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ABSTRACT

The γ-ray spectrum resulting from the reaction $\pi^- + D \rightarrow 2n + \gamma$ has been shown to depend strongly on the nature of the n-n interaction and has been calculated for several values of the n-n scattering length. A comparison of the theoretical cross section with preliminary experiments by Aamodt, Panofsky and Phillips indicates: (1) an upper limit of approximately 200 Kev can be put on the binding energy of the di-neutron; (2) photo-meson production involves an interaction with the nucleon spin.

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I. Introduction

Experimental studies of the capture of π^--mesons in deuterium[1] indicate that approximately 30% of the capture events lead to a high energy γ-ray. This has been interpreted, for instance, as implying that the π-meson is not scalar[2, 3]. However, quite apart from its implications as to the nature of the π-meson and as to meson-nucleon interactions, this experiment, as has been noted previously[2, 3], is of interest in that it offers a means of deducing something of the interaction between two neutrons.

This possibility arises through a measurement of the γ-ray spectrum resulting from the radiative decay and may be seen qualitatively as follows. Since the final state contains three particles--a γ-ray and two neutrons--the γ-ray spectrum is not monochromatic; on the otherhand, if there were only the γ-ray and one particle (say a bound di-neutron) in the final state, the γ-ray spectrum would be monochromatic. If the two neutrons in the final state are not actually bound, but interact through an attractive potential, we can expect a tendency for them to recoil in the same direction with about equal velocities and

thus cause the γ-ray spectrum to be more nearly monochromatic than if there were no n-n (neutron-neutron) force. That is, the effect of an attractive n-n potential should be such as to make the γ-ray spectrum show a pronounced peak near its high energy limit.

We can obtain in a simple manner the shape of the spectrum near this peak. Let us denote the transition matrix for the radiative capture to a singlet spin state for the two neutrons by M^s, and suppose that the singlet n-n wave function is $\psi^s(r)$. Then M^s will have the form (we use as units $\hbar = c = 1$)

$$M^s = \int d^3r \; \psi^{*s}(r) \quad x \left[\text{other factors}\right] \quad . \tag{1}$$

Let $\underline{p}$ be the relative momentum of the two outgoing neutrons, which is small near the high energy limit of the γ-ray spectrum. We may thus neglect all but the S-wave contribution to the integral (1). Furthermore, the absorption process is presumably an event which involves primarily just the proton, so we should expect the integral in Eq. (1) to show no singular behavior for close distances of approach of the two nucleons. Then the contribution to the integral will come from regions of space much larger than that for which the n-n potential is important, and we may replace $\psi^s(r)$ by

$$\psi_o^s(r) = \sin(pr + \delta)/pr \tag{2}$$

and Eq. (1) can be put into the form

$$M^s = \frac{1}{p}\left[I_1 \sin\delta + I_2 \cos\delta\right] \tag{3}$$

From Eq. (2) we see that $I_2 \longrightarrow 0$ and $I_1 \longrightarrow$ constant as $p \longrightarrow 0$; i.e., near the high energy limit of the γ-ray spectrum. Near this limit, we can neglect I_2 in Eq. (3). Squaring M^s and taking into account the phase space factor, we have for the singlet state γ-ray spectrum

$$\frac{dT^s}{dK} = \frac{1}{p}\left[\sin \delta\right]^2 \times \text{constant}, \tag{4}$$

where p is related to the p-ray energy K by energy conservation, and the phase shift δ is a function of p, depending on the assumed n-n potential. The characteristic shape of the spectrum expected from Eq. (4) is a rapid rise from zero at the high energy limit depending on the behavior of $\sin^2 \delta$. Near $p = 0$, $1/p$ is a rapidly decreasing function, so we expect the spectrum to fall off rapidly with farther decreasing γ-ray energy. The final triplet state contribution will be small near the high energy limit.

A correction for the finite range of the n-n force can be obtained as follows. The exact correction to Eq. (2) assuming M^s (Eq. (1)) is, of course,

$$\delta\psi^s = (\psi^s - \psi^s_o) .$$

We can write this as

$$\delta\psi^s = -\frac{\sin \delta}{p} \frac{f(p, r)}{r} \tag{5}$$

where $f(p,r) = \frac{p}{\sin\delta}(\psi^s_o - \psi^s)$. But $f(p,r)$ is to a very good approximation independent of p [4], so we can write $f(p,r) \simeq f(r)$.

Then the correction to M^s is

$$\delta M^s = -\frac{\sin\delta}{p} I_1' , \qquad (3')$$

where I_1' is very nearly a constant near the high energy limit of the spectrum. The correction (3') is of the same form as Eq. (3) with $I_2 = 0$, so the shape of the spectrum, as given by Eq. (4), is unchanged to a good approximation.

With sufficiently good energy resolution in the experimental measurement of the γ-ray spectrum, our problem would be essentially complete. The right hand side of Eq. (4) is proportional to $p\sigma_{n-n}$ times a constant, where σ_{n-n} is the low energy n-n scattering cross section. This single expression for the shape of the γ-spectrum permits, in principle, a direct comparison of the σ_{n-n} with the low energy p-p and n-p scattering. There is evidence from the binding energies of mirror nuclei, for instance, for believing that σ_{n-n} and σ_{p-p} should be quite similar, except for the Coulomb contribution to the latter.

Because of the limitation on present experimental accuracy and energy resolution in the measurement of the γ-ray spectrum, it is necessary to improve Eq. (4), by extending it to lower energies. This will be done in Section IV. The nature of this correction can be seen in Fig. (1), where $\frac{dT^s}{dK}$ as given by Eq. (4) and the S-wave component of the cross section as given by the more nearly complete theory are compared.

II. Numerical Results

The theory outlined in the Introduction and further developed in Sections IV, V and VI permits a determination of the ^1S phase shift, δ, for low energy n-n scattering from the shape of the γ-ray spectrum resulting from the reaction $\pi^- + D \rightarrow 2n + \gamma$. For this purpose it is convenient to relate δ to the familiar parameters[5] characteristic of low energy nucleon-nucleon scattering. The relationship is

$$p \cot \delta = -\alpha + \tfrac{1}{2} r_o p^2 , \tag{6}$$

where $\frac{1}{\alpha}$ is the "scattering length" and r_o the "effective range". The spectrum is not very sensitive to the exact value of r_o, so we have given it the value obtained from p-p scattering[6].

$$r_o = 2.65(10)^{-13} \text{ cm.} \tag{7}$$

The resulting γ-ray spectrum for various assumed values of α was calculated on the basis of the theory to be developed in the following sections. The results are given in Fig. (2) for several negative values of α (i.e., for a potential that has no bound state). Positive values of α correspond to a potential that has a bound di-neutron state. Thus, if α is positive, we have the possibility of forming a bound di-neutron, with a corresponding monochromatic component in the γ-ray spectrum. In Fig. (3) are given the continuous spectra corresponding to several positive values of α. In Table I are given the intensitives of the corresponding monochromatic spectra corresponding to formation

of a di-neutron. The actual spectrum for a positive value of α is then the sum of the continuous spectrum, as given in Fig. (3) and the monochromatic spectrum, as given in Table I.

III. Comparison with Experiment

Aamodt, Panofsky and Phillips[7] have recently improved earlier measurements[1] of the γ-ray spectrum resulting from π^- absorption in deuterium. In Fig. (4) is given the experimental result along with some of the theoretical curves[8], as given in Sections IV and V. Although these preliminary data have rather poor statistics, it seems clear that positive evidence for the n-n interaction is indicated.

An important qualitative conclusion to be drawn from this point is that a triplet-singlet spin transition does indeed occur[4], and thus that photo-meson production involves an interaction with the nucleon spin. If there were no singlet component present in the spectrum, the resulting triplet spectrum would show even worse agreement with the experimental results than that shown by the plane wave case in Fig. (3).

The results are also quite compatible with an equality of the n-n and p-p singlet forces, although present statistical uncertainties do not permit a quantitative conclusion to be drawn concerning this point. An upper limit, however, of approximately .200 Mev can probably be put on the binding energy of the di-neutron.

IV. Formulation of the Problem

The deuteron is a fairly loosely bound system, the neutron and proton spending a considerable part of the time outside the range of their mutual force. We may thus expect hypothetical non-linear effects from the overlapping of their meson fields to be small. This conclusion is strengthened when we recall that the absorption of a π-meson by a proton is a high energy process, which should be little affected by the presence of a weakly bound neutron. This suggests that the absorption with radiation in deuterium is essentially the same as the hydrogen[1], and is modified primarily by the momentum distribution of the proton in the deuteron and by the n-n interaction. We thus introduce the transition matrix, defined in the center-of-mass system,

$$R \equiv (\underline{K} \mid R \mid \underline{q}) \tag{8}$$

for the absorption in hydrogen. Here $\underline{q}$ is the relative momentum of the meson and proton in the initial state and $\underline{K}$ is that of the photon and neutron in the final state. Because the meson is absorbed essentially from rest under the experimental conditions for which this has been observed, the transition in hydrogen depends only on

$$R^{o} = (\underline{K} \mid R \mid 0) \quad . \tag{8'}$$

However, the complex conjugate of Eq. (8) is the transition matrix for photo-meson production, which has been observed under more general conditions.

The absorption in deuterium takes place from an initial triplet spin state of the n-p system, and it is important to know the relative admixture of singlet and triplet states for the final n-n system. This depends, of course, on the symmetry properties of R (Eq. (8)). Since, however, for the absorption in deuterium, q is non-vanishing only because of the small effect of the internal motion of the proton in the deuteron (this is discussed in more detail in Section VI) we can investigate the simpler symmetry properties of R^o (Eq. (8')). R can be assumed linear in the electromagnetic field strengths, so it must involve linearly either (or both) the electric field $\underline{E}$ or the magnetic field $\underline{H}$. The only remaining vectors available in R^o are $\underline{K}$ and $\underline{\sigma}'$, the proton spin. If we make the most likely assumption[2] that the meson is pseudoscalar, R must change sign under coordinate reflection, and we have the unique form for R^o:

$$R^o = \underline{\sigma}' \cdot \underline{E} \quad R'(K) \tag{9}$$

This determines the relative admixture of singlet and triplet states as 1/3 and 2/3, respectively.

Eq. (9) implies that, near threshold at least, photomeson production takes place through an interaction with the nucleon spin. Direct evidence for this would permit us to drop the restrictive assumption that the meson is pseudoscalar. On the otherhand, if there were no spin interaction, there would be no final singlet state (since the initial state is triplet), and thus no evidence for the n-n interaction would be found. Happily, we

have available the experimental results of Aamodt, Panofsky and Phillips (see Section III), which seem clearly to indicate a ^1S admixture in the final state and thus a spin interaction. If there are not two different competing mechanisms for photo-meson production near threshold, we can reasonably expect R^o to have the simple form, $R^o = \underline{\sigma}' \cdot \underline{A}\, R'(K)$, where $\underline{A}$ is some vector. This is all that we need to determine the above mixture of singlet and triplet states.

Eq. (8) is easily generalized to the coordinate system in which the meson-deuteron system is at rest, since the transformation is non-relativistic and thus R can change by a phase factor only.

The transition matrix for the absorption in deuterium is then

$$M^s = \frac{1}{(2\pi)^6}\left(e^{i\,\underline{P}\cdot\underline{x}}\; e^{i\,\underline{K}\cdot\underline{\rho}}\; \psi^s,\; R\;\omega\,\phi\right) \qquad (10)$$

for a final singlet state. Changing "s" to "t" defines it for a final triplet state. Here ω is the Coulomb wave function for the meson in the lowest Bohr orbit and ϕ is the deuteron wave function. The factors $e^{i\,\underline{P}\cdot\underline{x}}$ and $e^{i\,\underline{K}\cdot\underline{\rho}}$ represent the plane wave motion of the center-of-mass of the two neutrons and the propagation of the photon, respectively. Because the Coulomb wave function is very nearly constant over the radius of the deuteron, we can remove ω from the integrals in Eq. (10) and consider it to be a multiplicative factor, ω_o, evaluated at the position of

the deuteron. Then a straightforward manipulation of Eq. (8) leads to[9]

$$M^s = \sqrt{2}\,\omega_o\,(2\pi)^{-3/2} \int d^3g \int d^3r\; \psi^{*s}(r)\, e^{-i\frac{K}{2}\cdot \underline{r}}$$

$$\left(\underline{K} - \frac{\mu}{M+\mu}\,\underline{g}\,|R| - \frac{\mu}{M+\mu}\,\underline{g} \right) e^{i\,\underline{g}\cdot\underline{r}}\, \phi(\underline{g}) \tag{11}$$

where $\phi(\underline{g})$ is the momentum representation of ϕ and R is given by Eq. (6). μ is the meson mass and M is the nucleon mass.

The dependence of R on $\underline{g}$ is weakened because of the factor $\frac{\mu}{M+\mu}$, which is about 1/8. The values of g are also limited by the weak binding of the deuteron, whereas R supposedly varies only for changes of its variables of the order of μ, the characteristic energy for meson production. We thus set $g = 0$ in R and remove it from the integral in Eq. (11) (a more quantitative estimate of the error hereby incurred is made in Section VI).

We then obtain for M^s (see Eq. (9))

$$M^s = \sqrt{2}\,\omega_o\, R'(K) \int d^3r\; \psi^{*s}(r)\, e^{-i\frac{K}{2}\cdot r}\, \underline{\sigma}'\cdot\hat{e}\,\phi(r) \tag{12}$$

where we now use the polarization vector $\hat{e}$ of the photon, rather than $\underline{E}$ as in Eq. (9), absorbing numerical factor in $R'(K)$. $\phi(r)$ is the coordinate representation of ϕ. We replace the superscript "s" by "t" to define the transition matrix for final triplet states of the n-n system.

If we neglect the n-n interaction for all but the ^1S state (this will be justified in Section VI), we have the following wave functions[10]:

$$\phi(r) = \left[\chi_t^m \phi_o(r) + 2^{-3/2}\left(\frac{3\underline{\sigma}^1\cdot\underline{r}\ \underline{\sigma}^2\cdot\underline{r}}{r^2} - 1\right)\chi_t^m\ \phi_2(r)\right]$$

$$\psi^t(r) = \frac{1}{\sqrt{2}}\left[e^{i\,\underline{p}\cdot\underline{r}} - e^{-i\,\underline{p}\cdot\underline{r}}\right]\chi_t^{m'}$$

$$\psi^s(r) = \frac{1}{\sqrt{2}}\left[e^{i\,\underline{p}\cdot\underline{r}} + e^{-i\,\underline{p}\cdot\underline{r}} - 2\sin pr/pr + 2\,\psi_o\right]\chi_s \tag{13}$$

where ψ_o is the S-wave part of ψ^s, with due account taken for the n-n interaction. χ_t^m and χ_s are triplet and singlet spin wave functions, respectively. ϕ_o and ϕ_2 are the radial parts of the S- and D-wave components, respectively, of the deuteron wave function. We shall neglect the contribution of ϕ_2 to all but the integral involving the S-wave part of ψ^s, for which its effects will be the largest.

We introduce the following definitions:

$$I^t = \int d^3r\left[e^{-i\,\underline{p}\cdot\underline{r}} - e^{i\,\underline{p}\cdot\underline{r}}\right]e^{-i\frac{\underline{K}}{2}\cdot\underline{r}}\ \phi_o(r)$$

$$I_1 = \int d^3r\left[e^{-i\,\underline{p}\cdot\underline{r}} + e^{i\,\underline{p}\cdot\underline{r}} - 2\sin pr/pr\right]e^{-i\frac{\underline{K}}{2}\cdot\underline{r}}\ \phi_o(r)$$

$$I_2 = 2\int d^3r\ \psi_o^*(r)\ e^{-i\frac{\underline{K}}{2}\cdot\underline{r}}\ \phi_o(r)$$

$$I_3 = 2\int d^3r\ \psi_o^*(r)\ \left(\frac{\pi}{Kr}\right)^{\frac{1}{2}} J_{5/2}\left(\frac{Kr}{2}\right)\phi_2(r) \tag{14}$$

Then

$$M^s = \omega_o R'(K) \left\{ (\chi_s, \underline{\sigma}' \cdot \hat{e} \chi_t^m) \left[I_1 + I_2 \right] - 2^{-3/2} (\chi_s, \underline{\sigma}' \cdot \hat{e} \left[\frac{3\, \underline{\sigma}' \cdot \underline{K}\ \underline{\sigma}^2 \cdot \underline{K}}{K^2} - 1 \right] \chi_t^m) I_3 \right\} \tag{15}$$

and

$$M^t = \omega_o R'(K) (\chi_t^m, \underline{\sigma}' \cdot \hat{e}\ \chi_t^m) I^t \tag{16}$$

The volume in phase space per unit photon energy accessible to the particles in the final state is

$$dJ = 2K^2\, d\Omega_K\ \frac{Mp}{2}\ d\Omega_p \tag{17}$$

where the $d\Omega$ are elements of solid angle. Defining

$$G^t \equiv \frac{1}{4\pi} \int d\Omega_p\ |I^t|^2$$

$$G^s \equiv \frac{1}{4\pi} \int d\Omega_p\ |I_1 + I_2|^2 \tag{18}$$

we have for the transition rates per unit photon energy

$$\frac{dT^t}{dK} = \frac{2}{3}\ 2(2\pi)^3\, \omega_o^2\, |R'(K)|^2\, K^2 M p\, G^t$$

$$\frac{dT^s}{dK} = \frac{1}{3}\ 2(2\pi)^3\, \omega_o^2\, |R'(K)|^2\, K^2\, Mp \left\{ G^s - \sqrt{2}\ I_2\ I_3 \right\} \tag{19}$$

where in the latter expression we have kept only the largest term involving I_3--the D-state part of the deuteron wave function.

There is also the possibility that there exists a bound dineutron ^1S state. Denoting this wave function by $\psi^B \chi_s$, we

obtain the transition matrix to form a bound di-neutron by replacing $\sqrt{2}\,\psi_o^*$ in I_2 (Eq. (14)) by $(2\pi)^{3/2}\,\psi^{*B}$ to give a new integral, say I_2^B. The resulting spectrum is monochromatic, and its intensity is given to a sufficient approximation by

$$T^B = \frac{1}{3}\, 4(2\pi)^2 K^2 \omega_o^2 \,|R'(K)|^2\, |I_2^B|^2 \tag{20}$$

V. Evaluation of the integrals

For the S-wave part of the deuteron wave function, that is $\phi_o(r)$, we use the Chew-Goldberger[11] expression

$$\phi_o(r) = \frac{N}{r}\left[e^{-\alpha r} - e^{-\beta r}\right] \tag{21}$$

where $\alpha = (Me_D)^{\frac{1}{2}}$ and $\beta = 7\alpha \cdot e_D$ is the deuteron binding energy. As the recoil neutron energies in which we are interested are small, the integrals involving $\phi_o(r)$ do not depend critically on its form at close distances. Indeed, most of the contribution comes from values outside the range of the n-p potential, where the term $\frac{e^{-\alpha r}}{r}$ is rigorous. The correction arising from the term $\frac{e^{-\beta r}}{r}$ is small and thought to be fairly reliable. (The value of β is determined by the low energy n-p triplet scattering[11].)

The D-state part of the deuteron wave function, ϕ_2, occurs in our approximation only in I_3. Because of the smallness of $J_{5/2}$ near the origin, contributions to I_3 from distances within the nuclear potential are quite negligible, and we can use the asymptotic form for ϕ_2, the functional form of which is, of course, independent of the exact nature of the tensor force. The

ratio ϕ_2/ϕ_o in the asymptotic range is only approximately known[12], but seems sufficiently reliable for our purposes, since the D-state correction is small anyway. Evaluation of I_3 in the vicinity of the high energy limit of the γ-spectrum indicates that the term $\sqrt{2}\ I_2\ I_3$ in Eq. (19) amounts to about a three percent correction to G^s. The actual importance of this correction is considerably less, however, since according to the arguments given in the Introduction, the functional form of $I_2\ I_3$ is approximately the same as that of G^s. Thus the correction amounts primarily to about a three percent decrease in the magnitude of the transition rate rather than a change in the shape of the spectrum. In any case it is negligible as far as present requirements as to accuracy are concerned. It should be noted that the smallness of this correction is due more to the smallness of the overlap of $J_{5/2}$ and ϕ_2 than to the actual smallness of ϕ_2.

Using the form (21) for ϕ_o, the evaluation of I^t and I_1 (Eq. (14)) is straightforward.

For the evaluation if I_2, we proceed as outlined in the Introduction. Set

$$I_2 = I_2' + I_2'' ,$$

where

$$I_2' = 2\int d^3r \ \sin(pr+\delta)/pr \ e^{-i\frac{K}{2}\cdot\underline{r}} \ \phi_o(r)$$

$$I_2'' = 2\int d^3r \left[\psi_o^*(r) - \sin(pr+\delta)/pr\right] e^{-i\frac{K}{2}\cdot\underline{r}} \ \phi_o(r) \tag{22}$$

Again the evaluation of I_2' is elementary. For I_2'' the method described in the Introduction can be carried out in a fairly reliable manner, but for the additional correction of the order of 2-4 percent we have used the exact wave functions for a square well, whose depth and range were chosen to give the assumed phase shift, δ.

The bound di-neutron wave function has the form

$$\psi^B = \frac{N_B}{r} e^{-\alpha r} \tag{23}$$

outside the range of the n-n force. Corrections can be obtained as described in connection with Eq. (22). $\gamma = (ME_B)^{\frac{1}{2}}$, where E_B is the assumed binding energy of the di-neutron.

VI. Estimates as to the Validity of the Theory

We now inquire further into the justification for removing R from under the integral in Eq. (11) and into the expected behavior of $R'(k)$ in Eq. (12). In particular, both the absorption in hydrogen and photo-meson production depend upon R only on the energy shell, whereas in Eq. (11) we need more general values--although, indeed, these are not far from the energy shell, as argued above. In particular, the threshold for photo-meson production in the center-of-mass system (and the energy of the γ-ray for the absorption in hydrogen) is at a γ-ray energy of about 131 Mev. The peak of the γ-ray spectrum for the absorption in deuterium, on the otherhand, occurs at about 132 Mev. Thus in the vicinity of the peak of the spectrum, which is the region of most interest to us, the γ-ray energy lies very close to its value defined by the energy shell for photo-meson production.

In the hope of obtaining further information about the matrix R, we have analyzed the excitation function of Steinberger and Bishop[13] for photo-meson production. Transforming this to the center-of-mass system and removing the phase space factor, we have essentially

$$\Gamma \equiv \left| (K \mid R \mid q) \right|^2 \tag{24}$$

(see Eq. (8)) on the energy shell. The experiments measure this down only to within 20 Mev of threshold and are not very accurate in this region. Nevertheless, they were consistent with a constant Γ near threshold. This did not agree very well with the Γ as deduced from pseudoscalar meson theory, since this has a factor $1/K$ ($\frac{1}{\sqrt{K}}$ in $R'(K)$ of Eq. (12)). Again, a Γ, as deduced on the basis of an interaction with the nucleon magnetic moments, behaves as q^2 near threshold and is in violent disagreement with the experimental results.

In view of these considerations, we have taken $R'(K)$ as a constant in Eq. (18) for the numerical results given in Section II. A correction for any other assumed functional dependence on K can be easily obtained merely by multiplying each point on the spectra given by the corresponding value of $|R'(K)|^2$. The errors incurred are in any case small near the interesting part of the spectrum, and are very unlikely to be of much importance, since there are few γ-rays emitted at much lower energies. This uncertainty does, however, make it desirable to employ as sharp an energy resolution as possible in measuring the spectrum, so that the

low energy tail of the spectrum does not enter into the determination of the n-n force. More complete measurements of the excitation function for photo-meson production may also be of help in resolving this difficulty, which at present constitutes, perhaps, the greatest uncertainty in the theoretical analysis.

We have mentioned that the absorption in hydrogen does prove that $(\underline{K}) | R | \underline{q})$ (Eq. (8)) approaches a constant limit as q approaches zero. In order to remove R from the integral in Eq. (11), we must also assume that R does not have a singular behavior near $q = 0$. Aside from the rather inconclusive evidence from the excitation function for photo-meson production, the arguments that this is not the case are twofold: (1) There is a rather weak argument from pseudoscalar meson theory, which fits quite well the angular distribution for photo-meson production[14], that $(\underline{K} | R | \underline{q})$ varies smoothly with q as q approaches zero. (2) The relative amounts of observed absorption with and without radiation[1] as analyzed[2] on the basis of detailed balancing arguments from photo-meson production and meson production in nucleon-nucleon collisions are quite consistent with a smooth, slow variation of $(\underline{K} | R | \underline{q})$ to its value at threshold. On the otherhand, even a linear dependence of $(\underline{K} | R | \underline{q})$ on $\underline{q}$ near threshold would decrease the rate of radiative absorption to about one percent of its previously calculated value[2], and make it difficult to account for the observed ratio.

The meson rest-mass, μ, is the natural parameter to describe the structure of R. That is $\frac{K}{\mu} \simeq 1$ and $\frac{q}{\mu}$ would seem to be the proper dimensionless parameters to describe its functional

behavior. This was, indeed, seemingly borne out from the analysis of the experimental excitation function. Γ (Eq. (24)) was roughly constant for $q < \mu$ and showed a marked change in slope (decreasing rapidly) for $\underline{q} > \mu$. On this basis, we can now estimate the error incurred by removing R from the integral in Eq. (11). For purposes of the argument, we define a "modified deuteron wave function" as

$$\phi'(\underline{g}) = R\left(\frac{\mu}{M+\mu}\,\underline{g}\right)\,\phi(\underline{g}) \tag{25}$$

where R is the matrix element of Eq. (11). Since we have argued that R is approximately constant for small $\underline{q}$, we can expect the coordinate representation of ϕ' to differ from that of ϕ only in the vicinity of the origin. This is fortunate, as we have seen that our results do not depend much on the form of the deuteron wave function near the origin. Using the Chew-Goldberger wave function, we can write Eq. (25) as

$$\phi'(\underline{g}) = R\left(\frac{\mu}{M+\mu}\,\underline{g}\right)\frac{1}{(\alpha^2 + g^2)}\,\frac{1}{(\beta^2 + g^2)} \tag{25'}$$

As we have noted that the functional dependence of R on q is apparently such that it becomes important when $\frac{\mu}{M+\mu}\,g \simeq \mu$, we can define the following parameters to describe the values of g for which the three factors in Eq. (25') become important:

$$\alpha = 45.5 \text{ Mev}$$

$$\beta = 318 \text{ Mev}$$

$$\left(\frac{M+\mu}{\mu}\right)\mu = 1080 \text{ Mev.}$$

We have stated that the factor $(\beta^2 + g^2)^{-1}$ gives only a small correction to the results, so it follows that any correction due to the variation of R with g is likely to be much less. Indeed, this uncertainty may very well be of the same order as the uncertainty in the deuteron wave function.

As to uncertainty in the deuteron wave function, we note that the factor $(\beta^2 + g^2)^{-1}$ in Eq. (25'), or $\frac{e^{-\beta r}}{r}$ in Eq. (21), gives a correction of less than 10 percent to the leading term in Eqs. (14). Since this correction is probably fairly reliable, it would seem that the error resulting from the uncertainty in the deuteron wave function is, indeed, quite small.

The p-wave phase shifts for the final n-n state have been verified to be negligibly small over the energy range for which there are an appreciable number of γ-rays. Thus the use of plane waves for ψ^t in Eq. (13) is well justified.

To summarize our arguments, we note that the gross features of the spectrum seem to be well determined independently of the uncertainties in the calculations. The general shape of the tail of the spectrum at lower energies is determined by the deuteron wave function. According to the arguments given in the Introduction, the shape of the spectrum near the high energy limit is given quite accurately by Eq. (4). At lower energies we must rely for the detailed shape of the spectra on the arguments for removing R in Eq. (11) from under the integral sign. But here the exact identification of R with the matrix element for photo-meson production is not necessary[15]. We need only the argument that R

approaches a constant value smoothly as q approaches zero. If R is only to a fair approximation related to the matrix element for photo-meson production, we can still expect the detailed balancing arguments[2] quoted above to apply. In any case, it would be highly desirable to have a better determination of the excitation function near threshold for photo-meson production.

VII. Conclusion

It appears that the γ-ray spectrum resulting from the absorption of π^--mesons in deuterium offers considerable promise for a quantitative determination of the parameters characteristic of low energy n-n scattering. A better experimental determination of the excitation function for photo-meson production near threshold should make it possible to resolve the greatest of the present uncertainties in the theory. In any case, the calculated spectrum would seem to be quite accurate near its high energy limit (and is probably quite accurate over the entire energy range for which an appreciable number of γ-rays are emitted).

The authors are indebted to Professor R. Serber and to Dr. K. A. Brueckner, with whom the initial phases of the present work were done in connection with the considerations reported in reference (2). To Dr. L. Aamodt, Professor W. K. H. Panofsky and Mr. Robert Phillips we are indebted for encouragement concerning the feasibility of measuring the γ-ray spectrum and for permission to quote the experimental results in advance of their publication.

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8. The theoretical curves have been "folded" into the experimental energy resolution (see reference (1)). If this energy resolution is $S(\mathcal{E}_0 - K)$, then the "fold" is

$$\int S(\mathcal{E}_0 - K) \frac{dI(K)}{dK} dK ,$$

where $\frac{dI}{dK}$ is the spectrum as given in Figs. (2) and (3).

9. The occurrence of $\underline{g}$ in r (Eq. (9)) is due to the internal motion of the proton in the deuteron. This can be seen as follows: The velocity of the center-of-mass of the meson and proton is $\underline{v} = \underline{g}/(M+\mu)$, where $\underline{g}$ is the proton momentum. The momenta of the meson and proton in their mutual center-of-mass coordinate system are, respectively, $\underline{q}' = -\mu\underline{v}$ and $\underline{g}' = \underline{g} - M\underline{v}$. Their relative momentum is thus

$$1/2(\underline{q}' - \underline{g}') = -\frac{\mu}{M+\mu} \underline{g} .$$

But this is the coordinate system in which R is defined (cf. Eq. (6)).

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15. Deviation of the R in Eq. (11) from the transition matrix for photo-meson production can be expected to be of importance only when the neutron and proton are very close together in the deuteron. However, we have seen that our results are insensitive to the nature of the interactions at close distances.

TABLE I

Neutron Binding Energy E_B in Mev.	Intensity of Monochromatic Spike in Units of Fig. III.
.01	34.0
.05	62.6
.25	100.6

The monochromatic spikes are separated from the continuum by approximately the binding energy.

Fig. I. A comparison of the approximate γ-ray cross section $\frac{1}{p}[\sin\delta]^2$ and the corrected S-wave contribution to the cross section for $\alpha = -8$ Mev.

Fig. II. A plot of the γ-ray cross section for several negative values of α.

Fig. III. The γ-ray cross section for various positive values of α (positive binding energy). The total cross sections including the contribution of the monochromatic spike have been normalized to a constant.

Fig. IV. A comparison of several "folded" theoretical γ-ray cross sections for different n-n binding energies with the preliminary experimental data of Aamodt, Panofsky and Phillips.

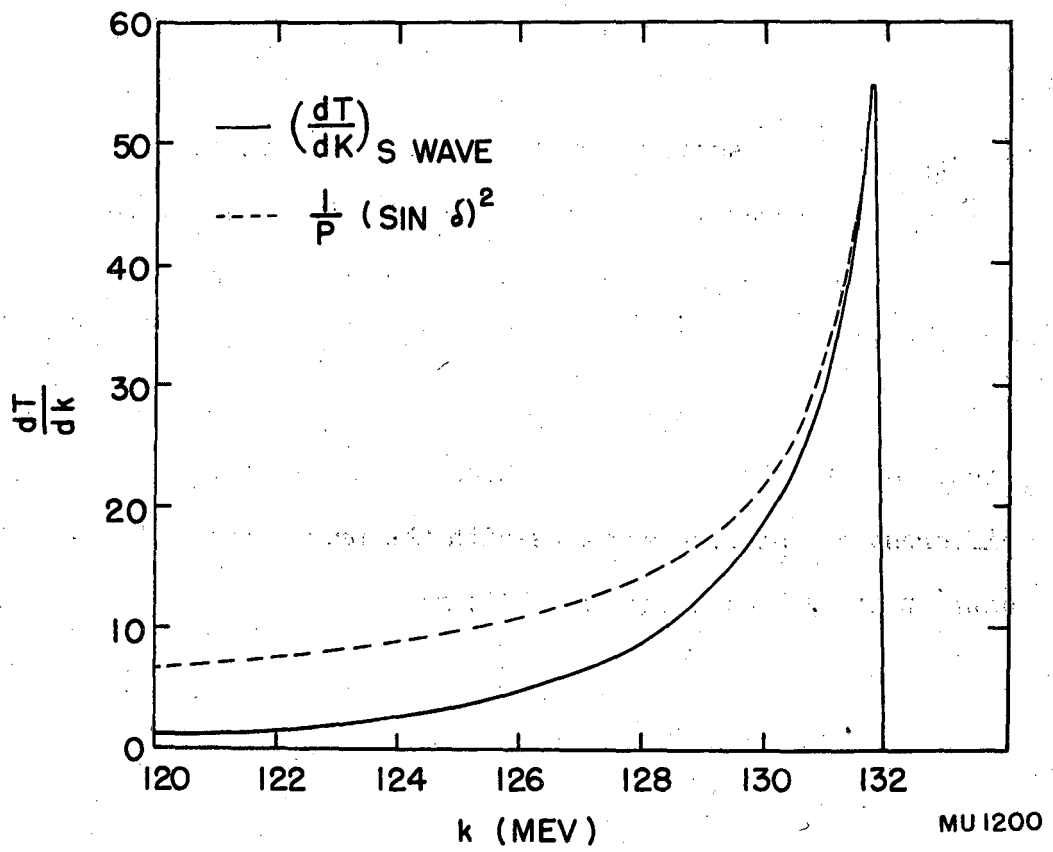
60
50
40
30
20
10
0
$\frac{dT}{dk}$
$\left(\frac{dT}{dK}\right)_{S\ WAVE}$
$\frac{1}{P}$ (SIN δ)^2
120
122
124
126
128
130
132
k (MEV)
MU 1200

Fig. I

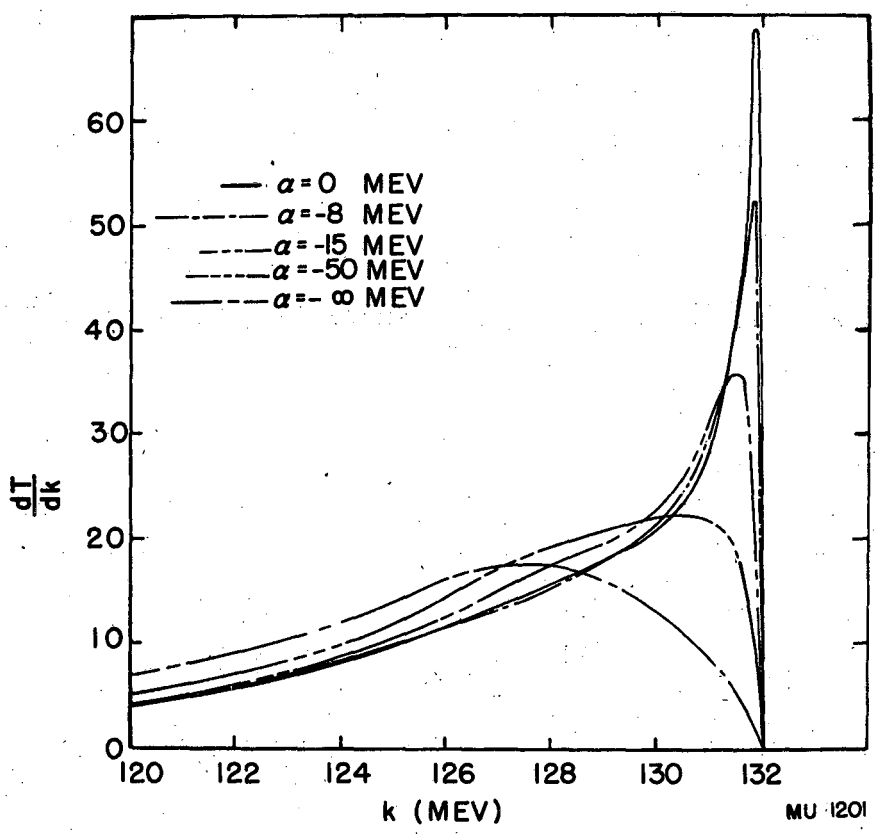
60
50
40
30
20
10
0
$\frac{dT}{dk}$
α = 0 MEV
α = -8 MEV
α = -15 MEV
α = -50 MEV
α = - ∞ MEV
120
122
124
126
128
130
132
k (MEV)
MU 1201

Fig. II

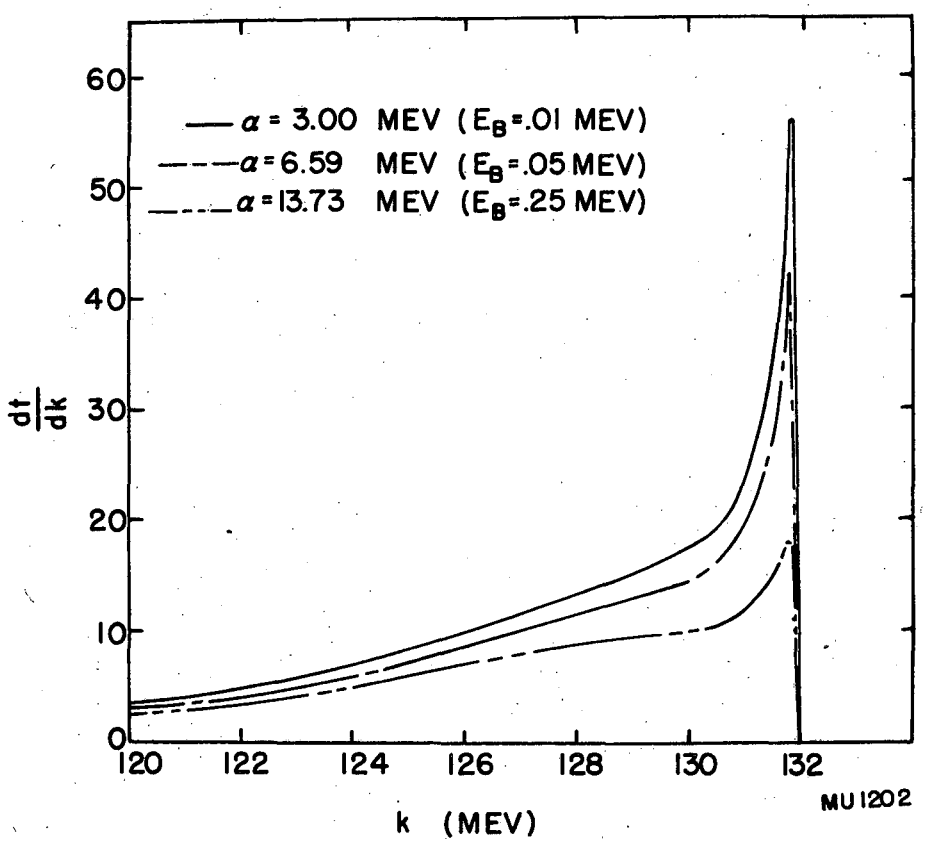
60
50
40
30
20
10
0
$\frac{dt}{dk}$
α = 3.00 MEV (E_B=.01 MEV)
α=6.59 MEV (E_B=.05 MEV)
α=13.73 MEV (E_B=.25 MEV)
120
122
124
126
128
130
132
k (MEV)
MU1202

Fig. III

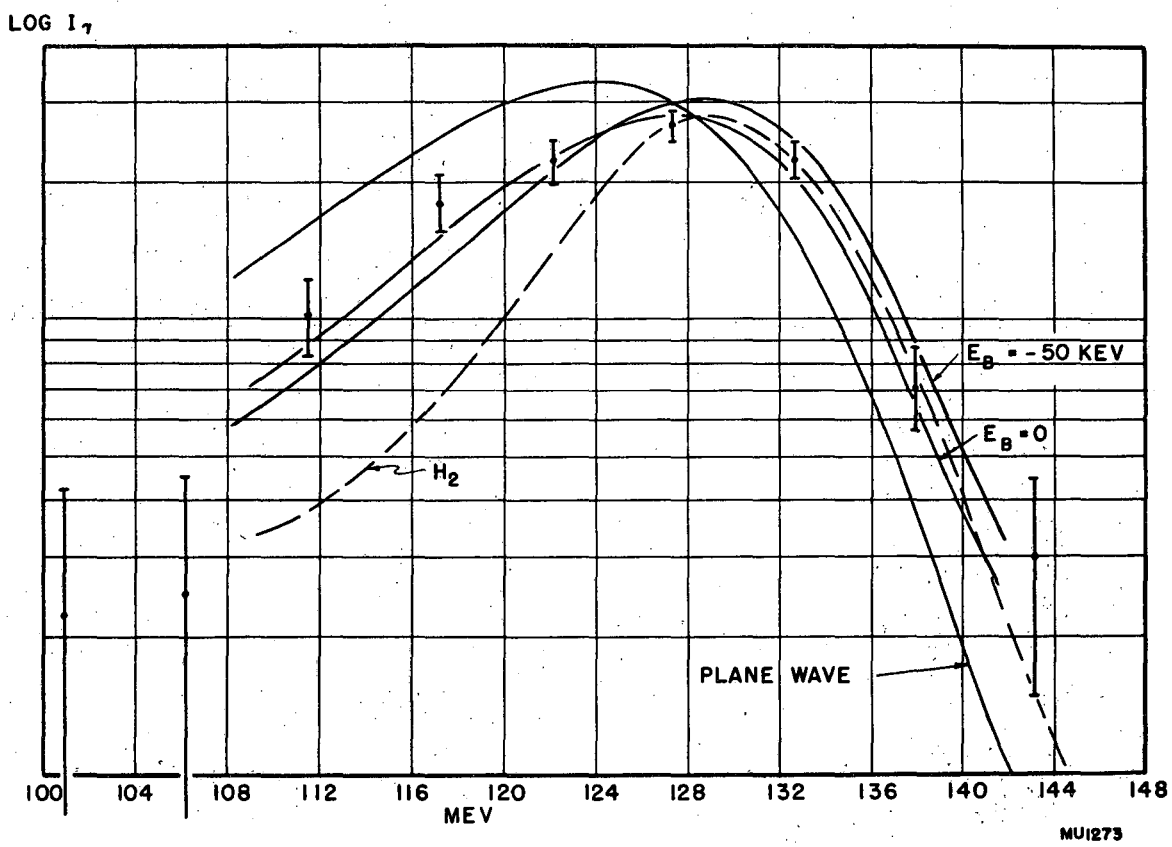
LOG I7
E_B = -50 KEV
E_B = 0
H2
PLANE WAVE
100
104
108
112
116
120
124
128
132
136
140
144
148
MEV
MU1273

Fig. IV