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Berkeley, California

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ABSTRACT

The problem of three simultaneous two-body, final-state interactions, in a three-body final state, is considered in the limiting case in which two of the interactions are long range and slowly varying. In this special case the effect of the long-range interactions is merely to shift the energy at which the short-range interaction is to be evaluated. Applications are made to the determination of the s-wave pion-pion scattering length from the He^3 experiment and to production of resonant states.

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Several authors[1] have considered the problem of simultaneous interactions between more than one pair of particles in many-particle final states. Approximate solutions have been found in cases in which the interactions present are of certain special forms. The purpose of this note is to point out still another special case in which three simultaneous interactions in a three-body final state may be treated approximately, and to apply the results of this treatment to several experiments.

We consider three two-body interactions, one a strong, short-range attraction, the other two, long-range. The case in which only the short-range attraction is present is the subject of two theoretical treatments. The first of these, by Watson,[2] is based on formal scattering theory, while the second, by Jacob et al.,[3] is based on dispersion-theory arguments. Jackson has shown that the two methods

lead to the same results for a wide class of problems.[4] Both methods lead to the result that the cross section for the overall process (initial state into final state) is enhanced by the cross section for the two strongly interacting particles evaluated at the energy at which they are produced. Thus, we have

$$\sigma = \sigma_{i\to abc} = \sigma^0 \sigma_{ab}(s_{ab}) , \tag{1}$$

where a and b are the strongly interacting particles, s_{ab} is the square of their total center-of-mass (c.m.) energy, and σ^0 is the cross section in the absence of the final-state interaction.

Suppose now that long-range interactions of the pairs (ac) and (bc) are present, and that these interactions can be approximated by effective potentials U_{ac} and U_{bc}. The modification of Eq. (1) to include the effect of the two additional final-state interactions then takes a particularly simple form. We need only recognize that the energy at which the cross section σ_{ab} is to be evaluated is not the observed energy s_{ab}, but rather s^*_{ab}, the observed energy shifted by the presence of the potentials U_{ac} and U_{bc} in the centers of mass of the pairs (ac) and (bc). Therefore, we have

$$\sigma = \sigma^0 \sigma_{ab} (s^*_{ab}) . \tag{2}$$

The validity of Eq. (2) depends on two requirements: (i) the potentials U_{ac} and U_{bc} are roughly constant over their range, and there is negligible scattering at their surfaces; (ii) the interaction

between a and b takes place only while a and b are within the ranges of their potentials with c . Whether (ii) is satisfied can be determined from the kinematics of the reaction under consideration and the ranges of the pertinent interactions. Requirement (i) is necessary in order that the optical model be valid for a-c and b-c scattering.

We now find the relationship between s_{ab} and s^*_{ab} . In doing so, we make the inessential approximation that m_c is large enough so that the centers of mass of (abc) , (ac) , and (bc) can be identified with the rest system of c . This approximation will be valid below in the examples to which we apply our results. In the rest system of c , it follows from (i) and (ii) that the angle χ between the three-momenta of particles a and b is the same inside and outside of the region in which the potentials U_{ab} and U_{bc} are present. Then we have

$$s_{ab} = M_a^2 + M_b^2 + 2\omega_a \omega_b - 2 \cos \chi \left[(\omega_a^2 - M_a^2)(\omega_b^2 - M_b^2)\right]^{1/2} \tag{3a}$$

and

$$s^*_{ab} = M_a^2 + M_b^2 + 2\omega^*_a \omega^*_b - 2 \cos \chi \left[(\omega^{*2}_a - M_a^2)(\omega^{*2}_b - M_b^2)\right]^{1/2}, \tag{3b}$$

with

$$\omega^*_{a(b)} = \omega_{a(b)} - V_{a(b),c} \tag{4}$$

and

$$\omega_{a(b)} = \left[(\nu + m^2_{a(b)})^{1/2} \pm \beta\, \nu^{1/2} \cos\theta\right](1 - \beta^2)^{-1/2}, \tag{5}$$

where ν is the square of the three-momentum in the a-b c.m. system:

$$\nu = (4\, s_{ab})^{-1} \left[s_{ab}{}^2 - 2(m_a{}^2 + m_b{}^2)s_{ab} + (m_a{}^2 - m_b{}^2)^2\right] .$$

In Eq. (5), β is the velocity of the (ab) center of mass in the rest system of c (in the absence of the potentials), and as usual we have chosen θ and ϕ to be the polar and azimuthal angles of a in the (ab) c.m. system, with respect to a coordinate system in which c travels in the negative z direction. Using Eqs. (3a), (4), and (5), we can solve (3b) for s^*_{ab}. The energy shift $\Delta s = s^*_{ab} - s_{ab}$ depends on $\cos\theta$, β, and ν. For β small, Δs is independent of $\cos\theta$. In addition, if we have $U_{ac} = U_{bc} = U$ and $m_a = m_b = m$, and U and ν are small compared to m, we obtain the "intuitive" nonrelativistic result, $s^* = (s^{1/2} - 2U)^2$. (Note that U is negative for an attractive potential.) In this limit a and b move in opposite directions in the rest system of c. The opposite limit is for large β and small ν. In this case the shift goes to zero with ν and is greatest for $\cos\theta = 0$.

As a first application of the above, we consider the determination of the s-wave, T = 0, π-π scattering length, a_0, from the experiment of Abashian et al. on the reaction

$$p + d \rightarrow He^3 + 2\pi .$$

The analyses of ABC and JMO lead to the values, $a_0 \cong 2.0$ (in units of the pion Compton wave length). On the other hand, consideration of low-energy pion-nucleon scattering[6] and single pion production near threshold[7,8] lead to values of a_0 that are in the neighborhood $a_0 \cong 1.0$. In spite of the uncertainties in the different determinations, there seems to be a discrepancy between values obtained from the ABC experiment and those obtained from other processes.[8]

In applying our method to this experiment, we may note that the radius of helium is about 2 fermis, while, if ρ exchange is the dominant force in producing the s-wave pion-pion interaction, the range of this interaction is around 1/4 fermi. In analysing their experiment, ABC use for $\sigma_{\pi\pi}$ in Eq. (1)

$$\sigma_{\pi\pi}(\nu) = \left\{1 + 2(2a_0/\pi)[\nu/(\nu+1)]^{1/2} \ln\left[(1+\nu)^{1/2} + \nu^{1/2}\right] + a_0^2\left[\nu/(\nu+1)\right]\right\}^{-1}$$

$$\approx \left[1 + (4a_0/\pi + a_0^2)\nu\right]^{-1}$$

$$\equiv \left[1 + C_0\,\nu\right]^{-1} \tag{6}$$

obtained from the Chew-Mandelstam effective-range formula[9]

$$\left[\nu/(\nu+1)\right]^{1/2} \cot \delta_0^{\,0} = A_0^{-1} + \frac{2}{\pi}\left[\nu/(\nu+1)\right]^{1/2} \ln\left[(1+\nu)^{1/2} + \nu^{1/2}\right]$$

$$\approx A_0^{-1} + 2\nu/\pi \; .$$

We have taken the small ν forms of the above expressions, since it is the region $\nu < 1/4$ in which a deviation from the distribution in ν predicted by the phase-space factors in σ^0 in Eq. (1) is observed, and an enhancement due to the factor $\sigma_{\pi\pi}$ is expected.

To include the effect of the interactions of the pions with the helium, we solve Eq. (3b) in the small ν limit and average the shift Δs over θ, ϕ, since the directions of the (s-wave) pions are not detected. We find for the average square of the pion three-momentum, in the pion-pion c.m. system and in the presence of the pion-helium potentials

$$\nu^* = \nu + \bar{U}\,\nu \,, \tag{7}$$

where

$$\bar{U} = -(1/3\mu)\left[U_{\pi^+} + U_{\pi^-}\right]\beta^{-2}(1-\beta^2)^{1/2} \,. \tag{8}$$

To find the correction to the pion-pion scattering length, we set

$$C_0^{ABC} = C_0(1+\bar{U}) \,. \tag{9}$$

We see that attractive potentials U_{π^+} and U_{π^-} decrease the result for the scattering length. If we use the results of Frank et al. for the pion-nucleus optical model potential,[10]

$$U_{\pi^+} \cong U_{\pi^-} \cong -30 \text{ MeV} \,,$$

the scattering length changes from 2.0 to 1.7. Although the change is in the proper direction, the effect does not seem large enough to resolve the discrepancy noted above.

We next apply our method to the cases of ρ and N^* production from complex nuclei. Consider first the process

$$\pi + A \rightarrow \rho + A \rightarrow \pi + \pi + A \ . \tag{10}$$

Solving equations (3) through (5) in the approximation $\nu \gg \mu^2$, we find

$$s^*_{\pi\pi} - s_{\pi\pi} = -8\, U_{\pi A}\, \nu^{1/2}(1 - \beta^2)^{1/2}(1 - \beta^2 \cos^2 \theta)^{-1}, \tag{11}$$

where we have taken the potentials for the two pions to be equal and independent of the charge. Averaging over pion orientations gives

$$s^*_{\pi\pi} - s_{\pi\pi} = -4\, U_{\pi A}\, F_1(\beta)\, \nu^{1/2} \ , \tag{12}$$

where

$$F_\ell(\beta) = (2\ell + 1)(1 - \beta^2)^{1/2} \int \left(P_\ell(z)\right)^2 \left(1 - \beta^2 z^2\right)^{-1} dz \ . \tag{13}$$

Here $F_\ell(\beta)$ is approximately equal to one for $\beta < 0.9$; the value of ℓ determines the rate at which F_ℓ decreases to zero near $\beta = 1$. Writing $s = W^2$, we thus have

$$W^*_{\pi\pi} - W_{\pi\pi} \cong 2\, U_{\pi A} \ . \tag{14}$$

From the results of Frank et al.,[10] we expect, for final-state pion kinetic energies around 350 MeV,

$$U_{\pi A} = +35 \text{ MeV} \ .$$

This corresponds to the initial pion kinetic energy in (10) being

$$T_\pi = 850 \text{ MeV} .$$

For this energy, we also have $\beta = 0.64$. Finally, at $T_\pi = 850$ MeV, the energy of the ρ is such that, if the above shift were absent, one would expect to detect the ρ at the energy of the maximum in the pion-nucleon phase-space factor.

As a final application, we consider the reaction

$$N + A \rightarrow N^* + A \rightarrow \pi + N + A . \tag{15}$$

We again solve Eq. (3) through (5), making the approximation

$$M^2 \gg \nu \gg \mu^2 ,$$

and obtaining

$$W^*_{\pi N} - W_{\pi N} = U_{\pi A}(1 - \beta^2)^{1/2}(1 + \beta z)^{-1} . \tag{16}$$

We observe that (16) is independent of the potential for the nucleon as long as U_{NA} is of the same order of magnitude as $U_{\pi A}$. Using the relation

$$\int (1 + \beta z)^{-1} \rho_\ell^{\,2} \, dz = \int (1 - \beta^2 z^2)^{-1} \rho_\ell \, dz$$

and recalling that $F_\ell(\beta) \approx 1$, we find, after averaging over nucleon directions in the pion-nucleon c.m. system,

$$W^*_{\pi N} - W_{\pi N} = U_{\pi A} \ . \tag{17}$$

If we again require the final-state pion kinetic energy to be around 350 MeV, we obtain a 35-MeV shift in the position of the N^* ; the initial nucleon kinetic energy is then

$$T_N = 640 \text{ MeV} ,$$

while $\beta \approx 0.46$, and the N^* occurs near the maximum in the pion-nucleon phase-space factor.

In concluding, we wish to point out that, if study of the optical model at high energies (several BeV) reveals an appreciable real part for, say, the pion potential, the present method would allow sizeable corrections to be made in the case of boson resonances found in very-high-energy pion nucleus experiments.

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REFERENCES

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1. N. N. Khuri and S. B. Treiman, Phys. Rev. 119, 467 (1960); R. F. Sawyer and K. C. Wali, Phys. Rev. 119, 1429 (1960); R. D. Amado, Phys. Rev. 122, 696 (1961); and R. F. Peierls and J. Tarski, Phys. Rev. 129, 981 (1963).
2. K. M. Watson, Phys. Rev. 88, 1163 (1953).
3. M. Jacob, G. Mahoux, and R. Omnes, Nuovo Cimento 23, 838 (1962). In the following, this paper will be denoted by JMO.
4. J. D. Jackson, Nuovo Cimento 25, 1038 (1962).
5. A. Abashain, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 7, 35 (1961). In the following, this paper will be denoted by ABC.
6. J. H. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Pion-Nucleon Scattering and Pion-Pion Interactions, Oxford University preprint. A complete list of references is given in this paper.
7. L. S. Rodberg, Phys. Rev. Letters 3, 58 (1959).
8. C. Ceolin and R. Stroffolini, Some Remarks about Low-Energy Pion-Pion Scattering (CERN, 2114-TH-212).
9. G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).
10. R. M. Frank, J. L. Gammel, and K. M. Watson, Phys. Rev. 101, 891 (1956).

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