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LIGHT INTERFERENCE IN TRANSPARENT THIN FILMS ON METAL SURFACES

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ABSTRACT

Interference fringes from transparent films on reflecting surfaces, which are usually very difficult to observe, are shown to be of optimum contrast (or color saturation) if observed with linearly polarized rather than natural light. For the direction of polarization parallel or perpendicular to the plane of incidence particular angles of incidence are calculated for which best contrast is expected. These "optimum" angles of incidence, computed for typical metal-film combinations are confirmed experimentally for a film of aqueous potassium hydroxide on a polished, optically flat nickel plate. A complete computer program for the calculations involved is contained in the appendix.

Interference phenomena in thin films which result from the superposition of light waves reflected at the two surfaces of a film have long been used[1] as convenient means for determining optical film thicknesses and different techniques, which usually employ unpolarized light at normal incidence have more recently been reviewed in monographs, such as those by Mayer,[2] Heavens[3] or Wolter.[4] While the reflectance of both surfaces is of the same magnitude for a film surrounded by air, such as a soap bubble, thus leading to easily observable interference phenomena, this is not the case for a film in contact with a reflecting metallic surface. Here, the reflection of light under close-to-normal incidence is so much higher at the film-metal interface than at the film-air interface that only partial extinction occurs in the case of destructive interference. As a result, the interference fringes observed are of such low contrast or whitish color that they are difficult to recognize and not suitable for quantitative evaluation. (Rayleigh Criterion[5]). This situation can be greatly improved by simple procedures which do not seem to be treated in the literature, although Tolansky[6] has discussed the influence of polarization on the appearance of silver-modified Newton rings. The method is based on the dependence of reflection and refraction on angle of incidence and direction of polarization. A theoretical analysis shows that particular angles of incidence exist for light polarized parallel ($\parallel$) and normal ($\perp$) to the plane of incidence under which the amplitudes of the interfering waves are matched, thus resulting in complete extinction under conditions of destructive interference. The film is assumed to be isotropic and non-absorbing. Experiments conducted with liquid films on polished

metal surfaces have confirmed the dependence of the interference phenomena on angle of incidence and mode of polarization and have yielded saturated, easily indentifiable interference colors under the conditions computed for amplitude matching.

THEORETICAL

Air-Film Interface

The amplitudes of the reflected and refracted waves in the film are identified in figure 1. For reflection and refraction of light at the air-film interface the amplitude ratios can be described[9] by the Fresnel equations [2 to 5] as a function of angle of incidence ϕ and refractive index n_1.*) They are different for light linearly polarized normal ($\perp$) and parallel ($\parallel$) to the plane of incidence.**) The angle of refraction ϕ' depends on ϕ and n_1 according to Snell's law (eq. 1).

$$\sin \phi = \frac{\sin \phi}{n_1} \qquad [1]$$

$$\perp \frac{R_o}{E} = -\frac{\sin(\phi - \phi')}{\sin(\phi + \phi')} \qquad [2]$$

$$\parallel \frac{R_o}{E} = -\frac{\tan(\phi - \phi')}{\tan(\phi + \phi')} \qquad [3]$$

$$\perp \frac{D_o}{E} = \frac{2 \sin \phi' \cos \phi}{\sin(\phi + \phi')} \qquad [4]$$

$$\parallel \frac{D_o}{E} = \frac{2 \sin \phi' \cos \phi}{\sin(\phi + \phi') \cos(\phi - \phi')} \qquad [5]$$

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*) The sign convention employed in recent texts such as Stone,[7] Francon,[8] Valasek[9] and Rossi[10] shall be used here.

**) Defined by the plane containing incident and reflected beams.

Plots of equations [2 to 5] are shown in Figure 2 for $n_1 = 1.365$. The negative sign of the reflected amplitude symbolizes a phase shift of $\lambda/2$ between incident and reflected wave. It can be seen that for the parallel mode of polarization this phase shift disappears for angles of incidence above Brewster's angle ϕ_p and does not exist for either mode of the refracted wave.

Film-metal Interface

The reflection at the film-metal interface can similarly be described by use of a complex refractive index n for the metal[10]

$$n = n_2 \ (1 - i\kappa_2)$$

Typical values of refractive index n_2 and absorption index κ_2 for a metal of low, medium and high reflectance are listed in Table 1. The reflectance for an air-metal interface as a function of refractive index, absorption index and angle of incidence has been given by König[11] and is shown in Eq. [6 to 9] modified[12] for contact with a medium of refractive index n_1.

$$\perp \left(\frac{M_1}{D_o}\right)^2 = \frac{a^2 + b^2 - 2a \cos \phi' + \cos^2 \phi'}{a^2 + b^2 + 2a \cos \phi' + \cos^2 \phi'} \qquad [6]$$

$$\parallel \left(\frac{M_1}{D_o}\right)^2 = \left(\frac{M_1}{D_o}\right)^2_{\perp} \frac{a^2 + b^2 - 2a \sin \phi' \tan \phi' + \sin^2 \phi' \tan^2 \phi'}{a^2 + b^2 + 2a \sin \phi' \tan \phi' + \sin^2 \phi' \tan^2 \phi'} \qquad [7]$$

where

$$a^2 = \tfrac{1}{2}\left[\sqrt{\left[\left(\frac{n_2}{n_1}\right)^2 (1 - \kappa_2^2) - \sin^2 \phi'\right]^2 + 4\left(\frac{n_2}{n_1}\right)^4 \kappa_2^2} + \left(\frac{n_2}{n_1}\right)^2 (1 - \kappa_2^2) - \sin^2 \phi'\right] \qquad [8]$$

$$b^2 = \tfrac{1}{2}\left[\sqrt{\left[\left(\frac{n_2}{n_1}\right)^2 (1 - \kappa_2^2) - \sin^2 \phi'\right]^2 + 4\left(\frac{n_2}{n_1}\right)^4 \kappa_2^2} - \left(\frac{n_2}{n_1}\right)^2 (1 - \kappa_2^2) + \sin^2 \phi'\right] \qquad [9]$$

Table I

Optical Constants of Metals for

Sodium Light ($\lambda = 0.589\ \mu$)

Metal	n_2	κ_2	Source
W	3.46	0.94	(14)
Ni	1.75	1.95	(14,15)
Ag	0.18	20.6	(14)

Numerical results of equations [6 to 9] using the data Table 1 and n_1 = 1.365 are shown in Figure 3 plotted vs. ϕ. The negative sign of the amplitude ratios has been introduced to indicate the phase shift of $\lambda/2$ due primarily to the metallic reflection combined with the effect of a very thin* film.

Film-air Interface

The internal reflection and refraction at the film-air interface is described by Eqns. [2 to 5] with ϕ and ϕ' exchanged in order to account for the reversed light propagation. Thus, Eqns. [10 to 13] are obtained, of which Figure 4 gives a graphical account.

$$\perp \frac{D_1}{M_1} = \frac{\sin(\phi - \phi')}{\sin(\phi + \phi')} = -\frac{R_o}{E} \qquad [10]$$

$$\parallel \frac{D_1}{M_1} = \frac{\tan(\phi - \phi')}{\tan(\phi + \phi')} = -\frac{R_o}{E} \qquad [11]$$

$$\perp \frac{R_1}{M_1} = \frac{2 \sin\phi \cos\phi'}{\sin(\phi + \phi')} \qquad [12]$$

$$\parallel \frac{R_1}{M_1} = \frac{2 \sin\phi \cos\phi'}{\sin(\phi + \phi')\cos(\phi - \phi')} \qquad [13]$$

Multiple reflection

The individual reflection and refraction coefficients $\frac{R_o}{E}$, $\frac{D_o}{E}$, $\frac{M_1}{D_o}$, $\frac{D_1}{M_1}$, $\frac{R_1}{M_1}$ are combined to obtain the amplitude of higher order reflections. This yields

$$\frac{R_1}{E} = \frac{D_o}{E} \frac{M_1}{D_o} \frac{R_1}{M_1}$$

---

*) Calculated e.g. for a film of n. = 1.365 on nickel to be 0.04 μ and 0.02 μ for ∥ and ⊥ polarization, respectively.

$$\frac{R_2}{E} = \frac{D_o}{E}\,\frac{M_1}{D_o}\,\frac{D_1}{M_1}\,\frac{M_2}{D_1}\,\frac{R_2}{M_2} \quad \text{etc.}$$

with

$$\frac{M_{n+1}}{D_n} = \frac{M_1}{D_o} \qquad n = 1, 2, 3 \ldots$$

$$\frac{R_{n+1}}{M_{n+1}} = \frac{R_1}{M_1}$$

$$\frac{D_{n+1}}{M_{n+1}} = \frac{D_1}{M_1}$$

one obtains

$$\frac{R_n}{E} = \frac{D_o}{E}\,\frac{R_1}{M_1}\left(\frac{D_1}{M_1}\right)^{n-1}\left(\frac{M_1}{D_o}\right)^{n} \qquad n = 1, 2, 3 \ldots \qquad [14]$$

For the systems under consideration, the first two factors of Eq. [14] are always positive and the last is always negative. The sign of the third factor depends on angle of incidence and mode of polarization, as indicated in Figure 4. The amplitudes of the reflected waves, expressed by Eq. 14, hold for transparent films of any thickness, while the phase expressed by the sign assumes a very small optical path length in the film (as defined on p. 6).

Destructive interference at very small film thickness

For parallel polarization (‖) under angles of incidence above Brewster's angle $\frac{D_1}{M_1}$ is negative and consequently $\frac{R_n}{E}$ in Eq. (14) is negative for any value of n. This means that at very small film thickness all higher order reflections are of opposite phase to the zero order reflection (since $\frac{R_o}{E}$ is positive under the same conditions) and destructive interference occurs. This phase relationship is sketched in Figure 5a. The total negative

amplitude is the sum of the progression [14].*)

$$\sum_{n=1}^{\infty} \frac{R_n}{E} = \frac{\frac{D_o}{E}\,\frac{R_1}{M_1}\,\frac{M_1}{D_o}}{1 - \frac{D_1}{M_1}\,\frac{M_1}{D_o}} \qquad [15]$$

In order to find the parameters for which positive and negative amplitudes are matched, the amplitude ratio [16] is formed

$$\frac{R_o}{E} \Big/ \sum \frac{R_n}{E} \qquad [16]$$

It is plotted in Figure 6 and designated $R_o/\Sigma R_n$. The ratio of -1 is reached at $\phi_{\parallel}$. At this "optimum angle of incidence" complete destructive interference is to be expected on a film of very small thickness ($d \ll \lambda$) with light polarized parallel ($\parallel$) to the plane of incidence.

Constructive interference at vanishing film thickness

For normal polarization ($\perp$) under any angle of incidence and parallel ($\parallel$) polarization below Brewster's angle $\frac{D_1}{M_1}$ is positive. Consequently $\frac{R_n}{E}$ in Eq. [14] is alternately negative and positive with the sum of the progression being negative. As the zero-order reflection $\frac{R_o}{E}$ under these conditions is also of negative sign, the interference is constructive. (Figure 5b). Since a matching of zero and higher order reflection amplitudes under constructive interference is not of particular experimental interest, the analysis is continued for a film of such thickness as to introduce an additional reversal of phase between successive

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*) A similar summation has been carried out by A. Vasicek[13] under the assumption that $D_o/E = R_1/M$.

reflections.*) As illustrated in Figure 5c, the resulting phase relationship between successive reflections is the same as that treated before (Figure 5a) except that all reflections have opposite phase. The amplitude ratio [Eq. 16] is plotted with a positive value in Figure 6 to indicate constructive interference at very small film thickness. At $\phi_{\perp}$, the other "optimum angle of incidence", where the interfering amplitude ratio is + 1 complete destructive interference is to be expected on a film of about $\lambda/4$ thickness with light polarized perpendicular ($\perp$) to the plane of incidence.

Film refractive index

The above computations were repeated for two other refractive incices of the film $n_1 = 1.20$ and $n_1 = 1.50$ to cover the range of aqueous solutions of interest in this study. Table II lists the Brewster's angle ϕ_p for the three film refractive indices as obtained from eq. [17][9]

$$\tan \phi_p = n_1 \qquad [17]$$

The final results for $\phi_{\perp}$ and $\phi_{\parallel}$ computed for the three metal surfaces covered with films of three different refractive indices are given in Table III.

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*) More precisely, the optical path in the film of thickness d between reflections

$$\Delta S = 2\, n_1\, d \cos \phi'$$

must be $\lambda/2$ larger than considered before. d is therefore about $\lambda/4$, depending on refractive index n_1 and angle of refraction ϕ'.

Table II

Brewster's angle ϕ_p for different film refractive indices n_1

n_1	ϕ_p
1.20	$50^\circ 12'$
1.365	$53^\circ 46'$
1.50	$56^\circ 19'$

Table III

Optimum angles of incidence $\phi_{\perp}$ and $\phi_{\parallel}$ for the observation of transparent thin films on different metal surfaces with Na light polarized normal ($\perp$) and parallel ($\parallel$) to the plane of incidence.

Metal Surface	Film Refractive Index n_1	$\phi_{\perp}$	$\phi_{\parallel}$
W	1.200	75°	82°
W	1.365	67°	82°
W	1.500	61°	82°
Ni	1.200	75°	84°
Ni	1.365	68°	84°
Ni	1.500	63°	84°
Ag	1.200	76°	89°
Ag	1.365	71°	89°
Ag	1.500	66°	89°

EXPERIMENTAL

The application of the "optimum angles of incidence" has proven very useful in a study on the formation of liquid electrolyte films of 0.5 to 5 micron thickness on polished metal surfaces. Experiments were conducted with a film of refractive index 1.365 (3.4 normal aqueous solution of potassium hydroxide) on nickel in sodium light ($\lambda = 0.589\ \mu$). The optical system employed is sketched in figure 7.*) The photographs obtained (figure 8) illustrate the changing appearance of the same thin film under different angles of incidence for both modes of polarization. It can be seen that, as predicted by the computations, with increasing angle of incidence the interference fringes from $\perp$ polarization remain qualitatively the same**) but increase in contrast. The fringes from $\parallel$ polarization, on the other hand, being the same as those of $\perp$ at normal incidence decrease in contrast with increasing ϕ until they vanish at Brewster's angle $\phi_p = 54^\circ$ (where no reflection occurs at the film air interface). For angles of incidence above ϕ_p the $\parallel$ picture increases in contrast with increasing ϕ while its fringe pattern is complementary to the $\perp$ pattern. Thus, under these conditions the use of non-polarized light, which can be described as a superposition of both modes of polarization, would result in a further decrease in contrast. At $\phi = 77.5^\circ$ (intermediate between $\phi_\perp$ and $\phi_\parallel$) the interfering

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*) For the larger values of ϕ the plate holder could not be tilted sufficiently to compensate for the geometrical distortion of the picture in the horizontal direction.

**) Except for a small shift due to the decreased optical path in the film as ϕ is increased. (see footnote on page 9).

amplitude ratios deviate for both modes of polarization by the same factor (1.9) from complete extinction, thus both pictures can be observed simultaneously with equal and satisfactory (although not maximum) contrast, as demonstrated in figure 8.

SUMMARY

1. Interference fringes (or colors) from transparent films on metal surfaces are best observed with light polarized normal or parallel to the plane of incidence.
2. For each mode of polarization an optimum angle of incidence $\phi_{\perp}$ or $\phi_{\parallel}$ exists at which the amplitudes of the interfering waves are matched (ratio $\pm$ 1).
3. The values of $\phi_{\perp}$ and $\phi_{\parallel}$ depend on the optical constants of the metal surface and the refractive index of the film. $\phi_{\perp}$ and $\phi_{\parallel}$ are larger with a more highly reflecting metal surface, and with a lower refractive index n_1 of the film, but $\phi_{\parallel}$ is practically independent of n_1.
4. At the respective optimum angles destructive interference occurs with the $\parallel$ polarization at film thicknesses close to 0, $\frac{\lambda}{2}$, λ etc., with the $\perp$ polarization at close to $\frac{\lambda}{4}$, $3\frac{\lambda}{4}$, $5\frac{\lambda}{4}$ etc.
5. An application of the above results has yielded very clear interference patterns which were almost unobservable before. Intensity minima and certain interferance colors could be identified corresponding to better than $\pm$ 0.05 micron film thickness.

Refinements of the theoretical analysis to include the exact phase change upon metallic reflection and the frequency dependence of the optical constants associated with the use of white light are under consideration.

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APPENDIX

A computer program in FORTRAN language capable of performing all the calculations contained in this report for any optical properties of film and metal surface is given below. In addition, the program incorporates the computation of the phase change δ occurring upon a single reflection at the film-metal interface according to:[11]

$$\perp \quad \tan \delta = \frac{-2b \cos \phi'}{a^2 + b^2 - \cos^2 \phi}$$

$$\parallel \quad \tan \delta = \frac{2b \cos \phi' \, (a^2 + b^2 - \sin^2 \phi')}{a^2 + b^2 - n^4 (1 + k_2^2)^2 \cos^2 \phi'}$$

where all the symbols have the previously defined meanings.

A tabulation of results for the nine cases investigated numerically and summarized in Table 3 is also contained in the appendix.

LIGHT INTERFERENCE IN TRANSPARENT THIN FILMS ON METAL SURFACES

NAMES OF INPUT AND AUTPUT VATABLES

TNF	n_1	
TNM	n_2	
TK	κ_2	
PD	ϕ	(deg.)
PHI	ϕ	(rad.)
PHIP	ϕ'	(rad.)
A	a	
B	b	
RS	$\perp$	$\frac{M_1}{D_o}$
RP	$\parallel$	$\frac{M_1}{D_o}$
TANDS	$\perp$	$\tan \delta$
TANDP	$\parallel$	$\tan \delta$
RATS	$\perp$	$\frac{R_o}{E} / \sum \frac{R_n}{E}$
RATP	$\parallel$	$\frac{R_o}{E} / \sum \frac{R_n}{E}$

FORTRAN PROGRAM

```
      DIMENSION PHI(7),PHIP(7),ASQ(7),A(7),BSQ(7),B(7),RPSQ(7),RP(7),
     CRSSQ(7),RS(7),TANDS(7),TANDP(7),RATS(7),RATP(7),PD(7)
    2 FORMAT (2F 10.0)
    3 FORMAT (F10.0)
    4 FORMAT (7F 10.0)
    5 FORMAT (1H0,20X,4HTNF=,F7.4,10X,4HTNM=,F7.4,10X,3HTK=,F7.4)
    6 FORMAT (1H0,4X,2HPD,8X, 3HPHI,7X,4HPHIP,7X,1HA,9X,1HB,8X,2HRS,8X,
     C2HRP,7X,5HTANDS,5X,5HTANDP,6X4HRATS,6X,4HRATP)
    7 FORMAT (11F10.5)
   20 READ 2,(TNM,TK)
      READ 3,TNF
      READ 4,(PD(M),M=1,7)
      DO 100 M=1,7
      PHI(M)=PD(M)*0.0174533
      SPHIP=SINF(PHI(M))/TNF
      PHIP(M)=0.0
      PJUMP=0.1
      GO TO 28
   26 IF(SINF(PHIP(M))-SPHIP)28,100,30
   28 PHIP(M)=PHIP(M)+PJUMP
```

```
      IF(1.57079632-PHIP)100,26,26
   30 PHIP(M)=PHIP(M)-PJUMP
      PJUMP=0.1*PJUMP
      PHIP(M)=PHIP(M)+PJUMP
      IF(PJUMP-0.0000002)100,26,26
  100 CONTINUE
      DO 200 M=1,7
      TN=TNM/TNF
      SM=PHI(M)+PHIP(M)
      DD=PHI(M)-PHIP(M)
      SP=SINF(PHI(M))
      CP=COSF(PHI(M))
      SX=SINF(PHIP(M))
      CX=COSF(PHIP(M))
      SD=SINF(DD)
      CD=COSF(DD)
      SS=SINF(SM)
      CS=COSF(SM)
      TX=SX/CX
      TM=SS/CS
      TD=SD/CD
      ASQ(M)=0.5*(SQRTF((TN*TN*(1.0-TK*TK)-SX*SX)**2.0+4.0*TN**4.0*TK*TK
     C)+TN*TN*(1.0-TK*TK)-SX*SX)
      IF(ASQ(M))110,112,112
  110 A(M)=0
      GO TO 114
  112 A(M)=SQRTF(ASQ(M))
  114 BSQ(M)=0.5*(SQRTF((TN*TN*(1.0-TK*TK)-SX*SX)**2.0+4.0*TN**4.0*TK*TK
     C)-TN*TN*(1.0-TK*TK)+SX*SX)
      IF(BSQ(M))116,118,118
  116 B(M)=0
      GO TO 120
  118 B(M)=SQRTF(BSQ(M))
  120 RSSQ(M)=(ASQ(M)+BSQ(M)-2.0*A(M)*CX+CX*CX)/(ASQ(M)+BSQ(M)+2.0*A(M)
     C*CX+CX*CX)
      IF(RSSQ(M))122,124,124
  122 RS(M)=0
      GO TO 126
  124 RS(M)=-SQRTF(RSSQ(M))
  126 RPSQ(M)=RSSQ(M)*(ASQ(M)+BSQ(M)-2.0*A(M)*SX*TX+SX*SX*TX*TX)/(ASQ(M)
     C+BSQ(M)+2.0*A(M)*SX*TX+SX*SX*TX*TX)
      IF(RPSQ(M))128,130,130
  128 RP(M)=0
      GO TO 132
  130 RP(M)=-SQRTF(RPSQ(M))
  132 TANDS(M)=-2.0*B(M)*CX/(ASQ(M)+BSQ(M)-CX*CX)
      TANDP(M)=2.0*B(M)*CX*(ASQ(M)+BSQ(M)-SX*SX)/(ASQ(M)+BSQ(M)-TN**4.0*
     C((1.0+TK*TK)**2.0)*CX*CX)
      ROS=-SD/SS
      DOS=2.0*SX*CP/SS
      RIS=2.0*SP*CX/SS
      SRNS=DOS*RIS*RS(M)/(1.0+ROS*RS(M))
      RATS(M)=ROS/SRNS
      ROP=-TD/TM
      DOP=2.0*SX*CP/(SS*CD)
      RIP=2.0*SP*CX/(SS*CD)
      SRNP=DOP*RIP*RP(M)/(1.0+ROP*RP(M))
      RATP(M)=ROP/SRNP
  200 CONTINUE
      PRINT 5,TNF,TNM,TK
```

```
      PRINT 6
      DO 300 M=1,7
  300 PRINT  7,PD(M),PHI(M),PHIP(M),A(M),B(M),RS(M),RP(M),TANDS(M),
     CTANDP(M),RATS(M),RATP(M)
      GO TO 20
      END
```

FIGURE CAPTIONS

Figure 1. Reflections from a transparent film on a metal surface, designation of amplitudes and angles.

Figure 2. External reflection and refraction of coefficients of the air-film interface (amplitude ratios, $n_1 = 1.365$).

Figure 3. Reflection coefficients for different film-metal interfaces (tungsten, nickel and silver surface, film refractive index $n_1 = 1.365$).

Figure 4. Internal reflection and refraction coefficients of the film-air interface ($n_1 = 1.365$).

Figure 5. Phase relationship of incident and reflected light

(a) For destructive interference with vanishing film thickness

(b) For constructive interference with vanishing film thickness

(c) For destructive interference with $\lambda/2$ path in the film

Figure 6. Amplitude ratio of interfering waves reflected by a transparent thin film ($n_1 = 1.365$) on different metal surfaces.

Figure 7. Optical system used for obtaining the photographs shown in figure 8. A = sodium vapor lamp; B = condenser; C = iris diaphragm, 2 m·m opening; D = collimator, f = 19.5 cm, d = 5 cm; E = heat filter, CORNING 1 - 59; F = POLAROID filter; G = polished nickel surface 1.7 x 3 cm with liquid film; H = objective lens, f = 19.5 cm, d = 5 cm; I = plate holder; ϕ = angle of incidence.

Figure 8. Appearance of a film of 3.4 normal potassium hydroxide solution ($n_1 = 1.365$) on a polished, optically flat nickel plate. Interference fringes of equal thickness are shown as a function of angle of incidence ϕ and mode of polarization ($|$ or $||$) in parallel sodium light.

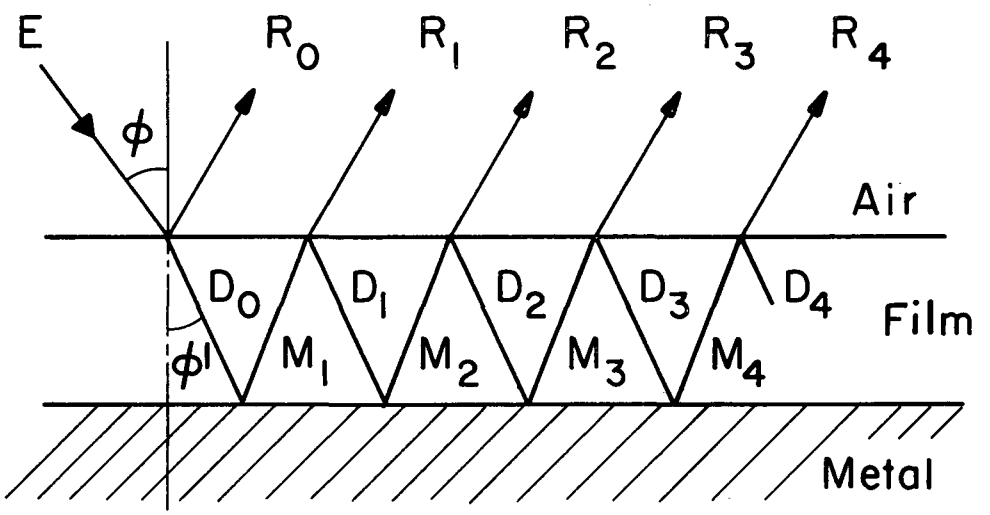

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Fig. 1

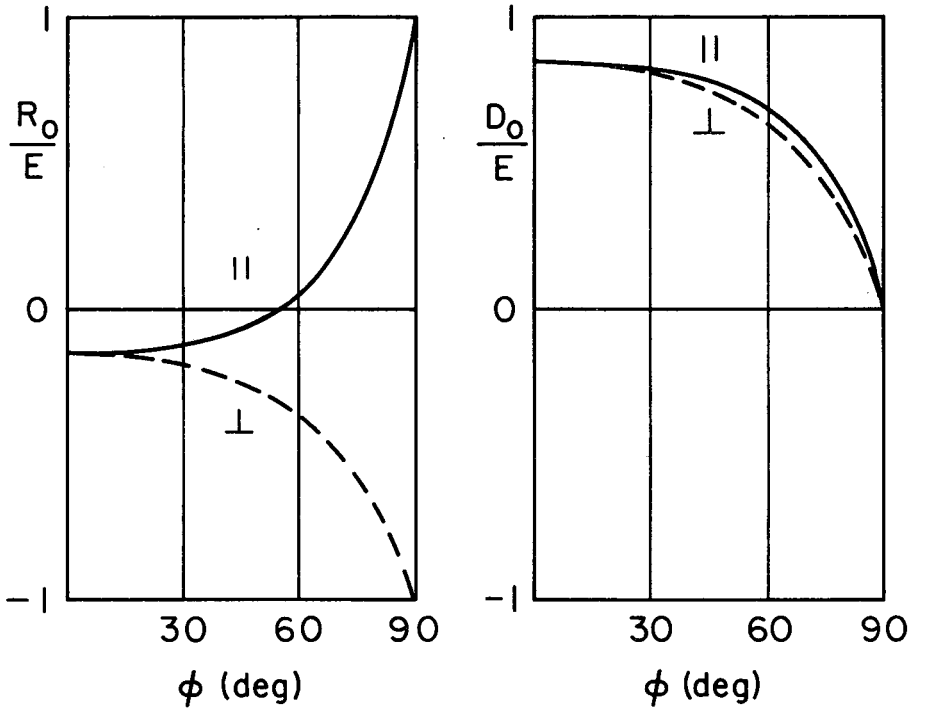

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Fig. 2

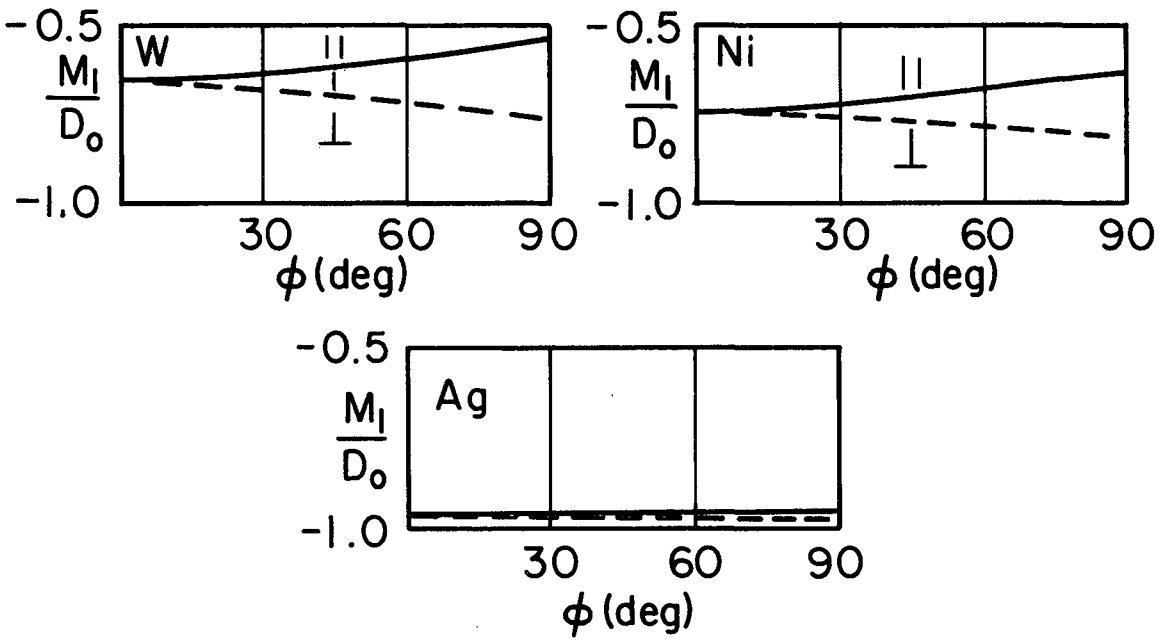

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Fig. 3

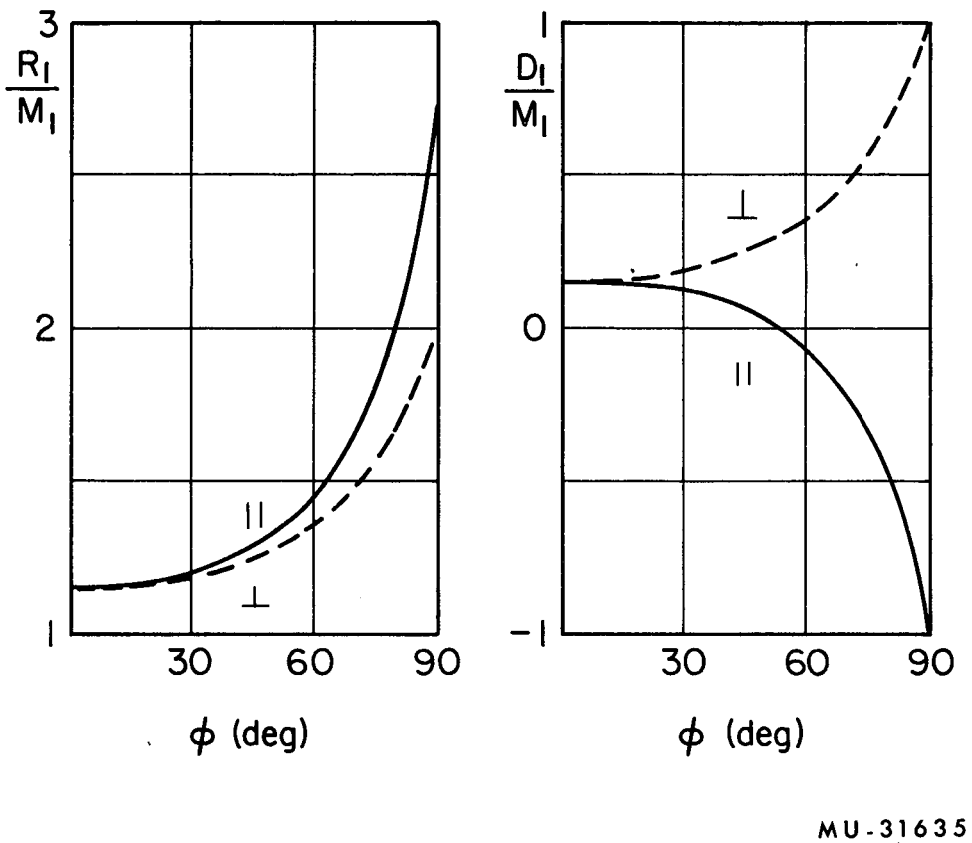
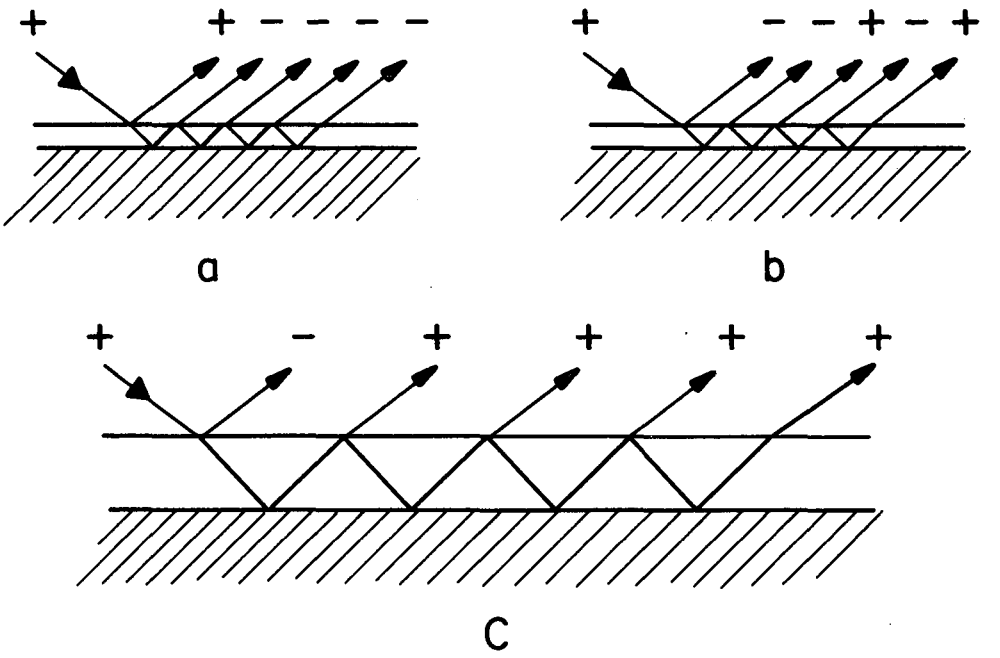

3
$\frac{R_l}{M_l}$
2
1
30
60
90
ϕ (deg)
||
⊥
1
$\frac{D_l}{M_l}$
0
−1
30
60
90
ϕ (deg)
⊥
||
MU-31635

Fig. 4



MU-31637

Fig. 5

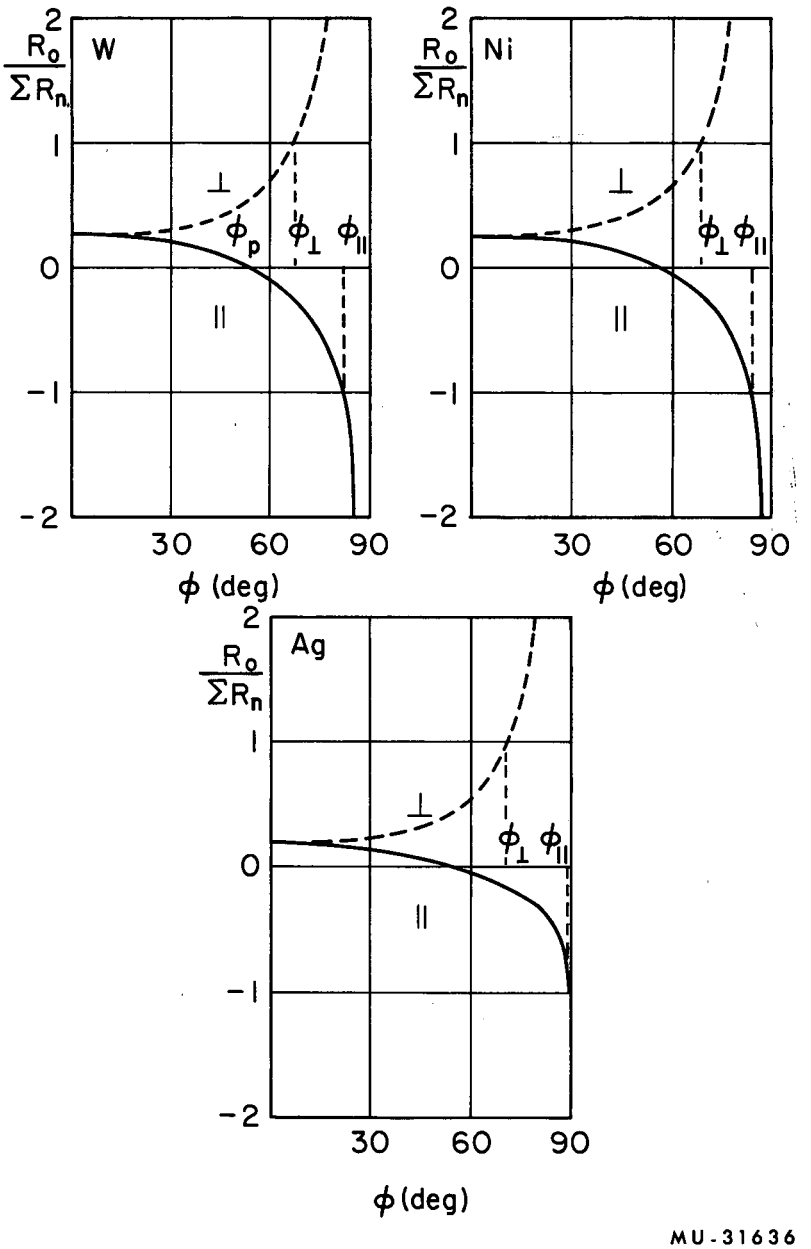

W
Ni
Ag
$\frac{R_o}{\Sigma R_n}$
2
1
0
-1
-2
30
60
90
ϕ (deg)
$\perp$
$\parallel$
ϕ_p
$\phi_\perp$
$\phi_\parallel$
MU-31636

Fig. 6

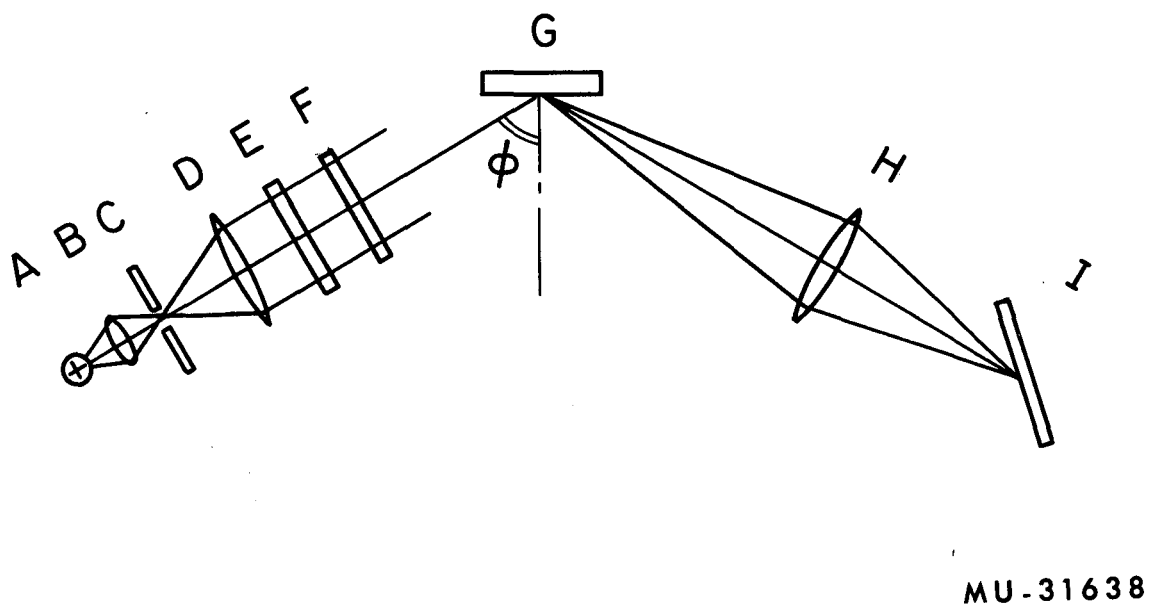
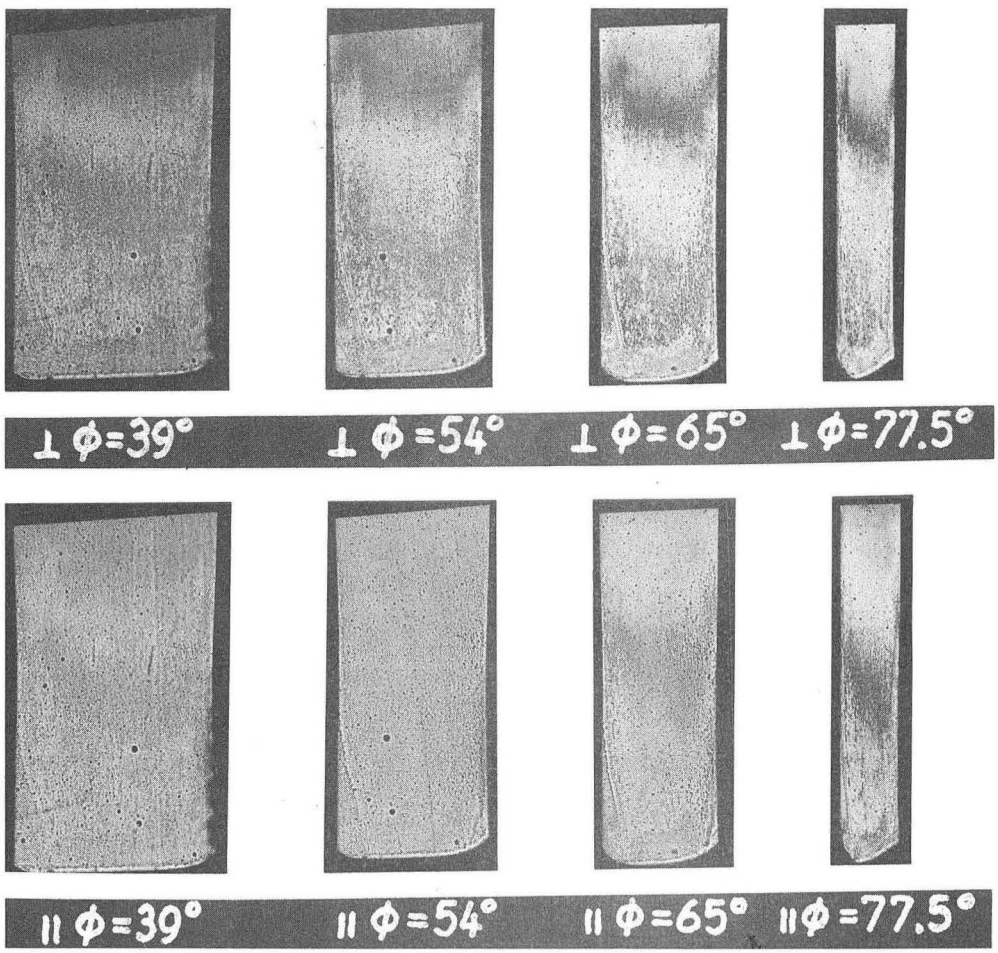


Fig. 7



ZN-3960

Fig. 8

TABULATION OF RESULTS

TNF= 1.2000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02909	2.88326	2.71041	-0.69704	-0.69683	-0.36960	-0.36994	0.13999	0.13973
10.00000	0.17453	0.14522	2.88141	2.71215	-0.69964	-0.69421	-0.36563	-0.37397	0.14323	0.13649
20.00000	0.34907	0.28902	2.87586	2.71737	-0.70765	-0.68596	-0.35351	-0.38697	0.15407	0.12557
30.00000	0.52360	0.42978	2.86739	2.72540	-0.72061	-0.67194	-0.33428	-0.41009	0.17496	0.10436
40.00000	0.69813	0.56528	2.85703	2.73529	-0.73785	-0.65192	-0.30939	-0.44559	0.21175	0.06699
50.00000	0.87266	0.69238	2.84603	2.74585	-0.75822	-0.62602	-0.28093	-0.49663	0.27757	0.00168
60.00000	1.04720	0.80624	2.83574	2.75582	-0.77988	-0.59550	-0.25172	-0.56612	0.40542	-0.11533

TNF= 1.2000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
65.00000	1.13446	0.85604	2.83125	2.76019	-0.79035	-0.57955	-0.23796	-0.60758	0.51831	-0.20719
70.00000	1.22173	0.89960	2.82738	2.76397	-0.79999	-0.56414	-0.22549	-0.65189	0.69878	-0.33526
73.00000	1.27409	0.92218	2.82540	2.76591	-0.80517	-0.55557	-0.21887	-0.67864	0.86528	-0.43779
73.50000	1.28282	0.92566	2.82510	2.76620	-0.80598	-0.55421	-0.21784	-0.68302	0.89952	-0.45741
74.00000	1.29154	0.92905	2.82480	2.76649	-0.80677	-0.55288	-0.21683	-0.68737	0.93608	-0.47789
74.50000	1.30027	0.93235	2.82452	2.76677	-0.80754	-0.55157	-0.21585	-0.69169	0.97519	-0.49928
75.00000	1.30900	0.93557	2.82424	2.76705	-0.80830	-0.55029	-0.21489	-0.69595	1.01711	-0.52166

TNF= 1.2000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
75.50000	1.31772	0.93870	2.82397	2.76731	-0.80903	-0.54904	-0.21396	-0.70017	1.06213	-0.54511
76.00000	1.32645	0.94173	2.82371	2.76757	-0.80975	-0.54782	-0.21305	-0.70433	1.11059	-0.56973
77.00000	1.34390	0.94753	2.82321	2.76805	-0.81112	-0.54546	-0.21131	-0.71245	1.21942	-0.62289
80.00000	1.39626	0.96259	2.82193	2.76931	-0.81473	-0.53920	-0.20677	-0.73476	1.68442	-0.82559
81.00000	1.41372	0.96680	2.82157	2.76966	-0.81575	-0.53742	-0.20549	-0.74133	1.91134	-0.91464
81.50000	1.42244	0.96875	2.82141	2.76982	-0.81623	-0.53659	-0.20490	-0.74443	2.04546	-0.96515
82.00000	1.43117	0.97060	2.82125	2.76997	-0.81667	-0.53580	-0.20433	-0.74738	2.19677	-1.02061

TNF= 1.2000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
82.50000	1.43990	0.97234	2.82111	2.77012	-0.81710	-0.53505	-0.20380	-0.75020	2.36875	-1.08197
83.00000	1.44862	0.97397	2.82097	2.77025	-0.81749	-0.53435	-0.20331	-0.75287	2.56581	-1.15045
83.50000	1.45735	0.97549	2.82084	2.77038	-0.81787	-0.53369	-0.20284	-0.75538	2.79375	-1.22766
84.00000	1.46608	0.97690	2.82072	2.77049	-0.81821	-0.53308	-0.20241	-0.75773	3.06029	-1.31574
85.00000	1.48353	0.97940	2.82051	2.77070	-0.81882	-0.53200	-0.20165	-0.76193	3.75556	-1.53733
86.00000	1.50098	0.98145	2.82034	2.77087	-0.81933	-0.53110	-0.20102	-0.76542	4.80220	-1.85706
87.00000	1.51844	0.98305	2.82021	2.77100	-0.81972	-0.53040	-0.20053	-0.76818	6.55166	-2.37257

TNF= 1.3650 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02557	2.53473	2.38277	-0.66496	-0.66477	-0.42907	-0.42937	0.26243	0.26196
10.00000	0.17453	0.12756	2.53310	2.38430	-0.66713	-0.66259	-0.42544	-0.43305	0.26816	0.25624
20.00000	0.34907	0.25326	2.52823	2.38890	-0.67380	-0.65578	-0.41440	-0.44474	0.28727	0.23730
30.00000	0.52360	0.37503	2.52078	2.39596	-0.68446	-0.64455	-0.39708	-0.46487	0.32376	0.20184
40.00000	0.69813	0.49032	2.51167	2.40465	-0.69836	-0.62921	-0.37504	-0.49418	0.38702	0.14298
50.00000	0.87266	0.59584	2.50201	2.41394	-0.71432	-0.61057	-0.35046	-0.53302	0.49766	0.04866
60.00000	1.04720	0.68730	2.49296	2.42270	-0.73066	-0.59030	-0.32605	-0.58005	0.70589	-0.10182

TNF= 1.3650 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
65.00000	1.13446	0.72610	2.48901	2.42654	-0.73829	-0.58040	-0.31490	-0.60516	0.88494	-0.20912
66.00000	1.15192	0.73322	2.48828	2.42725	-0.73973	-0.57849	-0.31281	-0.61017	0.93085	-0.23407
66.50000	1.16064	0.73669	2.48793	2.42760	-0.74044	-0.57755	-0.31178	-0.61267	0.95543	-0.24705
67.00000	1.16937	0.74010	2.48758	2.42794	-0.74114	-0.57662	-0.31077	-0.61515	0.98117	-0.26040
67.50000	1.17810	0.74345	2.48723	2.42828	-0.74184	-0.57570	-0.30977	-0.61763	1.00818	-0.27411
68.00000	1.18682	0.74675	2.48689	2.42861	-0.74252	-0.57479	-0.30879	-0.62009	1.03652	-0.28822
68.50000	1.19555	0.74998	2.48656	2.42893	-0.74319	-0.57389	-0.30782	-0.62254	1.06630	-0.30273

TNF= 1.3650 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
69.00000	1.20428	0.75315	2.48624	2.42925	-0.74385	-0.57300	-0.30687	-0.62497	1.09762	-0.31766
70.00000	1.22173	0.75931	2.48560	2.42987	-0.74515	-0.57126	-0.30501	-0.62978	1.16533	-0.34890
75.00000	1.30900	0.78615	2.48285	2.43257	-0.75091	-0.56340	-0.29679	-0.65218	1.65001	-0.54061
80.00000	1.39626	0.80592	2.48082	2.43456	-0.75528	-0.55732	-0.29060	-0.67034	2.64759	-0.84097
81.00000	1.41372	0.80898	2.48050	2.43487	-0.75597	-0.55637	-0.28964	-0.67328	2.98408	-0.92803
81.50000	1.42244	0.81039	2.48036	2.43501	-0.75628	-0.55592	-0.28919	-0.67465	3.18255	-0.97737
82.00000	1.43117	0.81172	2.48022	2.43514	-0.75658	-0.55550	-0.28877	-0.67595	3.40622	-1.03153

TNF= 1.3650 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
82.50000	1.43990	0.81297	2.48009	2.43527	-0.75687	-0.55510	-0.28837	-0.67719	3.66011	-1.09145
83.00000	1.44862	0.81415	2.47997	2.43539	-0.75713	-0.55473	-0.28800	-0.67835	3.95072	-1.15833
83.50000	1.45735	0.81524	2.47986	2.43550	-0.75738	-0.55438	-0.28765	-0.67944	4.28651	-1.23378
84.00000	1.46608	0.81626	2.47976	2.43560	-0.75761	-0.55405	-0.28733	-0.68045	4.67880	-1.31990
85.00000	1.48353	0.81805	2.47957	2.43578	-0.75802	-0.55348	-0.28676	-0.68225	5.70061	-1.53689
86.00000	1.50098	0.81952	2.47942	2.43593	-0.75835	-0.55301	-0.28629	-0.68374	7.23647	-1.85064
87.00000	1.51844	0.82066	2.47931	2.43604	-0.75861	-0.55264	-0.28593	-0.68490	9.80047	-2.35767

TNF= 1.2000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02909	0.14999	3.09014	-0.97202	-0.97200	-0.72065	-0.72149	0.10275	0.10252
10.00000	0.17453	0.14522	0.14984	3.09338	-0.97233	-0.97169	-0.71080	-0.73154	0.10552	0.09977
20.00000	0.34907	0.28902	0.14937	3.10309	-0.97327	-0.97072	-0.68121	-0.76413	0.11482	0.09067
30.00000	0.52360	0.42978	0.14866	3.11790	-0.97475	-0.96911	-0.63572	-0.82256	0.13283	0.07374
40.00000	0.69813	0.56528	0.14780	3.13598	-0.97666	-0.96687	-0.57926	-0.91386	0.16479	0.04576
50.00000	0.87266	0.69238	0.14690	3.15511	-0.97883	-0.96409	-0.51761	-1.04931	0.22250	0.00109
60.00000	1.04720	0.80624	0.14608	3.17298	-0.98104	-0.96101	-0.45711	-1.24350	0.33578	-0.06952

TNF= 1.2000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
65.00000	1.13446	0.85604	0.14572	3.18076	-0.98209	-0.95948	-0.42945	-1.36577	0.43657	-0.11860
70.00000	1.22173	0.89960	0.14541	3.18747	-0.98303	-0.95807	-0.40480	-1.50249	0.59847	-0.17940
74.00000	1.29154	0.92905	0.14521	3.19192	-0.98368	-0.95708	-0.38791	-1.61698	0.81223	-0.23781
75.00000	1.30900	0.93557	0.14517	3.19290	-0.98383	-0.95686	-0.38415	-1.64538	0.88535	-0.25392
75.50000	1.31772	0.93870	0.14514	3.19336	-0.98390	-0.95675	-0.38234	-1.65944	0.92600	-0.26221
76.00000	1.32645	0.94173	0.14512	3.19382	-0.98397	-0.95665	-0.38058	-1.67338	0.96978	-0.27068
76.50000	1.33518	0.94468	0.14510	3.19425	-0.98403	-0.95655	-0.37888	-1.68717	1.01702	-0.27931

TNF= 1.2000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
77.00000	1.34390	0.94753	0.14508	3.19468	-0.98410	-0.95645	-0.37723	-1.70079	1.06814	-0.28812
77.50000	1.35263	0.95028	0.14507	3.19508	-0.98416	-0.95635	-0.37563	-1.71422	1.12359	-0.29710
78.00000	1.36136	0.95294	0.14505	3.19548	-0.98422	-0.95626	-0.37409	-1.72743	1.18394	-0.30628
80.00000	1.39626	0.96259	0.14498	3.19689	-0.98444	-0.95593	-0.36848	-1.77752	1.48894	-0.34506
85.00000	1.48353	0.97940	0.14487	3.19934	-0.98483	-0.95534	-0.35868	-1.87388	3.36720	-0.46631
86.00000	1.50098	0.98145	0.14486	3.19963	-0.98488	-0.95527	-0.35748	-1.88651	4.31715	-0.50067
87.00000	1.51844	0.98305	0.14485	3.19986	-0.98491	-0.95521	-0.35655	-1.89651	5.90534	-0.54554

TNF= 1.2000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
88.00000	1.53589	0.98419	0.14484	3.20003	-0.98494	-0.95517	-0.35588	-1.90375	9.08924	-0.61637
88.83333	1.55043	0.98480	0.14484	3.20011	-0.98495	-0.95515	-0.35552	-1.90760	15.92180	-0.74311
89.00000	1.55334	0.98488	0.14484	3.20013	-0.98496	-0.95515	-0.35547	-1.90813	18.65620	-0.79060
89.16667	1.55625	0.98495	0.14484	3.20014	-0.98496	-0.95514	-0.35543	-1.90858	22.48494	-0.85581
89.33333	1.55916	0.98501	0.14484	3.20014	-0.98496	-0.95514	-0.35540	-1.90894	28.22835	-0.95200
89.50000	1.56207	0.98505	0.14484	3.20015	-0.98496	-0.95514	-0.35537	-1.90923	37.80252	-1.11020
89.66667	1.56498	0.98508	0.14484	3.20016	-0.98496	-0.95514	-0.35535	-1.90943	56.95336	-1.42339

TNF= 1.3650 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02557	0.13186	2.71660	-0.96908	-0.96906	-0.84893	-0.84976	0.18774	0.18735
10.00000	0.17453	0.12756	0.13172	2.71945	-0.96935	-0.96879	-0.83913	-0.85974	0.19252	0.18261
20.00000	0.34907	0.25326	0.13131	2.72799	-0.97016	-0.96795	-0.80987	-0.89160	0.20848	0.16719
30.00000	0.52360	0.37503	0.13069	2.74101	-0.97144	-0.96660	-0.76541	-0.94698	0.23908	0.13939
40.00000	0.69813	0.49032	0.12993	2.75691	-0.97306	-0.96482	-0.71123	-1.02897	0.29245	0.09576
50.00000	0.87266	0.59584	0.12915	2.77372	-0.97485	-0.96273	-0.65358	-1.14061	0.38648	0.03117
60.00000	1.04720	0.68730	0.12842	2.78943	-0.97662	-0.96057	-0.59893	-1.28114	0.56489	-0.06108

TNF= 1.3650 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
65.00000	1.13446	0.72610	0.12810	2.79627	-0.97743	-0.95955	-0.57472	-1.35893	0.71915	-0.11980
69.00000	1.20428	0.75315	0.12789	2.80108	-0.97801	-0.95882	-0.55756	-1.42179	0.90298	-0.17355
69.50000	1.21300	0.75626	0.12786	2.80163	-0.97808	-0.95873	-0.55557	-1.42954	0.93151	-0.18072
70.00000	1.22173	0.75931	0.12784	2.80217	-0.97815	-0.95864	-0.55362	-1.43725	0.96160	-0.18798
70.50000	1.23046	0.76229	0.12781	2.80270	-0.97821	-0.95856	-0.55171	-1.44491	0.99335	-0.19535
71.00000	1.23918	0.76521	0.12779	2.80322	-0.97828	-0.95848	-0.54983	-1.45252	1.02691	-0.20282
71.50000	1.24791	0.76806	0.12776	2.80373	-0.97834	-0.95840	-0.54800	-1.46005	1.06241	-0.21040

TNF= 1.3650 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
75.00000	1.30900	0.78615	0.12762	2.80694	-0.97874	-0.95788	-0.53632	-1.51043	1.38201	-0.26649
77.00000	1.34390	0.79494	0.12755	2.80851	-0.97894	-0.95763	-0.53062	-1.53666	1.64638	-0.30115
79.00000	1.37881	0.80256	0.12749	2.80986	-0.97911	-0.95740	-0.52565	-1.56039	2.01089	-0.33809
80.00000	1.39626	0.80592	0.12746	2.81046	-0.97919	-0.95731	-0.52346	-1.57116	2.24951	-0.35760
85.00000	1.48353	0.81805	0.12736	2.81260	-0.97947	-0.95695	-0.51553	-1.61171	4.90918	-0.47567
86.00000	1.50098	0.81952	0.12735	2.81286	-0.97950	-0.95690	-0.51457	-1.61681	6.24789	-0.50939
87.00000	1.51844	0.82066	0.12734	2.81307	-0.97953	-0.95687	-0.51382	-1.62081	8.48310	-0.55396

TNF= 1.3650 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
88.00000	1.53589	0.82148	0.12733	2.81321	-0.97954	-0.95685	-0.51328	-1.62369	12.95961	-0.62546
88.83333	1.55043	0.82191	0.12733	2.81329	-0.97955	-0.95683	-0.51300	-1.62522	22.56009	-0.75543
89.00000	1.55334	0.82197	0.12733	2.81330	-0.97956	-0.95683	-0.51296	-1.62543	26.40148	-0.80445
89.16667	1.55625	0.82202	0.12733	2.81331	-0.97956	-0.95683	-0.51293	-1.62561	31.77993	-0.87189
89.33333	1.55916	0.82206	0.12733	2.81331	-0.97956	-0.95683	-0.51290	-1.62575	39.84762	-0.97156
89.50000	1.56207	0.82209	0.12733	2.81332	-0.97956	-0.95683	-0.51288	-1.62586	53.29588	-1.13571
89.66667	1.56498	0.82212	0.12733	2.81332	-0.97956	-0.95683	-0.51286	-1.62594	80.19527	-1.46105

TNF= 1.3650 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02557	1.28200	2.50010	-0.74333	-0.74317	-0.72499	-0.72562	0.23734	0.23690
10.00000	0.17453	0.12756	1.28074	2.50256	-0.74522	-0.74128	-0.71748	-0.73325	0.24274	0.23151
20.00000	0.34907	0.25326	1.27697	2.50994	-0.75096	-0.73545	-0.69492	-0.75762	0.26077	0.21375
30.00000	0.52360	0.37503	1.27125	2.52123	-0.76007	-0.72597	-0.66021	-0.79994	0.29524	0.18082
40.00000	0.69813	0.49032	1.26431	2.53507	-0.77179	-0.71335	-0.61721	-0.86249	0.35515	0.12697
50.00000	0.87266	0.59584	1.25702	2.54978	-0.78508	-0.69852	-0.57065	-0.94732	0.46017	0.04264
60.00000	1.04720	0.68730	1.25025	2.56359	-0.79847	-0.68303	-0.52578	-1.05334	0.65834	-0.08747

TNF= 1.3650 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
65.00000	1.13446	0.72610	1.24731	2.56963	-0.80466	-0.67572	-0.50568	-1.11160	0.82904	-0.17723
67.00000	1.16937	0.74010	1.24625	2.57182	-0.80697	-0.67297	-0.49829	-1.13512	0.92086	-0.21929
67.50000	1.17810	0.74345	1.24600	2.57234	-0.80753	-0.67231	-0.49651	-1.14098	0.94664	-0.23045
68.00000	1.18682	0.74675	1.24574	2.57286	-0.80808	-0.67165	-0.49476	-1.14682	0.97369	-0.24189
68.50000	1.19555	0.74998	1.24550	2.57337	-0.80862	-0.67100	-0.49304	-1.15263	1.00212	-0.25362
69.00000	1.20428	0.75315	1.24526	2.57387	-0.80915	-0.67036	-0.49134	-1.15842	1.03201	-0.26565
69.50000	1.21300	0.75626	1.24502	2.57435	-0.80968	-0.66974	-0.48968	-1.16418	1.06349	-0.27799

TNF= 1.3650 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
70.00000	1.22173	0.75931	1.24479	2.57483	-0.81019	-0.66912	-0.48805	-1.16991	1.09667	-0.29066
75.00000	1.30900	0.78615	1.24275	2.57906	-0.81482	-0.66356	-0.47353	-1.22406	1.55971	-0.44013
80.00000	1.39626	0.80592	1.24126	2.58217	-0.81831	-0.65934	-0.46269	-1.26876	2.51350	-0.66197
82.00000	1.43117	0.81172	1.24082	2.58308	-0.81935	-0.65809	-0.45949	-1.28274	3.23910	-0.79672
82.50000	1.43990	0.81297	1.24072	2.58327	-0.81958	-0.65782	-0.45879	-1.28581	3.48196	-0.83838
83.00000	1.44862	0.81415	1.24064	2.58346	-0.81979	-0.65756	-0.45815	-1.28872	3.75996	-0.88457
83.50000	1.45735	0.81524	1.24055	2.58363	-0.81999	-0.65732	-0.45754	-1.29144	4.08119	-0.93630

TNF= 1.3650 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
84.00000	1.46608	0.81626	1.24048	2.58379	-0.82017	-0.65710	-0.45698	-1.29398	4.45648	-0.99495
84.50000	1.47480	0.81719	1.24041	2.58394	-0.82034	-0.65690	-0.45646	-1.29633	4.90055	-1.06239
85.00000	1.48353	0.81805	1.24034	2.58407	-0.82049	-0.65671	-0.45598	-1.29849	5.43407	-1.14124
85.50000	1.49226	0.81882	1.24028	2.58419	-0.82063	-0.65654	-0.45555	-1.30045	6.08682	-1.23527
86.00000	1.50098	0.81952	1.24023	2.58430	-0.82076	-0.65639	-0.45517	-1.30222	6.90354	-1.35017
87.00000	1.51844	0.82066	1.24014	2.58448	-0.82097	-0.65614	-0.45453	-1.30515	9.35682	-1.68419
88.00000	1.53589	0.82148	1.24008	2.58461	-0.82111	-0.65596	-0.45408	-1.30725	14.26968	-2.33072

TNF= 1.5000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02327	1.16662	2.27509	-0.72615	-0.72601	-0.82147	-0.82210	0.32891	0.32831
10.00000	0.17453	0.11603	1.16547	2.27733	-0.72782	-0.72433	-0.81397	-0.82971	0.33610	0.32114
20.00000	0.34907	0.23004	1.16205	2.28404	-0.73291	-0.71919	-0.79153	-0.85380	0.36009	0.29772
30.00000	0.52360	0.33984	1.15684	2.29432	-0.74091	-0.71098	-0.75722	-0.89492	0.40572	0.25519
40.00000	0.69813	0.44286	1.15053	2.30692	-0.75112	-0.70030	-0.71515	-0.95400	0.48434	0.18781
50.00000	0.87266	0.53599	1.14389	2.32030	-0.76252	-0.68812	-0.67021	-1.03085	0.62057	0.08676
60.00000	1.04720	0.61548	1.13773	2.33287	-0.77381	-0.67583	-0.62760	-1.12160	0.87391	-0.06112

TNF= 1.5000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
61.00000	1.06465	0.62251	1.13717	2.33402	-0.77488	-0.67466	-0.62367	-1.13109	0.91048	-0.07920
62.00000	1.08210	0.62936	1.13662	2.33514	-0.77593	-0.67350	-0.61981	-1.14059	0.94998	-0.09800
62.50000	1.09083	0.63272	1.13635	2.33570	-0.77644	-0.67293	-0.61792	-1.14534	0.97093	-0.10767
63.00000	1.09956	0.63603	1.13608	2.33624	-0.77696	-0.67237	-0.61605	-1.15009	0.99275	-0.11753
63.50000	1.10828	0.63929	1.13582	2.33678	-0.77746	-0.67181	-0.61419	-1.15483	1.01547	-0.12759
64.00000	1.11701	0.64250	1.13556	2.33732	-0.77797	-0.67126	-0.61237	-1.15957	1.03916	-0.13785
65.00000	1.13446	0.64877	1.13505	2.33836	-0.77895	-0.67017	-0.60878	-1.16900	1.08965	-0.15899

TNF= 1.5000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
70.00000	1.22173	0.67700	1.13276	2.34310	-0.78350	-0.66514	-0.59242	-1.21476	1.42517	-0.27929
75.00000	1.30900	0.69965	1.13090	2.34694	-0.78728	-0.66095	-0.57905	-1.25576	2.00128	-0.43392
80.00000	1.39626	0.71622	1.12954	2.34977	-0.79010	-0.65782	-0.56915	-1.28855	3.18041	-0.65903
82.00000	1.43117	0.72106	1.12914	2.35060	-0.79094	-0.65689	-0.56624	-1.29861	4.07451	-0.79457
82.50000	1.43990	0.72210	1.12906	2.35078	-0.79112	-0.65668	-0.56561	-1.30081	4.37349	-0.83639
83.00000	1.44862	0.72308	1.12898	2.35095	-0.79129	-0.65650	-0.56501	-1.30289	4.71559	-0.88272
83.50000	1.45735	0.72399	1.12890	2.35110	-0.79145	-0.65632	-0.56446	-1.30483	5.11075	-0.93458

TNF= 1.5000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
84.00000	1.46608	0.72484	1.12883	2.35125	-0.79160	-0.65616	-0.56395	-1.30664	5.57228	-0.99333
84.50000	1.47480	0.72562	1.12877	2.35138	-0.79173	-0.65601	-0.56348	-1.30831	6.11822	-1.06086
85.00000	1.48353	0.72633	1.12871	2.35150	-0.79186	-0.65587	-0.56305	-1.30984	6.77394	-1.13977
85.50000	1.49226	0.72697	1.12866	2.35161	-0.79197	-0.65574	-0.56266	-1.31124	7.57601	-1.23385
86.00000	1.50098	0.72755	1.12861	2.35171	-0.79207	-0.65563	-0.56231	-1.31249	8.57933	-1.34877
87.00000	1.51844	0.72850	1.12853	2.35188	-0.79224	-0.65545	-0.56174	-1.31456	11.59222	-1.68272
88.00000	1.53589	0.72918	1.12848	2.35199	-0.79236	-0.65531	-0.56133	-1.31604	17.62384	-2.32889

TNF= 1.5000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02327	2.30660	2.16833	-0.64030	-0.64014	-0.48051	-0.48080	0.36741	0.36677
10.00000	0.17453	0.11603	2.30512	2.16972	-0.64221	-0.63823	-0.47708	-0.48427	0.37511	0.35907
20.00000	0.34907	0.23004	2.30069	2.17390	-0.64803	-0.63233	-0.46668	-0.49522	0.40079	0.33387
30.00000	0.52360	0.33984	2.29391	2.18032	-0.65726	-0.62273	-0.45045	-0.51372	0.44955	0.28768
40.00000	0.69813	0.44286	2.28562	2.18823	-0.66917	-0.60994	-0.43000	-0.53991	0.53342	0.21349
50.00000	0.87266	0.53599	2.27683	2.19668	-0.68265	-0.59489	-0.40746	-0.57324	0.67841	0.09984
60.00000	1.04720	0.61548	2.26859	2.20466	-0.69620	-0.57912	-0.38544	-0.61149	0.94743	-0.07163

TNF= 1.5000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
59.00000	1.02974	0.60826	2.26936	2.20391	-0.69489	-0.58067	-0.38754	-0.60756	0.91144	-0.05111
60.50000	1.05592	0.61902	2.26821	2.20503	-0.69685	-0.57836	-0.38441	-0.61346	0.96645	-0.08222
61.00000	1.06465	0.62251	2.26784	2.20539	-0.69749	-0.57759	-0.38338	-0.61542	0.98622	-0.09303
61.50000	1.07338	0.62596	2.26747	2.20575	-0.69813	-0.57683	-0.38236	-0.61738	1.00675	-0.10408
62.00000	1.08210	0.62936	2.26710	2.20611	-0.69876	-0.57607	-0.38135	-0.61934	1.02811	-0.11538
63.00000	1.09956	0.63603	2.26638	2.20681	-0.70001	-0.57458	-0.37936	-0.62325	1.07345	-0.13872
65.00000	1.13446	0.64877	2.26500	2.20816	-0.70243	-0.57165	-0.37552	-0.63099	1.17617	-0.18865

TNF= 1.5000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
70.00000	1.22173	0.67700	2.26190	2.21118	-0.70798	-0.56489	-0.36678	-0.64951	1.53157	-0.33673
75.00000	1.30900	0.69965	2.25939	2.21364	-0.71259	-0.55917	-0.35958	-0.66584	2.14134	-0.53467
80.00000	1.39626	0.71622	2.25754	2.21545	-0.71607	-0.55482	-0.35421	-0.67873	3.38854	-0.83898
80.50000	1.40499	0.71753	2.25740	2.21559	-0.71635	-0.55447	-0.35378	-0.67978	3.58712	-0.88105
81.00000	1.41372	0.71877	2.25726	2.21573	-0.71661	-0.55413	-0.35337	-0.68079	3.80809	-0.92658
81.50000	1.42244	0.71995	2.25713	2.21586	-0.71686	-0.55382	-0.35298	-0.68174	4.05539	-0.97616
82.00000	1.43117	0.72106	2.25700	2.21598	-0.71710	-0.55352	-0.35262	-0.68265	4.33398	-1.03054

TNF= 1.5000 TNM= 3.4600 TK= 0.9400

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
82.50000	1.43990	0.72210	2.25688	2.21609	-0.71732	-0.55324	-0.35228	-0.68351	4.65009	-1.09067
83.00000	1.44862	0.72308	2.25678	2.21620	-0.71753	-0.55297	-0.35195	-0.68431	5.01179	-1.15775
83.50000	1.45735	0.72399	2.25667	2.21630	-0.71773	-0.55272	-0.35165	-0.68507	5.42957	-1.23338
84.00000	1.46608	0.72484	2.25658	2.21639	-0.71791	-0.55249	-0.35137	-0.68577	5.91750	-1.31968
85.00000	1.48353	0.72633	2.25641	2.21656	-0.71823	-0.55209	-0.35088	-0.68701	7.18787	-1.53699
86.00000	1.50098	0.72755	2.25627	2.21669	-0.71849	-0.55175	-0.35048	-0.68804	9.09639	-1.85108
87.00000	1.51844	0.72850	2.25617	2.21680	-0.71870	-0.55149	-0.35016	-0.68884	12.28127	-2.35850

TNF= 1.2000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02909	1.45827	2.84387	-0.76637	-0.76618	-0.61697	-0.61762	0.12808	0.12783
10.00000	0.17453	0.14522	1.45684	2.84666	-0.76858	-0.76396	-0.60923	-0.62550	0.13117	0.12475
20.00000	0.34907	0.28902	1.45256	2.85506	-0.77536	-0.75699	-0.58585	-0.65104	0.14151	0.11441
30.00000	0.52360	0.42978	1.44605	2.86790	-0.78624	-0.74536	-0.54947	-0.69683	0.16147	0.09453
40.00000	0.69813	0.56528	1.43816	2.88364	-0.80053	-0.72918	-0.50360	-0.76828	0.19671	0.06008
50.00000	0.87266	0.69238	1.42986	2.90038	-0.81716	-0.70902	-0.45265	-0.87380	0.25995	0.00148
60.00000	1.04720	0.80624	1.42216	2.91608	-0.83456	-0.68644	-0.40183	-1.02363	0.38317	-0.09938

TNF= 1.2000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
65.00000	1.13446	0.85604	1.41882	2.92295	-0.84288	-0.67518	-0.37835	-1.11693	0.49222	-0.17550
70.00000	1.22173	0.89960	1.41595	2.92887	-0.85048	-0.66467	-0.35729	-1.22022	0.66680	-0.27793
73.00000	1.27409	0.92218	1.41449	2.93190	-0.85455	-0.65900	-0.34619	-1.28451	0.82803	-0.35721
74.00000	1.29154	0.92905	1.41405	2.93281	-0.85580	-0.65725	-0.34280	-1.30585	0.89662	-0.38762
74.50000	1.30027	0.93235	1.41384	2.93325	-0.85641	-0.65640	-0.34116	-1.31644	0.93452	-0.40372
75.00000	1.30900	0.93557	1.41363	2.93368	-0.85700	-0.65557	-0.33956	-1.32696	0.97514	-0.42047
75.50000	1.31772	0.93870	1.41343	2.93409	-0.85757	-0.65476	-0.33801	-1.33739	1.01878	-0.43792

TNF= 1.2000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
76.00000	1.32645	0.94173	1.41324	2.93449	-0.85813	-0.65398	-0.33650	-1.34772	1.06575	-0.45613
76.50000	1.33518	0.94468	1.41305	2.93488	-0.85868	-0.65321	-0.33503	-1.35793	1.11643	-0.47517
77.00000	1.34390	0.94753	1.41287	2.93525	-0.85921	-0.65247	-0.33361	-1.36801	1.17125	-0.49511
80.00000	1.39626	0.96259	1.41193	2.93721	-0.86202	-0.64852	-0.32607	-1.42453	1.62221	-0.63978
82.00000	1.43117	0.97060	1.41143	2.93825	-0.86353	-0.64641	-0.32204	-1.45705	2.11929	-0.77421
83.00000	1.44862	0.97397	1.41122	2.93868	-0.86417	-0.64551	-0.32034	-1.47130	2.47738	-0.86179
83.50000	1.45735	0.97549	1.41113	2.93888	-0.86446	-0.64511	-0.31957	-1.47784	2.69858	-0.91330

TNF= 1.2000 TNM= 1.7500 TK= 1.9500

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
84.00000	1.46608	0.97690	1.41104	2.93906	-0.86473	-0.64473	-0.31886	-1.48399	2.95726	-0.97162
84.50000	1.47480	0.97821	1.41096	2.93923	-0.86498	-0.64438	-0.31820	-1.48971	3.26364	-1.03860
85.00000	1.48353	0.97940	1.41089	2.93938	-0.86521	-0.64407	-0.31760	-1.49499	3.63205	-1.11678
85.50000	1.49226	0.98048	1.41082	2.93952	-0.86541	-0.64378	-0.31705	-1.49982	4.08314	-1.20986
86.00000	1.50098	0.98145	1.41076	2.93964	-0.86560	-0.64352	-0.31656	-1.50419	4.64795	-1.32341
87.00000	1.51844	0.98305	1.41066	2.93985	-0.86590	-0.64309	-0.31575	-1.51146	6.34614	-1.65266
88.00000	1.53589	0.98419	1.41059	2.93999	-0.86612	-0.64279	-0.31517	-1.51672	9.75016	-2.28801

TNF= 1.5000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
2.00000	0.03491	0.02327	0.11999	2.47211	-0.96688	-0.96686	-0.96423	-0.96507	0.25741	0.25689
10.00000	0.17453	0.11603	0.11987	2.47470	-0.96712	-0.96661	-0.95423	-0.97523	0.26369	0.25065
20.00000	0.34907	0.23004	0.11949	2.48247	-0.96786	-0.96586	-0.92450	-1.00743	0.28467	0.23053
30.00000	0.52360	0.33984	0.11893	2.49432	-0.96901	-0.96467	-0.87964	-1.06244	0.32466	0.19489
40.00000	0.69813	0.44286	0.11824	2.50878	-0.97045	-0.96313	-0.82554	-1.14163	0.39377	0.14052
50.00000	0.87266	0.53599	0.11752	2.52409	-0.97202	-0.96138	-0.76878	-1.24506	0.51401	0.06303
60.00000	1.04720	0.61548	0.11686	2.53838	-0.97356	-0.95964	-0.71587	-1.36803	0.73860	-0.04251

TNF= 1.5000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
64.00000	1.11701	0.64250	0.11663	2.54343	-0.97412	-0.95899	-0.69716	-1.41979	0.88551	-0.09357
65.00000	1.13446	0.64877	0.11658	2.54461	-0.97425	-0.95884	-0.69277	-1.43269	0.93045	-0.10717
65.50000	1.14319	0.65183	0.11655	2.54519	-0.97432	-0.95876	-0.69062	-1.43911	0.95443	-0.11410
66.00000	1.15192	0.65484	0.11652	2.54576	-0.97438	-0.95869	-0.68850	-1.44551	0.97950	-0.12111
66.50000	1.16064	0.65779	0.11650	2.54632	-0.97444	-0.95861	-0.68642	-1.45188	1.00574	-0.12821
67.00000	1.16937	0.66070	0.11647	2.54687	-0.97451	-0.95854	-0.68437	-1.45822	1.03322	-0.13539
67.50000	1.17810	0.66355	0.11645	2.54741	-0.97457	-0.95847	-0.68235	-1.46453	1.06202	-0.14266

TNF= 1.5000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
68.00000	1.18682	0.66635	0.11642	2.54794	-0.97463	-0.95840	-0.68037	-1.47079	1.09224	-0.15001
70.00000	1.22173	0.67700	0.11633	2.54997	-0.97486	-0.95813	-0.67280	-1.49538	1.22938	-0.18031
75.00000	1.30900	0.69965	0.11613	2.55432	-0.97536	-0.95755	-0.65658	-1.55184	1.74358	-0.26250
80.00000	1.39626	0.71622	0.11599	2.55751	-0.97573	-0.95711	-0.64461	-1.59718	2.79742	-0.35591
85.00000	1.48353	0.72633	0.11590	2.55947	-0.97596	-0.95684	-0.63726	-1.62671	6.01226	-0.47508
86.00000	1.50098	0.72755	0.11589	2.55971	-0.97599	-0.95681	-0.63637	-1.63038	7.62788	-0.50891
87.00000	1.51844	0.72850	0.11588	2.55989	-0.97601	-0.95678	-0.63568	-1.63325	10.32428	-0.55355

TNF= 1.5000 TNM= 0.1800 TK=20.6000

PD	PHI	PHIP	A	B	RS	RP	TANDS	TANDP	RATS	RATP
88.00000	1.53589	0.72918	0.11587	2.56002	-0.97603	-0.95676	-0.63518	-1.63532	15.72264	-0.62505
88.83333	1.55043	0.72954	0.11587	2.56009	-0.97603	-0.95675	-0.63492	-1.63641	27.29797	-0.75489
89.00000	1.55334	0.72959	0.11587	2.56010	-0.97604	-0.95675	-0.63489	-1.63656	31.92924	-0.80385
89.16667	1.55625	0.72963	0.11587	2.56011	-0.97604	-0.95675	-0.63486	-1.63669	38.41347	-0.87120
89.33333	1.55916	0.72967	0.11587	2.56012	-0.97604	-0.95675	-0.63483	-1.63679	48.13973	-0.97072
89.50000	1.56207	0.72969	0.11587	2.56012	-0.97604	-0.95675	-0.63481	-1.63687	64.35260	-1.13461
89.66667	1.56498	0.72971	0.11587	2.56012	-0.97604	-0.95675	-0.63480	-1.63693	96.78123	-1.45944

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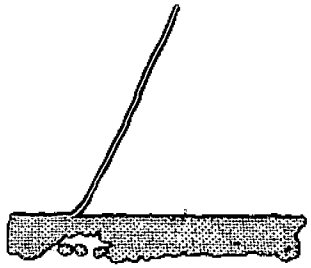
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