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Berkeley, California

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December 5, 1962

An Approximation Method for Regge Trajectories and its Implications on the Spin-2 Particle of the Chew-Frautschi Diagram*

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The Chew-Frautschi diagram[1] suggests the possibility of a spin-2 particle of mass of about 50 m_π^2 on the Pomeranchuk trajectory. Two experimental groups[2,3] have found such a particle at 80 m_π^2 and at first glance it is tempting to assume that this particle belongs to the Pomeranchuk trajectory. In this letter we suggest a method of approximation consistent with all the generally accepted properties and apply this to the Pomeranchuk trajectory.[4] Our results show that this particle cannot belong to the Pomeranchuk trajectory unless one of the generally accepted assumptions is wrong or the conclusions drawn from the high-energy p-p scattering experiments[5,6] are not correct.

The following features are either proved or conjectured on general grounds:

(a) $\mathrm{Im}\,\alpha(t) = 0 \qquad \text{for } t < t_o,$

and

$$\mathrm{Im}\,\alpha(t) > 0 \qquad \text{for } t > t_o \qquad (t_o = 4\,m_\pi^2)\,, \tag{1}$$

where $\sqrt{t}$ is the center-of-mass energy and $\sqrt{t_o}$ is the two-particle threshold energy;

$$\text{(b)}\quad \lim_{t\to\infty} \alpha(t) = -1\,, \tag{2}$$

$$\text{(c)}\quad \mathrm{Im}\,\alpha(t) \underset{t\approx t_o}{\sim} (t-t_o)^\lambda\,, \qquad \text{where } \lambda = \alpha(t_o) + \frac{1}{2}\,, \tag{3}$$

$$\text{(d)}\quad \mathrm{Re}\,\alpha(t) = \mathrm{Re}\,\alpha(\infty) + \frac{P}{\pi}\int_{t_o}^{\infty} \frac{\mathrm{Im}\,\alpha(t')\,dt'}{t'-t} \quad , \tag{4}$$

where from (a) and (d) $\mathrm{Re}\,\alpha(t)$ is monotonic for $t < t_o$;

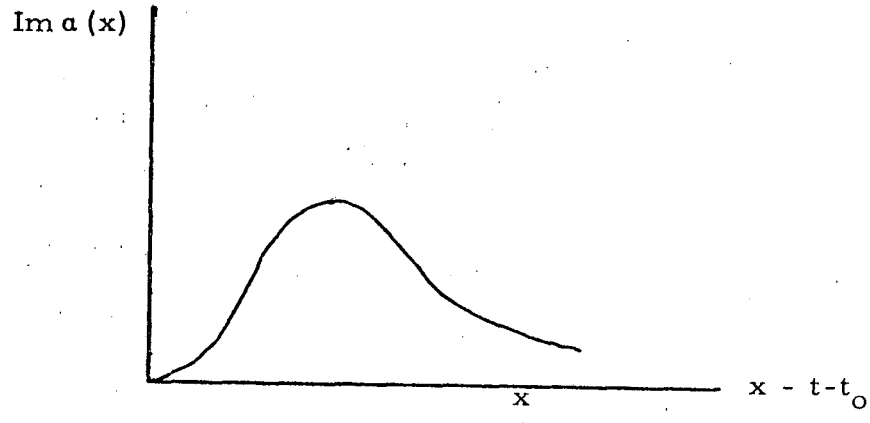
and

$$\text{(e)}\quad \Gamma = \frac{\mathrm{Im}\,\alpha(t)}{\dfrac{d\,\mathrm{Re}\,\alpha(t)}{dt}\sqrt{t}} \tag{5}$$

where Γ is the width of the particle.[7]

Statements (a), (b) and (c) are satisfied at least for the first Regge trajectory of a Yukawa potential.[8] In this case, $\mathrm{Im}\,\alpha$ has a maximum and its derivative vanishes only at one point. Gribov[9] has shown that $\alpha(t)$ for a fermion trajectory has a left-hand cut; therefore, assumptions (a) and (d) should be modified for a fermion. Assumptions (a), (b), and (d) are also satisfied for Coulomb potential;[10] (c) has been proved[11] and (d) has already been conjectured[12] in the relativistic case.

Our aim is to find a plausible form for $\mathrm{Im}\,\alpha$ satisfying the conditions discussed. From experience with the Yukawa potential[8] we expect that for a boson trajectory $\mathrm{Im}\,\alpha$ should have the general shape of Sketch 1.



Sketch 1.

This shape suggests a formula of the type

$$\mathrm{Im}\,\alpha(x) = \frac{c\,x^{\lambda}}{c_1 + (x-c_2)^2} \quad . \tag{6}$$

Such a formula can represent a large variety of curves ranging from a curve with a narrow peak like a δ function to an extremely flat curve. Parameters c, λ, c_1, and c_2 are to be determined from the conditions and experimental information about the trajectory. More generally, we can take $\mathrm{Im}\,\alpha = x^{\lambda}\frac{N}{D}$ where N and D are polynomials in x with no positive roots and the degree of x^{λ}. N is less than that of D for convergence reasons. The number of parameters one can introduce depends on the available information about the trajectory. Using expression (6) for the $\mathrm{Im}\,\alpha(t')$, the integral in Eq. (4) can be evaluated[13] exactly. With the help of the four conditions

$$\alpha(0) = 1 \quad , \quad \text{(see Ref. 1)}$$

$$\lambda = \alpha(t_0) + \frac{1}{2} \quad ,$$

$$\mathrm{Re}\,\alpha(80) = 2 \quad ,$$

and

$$\Gamma(80) = \left(\frac{\mathrm{Im}\,\alpha(t)}{\frac{d\,\mathrm{Re}\,\alpha(t)}{dt}\sqrt{t}}\right)_{t=80} \leq 200 \text{ MeV} \quad ,$$

the four parameters c_1, c_2, c, and λ can be determined numerically from four nonlinear equations. This calculation was carried out at the IBM 7090 computer of the Lawrence Radiation Laboratory. The resulting solution is given by curve (1) of Fig. 1. The values obtained for the parameters are $c_1 = 259{,}66$, $c_2 = 55{,}243$, $c = 3{,}79$, and $\lambda = 1.533$. Parameter c_2 gives approximately the position of the maximum of the $\mathrm{Im}\,\alpha(t)$, and λ controls the slope of the $\mathrm{Re}\,\alpha(t)$ at $t = 0$ which turns out to be 2/5 BeV^{-2}. The width Γ is about 180 MeV. Figure 2 shows $\mathrm{Im}\,\alpha(t)$ for this solution. The differential p-p cross section at high energies has been measured by two experimental groups,[5,6] and from this $\mathrm{Re}\,\alpha(t)$ for negative t is found. Their analysis is based on the generally believed assumption that at these

energies only the contribution of the Pomeranchuk trajectory is significant. For negative t, our solution is in clear contradiction with these experimental points. Taking the experimental points seriously, we conclude that if the aforementioned assumptions are correct the 1250 MeV particle can not belong to the first Pomeranchuk trajectory. There is of course the possibility that it might belong to the second vacuum trajectory hypothesised by Igi.[14] Using the general assumptions (a) through (e) and $\alpha(0) = 1$, we find that a trajectory in reasonable agreement with the experimental data for negative t reaches $\mathrm{Re}\,\alpha = 2$ at an energy much lower than 80 m_π^2. Curve (2) is an example of such a trajectory, which fits the negative t region and predicts a spin-2 particle at 400-750 MeV. The exact prediction of the location of a spin-2 particle is difficult since the experimental data for negative t contain rather large errors. Aside from the 1250-MeV particle there has been recent experimental evidence of a resonance at about 500 MeV having a width of between 50 and 100 MeV, which might be of spin 2 and isospin zero.[15] The existence of such a particle is compatible with our results. From our results it was suggested to us by A. Pignotti that the spin-2 particle of the Pomeranchuk trajectory may be hidden under the ρ resonance.[16] This conjecture too is not incompatible with our result. We leave the question open although from our curves a 500 MeV resonance may be favored. It is interesting to note that if we relax the constraint $\alpha(\infty) = -1$ and find a solution which coincides with a "visual least-squares fit" of the experimental data it would still predict a mass much lower than 1250 MeV (Curve 3). For Curve (3), $\alpha(\infty) = -3.25$

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FOOTNOTES AND REFERENCES

*Work done under the auspices of the U.S. Atomic Energy Commission.

†American Colleges Fellow.

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3. J. J. Veillet, J. Hennessy, H. Bingham, M. Bloch, D. Drijard, A. Lagarrigue, P. Mittner, A. Rousset, Ecole Polytechnique, Paris; G. Bellini, M. di Corato, E. Fiorini, P. Negri, Istituto Nazionale di Fisica Nucleare Milano.
4. G. Domokos has already used an approximation with a different Ansatz for Im α (t) which gives a set of linear equations (CERN Proceedings). Although our Ansatz leads to nonlinear equations which are more difficult to solve we feel that it represents a more economical use of parameters.
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13. We would like to thank A. Pignotti for pointing out that the integration can be carried out if we apply Cauchy's theorem to the imaginary part of $a(t)$. The same integration has been evaluated by Domokos using Feynman's trick.

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16. We would like to mention that A. Pignotti, using assumptions similar to ours, has again arrived at the conclusion that the 1250-MeV resonance can not be on the Pomeranchuk trajectory; see A. Pignotti, Lawrence Radiation Laboratory Report UCRL-10600, December 1962 (unpublished).

FIGURE CAPTIONS

Fig. 1. Re α vs t.

Fig. 2. Im α vs t corresponding to Curve (1) of Fig. 1.

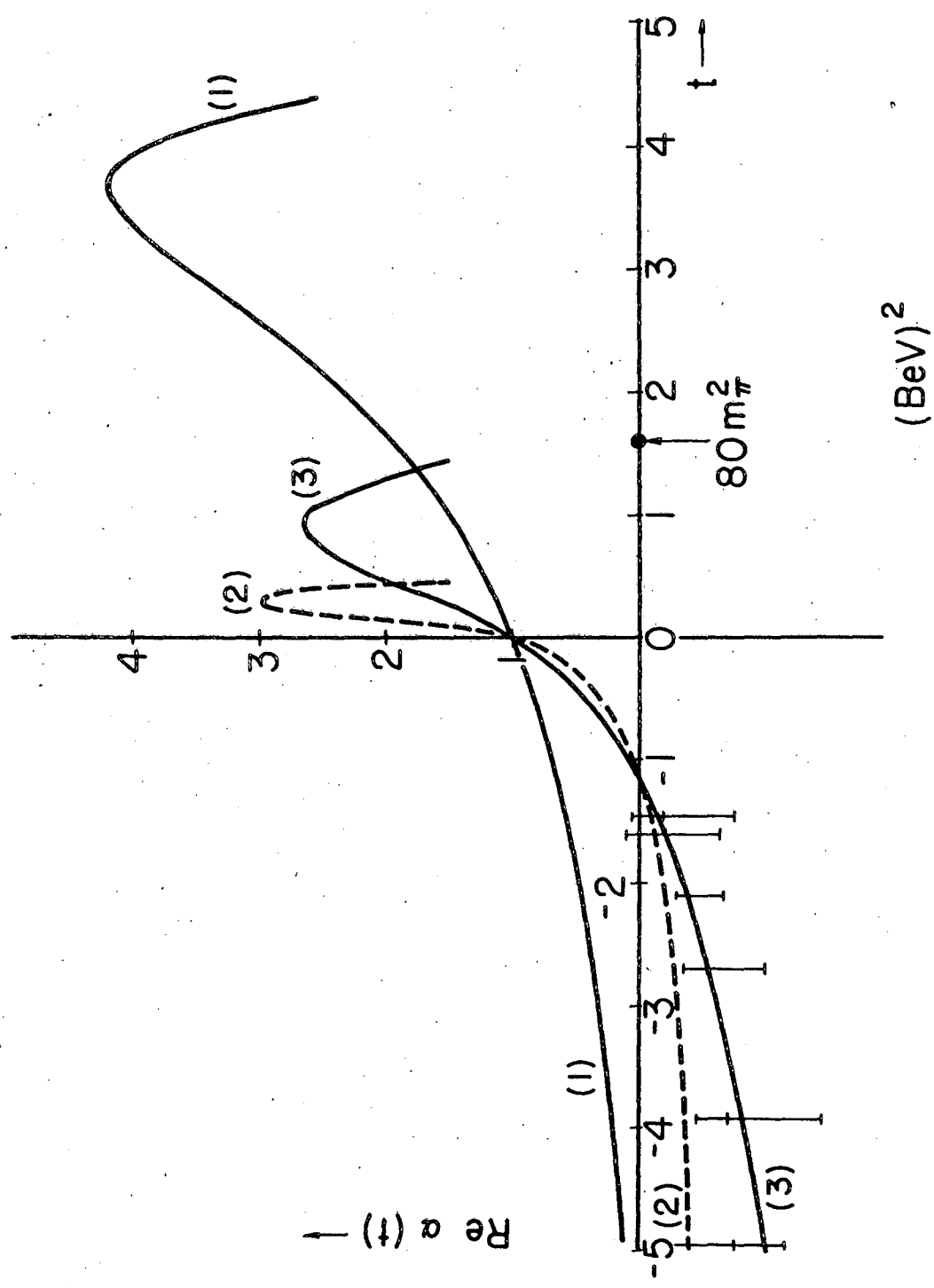

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Fig. 1.

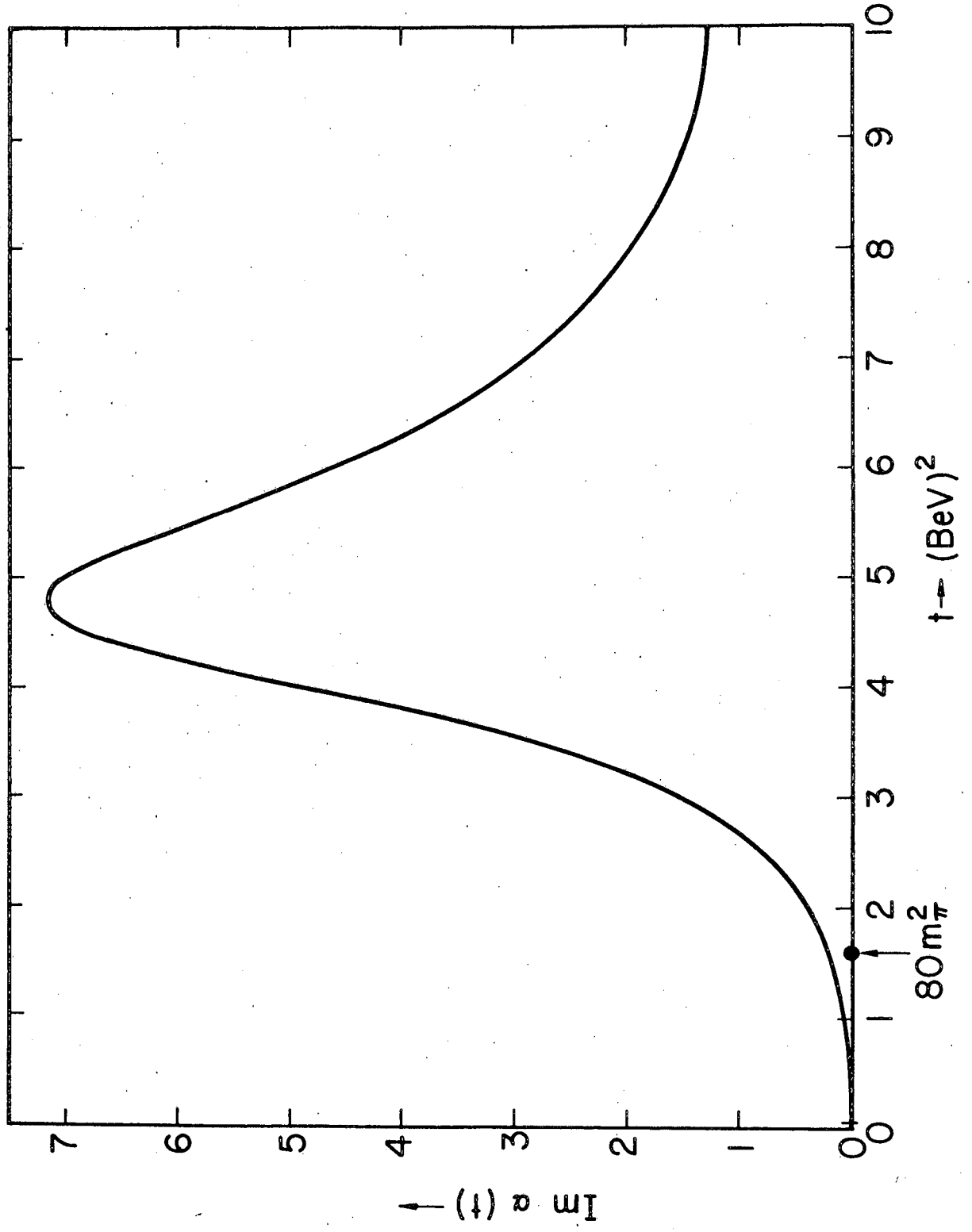

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Fig. 2.

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