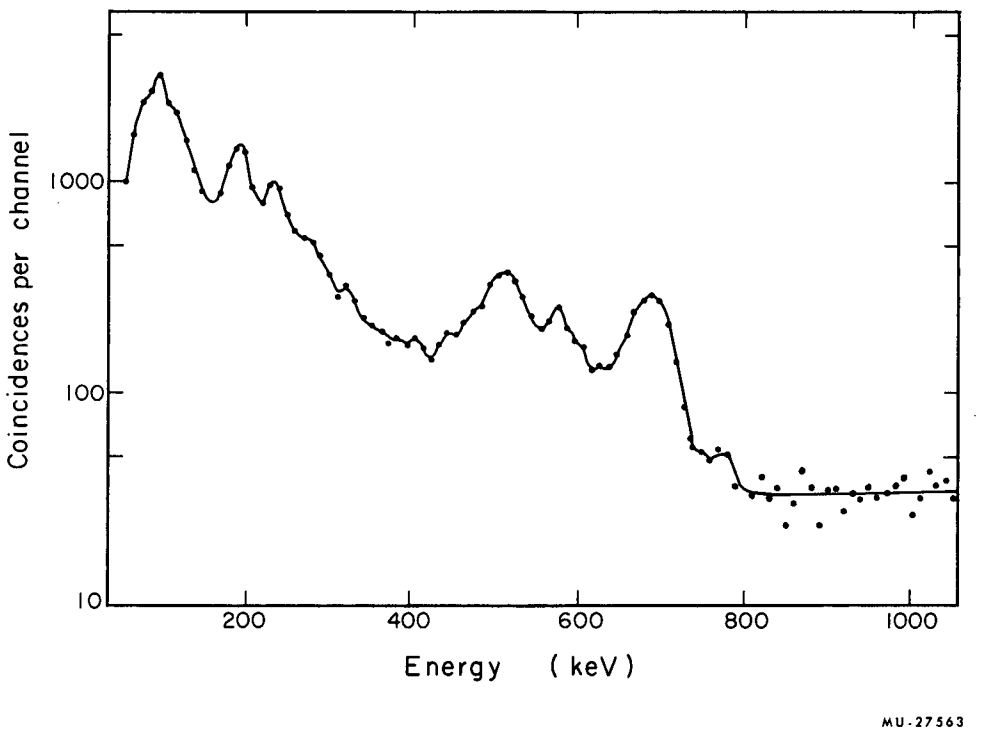


Fig. 2. Gamma-ray spectrum of Ho^{165} obtained in coincidence with back-scattered O^{16} ions.

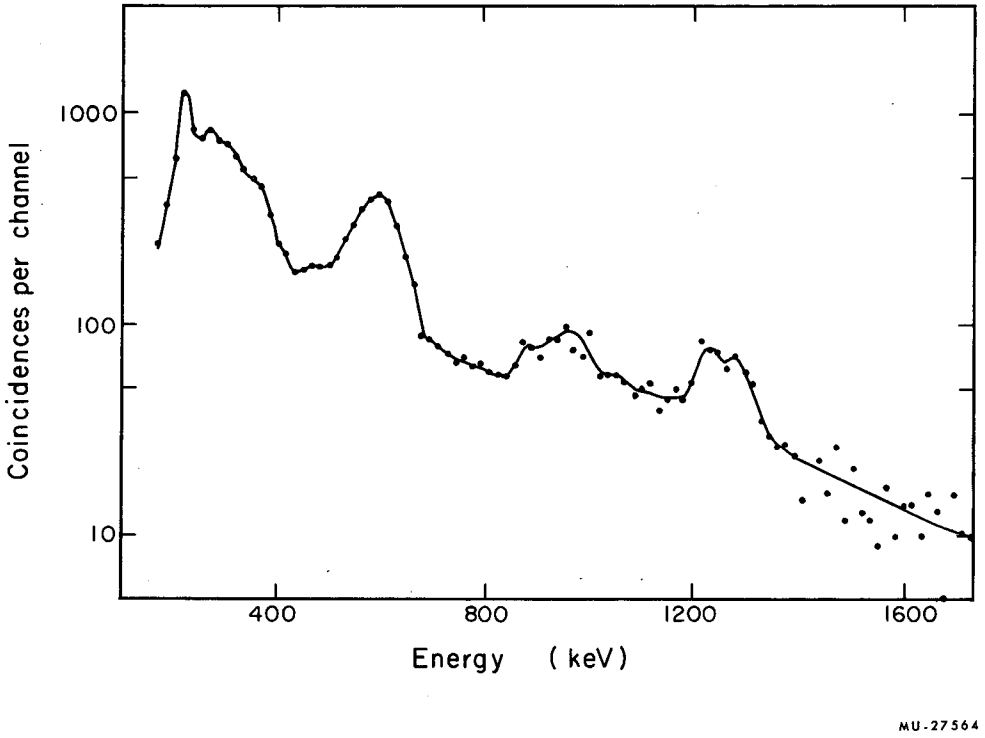


Fig. 3. Gamma-ray spectrum of Tb^{159} obtained in coincidence with back-scattered O^{16} ions.

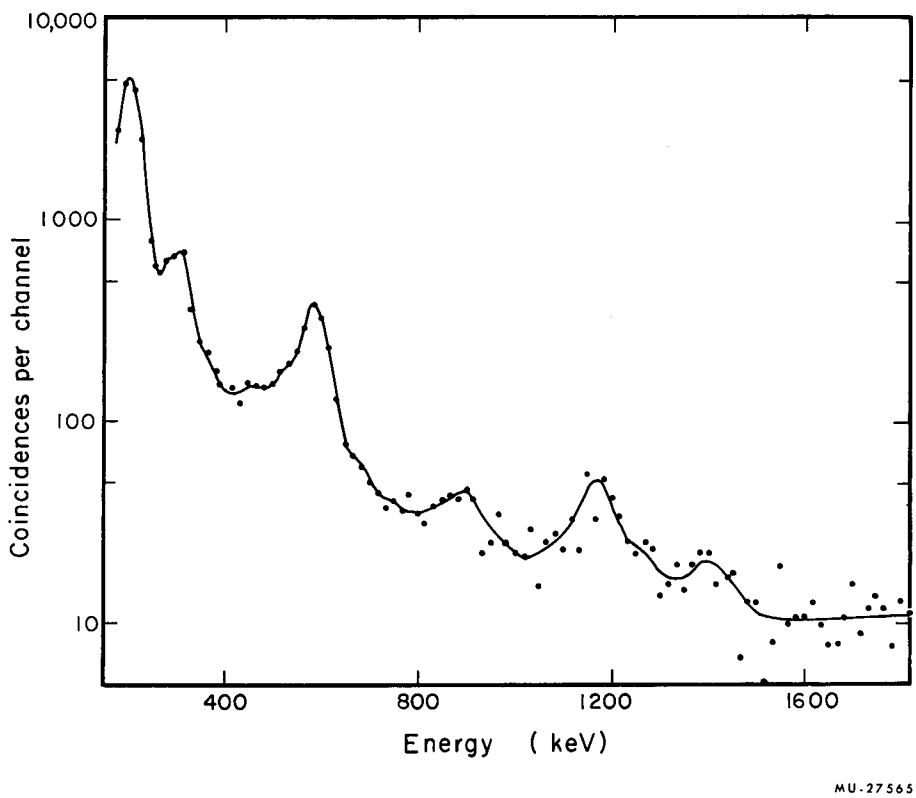


Fig. 4. Gamma-ray spectrum of Tm^{169} obtained in coincidence with back-scattered O^{16} ions.

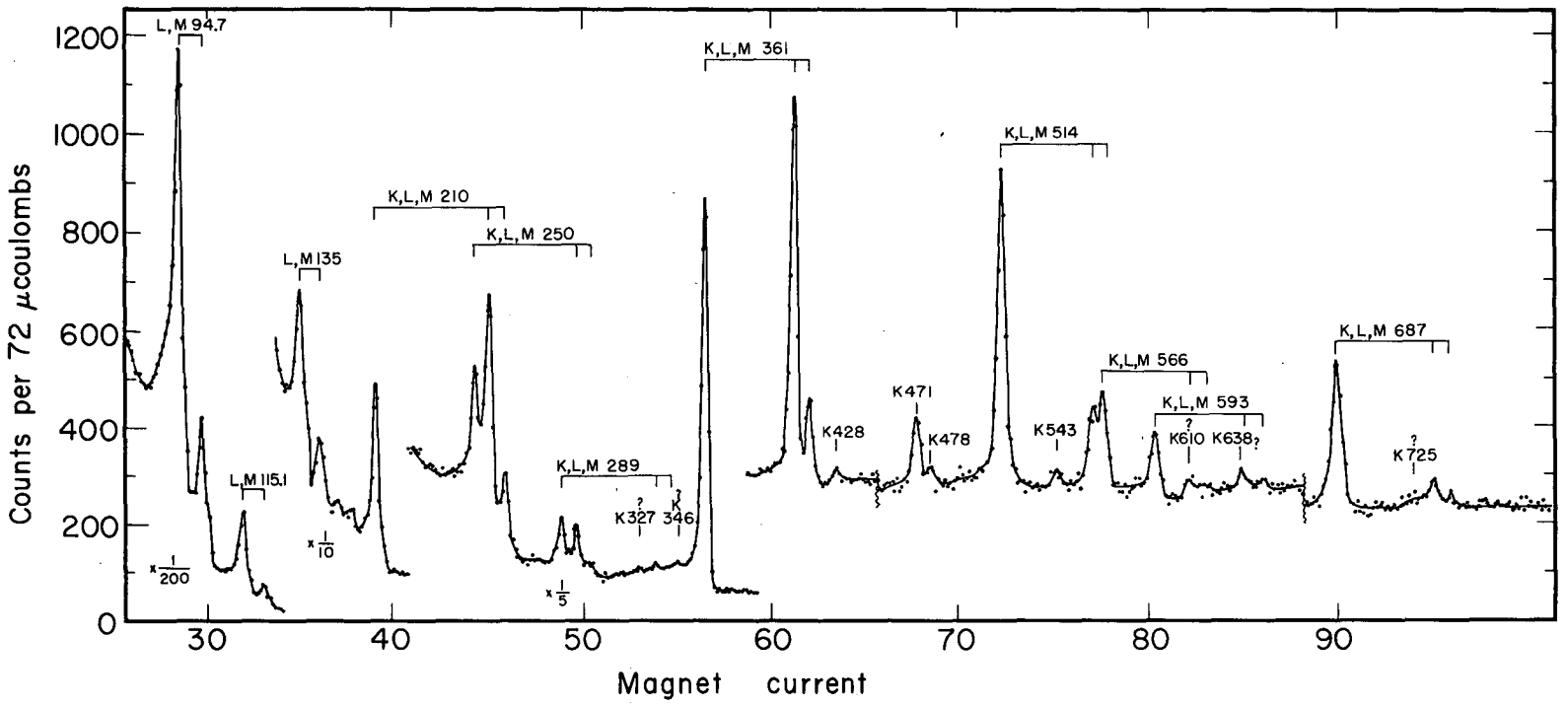


Fig. 5. Conversion-electron spectrum of Ho^{165}. The symbol ≀ indicates the junction between different runs having slightly different back-grounds.

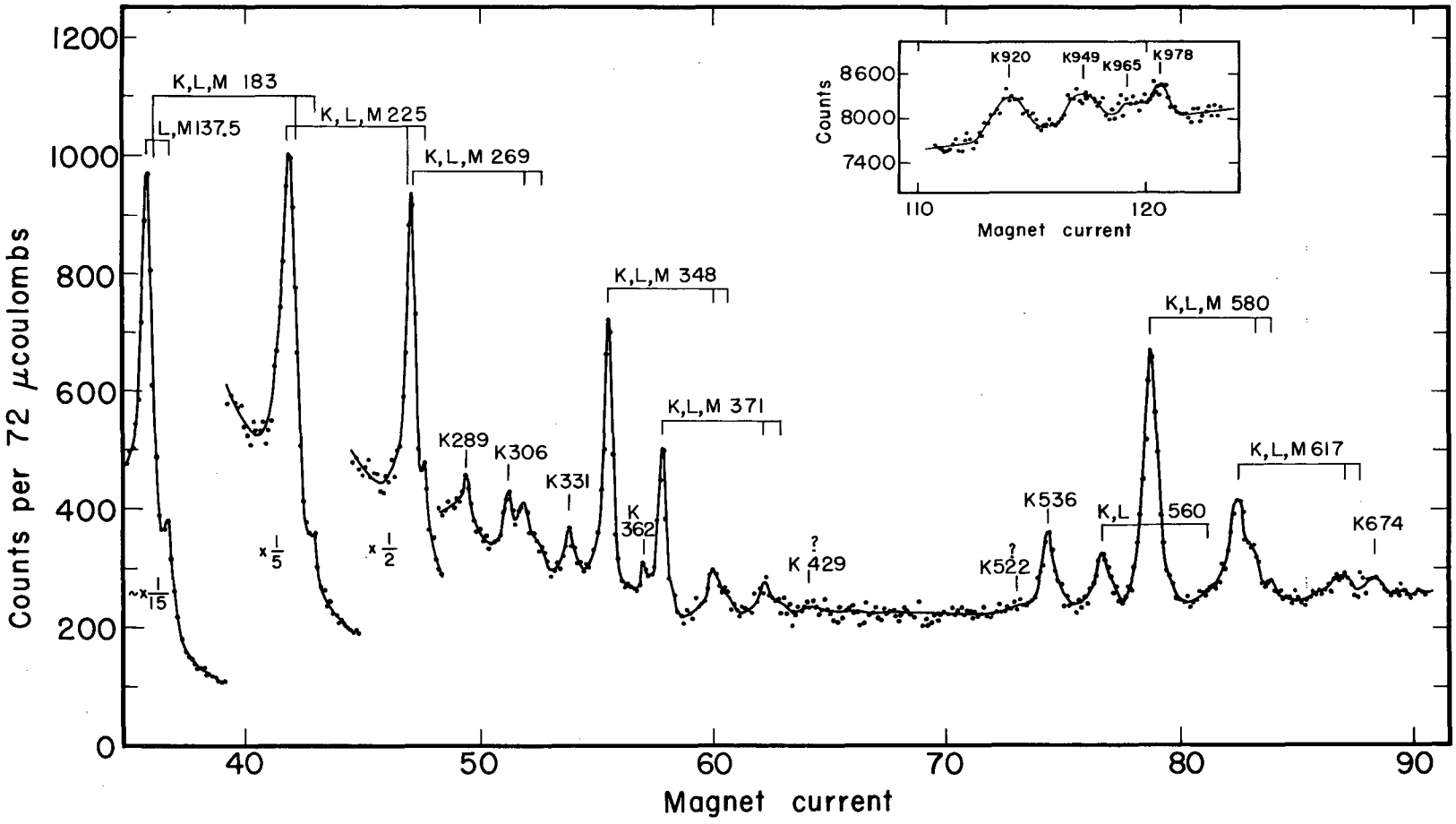
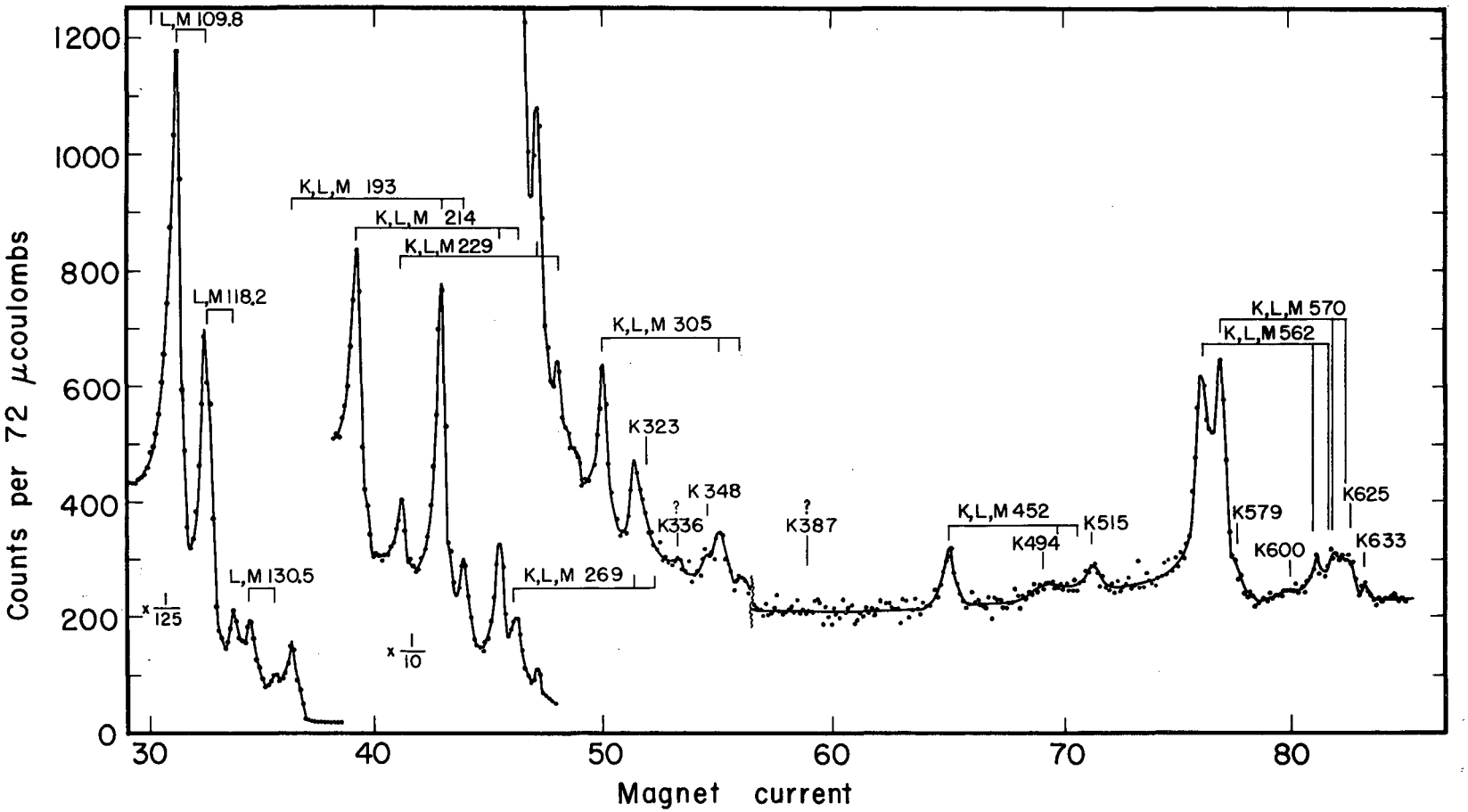


Fig. 6. Conversion-electron spectrum of Tb^{159}.



MUB-908

Fig. 7. Conversion-electron spectrum of Tm^{169}. The symbol ⸗ indicates the junction between different runs having slightly different back-grounds.

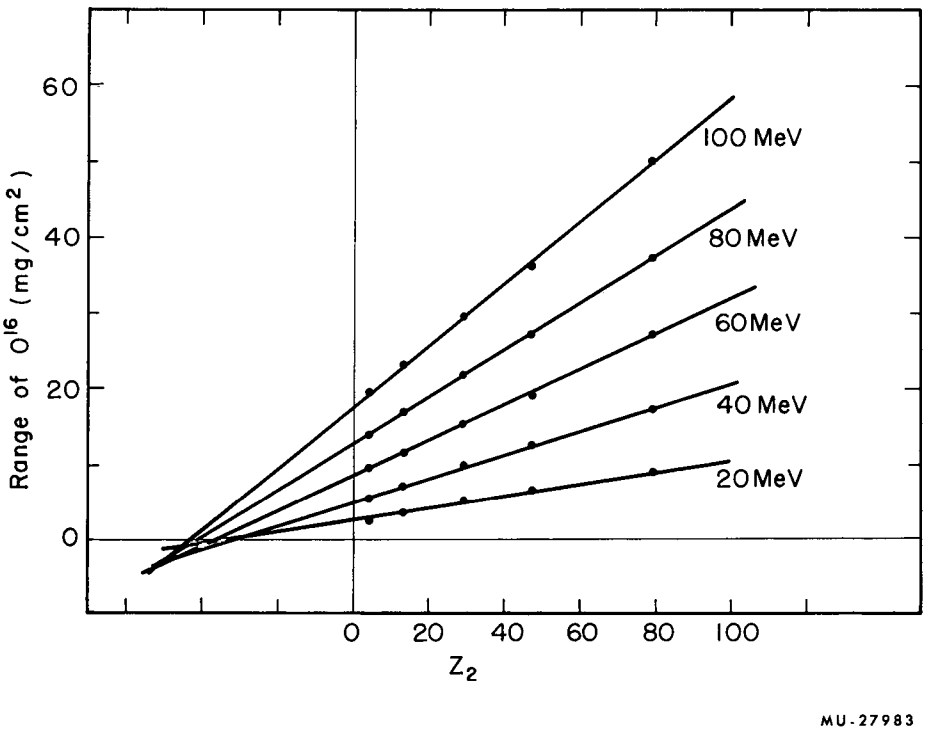


Fig. 8. Plot of the range of O^{16} ions of indicated energy vs the atomic number of the absorbing material.

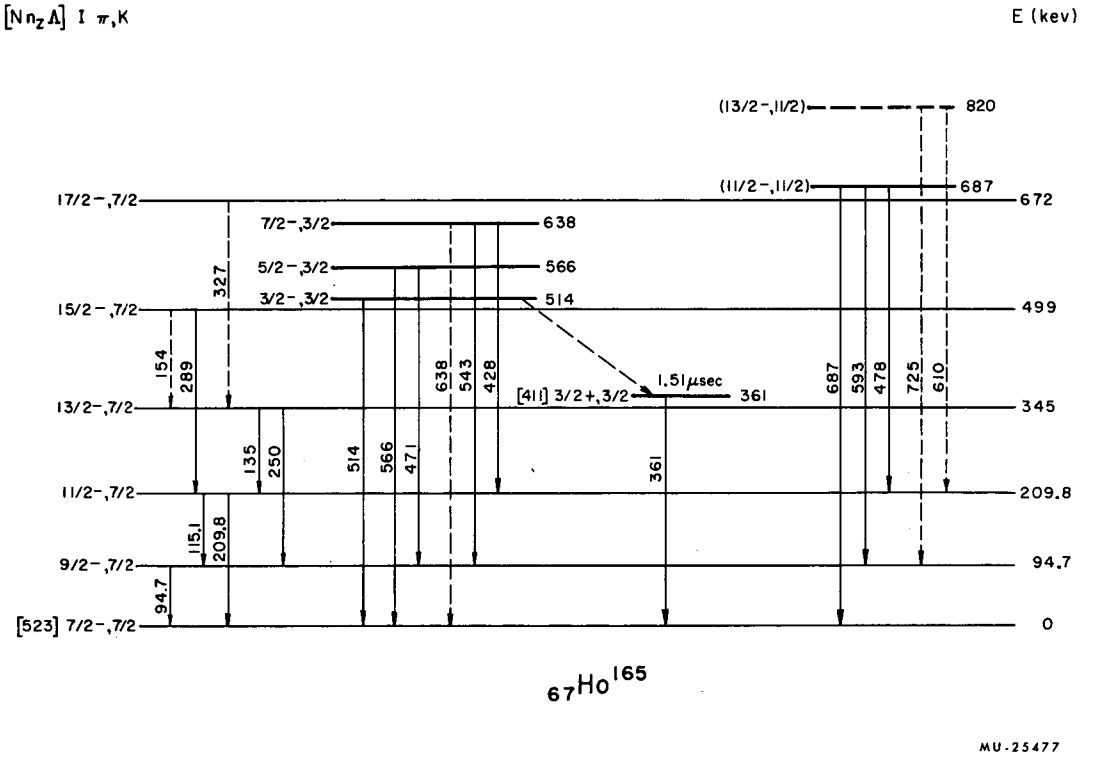


Fig. 9. Level scheme of Ho^{165}.

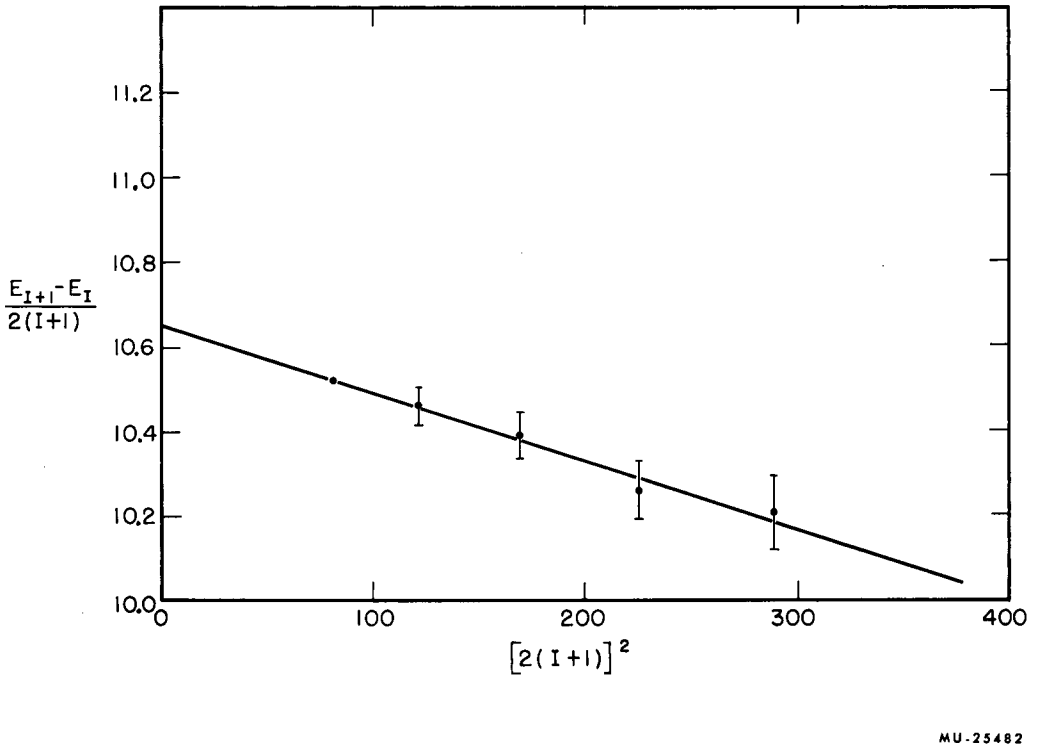


Fig. 10. Analysis of the energies of the ground-state rotational-band members for Ho^{165}.

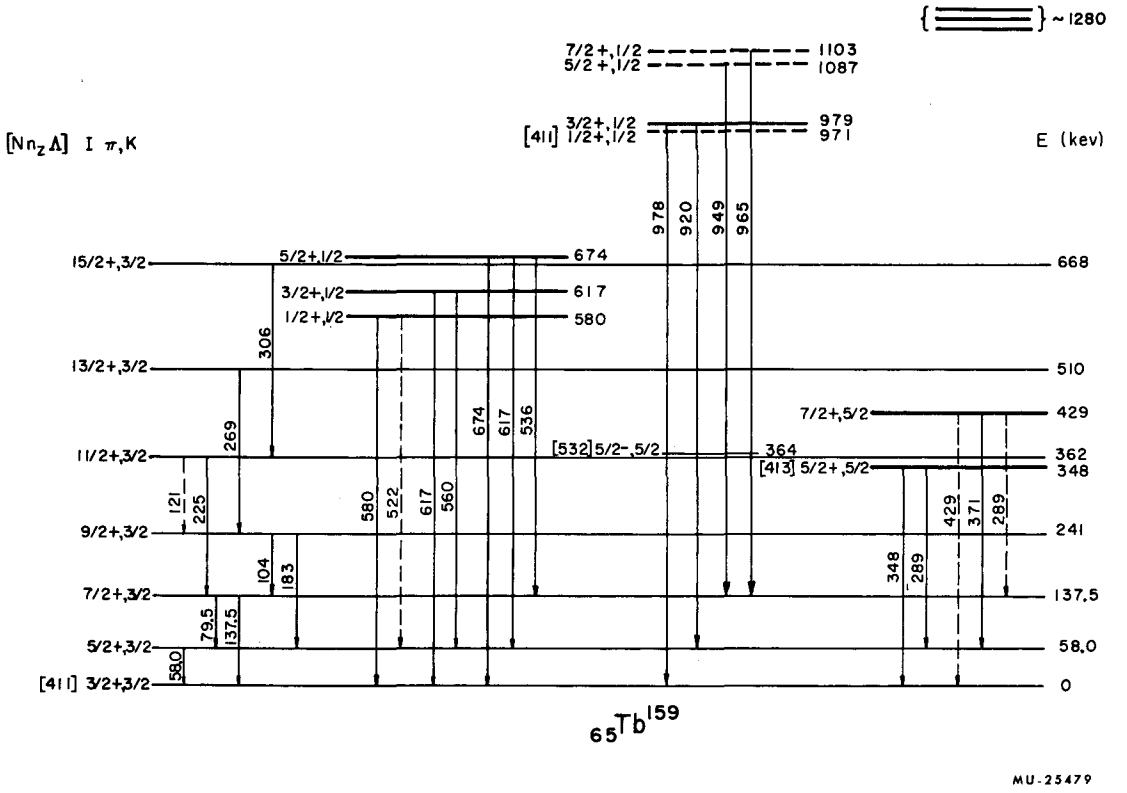


Fig. 11. Level scheme of Tb^{159}. The bands indicated by heavy lines were Coulomb-excited in the present study.

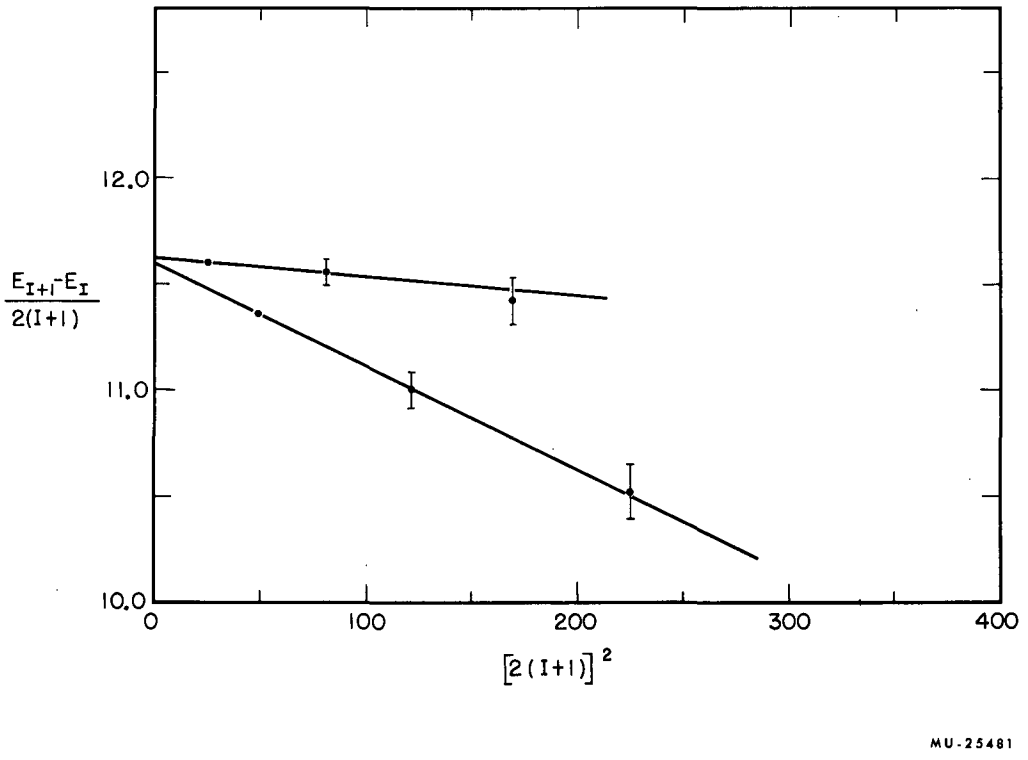


Fig. 12. Analysis of the energies of the ground-state rotational-band members for Tb^{159}.

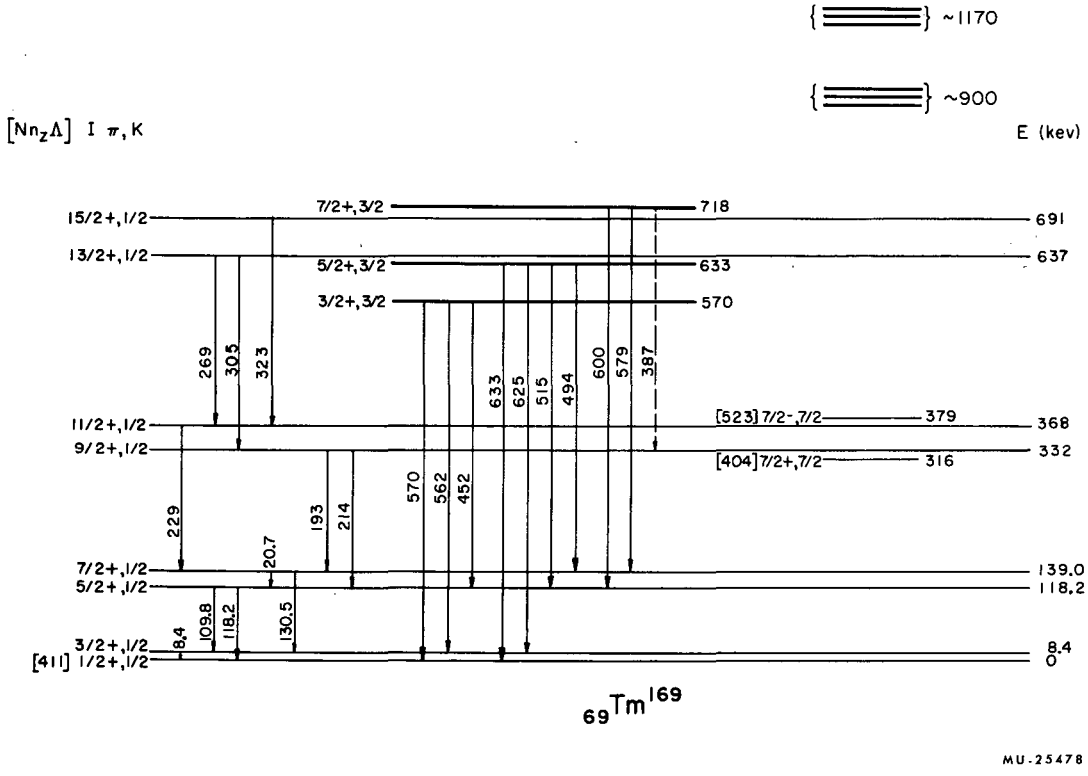


Fig. 13. Level scheme of Tm^{169}. The bands indicated by heavy lines were Coulomb-excited in the present study.

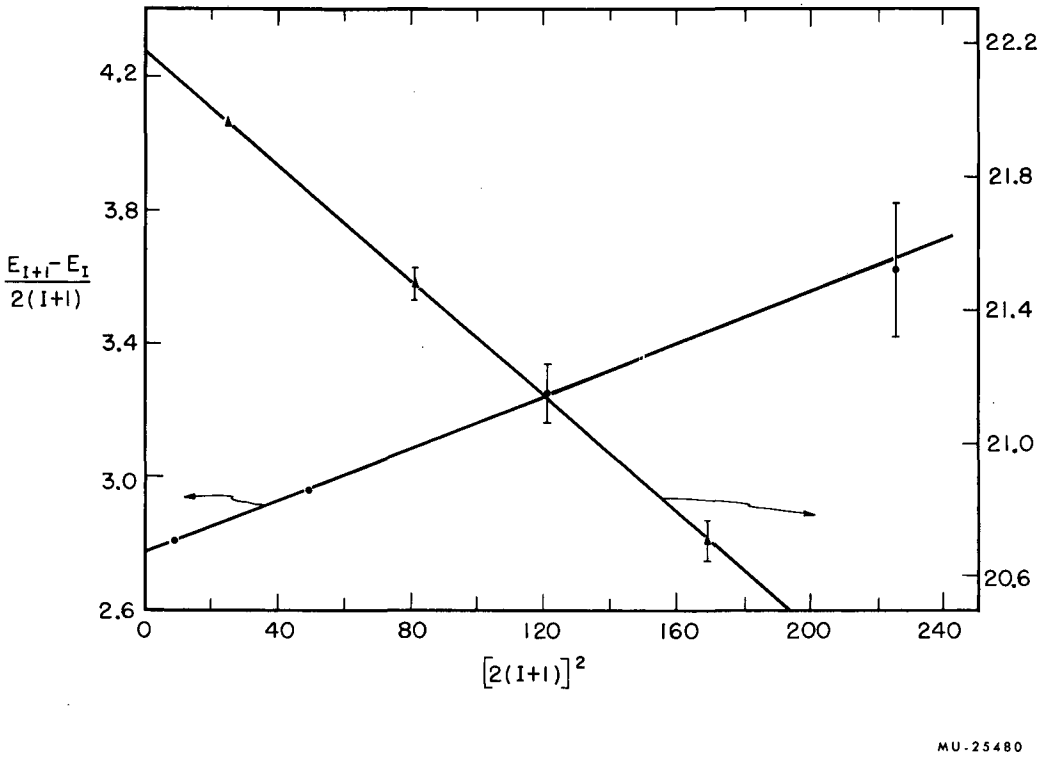


Fig. 14. Analysis of the energies of the ground-state rotational-band members of Tm^{169}.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.