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IN A MAGNETIC FIELD

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ABSTRACT

The positive column of a glow discharge in a magnetic field is known to become macroscopically unstable at some critical magnetic field. The driving mechanism proposed by Kadomtsev and Nedospasov for the growth of a helical ( $m = 1$ ) perturbation is such that the periodic reversal of the axial electric field at a frequency greater than a critical frequency would be expected to lead to stability. This critical frequency is expected to be approximately the e-folding rate. An experimental study has been made in dc and half-wave rectified-current discharges and in square and sinusoidal ac glow discharges in  $H_2$ ,  $D_2$ , He, Ne, and Ar at frequencies as high as 70 kc. Over a wide range of frequencies the half-wave rectified discharges yielded instabilities whose onset and helical nature duplicated those of the dc cases. However, the instability in the ac discharges was indeed suppressed as the frequency of electric field reversal was increased. The frequency-dependent suppression of the instability is compared with theoretical growth rates. The effect of including several  $m$  modes is calculated and good qualitative agreement is obtained with the Kadomtsev and Nedospasov model. However, better quantitative agreement is obtained when the Johnson and Jerde modification is employed.

## I. INTRODUCTION

In 1955 Bickerton and von Engel investigated the positive column of a helium discharge in a longitudinal magnetic field, making the discharge long enough to avoid end effects.<sup>1</sup> The maximum field strength was approximately 600 gauss. As the magnetic field increased in strength, the loss of charged particles to the wall, and hence the electric field necessary to maintain the discharge, was observed to decrease monotonically, as expected. This work was extended by Lehnert<sup>2</sup> and Hoh and Lehnert,<sup>3</sup> who applied larger magnetic fields in a similar geometry. Above a critical magnetic field,  $B_c$ , the axial electric field increased markedly, indicating an increased loss of particles to the wall.

Examination of this phenomenon by Allen et al.,<sup>4</sup> and by Paulikas and Pyle,<sup>5</sup> showed that at  $B_c$  a macroscopic instability appears in the column, transforming the azimuthally symmetric discharge into a rotating helix. The rotational frequency and wavelength of the helix as well as the critical magnetic field strengths were documented as functions of the discharge parameters.

The generally accepted theory has been given by Kadomtsev and Nedospasov (hereafter referred to as K-N), who considered the growth of a screw-shaped perturbation superimposed on a stationary electron density and potential distribution.<sup>6</sup> Using the same basic model, others have investigated the effect of different assumed distributions and boundary conditions.<sup>7-9</sup>

According to the K-N model, the driving term of the instability is such that one would expect that the instability could not grow if the axial electric field were reversed at a sufficiently high frequency, for example before the perturbation could more than e-fold. Because the growth rate of the instability increases with increasing magnetic field, we would expect that higher and higher reversal frequencies would be required as  $B$  becomes larger. Our experiment documents

the onset of the instability as a function of the discharge parameters, including the ac frequency. Sine-wave and square-wave currents were employed, the latter being most amenable to a theoretical comparison, i. e., producing a discharge in which the plasma parameters (except for the direction of the electric field) are approximately independent of time. A comparison is then made between experimental results and quantities calculated from the dc theory of Kadomtsev and Nedospasov and a modification of this theory by Johnson and Jerde (J-J).<sup>8</sup>

The basic mechanism may not be restricted to a positive column. Simon and Guest have shown that instability can develop in the absence of an externally applied electric field, because of unequal streaming rates of ions and electrons in the direction of the magnetic field.<sup>10</sup> Since the proposed instability mechanism is not very sensitive to the degree of ionization, Hoh suggests it may also be operative in highly ionized plasmas such as those in the Stellarator and Zeta machines.<sup>7</sup> The applicability of the model to fully ionized gases has also been demonstrated by Lehnert.<sup>11</sup> Similar instabilities have been observed in a highly ionized cesium plasma<sup>12</sup> and in the electron-hole plasma of a semiconductor.<sup>13</sup> The theory of the latter has been given by Glicksman.<sup>14</sup>

Previous ac glow-discharge work in this frequency range (below 100 kc) is very sparse; a fairly complete summary of the existing information is given by Francis.<sup>15</sup>

## II. THEORY

The axial electric field in a uniform steady-state positive column is such that the number of electrons and ions produced per second exactly balances the loss rate to the walls. Making the usual assumptions,<sup>16</sup> the radial density distribution is given by

$$n(r) = n(0)J_0 \left( \frac{2.4}{R} r \right). \quad (1)$$

The argument of the Bessel function has been determined by assuming the Schottky condition ( $n = 0$  at the wall). Similarly the potential distribution as a function of radius is given by

$$V(r) = \frac{kT_-}{e} \frac{1 + \mu_+ \mu_- B^2 \left( \frac{\mu_+}{\mu_-} - \frac{T_+}{T_-} \right)}{1 + \mu_+ \mu_- B^2} \ln \left[ J_0 \left( \frac{2.4}{R} r \right) \right]. \quad (2)$$

In the above expressions  $\mu_{\pm}$  are the electron and ion mobilities at  $B = 0$ ,  $R$  is the radius of the discharge chamber,  $J_0$  is the zero-order Bessel function of the first kind,  $V$  is the electric potential, and  $T$  is temperature.

Kadomtsev and Nedospasov use a perturbation analysis in the approximation that  $\Omega_- \tau_- \gg 1 \gg \Omega_+ \tau_+$ ,  $D_+ = 0$ , and  $1/\tau_+ \gg \omega$ , where  $\Omega_{\pm}$  and  $\tau_{\pm}$  are respectively the ion and electron cyclotron frequencies and the mean times between collisions with neutrals,  $\omega$  is the frequency of the oscillations, and  $D$  is the diffusion coefficient. They assumed perturbations of the form

$$n(r, t) = n(0)J_0 \left[ (2.4/R)r \right] + n_1 J_1 \left[ (3.83/R)r \right] e^{i(m\phi + kz - \omega t)} \quad (3)$$

and

$$V(r, t) = V(r) + V_1 J_1 \left[ (3.83/R)r \right] e^{i(m\phi + kz - \omega t)}. \quad (4)$$

Expressions (3) and (4) were inserted in the equations of motion and continuity for electrons and ions, the resulting equations were linearized, multiplied by weighting factors  $J_1(3.83 r/R)$ , and integrated over  $r$  to obtain the dispersion relation for the frequency  $\omega$  as a function of the wave number,  $k = 2\pi/\lambda$ . The real part at  $B = B_c$  is given by

$$\text{Re}(\omega) \approx [3\mu_+ D_- (3.83/R)] / [\mu_- (\Omega_- \tau_-)] (0.6 + x^2). \quad (5)$$

The expression for the imaginary part of  $\omega$  is

$$\text{Im}(\omega) = \left(\frac{3.83}{R}\right)^2 \frac{D_-}{y(\Omega_- \tau_-)^2} \frac{1}{\left(\frac{0.33}{\Omega_- \tau_-} m \frac{\mu_-}{\mu_+}\right)^2 + \left[\frac{1+y}{y} + \frac{k^2}{0.79} \frac{\mu_-}{\mu_+} \left(\frac{R}{3.83}\right)^2\right]^2} \left\{ -\frac{1.28+y}{0.8(y+1)} x^4 - (2+y)x^2 - 0.12 \left(\frac{\mu_-}{\mu_+} \frac{ym^2}{y+1}\right) - 0.6(1+y) + 0.2xyv^* m \frac{\mu_-}{\mu_+} \right\}, \quad (6)$$

where  $x \equiv k\Omega_- \tau_- R/3.83$ ,  $v^* \equiv v_{-z} R/2.4D_-$ , and  $y \equiv (\mu_+/\mu_-) (\Omega_- \tau_-)^2$ . The instability growth rates that are given later in this paper were calculated from the Eq. (6). For stability we have  $\text{Im}(\omega) < 0$ .

The sign of the right side of Eq. (6) depends only on the terms in the braces. Instability will occur when the driving term that contains the longitudinal electric field [ $v^* = \mu_- E_z R/2.4D_-$ ] exceeds the four stabilizing terms. Note that the last term can always be positive, independent of the direction of  $E_z$ , because  $x(k)$  can be positive or negative. Equation (6) shows that an instability can occur only for  $m > 0$ . A qualitative picture of the onset of this instability has been given by Hoh and Lehnert, who discussed the charge separation and drifts of a screw-shaped ( $m = 1$ ) perturbation. 7, 17

The choice of  $V_1(r)$ , which leads to  $V_1(R) = 0$  at the wall [Eq. (4)], was made to simplify the calculation, although it was known to be physically unrealistic. Hoh has assumed a squared-parabolic density distribution, which is very similar to the K-N perturbed distribution, and has showed how the perturbed radial potential can be related to the perturbed density.<sup>7</sup> The resulting value of  $V_1$  is finite at the wall, thus eliminating an objection to the K-N analysis. Hoh obtained values of  $B_c$  quite close to those calculated from the K-N theory.

Johnson and Jerde have attempted to put the K-N analysis on a firmer physical and mathematical basis by including the effect of the magnetic field on the ions and solving the resulting equations rigorously.<sup>8</sup> No assumptions are made regarding the form of the perturbed radial density distribution or the potential distribution, except that  $n_0$  and  $n_1$  are zero at the wall. The formalism is too complex to reproduce here and the reader is referred to Reference 8. They obtained an infinite determinant of eigenvalues,  $\omega(k)$ , for the dispersion relation, of which only the first term is retained to give an approximate solution. They were unable to show the convergence of the whole series, but estimated that the next-order terms amount to about a 6% correction. The effect of including the 6% correction terms would be to reduce slightly the calculated values of  $B_c$ . They obtained a density perturbation identical to that of Kadomtsev and Nedospasov, but their perturbed potential has the form  $J_1(3.83r/R)/J_0(2.4r/R)$ , which is finite at the wall.

Ecker has treated the problem of the critical magnetic field<sup>9</sup> by a method similar to that of K-N. However, he avoids the use of a weighting factor for averaging and applies the physically meaningful boundary condition that the positive and negative particle currents must be equal at the wall. This prescribes the potential gradient at the wall. Introducing different forms for the trial solution, he demonstrates that the critical magnetic field is quite sensitive to the trial function.

### III. EXPERIMENTAL PROCEDURE

The experimental arrangement is shown in Fig. 1. The discharges were produced in Pyrex tubes 300 cm in length and 1.27 and 2.75 cm in radius. Electrodes containing tungsten filaments were inserted in each end of the tube and were run space-charge limited. Two variable-frequency ac power sources were used in the experiment. One was a 5-kW sine-wave oscillator that could supply 100 mA to 2 A rms in the frequency range from 7 to 70 kc; the other was a square-wave amplifier that produced 100 mA to 1 A in the frequency range from 0 to 50 kc. The magnetic field was provided with ten water-cooled coils 9-in. i. d. and 6 in. wide, spaced 2.25 in. apart to permit space for streak photography and photomultiplier monitoring of the plasma. The field strengths could be continuously varied from 0 to 7 kG. The base pressure of the vacuum was  $10^{-7}$  mm Hg, and gases were continuously bled through the tube during an experimental run. Prior to a run the tube was cleaned by flushing the system with the proper gas and running the discharge for a few hours.

Two Langmuir-type wall probes were used to monitor the axial electric field. They were 20 cm apart and were made from 0.5-mm-diameter tungsten rod protruding approximately 1 mm into the discharge. The probe signals were amplified in a high-gain differential preamplifier and displayed on an oscilloscope. In addition, the differential signal was rectified and the amplitude was plotted on an X-Y recorder as a function of magnetic field strength. The total voltage across the tube and the discharge current were continuously monitored on an oscilloscope, the current being kept constant during an experimental run. The total voltage across the tube vs magnetic field could be plotted on the X-Y recorder in the same manner as the probe signals. Space- and time-resolved information on the luminous structure of the plasma were obtained with stereo rotating-mirror streak-camera photographs. The time-varying light intensity was monitored with a photomultiplier and the signal displayed on an oscilloscope. This enabled us to determine the wave shape of the emitted light below the critical magnetic field

for comparison with the tube voltage and discharge-current wave shapes. The onset of the instability could be seen quite plainly as a sharp increase in the light intensity, and was accompanied by a signal at the rotation frequency of the spiral.

#### IV. RESULTS

After making the qualitative observation that the critical magnetic field indeed increased as the discharge frequency increased, it was necessary to do some auxiliary experiments to lay the ground work for a quantitative comparison with theory. Measurements of critical magnetic field and rotation frequency were obtained in dc discharges as a function of gas, tube radius, and pressure; these measurements agreed very well with previous results.<sup>5</sup>

Electron temperatures just below the critical magnetic field must be known in order to calculate properly the onset of the instability, because  $B_c$  is quite sensitive to  $T_e$  — approximately directly proportional. Previous experimenters who have compared their results with the K-N theory have used temperatures that were calculated from the electric-field values at  $B = B_c$ ,<sup>2,3</sup> or were calculated from the modified Schottky theory,<sup>5</sup> or have been extrapolated from previous experimental measurements at  $B = 0$ .<sup>6</sup> In the present work many temperature measurements were made with single and double probes in magnetic fields slightly weaker than the critical magnetic field. These values agreed to within 10 to 20% with calculations from the modified Schottky theory; the uncertainty in  $B_c$  should be of approximately the same magnitude. The modified Schottky theory is used in later calculations for simplicity and consistency.

The question as to whether the ac column can be described properly by the dc theory must be considered. The ac case is very complex, but some insight can be obtained from a comparison of the electron density and temperature-relaxation times with the period of the applied ac current. For example, if the

half-cycle time is much larger than the relaxation times we have a quasi-stationary state, and the K-N (dc theory) results are directly applicable. In the other limit, at very high frequencies, the stationary solution of the column gives a practically time-independent density distribution that is very close to the Bessel-function solution for the dc case,  $J_0(2.4r/R)$ . Here, the relation between the effective electric field and  $T_e$  is more complicated than in the dc column, but the necessary conditions for the instability mechanism are met and the K-N model should again apply.

In our experiments, the oscillation times are often comparable to the ambipolar diffusion times at magnetic fields close to  $B_c$ . For square-wave currents, the density distribution cannot change appreciably during the short time required for current reversal, and the dc theory should apply. In the sine-wave case we can expect the K-N analysis to be at least semiquantitatively correct, but appeal to the following auxiliary experiment to reinforce this conclusion.

We investigated experimentally whether the critical magnetic field was affected by the modulation of a unidirectional current. The discharges were made to run in a half-wave rectified mode by turning off the heating current to the filament at one end.  $E_z$  vs  $B$  curves showed that at all but the highest frequencies the critical magnetic field was the same as the dc critical field. Streak photographs taken with these pulsed unidirectional currents showed that during the current pulse instabilities develop with the same characteristics as those developed in the dc case. The agreement between the dc and half-wave rectified results indicates that time-dependent fluctuations do not appreciably affect the properties of this instability, and tends to confirm the validity of the comparison between the predictions of the dc theory and the ac measurements.

Alternating current measurements of  $B_c$  vs frequency were made in the 1.27 and 2.75-cm radius tubes with  $H_2$ ,  $D_2$ , He, Ne, and Ar at pressures ranging from 0.05 to 2.0 torr. Examples of results obtained in the larger tube with square-wave currents are shown in Fig. 2. Similar results were obtained with sine waves, and many other examples are given in Ref. 18.

The critical magnetic field at which the instability occurs could be measured by recording the axial electric-field dependence on  $B$ , or the total voltage across the discharge vs  $B$ , or the variation in the time-averaged light intensity as the magnetic field was changed. The three methods were not always employed simultaneously, but when they were, exact agreement as to the onset of the instability was obtained.

The  $E_z$  vs  $B$  curves, as extended above the dc instability magnetic fields by operation at sufficiently high frequencies, are seen from Fig. 2(a) and (c) to be in fairly good agreement with dc( $E_z$  vs  $B$ ) curves based on calculations of  $E_z$  vs  $T_-$  for He and Ar (Ref. 19) and electron temperatures calculated from the modified Schottky theory. It might seem more desirable to base the  $E_z$  vs  $B$  calculations on experimentally obtained dc relationships between  $T_-$ , the ionization rate as a function of  $T_-$ ,  $v_1(T_-)$ , and  $E_z/p$ , but such data are not available for the conditions of our experiment.

The streak-camera photographs show that once the instability sets in the rotational frequency of the spiral is in good agreement with the value obtained from the dc expression, Eq. (5), even though there is no a priori reason to expect the frequency of the fully developed helix to be predicted by the small-amplitude theory. At magnetic fields much stronger than the critical field, the streak photographs indicate a chaotic light distribution, and photomultiplier traces also show

that the light emission is hashy. This may be compared with the K-N theory, which shows that many higher  $m$ -value modes are unstable above  $B_c$ .

In order to verify that the onset of the instability in the ac discharges is suppressed until the growth rate exceeds the current-reversal frequency, we compare (a) the calculated growth rates vs magnetic field with (b) the experimental current-reversal frequency vs critical-magnetic field in Fig. 3. The  $m$  values are shown on the curves and the experimental points are indicated by dots.

By means of an IBM 7090 computer, the theoretical growth rates from both theories were calculated for several values of  $m$ . For the K-N case, Eq. (6) gave the growth rate,  $\text{Im}(\omega)$ . The wave numbers  $k(B)$  were obtained from Eq. (6) and the conditions that  $\text{Im}(\omega) = 0$  and  $\partial/\partial k \text{Im}(\omega) = 0$  at  $B_c$ . In the J-J case the growth rate was calculated from Eqs. (19), (25), (26), (28), and subsidiary equations of Ref. 8.<sup>20</sup> The wave numbers were obtained from Eq. (33).

The solution of the various equations required values of axial electric field, electron temperature, and electron and ion mobilities. The axial electric fields were obtained from the ac experiments or from the corresponding dc values, which generally agreed to within about 10%. The electron temperatures as a function of magnetic field were calculated from the modified Schottky theory, which had been shown to agree well with the experimental temperatures obtained from the dc experiments. Ion and electron mobilities were obtained from the summary by Brown,<sup>21</sup> except for  $H_2^+$  and  $D_2^+$ , which were obtained from Chanin.<sup>22</sup> The same values of these parameters were used in both sets of calculations. In each case calculations were made for both atomic and molecular ions, but the K-N model is relatively insensitive to the value of the ion mobility. (The dc critical magnetic field turns out to be approximately 10% weaker for the molecular ion case than for the atomic ion case, whereas the growth rates are somewhat larger.)

It can be seen from Fig. 3 that the  $m = 1$  mode becomes unstable first, followed at higher magnetic fields by higher  $m$  modes. In addition, above the

dc critical magnetic field the theoretical growth rates of the higher  $m$ -value modes may exceed those for  $m = 1$ . Hence if for a given frequency the onset of instability is suppressed up to a magnetic field at which a mode with  $m > 1$  has the largest growth rate, this mode should become unstable first, possibly followed closely by the other modes that correspond to values of  $\text{Im}(\omega) > 0$ . The streak-camera photographs do not contradict this idea, but individual  $m > 1$  modes have not been identified. Following this line of reasoning we would expect the experimental points, plotted as current-reversal frequency vs <sup>critical</sup> initial-magnetic field, to fall on the envelope of the theoretical growth rates vs magnetic field for all values of  $m$ . We assume here that the quasi-dc approach is valid, and that an instability occurs if the density perturbation grows by a factor of  $e$  in one-half cycle of the discharge. This particular criterion seems reasonable although somewhat arbitrary. We see from Fig. 3 that this hypothesis seems to be borne out quite well.

The results for the  $H_2$  and  $D_2$  discharges (not shown) are in general agreement with the K-N or J-J analyses, the experimental and theoretical growth rates being an order of magnitude larger than they are for He. The rapid growth rates and the large power required to operate the hydrogen discharges made it impossible to make measurements over a large range in  $B$ .

Stabilization of a positive column above the dc critical-magnetic field can be achieved in other ways. Johnson and Jerde have demonstrated experimentally and theoretically that the azimuthal electric field induced by a rising longitudinal magnetic field can delay the onset of the helical instability by an arbitrarily large factor.<sup>23</sup> In addition, von Gierke and Wöhler have changed  $B_c$  by superimposing a 4 mc rf field on a dc discharge.<sup>24</sup> The resulting heating of the electrons decreased the axial electric field and increased  $B_c$ , in agreement with the Kadomtsev and Nedospasov model.

## V. CONCLUSIONS

It has been shown that the instability of a positive column in a strong magnetic field is suppressed when the discharge current is reversed at a frequency greater than the calculated growth rate for the instability. The onset of the instability is pushed to higher magnetic fields as the ac frequency is increased, in agreement with our interpretation of the Kadomtsev-Nedospasov instability model. Quantitatively the Johnson and Jerde theory gives better agreement with both the dc critical magnetic fields and for the growth rates. It might be expected that the K-N calculation would be in poorer agreement in general, because of the physically unrealistic form of the perturbed potential, and because of the neglect by the magnetic-field effect on the ions; this effect is not negligible at the stronger magnetic fields employed in this experiment.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. Schematic diagram of the experimental apparatus.

Fig. 2. Experimental ( ——— ) and theoretical ( - - - - ) curves for various gases of  $E_z$  vs  $B$ , all at  $R = 2.75$  cm and  $I = 500$  mA square wave.

(a) He at 0.6 torr; (b) Ne at 0.18 torr (no data available for a theoretical curve); (c) Ar at 0.20 torr.

Fig. 3. Curves showing growth rate,  $\text{Im}(\omega)$ , and current-reversal frequency,  $f$ , vs  $B$  for various gases, all at  $R = 2.75$  cm,  $I = 500$  mA square wave.

(a)  $\text{He}^+$  ions in He at  $p = 0.6$  torr; (b)  $\text{Ne}^+$  ions in Ne at  $p = 0.18$  torr; (c)  $\text{Ar}^+$  ions in Ar at  $p = 0.20$  torr.

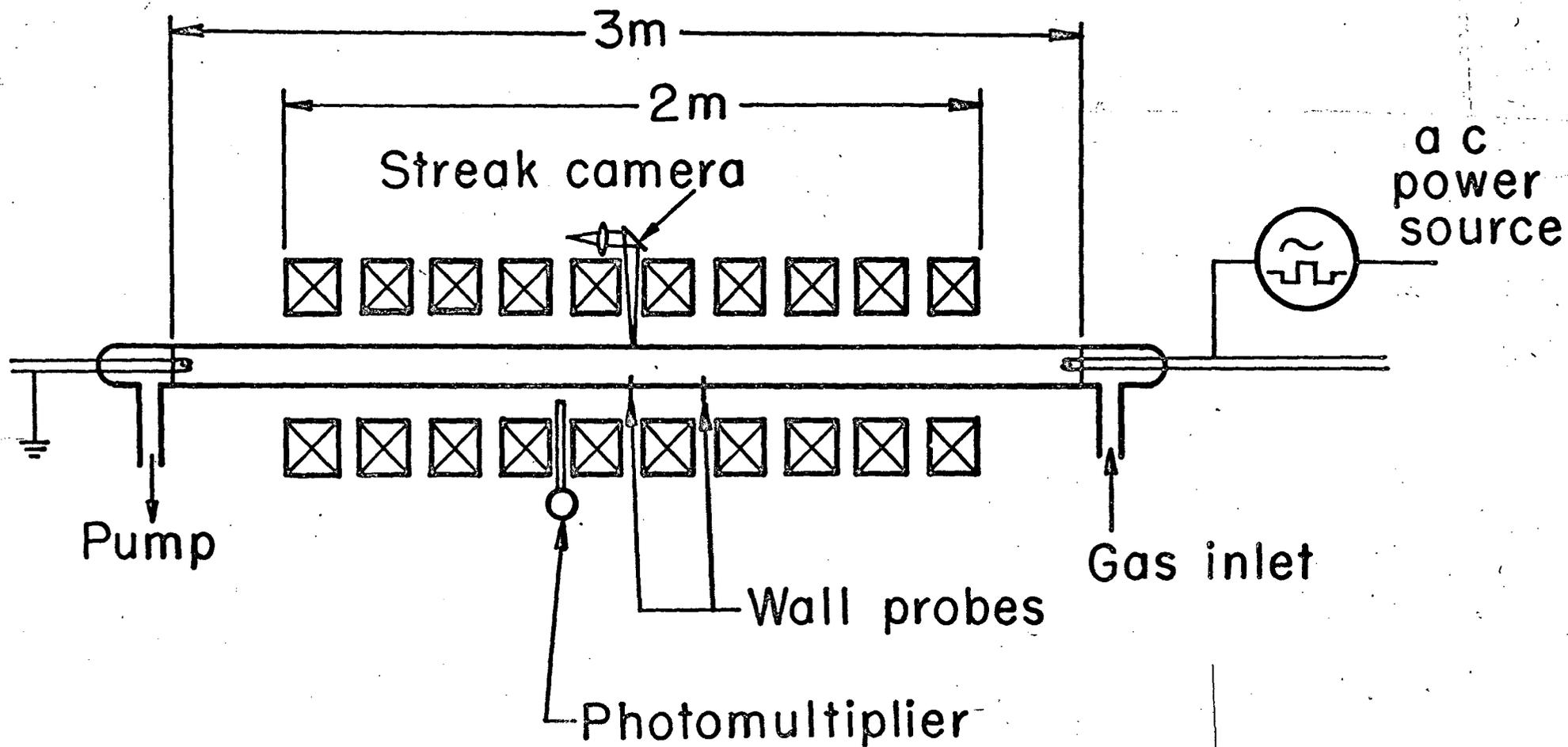


Fig 1

MU-27752

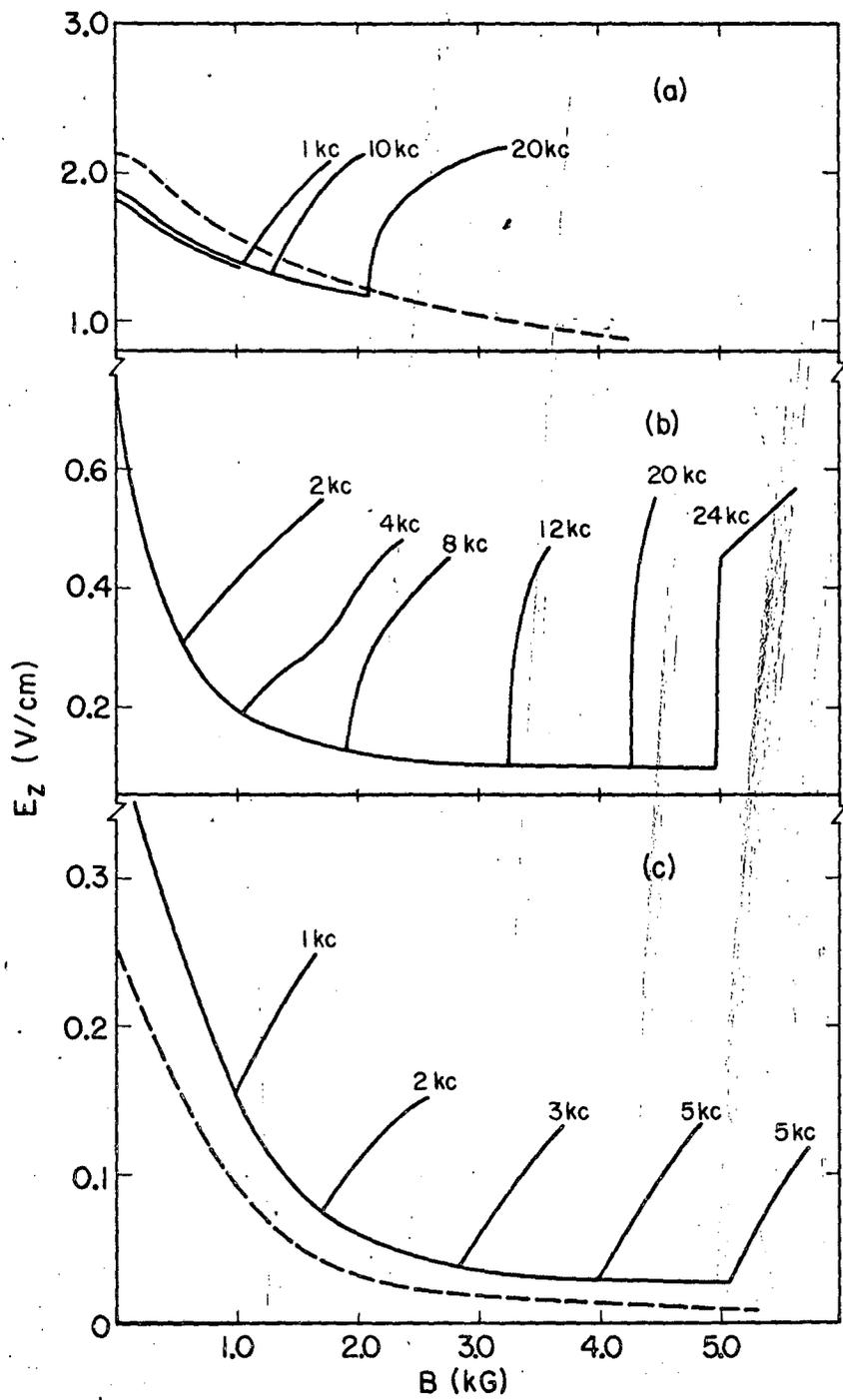


Fig 2

MUB-2233

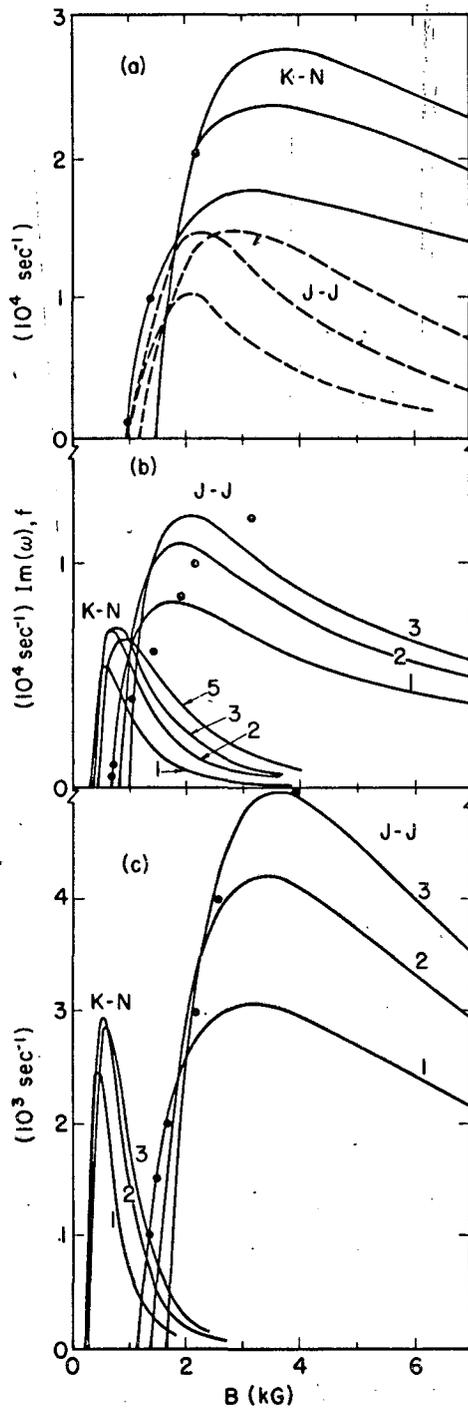


Fig 3

MUB-2234