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ESTIMATES OF FISSION-FRAGMENT KINETIC-ENERGY DISTRIBUTIONS ON THE BASIS OF THE LIQUID-DROP MODEL

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ABSTRACT

A model is discussed in which a fissioning nucleus is represented by an idealized charged liquid drop whose shape is constrained to the family of shapes generated by two overlapping or separated spheroids. On the basis of this model, one can calculate, among other things, the distribution of total translational kinetic energy of fission fragments. The dynamics of the separation of two completely symmetrical drops was studied using an IBM 7090 computer to integrate the classical equations of motion. For this restricted case it was found that the final translational kinetic energy is approximately related in a very simple way to the initial conditions at the saddle point. The distribution of initial conditions at the saddle is determined by transforming to normal coordinates and assuming statistical equilibrium. This results in a Gaussian probability distribution for each coordinate and conjugate momentum, with a temperature-dependent width. From this and the above-mentioned relationship between kinetic energy and initial conditions, the distribution of final kinetic energy is determined. For a range of nuclei, the experimentally observed most probable kinetic energy is compared with

that predicted by the model. Also, the predicted variation of the width of the kinetic-energy distribution with nuclear temperature is compared with preliminary experimental data.

ESTIMATES OF FISSION-FRAGMENT KINETIC-ENERGY DISTRIBUTIONS ON THE BASIS OF THE LIQUID-DROP MODEL*

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I. INTRODUCTION

I will discuss a model first suggested by Swiatecki in which a fissioning nucleus is represented by an idealized charged drop whose shape is constrained to the family of shapes generated by two overlapping or separated spheroids (ellipsoids of revolution).[1] The model is described here from the point of view of calculating the distribution of total translational kinetic energy (KE) of fission fragments. Various other distributions associated with the fission process, such as the distributions of excitation energy and rotational energy of the fragments, are also being calculated on the basis of the model.

Previous discussions of the liquid-drop model have been largely concerned with the static properties of charged drops; i.e., the calculation of their potential energy (PE) as a function of deformation. However, the properties of the division of a drop are not determined by the PE alone; a complete theory must include dynamics as well as statics.[2]

In the model discussed here, dynamics is included by calculating the KE of the system and solving the corresponding equations of motion. Statistical mechanics is employed to determine the initial conditions

for the solutions to the equations of motion. It should be emphasized from the beginning that the present formulation of the theory is essentially classical, with quantum mechanics being introduced only to determine the initial conditions.

II. POTENTIAL ENERGY OF THE SYSTEM

The model is actually a sub-model within the framework of existing liquid-drop theory; the nucleus is consequently assumed to be uniformly charged and to possess a sharp surface, with hydrodynamic flow that is incompressible, nonviscous, and irrotational. The PE is then simply the sum of a surface energy and a Coulomb energy. For the restricted case of complete symmetry considered here, the system (whose shape, recall, is generated by two overlapping or separated spheroids) is completely specified by two coordinates.[3] The separation coordinate q_1 is the distance between the centers of the two spheroids, while q_2, the semi-major axis of either of the spheroids, measures fragment distortion. (The semi-minor axis of either spheroid is determined by volume conservation.) Figure 1 illustrates the configuration of the system corresponding to various values of the coordinates. (The unit of length is R_0, the radius of the original sphere. Here $R_0 = r_0 A^{1/3}$, where A is the number of nucleons and $r_0 \simeq 1.216$ cm.[4]) The surface energy is easy to calculate, being expressible in a closed form for all values of q_1 and q_2. In order to calculate the Coulomb energy, on the other hand, a double multipole summation is required for the separated region,[1] while a double integral must be evaluated numerically for the overlapping region.

Figure 2 shows the PE surface corresponding to a selected value of the fissionability parameter x, given by[5]

$$x = (Z^2/A)/(Z^2/A)_{crit} , \qquad (1)$$

where Z is the number of protons and $(Z^2/A)_{crit} \simeq 50.13$.[4] (The unit of energy is $E_S^{(0)}$, the surface energy of the original sphere, where $E_S^{(0)} = a_S A^{2/3}$, with $a_S \simeq 17.80$ MeV.[4]) Note the presence of the cusp at the scission line $q_1 = 2q_2$. This results from the discontinuous rate of change of the surface energy as the system passes from the configuration of overlapping to separated spheroids. Note also that for this case the saddle point (SP) lies on the scission line $q_1 = 2q_2$, possessing the shape of two <u>tangent</u> spheroids. This is a characteristic feature of all PE surfaces in this model for $x \lesssim 0.80$; as x decreases from 0.80, the SP moves downward along the scission line until the configuration of two tangent spheres is reached at $x = 0$. For values of $x \gtrsim 0.80$, the SP breaks away from the scission line and occurs for some value of $q_1 < 2q_2$; i.e., the SP shape is the configuration of two overlapping spheroids. As x approaches 1.0, the SP moves toward the spherical configuration.

This behavior of the SP shapes agrees <u>qualitatively</u> with that observed by Cohen and Swiatecki in a calculation employing a large number of degrees of freedom.[6] They found that for $x \lesssim 0.67$ the SP shapes are essentially dumbbell-like (approximated by two tangent spheroids), whereas for $x \gtrsim 0.67$ they are cylinder-like (approximated by two overlapping spheroids). Although the qualitative behavior of the model's

SP shapes is correct, the true transition region occurs at $x \simeq 0.67$, whereas in the two-spheroid model it is at $x \simeq 0.80$. Thus, for describing phenomena associated with SP shapes, the two-spheroid model is not valid for $0.67 \lesssim x \lesssim 0.80$. Nevertheless, the model is still useful for this range of x for discussing phenomena not related to the SP, such as the separation of the fragments after scission. For $x \gtrsim 0.80$, the model again yields SP shapes that are qualitatively correct. The two-spheroid model cannot, of course, be used to describe ternary fission, which in actual nuclei may become important at high x.

III. SOLUTION OF EQUATIONS OF MOTION

The total KE of the system is given by an integral over the nuclear volume of one-half the mass density times the square of the local fluid velocity. For incompressible, nonviscous, irrotational flow, the integration may be performed approximately by using Wheeler's method.[7] This yields a closed expression for the KE as a function of the coordinates, q_1 and q_2, and their time derivatives, $\dot{q}_1$ and $\dot{q}_2$. In the region of separated spheroids, Wheeler's method gives the exact hydrodynamic result. The total KE in this region is then simply the KE of separation of the centers of mass of the spheroids (translational KE) plus the KE of oscillation of each spheroid about its own center of mass (vibrational KE).

From the equations for the potential and kinetic energies the classical equations of motion are determined in a straightforward way.[8] Using a Hamiltonian formulation, one obtains four first-order differential equations, which are solved numerically on an IBM 7090 computer. Thus,

for a given set of initial conditions, a unique solution to the equations of motion is obtained.

The solution for $x = 0.70$ corresponding to starting from rest at the SP is indicated in Figs. 1 and 2, where the points along the path are equally spaced in time at intervals of 5×10^{-23} sec. The motion of the system is a fairly rapid oscillation of the fragments, superimposed on a separation of their centers. Carrying the solution out to infinity, one finds what part of the initial interaction energy goes into translational KE and what part into vibrational KE. We will see later that under certain assumptions the most probable set of initial conditions is the one from rest at the SP; the translational KE resulting from this path will be close to the most probable KE. If the initial conditions specified starting from rest on the scission line at a point above the SP, then the path would oscillate with greater amplitude, resulting in greater vibrational KE and less translational KE. Conversely, starting from rest from a point on the scission line below the SP leads to less oscillation and therefore greater translational KE. Initial conditions corresponding to starting from the scission line with nonzero velocities $\dot{q}_1$ and $\dot{q}_2$ lead to paths that are qualitatively similar, but yet modified somewhat by the additional velocities.

A consideration of the solutions corresponding to a large number of combinations of initial conditions indicates that, when the SP is on the scission line, the final translational KE is approximately related in a very simple way to the initial conditions. For a

given value of x, this relationship is

$$E = E_0/(1 + a\, s_2) + \frac{1}{2}\mu\, \dot{q}_1^{\,2} \quad , \qquad (2)$$

where E is the final translational KE of both fragments, E_0 is its most probable value, a is a constant, μ is the reduced mass of the two spheroids, $\dot{q}_1$ is the initial value of the translational velocity, and s_2 is the initial value of the distance from the SP measured along the scission line (see Fig. 1). To a very good approximation, E is independent of the initial vibrational velocity $\dot{q}_2$. Note the reasonableness of this expression; E is just the final translational KE that would result from two effective point charges initially separated a certain distance and moving with relative velocity $\dot{q}_1$.

The considerations thus far have been limited to the case of nonviscous irrotational flow. Although it is difficult to discuss the dynamics of a fluid with arbitrary viscosity, the other limiting case of infinite viscosity is very simple. Infinitely viscous fragments would simply separate to infinity along a line of constant q_2 without oscillating. The final translational KE would then equal the initial interaction energy of the tangent spheroids. This limiting case would be approached physically if the fragments were sufficiently viscous such that their period of oscillation was large in comparison with the time of separation to a few nuclear diameters.

These two limiting situations are compared in Fig. 3, which shows for each case the dependence on x of the most probable translational KE. Note that the translational KE that would result if the

fragments are extremely viscous (top curve) is larger than the KE corresponding to nonviscous fragments with irrotational flow (solid curve). The difference between these two curves represents the portion of original interaction energy which, for the nonviscous irrotational case, is converted into vibrational KE rather than translational KE . The short-dashed curve is the result obtained using the very simple approximation that the KE is equal to the product of the charges of the spheroids divided by the initial distance between their centers. Of course, this is equivalent to replacing the oscillating spheroids by two rigid spheres whose centers initially coincided with the spheroid centers. That this procedure should give a result which is close to the nonviscous irrotational limit is physically very reasonable, since the fairly rapid oscillations of the fragments tend to cancel the opposing effects of the prolate and oblate shapes. For comparison, the recent experimental points of Viola and Sikkeland are also shown.[9]

IV. DISTRIBUTIONS OF INITIAL CONDITIONS

We have approximately solved the equations of motion in terms of given initial conditions at the SP . The question now arises of what initial conditions to use. The conditions at the SP will in general depend upon the past history of the system in a way that cannot be predicted from SP considerations alone; it would then be necessary to consider the pre-SP motion of the system. Studies along these lines were performed by Hill,[2] and have also been considered in

the two-spheroid model. A simplifying limiting case that eliminates the need for discussing the pre-SP motion is the assumption of statistical equilibrium at the SP. Without committing ourselves as to the validity of this assumption, we will trace out its consequences with regard to KE distributions.

A normal coordinate transformation is performed at the SP which defines a new set of coordinates s_1 and s_2 in terms of q_1 and q_2. The relationship is indicated in Fig. 1; s_1 is the coordinate in the "fission direction" (approximately an overall separation of the fragments), with s_2 corresponding to vibrations about the SP in which the spheroids remain tangent. In terms of the normal coordinates and their time derivatives $\dot{s}_1$ and $\dot{s}_2$, both the PE and the KE are simultaneously diagonalized (i.e., the cross terms $s_1 s_2$ and $\dot{s}_1 \dot{s}_2$ do not appear) at the SP. The Hamiltonian for the system thus separates into a sum of two terms, one involving only s_1 and $\dot{s}_1$, with the other being a function of only s_2 and $\dot{s}_2$. Physically, the motion of the system at the saddle separates into two independent modes which may be discussed separately: motion in the fission direction s_1 and vibrational motion in the direction s_2.

The time has been chosen to be zero when the fission coordinate s_1 is zero. This leaves the distribution of initial values of $\dot{s}_1$, s_2, and $\dot{s}_2$ to be determined. If we assume statistical equilibrium at the saddle, the classical probability distribution of $\dot{s}_1$ is proportional to $\exp(-\frac{1}{2} M_1 \dot{s}_1^2/\theta)$, where M_1 is the effective mass for motion in the s_1 direction, and θ is the nuclear temperature at the SP (measured in units of energy).

Recall that the entire discussion up to this point has been purely classical. Quantum mechanics is now introduced to determine the distributions of initial conditions for the vibrational coordinate and velocity s_2 and $\dot{s}_2$.[10] We thus consider the quantum-mechanical solution of the vibrational motion at the SP. To lowest order in s_2, the term in the Hamiltonian corresponding to the s_2 motion is

$$H_2 = \frac{1}{2} K_2 s_2^2 + \frac{1}{2} M_2 \dot{s}_2^2 , \tag{3}$$

where K_2 is the second derivative of the PE with respect to s_2 evaluated at the SP, and M_2 is the effective mass for motion in the s_2 direction. Of course, this is just the Hamiltonian for a simple harmonic oscillator. The solution of this part of the problem thus yields the harmonic-oscillator wave functions $\psi_n(s_2)$ with corresponding energy levels $E_n = (n + \frac{1}{2})\hbar\,\omega_2$, where $\hbar$ is Planck's constant divided by 2π, and ω_2 is the angular frequency, given by $(K_2/M_2)^{1/2}$.

The quantum-mechanical probability that the system initially is at position s_2 is then given by

$$P(s_2) = \sum_{n=0}^{\infty} P_n \, |\psi_n(s_2)|^2 , \tag{4}$$

where P_n is the probability that the oscillator is in the quantum-mechanical state n.[11] The factor $|\psi_n(s_2)|^2$ gives the probability that an oscillator known to be in the state n has position coordinate s_2. The assumption of statistical equilibrium at the SP implies

that P_n is proportional to $\exp(-E_n/\theta)$. If we substitute this for P_n and use properties of the harmonic-oscillator wave functions ψ_n, the infinite summation (4) yields[12,13]

$$P(s_2) = (\pi C)^{-1/2} \exp(-s_2^2/C) , \tag{5}$$

where the temperature-dependent constant C is given by

$$C = \frac{\hbar \omega_2}{K_2} \coth(\hbar \omega_2/2\theta) \longrightarrow \begin{cases} 2\theta/K_2 , & \theta \gg \hbar \omega_2 \\ \hbar \omega_2/K_2 , & \theta \ll \hbar \omega_2 . \end{cases} \tag{6}$$

Note that for high temperatures $P(s_2)$ reduces to the classical probability distribution proportional to $\exp(-\frac{1}{2} K_2 s_2^2/\theta)$; in the low-temperature limit, it reduces to the distribution for the quantum-mechanical zero-point motion of a harmonic oscillator, proportional to $\exp\left[-\frac{1}{2} K_2 s_2^2/(\frac{1}{2}\hbar \omega_2)\right]$.

An analogous probability distribution for $\dot{s}_2$ is obtained by solving Schrödinger's equation for the s_2 motion in the momentum representation. The result is[12]

$$P(\dot{s}_2) = (\pi C')^{-1/2} \exp(-\dot{s}_2^2/C') , \tag{7}$$

where

$$C' = \frac{\hbar \omega_2}{M_2} \coth(\hbar \omega_2/2\theta) , \tag{8}$$

with corresponding high- and low-temperature limits.

V. DISTRIBUTION OF TRANSLATIONAL KINETIC ENERGY

We have now determined two things: (1) an approximate formula for the final translational KE in terms of the initial conditions at the SP [Eq. (2)], and (2) the probability distributions for these initial conditions. The distribution of translational KE is then given by an appropriate multiple integral over the distributions of initial conditions. Through a change of variables, the expression is reduced to a single integration, which must be evaluated numerically.

Before examining distributions calculated with this complicated result, let us derive a simple expression for the KE distribution in which only the lowest-order contribution is considered. If we neglect the dependence of E on the initial velocity $\dot{q}_1$ entirely, a Taylor expansion about the position of the SP gives

$$E \simeq E_0 + \left(\frac{dE}{ds_2}\right)_0 s_2 \quad . \tag{9}$$

The probability distribution for the final total translational KE is then independent of $P(\dot{s}_1)$ and $P(\dot{s}_2)$, being determined solely by $P(s_2)$. One obtains, upon substituting for s_2 in (5),

$$P(E) = P(s_2)\left|\frac{ds_2}{dE}\right| \simeq \frac{1}{\left[\pi\, C\left(\frac{dE}{ds_2}\right)_0^2\right]^{1/2}} \exp\left[-\frac{(E - E_0)^2}{C\left(\frac{dE}{ds_2}\right)_0^2}\right] \quad . \tag{10}$$

Thus, to lowest order, the KE distribution is a Gaussian centered about the most probable value E_0, with a full width at half maximum (FWHM) proportional to $C^{1/2}(dE/ds_2)_0$.

The temperature dependence of the constant C reflects itself

in a FWHM of the KE distribution that is also temperature-dependent. As the temperature approaches zero, the FWHM approaches a finite value determined by the quantum-mechanical zero-point vibrations of the oscillator in the s_2 direction. Although the magnitude of the zero-point FWHM follows directly from our equations, the physical reason for its largeness can be more easily seen from Fig. 4. The solid curve indicates the dependence of the KE on the initial value of s_2 (for $\dot{q}_1 = 0$), while the dot-dashed curve is the lowest-order approximation to the PE of the system (along the scission line $s_1 = 0$). Note the relative flatness of the PE ; this results from the near cancellation of the opposing effects of the surface and Coulomb energies near the SP . The value of the PE at which the probability of initially finding the system falls to one-half its maximum value is shown by the dotted line. (This value of PE is an increase of $(\ln 2)(\frac{1}{2}\hbar\omega_2) \simeq 0.35\,\hbar\omega_2$ from the minimum. The calculated vibrational frequency $\hbar\omega_2$ is ~ 0.8 MeV .) The relative steepness of the KE curve means that this very small uncertainty ~ 0.3 MeV in the PE is "amplified" into a rather large zero-point FWHM ~ 12 MeV in the KE.[14] As the nuclear temperature increases, the uncertainty in PE increases, resulting in a larger FWHM.

Shapes of KE distributions calculated using the original expression (2) for E and taking into account all three distributions of initial conditions are shown in Fig. 5. These curves are very similar to curves calculated using the simple expression (10), indicating that the lowest-order approximation is fairly good. Note the increase

in width as the nuclear temperature increases.

The predicted variation of the FWHM of the KE distribution with nuclear temperature is compared with preliminary experimental data in Fig. 6.[15] The solid circles are the data of Plasil[16] for the compound nucleus ${}_{82}Pb^{198}$, formed by the heavy-ion bombardment of ${}_{74}W^{182}$ by ${}_{8}O^{16}$. Data for the compound nucleus ${}_{85}At^{213}$, formed from the reaction ${}_{2}He^{4} + {}_{83}Bi^{209}$, are indicated by the open circle (Burnett)[17] and the open square (Unik, et al.).[18] The value of the fissionability parameter x for both compound nuclei is 0.677.

The reported experimental values have been corrected for the effects of dispersion due to neutron emission. For the higher temperatures, where several neutrons are emitted from each fragment, this correction is rather large (~ 8 MeV). The nuclear temperature is determined in terms of the excitation energy at the SP from the semi-empirical nuclear equation of state[19]

$$E_{ex}^{SP} = \frac{1}{8} A \theta^2 - \theta \qquad (11)$$

The excitation energy at the SP is in turn given by the total bombarding energy in the center-of-mass system, plus the binding energy of the projectile to the target, minus the liquid-drop fission-threshold energy. The effect of angular momentum on the threshold energy has been taken into account for the heavy-ion-induced reactions.[20] The error bars reflect, in addition to experimental errors, uncertainties in the conversion from bombarding energy to nuclear temperature and in the correction for neutron-emission effects.

Experimental points at lower and at higher excitation energies would be extremely valuable. If the "flattening" of the curve to a constant value at low temperatures and the linearity of the curve at high temperatures could be verified experimentally, two of the theory's major predictions would be confirmed. At extremely low excitation energies, fission cross sections, of course, become so small that experimental difficulties are encountered.

VI. DISCUSSION AND SUMMARY

We have discussed a very simple sub-model of the liquid-drop model in connection with the calculation of fission-fragment KE distributions. A step beyond the usual liquid-drop "static" discussion of PE surfaces has been taken by considering the dynamics of the separation of the fragments. From results of this study, we have calculated the most probable KE and also, for the limiting case of statistical equilibrium at the SP, widths of KE distributions. The preliminary comparisons of calculations with experiment suggest the following conclusions. The order of magnitude of the experimental most probable kinetic energies and widths are reproduced by the calculations. This is perhaps the most significant result, since there are <u>no adjustable parameters</u> in the theory. (The constants of the Bethe-Weizsäcker semi-empirical mass formula have been taken from Green's analysis.[4]) The experimental trend of the most probable KE with fissionability parameter in Fig. 3 also seems to be approximately reproduced, but there are indications of systematic deviations outside the experimental errors. Although the experimental increase in widths

with nuclear temperature in Fig. 6 is roughly in accord with theory, the trend is not sufficiently well defined to make the approximate agreement very significant.[21]

On the whole, preliminary comparison with experiment suggests that the limitations of the liquid-drop model — in its simplified two-spheroid approximation — are not yet in evidence to a serious degree. The kinetic-energy distribution discussed in this paper is only one of some half-dozen distributions of fission properties that may be deduced from the two-spheroid model (in its more general asymmetric noncollinear form). It is hoped that by comparing the predictions of several different types of fission distributions with experiment, some idea of the relevance of the liquid-drop model for discussing fission phenomena can be obtained. It is also hoped that some conclusions can be reached regarding such questions as the viscosity of nuclear matter and the establishment of statistical equilibrium at the SP .

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Stimulating discussions with several individuals, particularly Drs. Stanley G. Thompson and Eldon Lee Haines and Messrs. Donald S. Burnett and Frank Plasil, are acknowledged. Special thanks are due the latter two for the use of their preliminary data.

Thanks are also extended to Dr. J. C. D. Milton for the opportunity to compare results of some of my computer codes with his tables, and to Drs. Stanley Cohen and W. J. Swiatecki for the use of their saddle-point-searching computer code.

FOOTNOTES AND REFERENCES

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3. The more general case in which the two spheroids are of unequal volume, have unequal semi-major axes, and are not collinear is currently being studied.

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6. S. Cohen and W. J. Swiatecki, The Deformation Energy of a Charged Drop: Part V: Results of Electronic Computer Studies, Lawrence Radiation Laboratory Report UCRL-10450, Aug. 1962.

7. John Archibald Wheeler (Princeton University), unpublished work.

8. The use of classical equations of motion may be partially justified on the grounds that a short distance from the SP the de Broglie wavelength for translational motion has become relatively small, and that the oscillations about each fragment's center of mass involve several quanta of energy.

9. V. E. Viola, Jr. and T. Sikkeland, Kinetic Energy Release in Heavy-Ion-Induced Fission Reactions, Lawrence Radiation Laboratory Report UCRL-10284, Oct. 1962, p. 30.

10. A completely consistent calculation would, of course, treat the system quantum mechanically both in the SP region and in the separated region. The present mixture of classical and quantum mechanics represents the first step in a series of refinements aimed at a consistent quantum-mechanical solution.

11. The same symbol P is used to denote each of several probability distributions; the argument or subscript indicates which explicit function is being referred to.

12. L. D. Landau and E. M. Lifshitz, Statistical Physics, E. Peierls and R. F. Peierls, translators (Addison-Wesley, Reading, Mass., 1958), pp. 86-89.

13. An analogous formula has been used by M. Blann (University of Rochester) and W. J. Swiatecki (Lawrence Radiation Laboratory, Berkeley) in connection with fission-fragment charge distributions (unpublished work).

14. Similar ideas regarding an amplification mechanism have been described by Robert Vandenbosch, Dependence of Fission Fragment Kinetic Energies and Neutron Yields on Nuclear Structure, Institute for Theoretical Physics, University of Copenhagen, Denmark, and Argonne National Laboratory, Argonne, Illinois (n.d.), Preprint.

15. The predicted FWHM refers to symmetric-mass fission, while the reported experimental values include all mass ratios. The comparison is possible because for this low value of x, there is experimentally little difference, if any, between the FWHM for symmetric-mass fission and for all mass ratios.

16. Frank Plasil (Lawrence Radiation Laboratory, Berkeley), private communication.

17. Donald S. Burnett (Lawrence Radiation Laboratory, Berkeley), private communication.

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21. In addition to experimental uncertainties, the present techniques employed in tracing out the implications of the model are not entirely satisfactory. Either the mixture of quantum-mechanical

initial conditions with solutions to classical equations of motion must be fully justified from first principles, or a complete quantum-mechanical calculation must be performed. On the other hand, for most of the data compared here, the nuclear temperature is sufficiently high that classical statistical mechanics is valid for determining the initial conditions, and the ambiguities associated with the mixture of classical and quantum mechanics are not present.

FIGURE LEGENDS

Fig. 1. Symmetric two-spheroid configurations for selected coordinates (marked by +). The solution of the equations of motion for $x = 0.70$ corresponding to starting from rest at the saddle point is indicated by the points. The normal coordinates at the saddle for this value of x are designated by s_1 and s_2 .

Fig. 2. Map of the potential energy for $x = 0.70$. The solution of the equations of motion corresponding to starting from rest at the saddle point is indicated by the points.

Fig. 3. Most probable fission-fragment translational kinetic energy as a function of x. The three curves shown are for infinitely viscous fragments (top curve), fragments with nonviscous irrotational flow (solid curve), and a simple approximation to the latter (short-dashed curve). The data are those of Viola and Sikkeland.[9]

Fig. 4. Illustration of the effect of zero-point vibrations on the width of the KE distribution. The relative flatness of the PE (dot-dashed curve, right ordinate) means that an extremely small uncertainty in PE ~ 0.3 MeV is amplified into a rather large zero-point FWHM ~ 12 MeV in the KE (solid curve, left ordinate). The normal coordinate s_2 is given along the scission line by $s_2 = \sqrt{5}\,(q_2 - q_2^{SP}) = (\sqrt{5}/2)(q_1 - q_1^{SP})$. The zero of PE is at the initial sphere.

Fig. 5. Calculated fis n-fragment translational kinetic-energy distributions for ious nuclear temperatures.

Fig. 6. Full width at half maximum of fission-fragment translational kinetic-energy distributions as a function of nuclear temperature. The data are for the compound nuclei: $_{82}Pb^{198}$ (solid circles, Plasil[16]) and $_{85}At^{213}$ (open circle, Burnett[17]; open square, Unik, et al.[18]). The abscissa is scaled linearly with respect to the square root of the temperature.

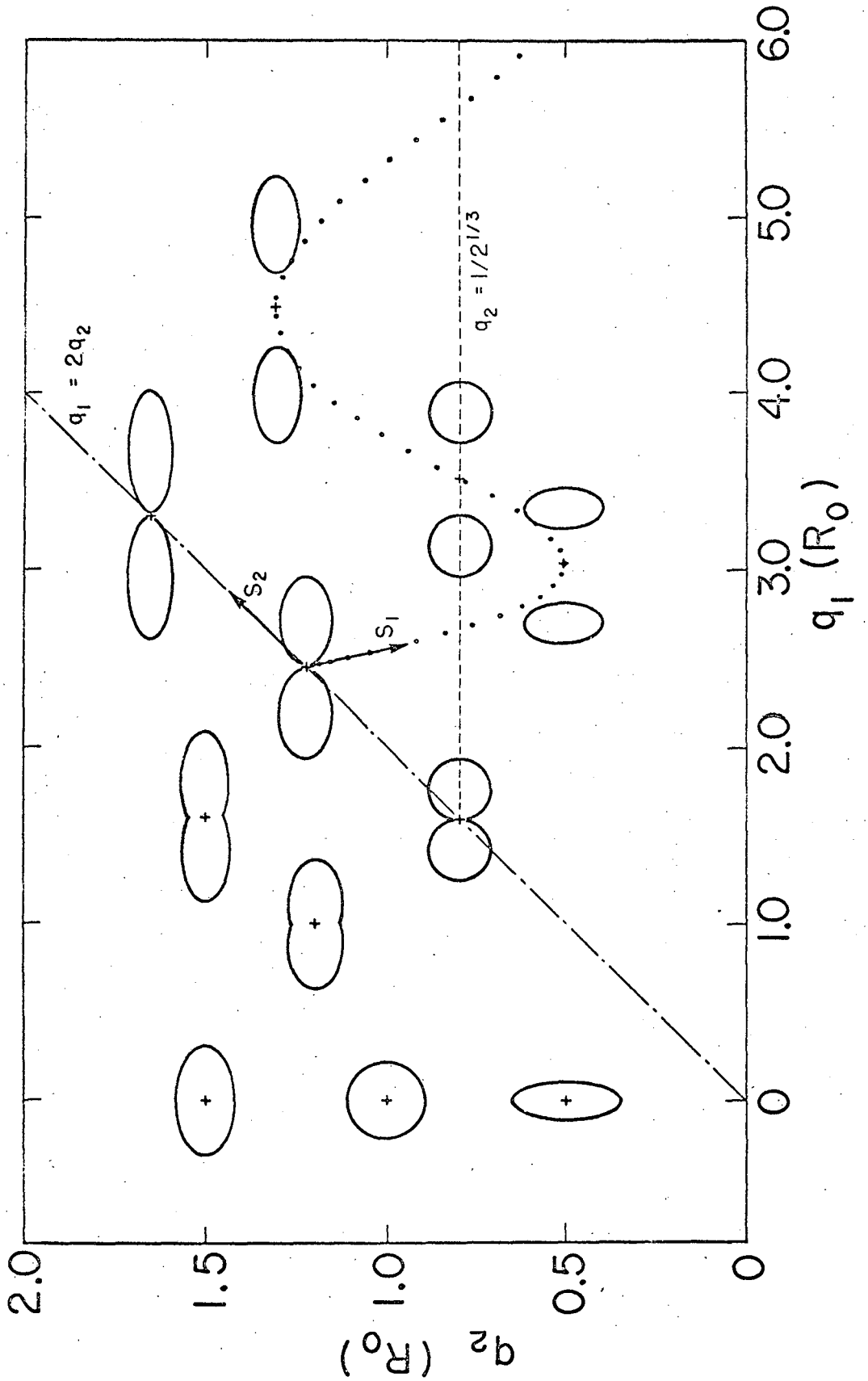
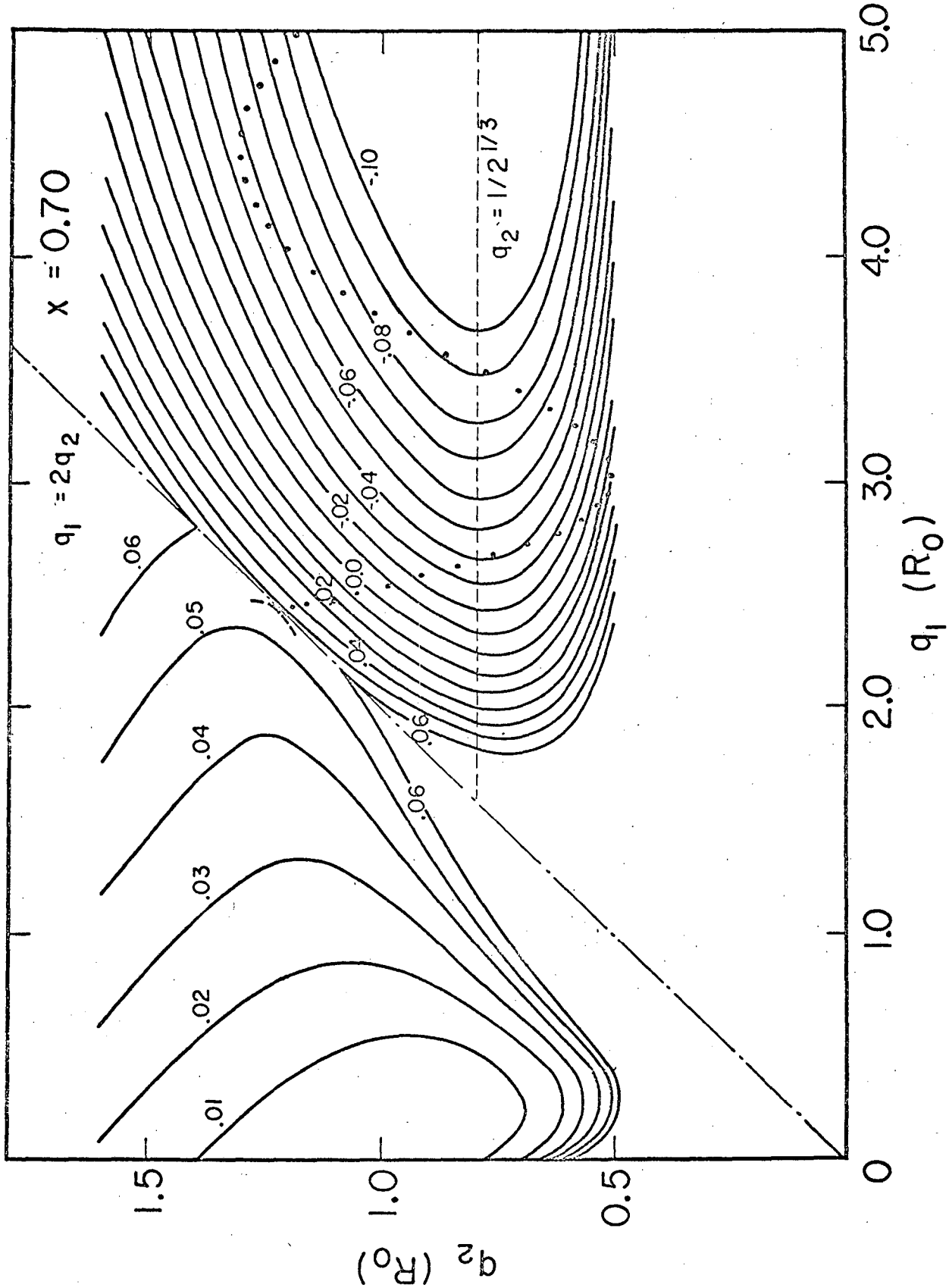

MU-29987
$q_1 = 2q_2$
$q_2 = 1/2^{1/3}$
s_2
s_1
q_1 (R_o)
q_2 (R_o)
0
1.0
2.0
3.0
4.0
5.0
6.0
0.5
1.0
1.5
2.0

Fig. 1.



MU-29479-A

Fig. 2.

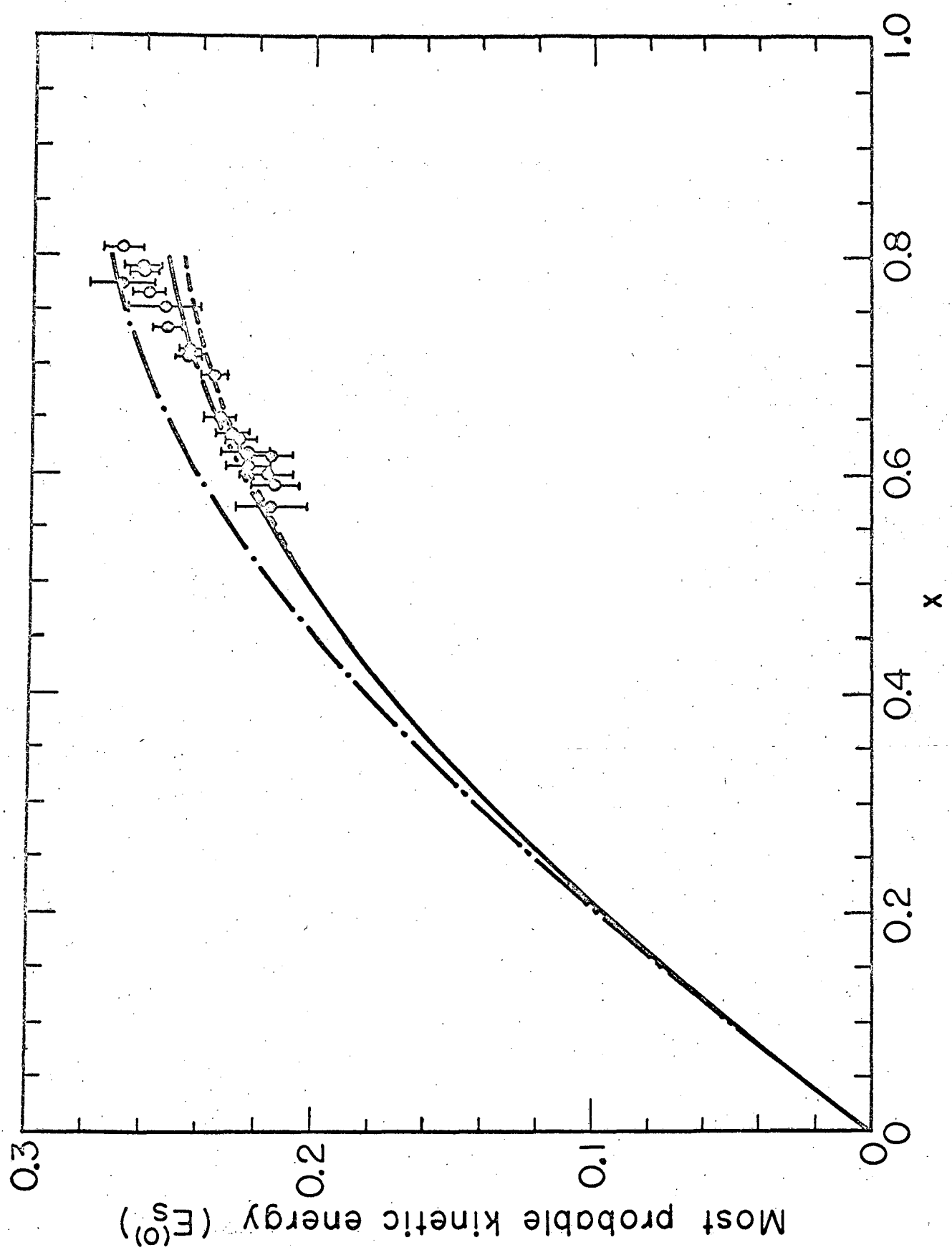


Fig. 3.

MU-29983

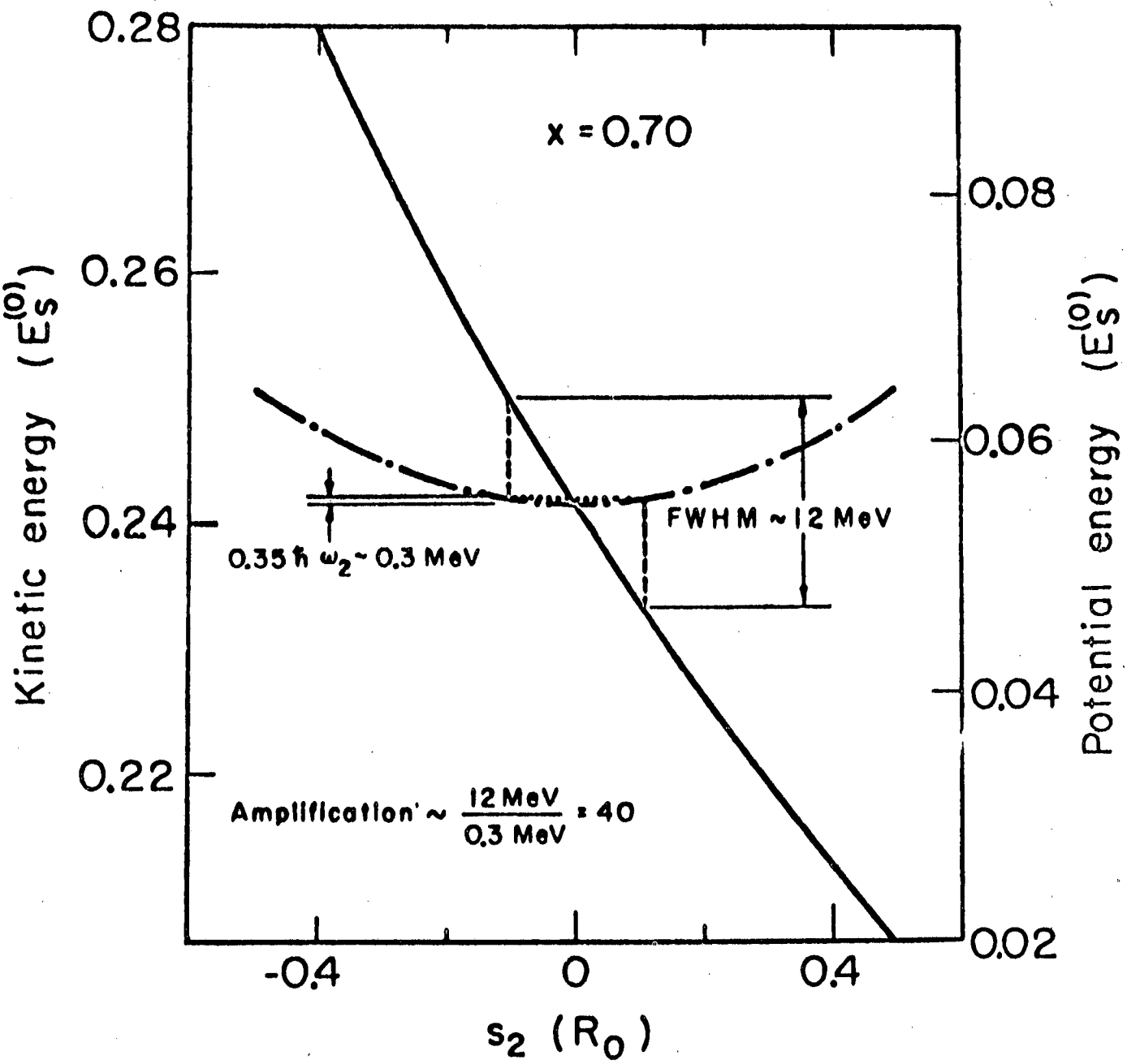


Fig. 4.

MU-29986

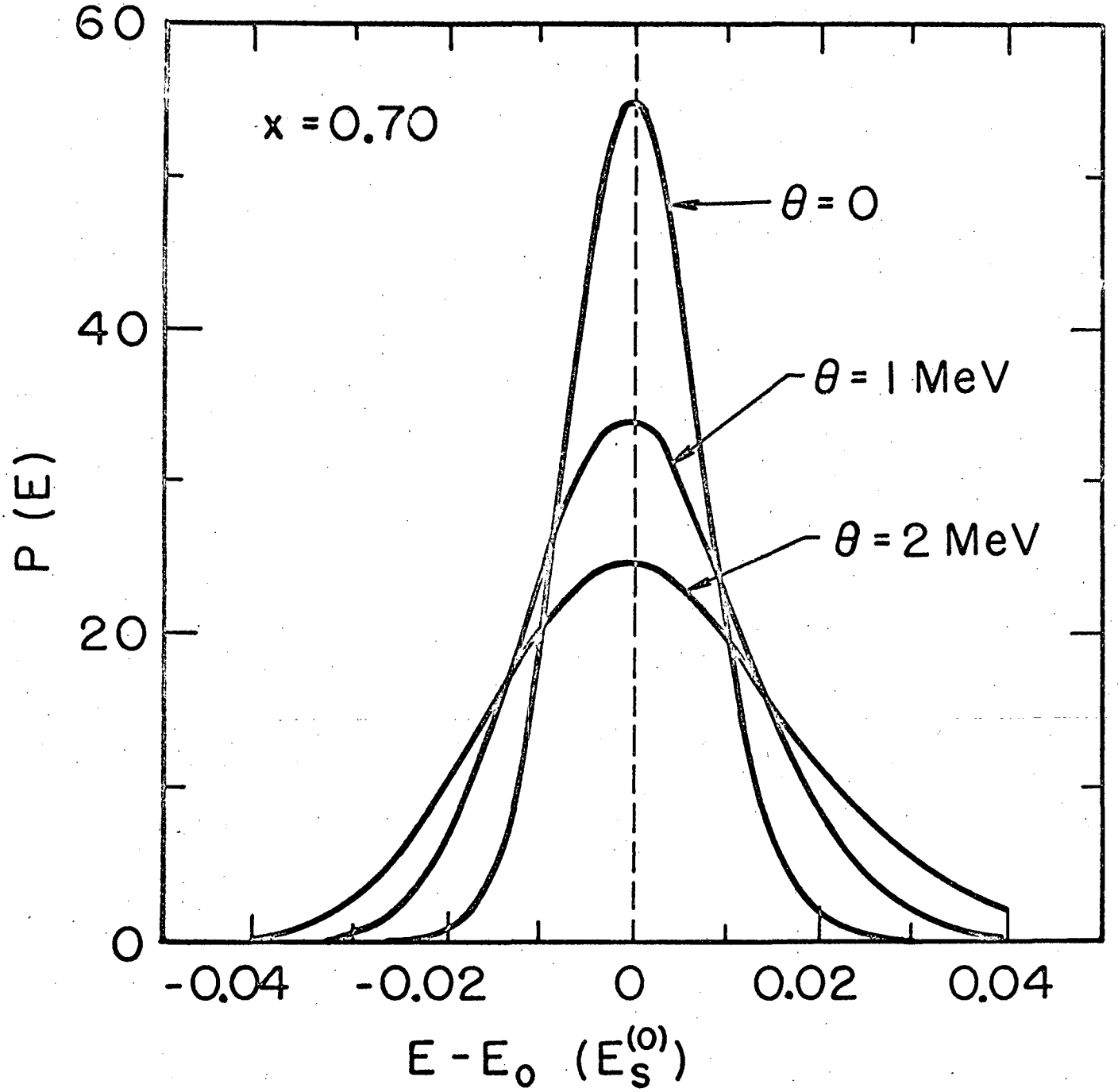
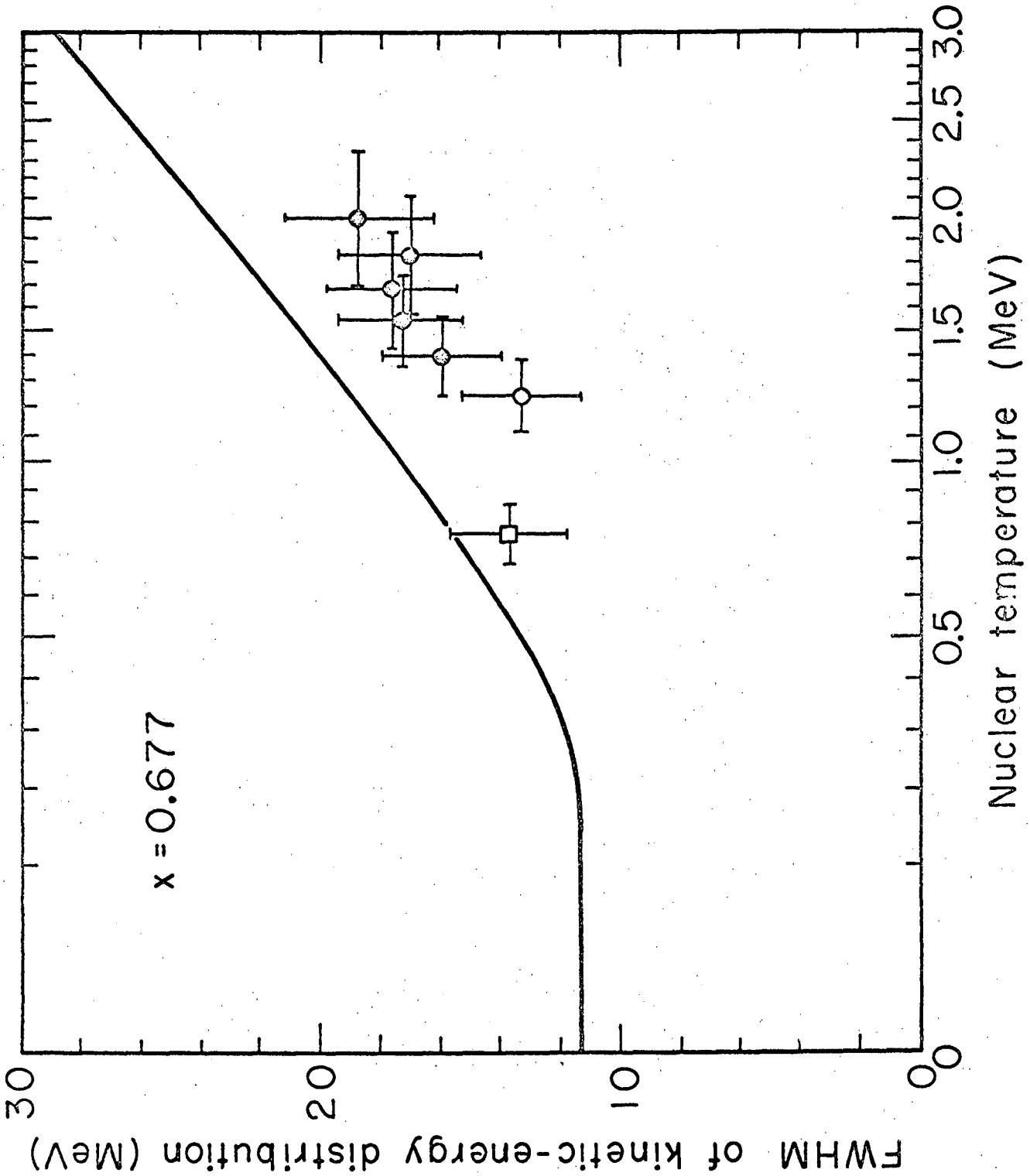


Fig. 5

MU-29985



MU-29984

Fig. 6.

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