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CYCLOTRON ORBITS: A 709 RAY-TRACE PROGRAM FOR CYLINDRICALLY SYMMETRIC MAGNETIC FIELDS

Berkeley, California

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Joe Good, Morris Pripstein, and Howard S. Goldberg

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ABSTRACT

CYCLOTRON ORBITS is a 709 program for tracing secondary beams out of a cylindrically symmetric magnetic field. Development of a 6-by-6 transfer matrix allows one to couple to external quadrupole and bending magnets.

INTRODUCTION

In designing beams at the 184-inch cyclotron it is often necessary to know the path of the secondary beam as it traverses the direction between the internal target and external target area. For a point-by-point description of the ray path, the radius r and the azimuthal angle ϕ are sufficient to determine the trajectory. If one also desired to project the ray, then θ, the angle between the tangent to the ray and the tangent to the radius circle where it intersects the radius circle and trajectory, must be given. The radius circle here means the loci of constant values of magnetic field. These quantities are illustrated in Fig. 1 below.

For a given set of initial conditions, $(r, \theta, \phi)_0$, the program will calculate, for a series of finite steps in radius, the θ and ϕ at each new radius. This new set of numbers $(r, \theta, \phi)_i$ will be printed at any desired interval. The program will stop at a specified final radius. The program works equally well when the initial radius is less than or greater than the final radius. This allows one to trace rays out of or into the cyclotron.

In the Appendix a method of determining the 6-by-6 transfer matrix by using cyclotron orbits is described. This enables one to "hook" onto an external beam setup consisting of quadrupoles and bending magnets through the use of Devlin's program OPTIK.[1]

DISCUSSION

CYCLOTRON ORBITS is set up for a radial symmetric field. It was written for use at the 184-inch cyclotron; therefore the coordinate systems used were set up in a convenient manner for that machine. However, the program is valid for any magnetic field with radial symmetry, and the coordinate systems described should be general enough for any application.

The coordinate system for ϕ, as set up for the 184-inch cyclotron, is fixed in the following way: The angle ϕ is equal to 270 deg-ϕ^*, where ϕ^* is the conventional azimuthal angle used by the 184-inch crew. This would put the center of the meson cave wheel at $\phi = 270 - 266.55 = 3.45$ deg. The center of the meson wheel is at a radius of 204 in.

The 0-degree line for θ is taken along the tangent to the radius circle in the direction of increasing ϕ, where ϕ increases counterclockwise looking from above. The θ's are positive when measured in a clockwise direction from $\theta = 0$, again looking from above the cyclotron. However, to keep a continuous function, angles greater than 270 deg are taken as the angle minus 360 deg. That is, angles about zero degrees are taken as +5 deg and -5 deg and not as 5 deg and 355 deg. These conventions are illustrated in Fig. 2.

To avoid the possibility of choosing the wrong magnetic-field polarity, a quantity called H Field Sign must be stipulated.

H Field Sign is defined as the product of the sign of B (the cyclotron field polarity) and e (the sign of the charge of the secondary particle); B and e are determined in the following way:

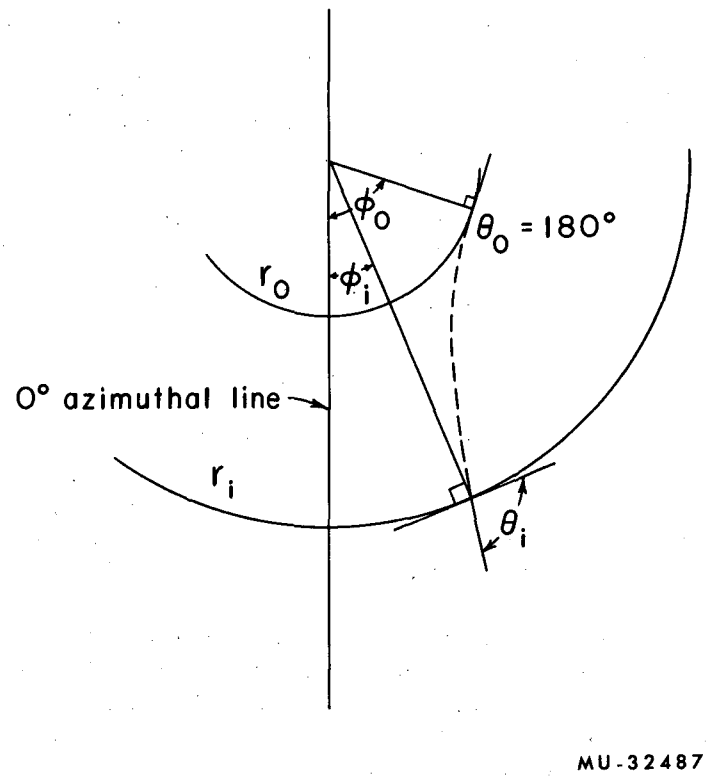

Fig. 1. Geometry for θ and ϕ (looking down onto cyclotron).

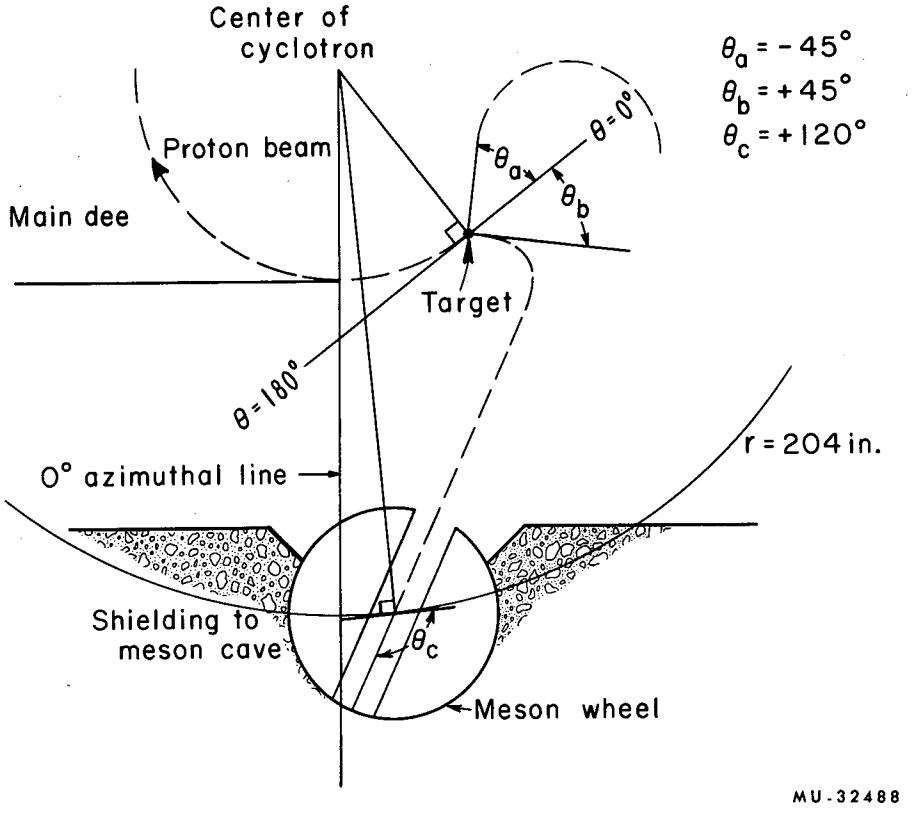

Fig. 2. The coordinate system as defined relative to the 184-inch cyclotron and meson cave wheel.

For a clockwise circulating internal proton beam, the sign of the cyclotron field polarity, B, is taken as +1.0. For a counterclockwise circulating proton beam it is -1.0. (For the above we are always looking down onto the cyclotron.)

For a positively charged secondary particle, the particle sign e is +1.0. For a negatively charged secondary particle it is -1.0.

The product of these, B and e determines the H Field Sign. (See Fig. 3.)

The program calculates θ_i and ϕ_i for a series of radii, r_i, moving in steps of Δr, starting from some initial radius r_0. The increments in radius, Δr, must be specified. Here, owing to programming approximations, two cases must be distinguished. One is for the interval between the internal target radius and a radius, R FINE, which should be 2 or 3 inches greater than the target radius. The second case is for the interval between R FINE and the outermost radius R FINAL. For the first case the interval is called DR FINE, and should be about 0.01 in. For the second case it is called DR COARSE, and should be about 0.05 in. The programmer may change these quantities because they are fed in as input data. Finer increments will give greater accuracy. With the suggested intervals the accuracy is about 5%. Although the program calculates θ_i and ϕ_i at relatively fine radial increments, one can print out the data at any specified interval, say, 1-inch steps.

MAGNETIC FIELD INPUT CARDS

To specify the initial and final magnetic field radii, a special card must be prepared. It is placed immediately after the machine DATA card. The special card contains:

Columns	
3 to 10	smallest magnetic field radius
11 to 20	largest magnetic field radius
21 to 30	8.0 (signifying the number of title cards that come with the binary deck)
31 to 80	blank

The eight title cards follow immediately.

Following the title cards are the magnetic field cards. Each card contains fourteen values of the magnetic field, one value in each of the first 14 fields of five spaces. The units number always appears in column 5 of each five-number field. The magnetic field at the smallest radius is always taken as plus. When the field changes sign, the minus always appears just before the leftmost integer. The card in Fig. 4 is an example illustrating this. All values are given in gauss: 22 105; 22 000; 21 500; 19 500; 17 000; 12 000; 9 500; 7 521; 5 740; 962, 75, 6, -2, -35.

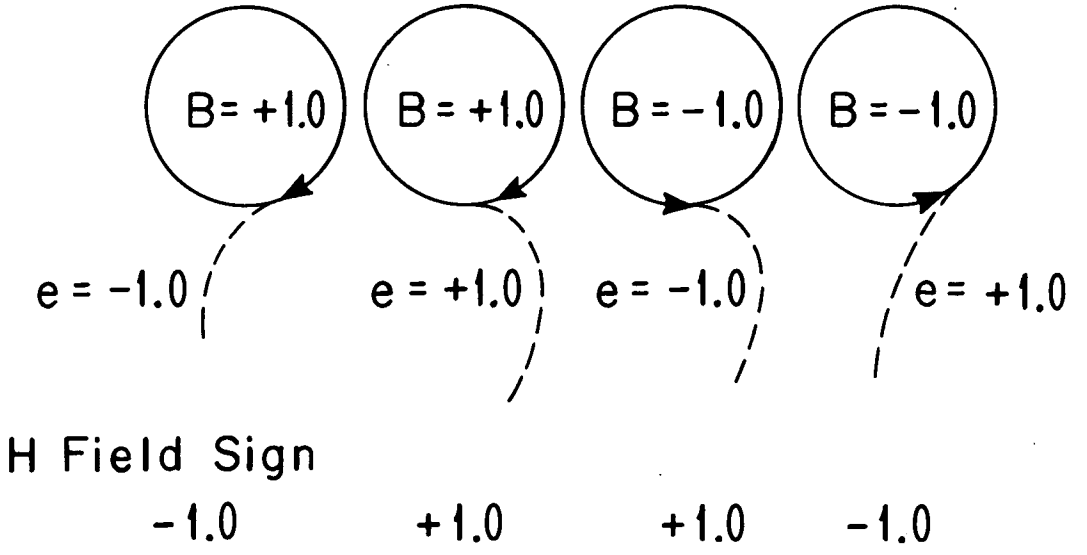

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Fig. 3. Determination of H Field Sign. Arrow indicates direction of internal beam (as viewed looking down onto the cyclotron).

Fig. 4. Sample magnetic field card.

DATA INPUT FORMAT

Two input cards are required for each ray. Momentum is in MeV/c; all angles are in degrees, and all linear dimensions are in inches. Data are in floating point--that is, they can be written decimally with the decimal point appearing anywhere in the ten-number field.

First Card

Columns		Information
1 to 10	P	Momentum of the secondary beam
11 to 20	THETA INIT.	The initial angle
21 to 30	R INITIAL	Initial radius
31 to 40	R FINAL	Final radius
41 to 50	PHI INIT.	Initial azimuthal angle
51 to 60	0.0	Write 0.0
61 to 70	PRINT	Determines at what radial intervals the data are to be printed. They are expressed in inches.
71 to 80		Leave blank

Second Card

Columns		Information
1 to 10	DR FINE	Determines radial intervals near internal target
11 to 20	DR COARSE	Determines radial interval away from target.
21 to 30		Write as 1.0
31 to 40	H FIELD SIGN	Explained in Fig. 3
41 to 50	R FINE	Radius where DR FINE changes to DR COARSE
51 to 60		Leave blank
61 to 70		Leave blank
71 to 80		Leave blank

An example might be appropriate.

We desire to trace a beam out of the cyclotron. We are interested in 250-MeV pions with momentum of 364 MeV/c. We want the internal target at a radius of 82 in. We guess an azimuth of 17 deg. We take our final radius to be 204 in., the center of the meson wheel. If our final azimuthal angle is 3.45 deg, as given by CYCLOTRON ORBITS, then we have guessed correctly; we are passing through the center of the meson wheel. If our final azimuth had turned out to be 5.45 deg, then, because of the radial symmetry of the cyclotron fields, our initial azimuth should have been 15 deg. Note that only one central ray solution is needed even if it is a bad "guess."

The input cards for the above set of data would be:

Card 1

1	11	21	31	41	51	61	71
364.0	180.0	82.0	204.0	17.0	0.0	1.0	

Card 2

1	11	21	31	41	51	61	71
0.01	0.05	1.0	-1.0	84.0			

OUTPUT FORMAT

For n sets of data cards the cyclotron magnetic field at each radius is printed first and only once.

For each ray the input data are printed with the following format headings;

H FIELD SIGN
Z INITIAL
P - THETA INIT. - R INITIAL - R FINAL - PHI INIT. - THETA Z-PRINT
DR FINE - DR COARSE - NOT USED - H FIELD SIGN - R FINE - TEST - TRAP

Immediately following is the output under the headings

R PHI THETA Z DZ/DT D(DZ/DT)/DT DR CI*INTEGRAL

All headings using Z should be ignored.

PROGRAM PACKAGE

Programmers I. D. card	one card
*Machine XEQ card	one card
*Binary deck	
*Machine DATA card	one card
*Magnetic field radius card	one card
*Title and heading cards	eight cards
*Cyclotron magnetic field	six cards, or as many as necessary
Input data	two cards for each ray. The second ray immediately follows the first, etc.
EOF card	one card

*Included in the complete program package

Note: A complete program package can be obtained for reproduction from the Mathematics and Computing Group, Bldg. 50A, Room 1148. Included in the package deck will be June 1963 values of the 184-inch cyclotron magnetic field.

APPENDIX

A method is described for obtaining, by use of CYCLOTRON ORBITS, the transfer matrix from an internal target to some point external to the main magnetic field, say an internal quadrupole. (The following assumes a knowledge of the matrix method for designing beam transport systems; for an explanation of this technique see Pener.[2])

The distance from the target to the quadrupole is broken up into an initial straight section, an equivalent bending magnet, and a final straight section.

In the horizontal plane we will have

$$A = L_2 \, a \, L_i \, ,$$

and in the vertical plane,

$$B = L_2 \, b \, L_1 \, ,$$

where a and b are the horizontal and vertical submatrices representing a bending magnet with parameters α,[2] the angle of bend, ρ, the radius of curvature; and ϵ_1 and ϵ_2 the entrance and exit angles, respectively.[2]

The final matrix will have the form

$$\begin{pmatrix} X_f \\ X'_f \\ X_f \\ Y'_f \\ \frac{\Delta p}{p} \\ 0 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & 0 & 0 & A_{15} & 0 \\ A_{21} & A_{22} & 0 & 0 & A_{25} & 0 \\ 0 & 0 & B_{11} & B_{12} & 0 & 0 \\ 0 & 0 & B_{21} & B_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_0 \\ X'_0 \\ Y_0 \\ Y'_0 \\ \frac{\Delta p}{p} \\ 0 \end{pmatrix}$$

In order to save space we represent A and B as 3-by-3 matrices. Following is a breakdown of A and B into a product of matrices, along with a definition of the matrix elements in terms of the bending magnet parameters:

$$A = \begin{pmatrix} 1 & L_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} + L_2 a_{21} & a_{12} + L_1 a_{11} + L_2(a_{22} + L_1 a_{21}) & a_{13} + L_2 a_{23} \\ a_{21} & a_{22} + L_1 a_{21} & a_{23} \\ 0 & 0 & 1 \end{pmatrix},$$

where[2]

$$a_{11} = \frac{\cos(\alpha - \epsilon_1)}{\cos \epsilon_1}; \quad a_{12} = \rho \sin\alpha; \quad a_{13} = 2\rho \sin^2(\alpha/2),$$

$$a_{21} = -\frac{\sin(\alpha - \epsilon_1 - \epsilon_2)}{\rho \cos \epsilon_1 \cos \epsilon_2}; \quad a_{22} = \frac{\cos(\alpha - \epsilon_2)}{\cos \epsilon_2}; \quad a_{23} = \frac{2 \sin(\alpha/2) \cos(\epsilon_2 - \alpha/2)}{\cos \epsilon_2}.$$

Note that the elements A_{13} and A_{23} are the momentum-dependent ones in that they operate on the momentum components of the ray vector. Similarly,

$$B = \begin{pmatrix} b_{11} + L_2 b_{21} & b_{12} + L_1 b_{11} + L_2(b_{22} + L_1 b_{21}) & 0 \\ b_{21} & b_{22} + L_1 b_{21} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where[2]

$$b_{11} = 1 - \alpha \tan \epsilon_1, \quad b_{12} = \alpha\rho,$$

$$b_{21} = -\frac{1}{\rho}\left[\tan \epsilon_1 + \tan \epsilon_2 - \alpha \tan \epsilon_1 \tan \epsilon_2\right],$$

$$b_{22} = 1 - \alpha \tan \epsilon_2.$$

The procedure for solving for the bending matrix parameters is as follows.

We will work with the geometry as shown in Fig. 5.

T is the target position. The ray shown is meant to be the central ray (0, 0, 0). We wish to obtain the transfer matrix at some final radius r_f. From CYCLOTRON ORBITS we have, at r_f, θ_f and ϕ_f. First we find α, the angle of bend, the first of our bending parameters:

$$|\alpha| = \left|90° - |\phi_i| + |\psi_f|\right|,$$

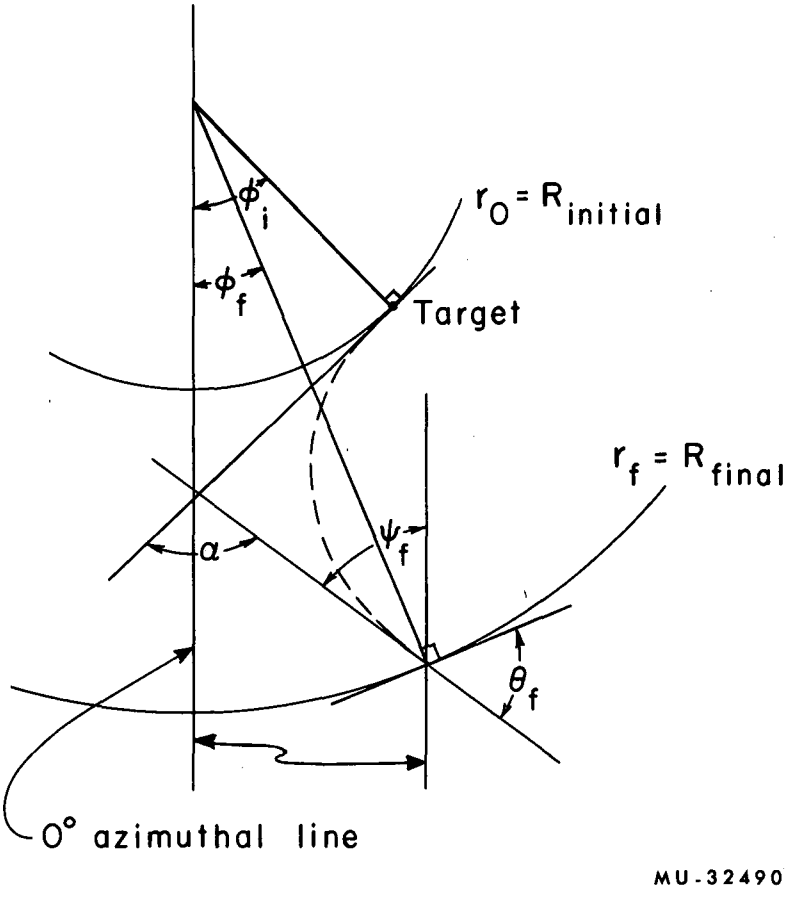

Fig. 5. Geometry for α and ψ.

where

$$|\psi_f| = |90^\circ - \theta_f + \phi_f| .$$

The sign of α depends on the sign convention for measuring plus and minus coordinates from the central ray. We have taken the orientation of our coordinate system in the following way. When $\tan\theta_0$ (defined as x_0', the slope on leaving the target) is positive and in the direction of the central ray, that side of the central ray that the vector $(0, X_0', 0)$ falls on is the +x side. Since $(0, X_0', 0)$ will never cross the central ray, this uniquely determines the +x direction. Now, A_{13} and A_{23} change sign as α changes sign. If we take the vector $(0, 0, +\Delta p/p)$ and it falls on the +x side α must be positive, if on the -x side α will be negative.

If further external bending magnets are used, their sign of bend will be the same as that of α if the bend is in the same direction, and opposite to that of α if the bend is opposite.

The rest of the parameters, as functions of the A_{ij}, are as follows:

$$\tan\epsilon_2 = \frac{\dfrac{A_{23}}{2\sin(\alpha/2)} - \cos(\alpha/2)}{\sin(\alpha/2)} , \tag{1}$$

$$L_1 = \frac{A_{22} - \dfrac{\cos(\alpha-\epsilon_2)}{\cos\epsilon_2}}{A_{21}} , \tag{2}$$

$$\rho = \frac{\begin{vmatrix} A_{12} - L_1 A_{11} & \dfrac{\cos(\alpha-\epsilon_2)}{\cos\epsilon_2} \\ A_{13} & A_{23} \end{vmatrix}}{\Delta} , \tag{3}$$

$$L_2 = \frac{\begin{vmatrix} \sin\alpha & A_{12} - L_1 A_{11} \\ 2\sin^2(\alpha/2) & A_{13} \end{vmatrix}}{\Delta} , \tag{4}$$

where

$$\Delta = \begin{vmatrix} \sin(\alpha) & \dfrac{\cos(\alpha-\epsilon_2)}{\cos\epsilon_2} \\ 2\sin^2(\alpha/2) & A_{23} \end{vmatrix} ,$$

and, finally,

$$\tan\epsilon_1 = \frac{A_{11} - L_2 A_{21} - \cos\alpha}{\sin\alpha} . \tag{5}$$

As a check take

$$A_{21} = -\frac{1}{\rho}\,\frac{\sin(\alpha-\epsilon_1-\epsilon_2)}{\cos\epsilon_1\cos\epsilon_2}\,.$$

Plug in α, ρ, ϵ_1, and ϵ_2 and see how A_{21} calculated compares with A_{21} from CYCLOTRON ORBITS.

Once $\alpha, \rho, \epsilon_1, \epsilon_2, L_1$, and L_2 are known we can solve directly for the vertical matrix elements b given on page 10. The elements for B are then determined from the relationships on the same page.

Finally we have to determine the various A_{ij} to plug into the previous equations for $\alpha, \rho, \epsilon_1, \epsilon_2, L_1$, and L_2.

This is done in the following way. First we define three vectors (at the target),

$$\begin{pmatrix} X_0 \\ 0 \\ 0 \end{pmatrix} = a, \quad \begin{pmatrix} 0 \\ X' \\ 0 \\ 0 \end{pmatrix} = b, \quad \begin{pmatrix} 0 \\ 0 \\ \Delta p/p \end{pmatrix} = c,$$

where the sign conventions on X and X' have been previously described. Each of these vectors determines a final vector at the face of our first element $(X_f, X_f', \Delta p/p)_{a,b,\text{ or } c}$.

Therefore we must have

$$A_{11} = \left(\frac{X_f}{X_0}\right)_a, \quad A_{12} = \left(\frac{X_f}{X_0'}\right)_b, \quad A_{13} = \left(\frac{X_f}{\Delta p/p}\right)_c,$$

$$A_{21} = \left(\frac{X_f'}{X_0}\right)_a, \quad A_{22} = \left(\frac{X_f'}{X_0'}\right)_b, \quad A_{23} = \left(\frac{X_f'}{(\Delta p/p)}\right)_c.$$

The values of (X_f, X_f') can be obtained in two ways. First, a plot of each initial ray can be made and the corresponding values of X_f and X_f' read off with a ruler and a protractor.

Second, X_f and X_f' can be calculated trigonometrically. Here one starts with the central ray. At r_f, the quantity $\psi_f = \theta_f^c - \frac{\pi}{2} - \phi_f^c$ is evaluated, where the superscript c refers to the central ray. The ψ_f is the angle the ray makes with the 0-deg azimuthal line. This is shown in Fig. 6. Since the central ray, by definition, enters our first element with $X' = 0$, that element must have its face rotated by the angle ψ_f^c. This is the line A - A' in Fig. 5, which now defines a plane perpendicular to the central ray. Now one of our three vectors, a, b, or c is chosen, say a. Its path is also shown in Fig. 5. Although the ray a crosses the quad face at some $r_f + \Delta r$, the distance r_f is so large compared with the target and quad aperture dimensions that one can ignore Δr. Therefore, for a, at r_f one computes

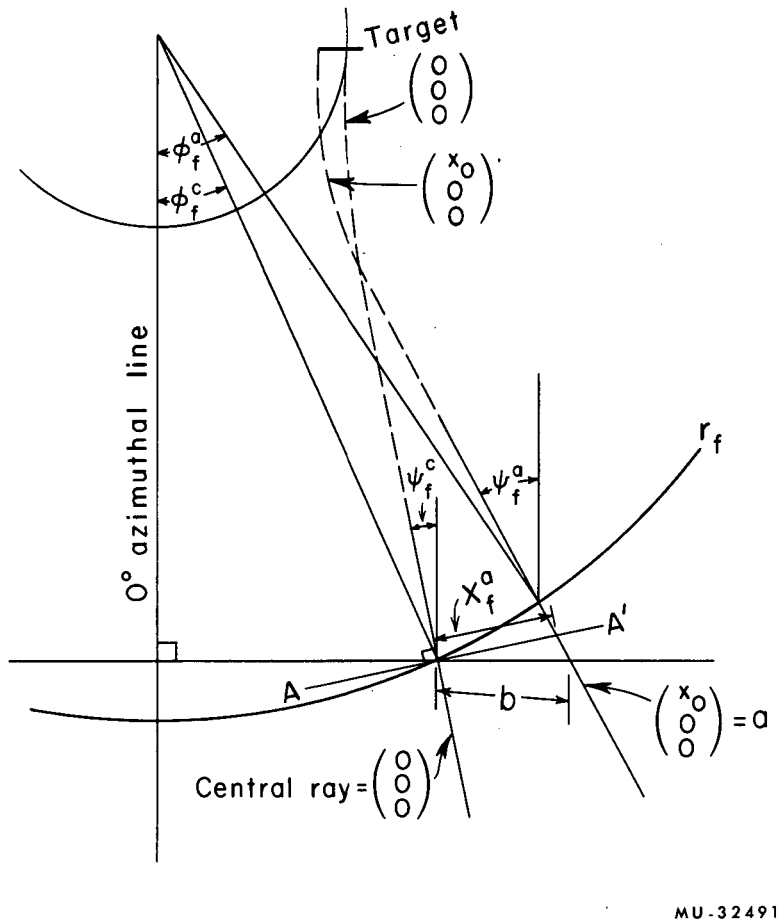

Fig. 6. Geometry for describing x_f^a and $x_f'^a$.

$$\psi_f^a = \theta_f^a - \frac{\pi}{2} - \phi_f^a .$$

Since we have rotated the quad face by ψ_f^c we have

$$x'^a_f = \tan(\psi_f^a - \psi_f^c) .$$

The sign of x_f' will be consistent with the convention described on page 12

To calculate the x_f^a coordinate, shown in Fig. 5, we need the quantity

$$b = r_f^c \left[(\sin\phi_f^a - \sin\phi_f^c) + (\cos\phi_f^a - \cos\phi_f^c)\tan\psi_f^a\right].$$

Then, as a first estimate,

$$x_f^a = |b| .$$

For a little better estimate,

$$x_f^a = \left|\frac{b\cos\psi_f^a}{\cos(\psi_f^c + \psi_f^a)}\right| .$$

The sign of x_f^a should be taken consistent with the previously described convention.

Using the trigonometric method, we plotted the value of A_{11}, A_{12}, A_{15}, A_{21}, A_{22}, A_{25} in Fig. 7 for a π^- beam at 220, 250, 260, and 380 MeV with the central ray coming off the target at 180 deg. These matrix elements should be referred to only when the above energy range, central ray conditions, and 184-inch magnetic field are duplicated.

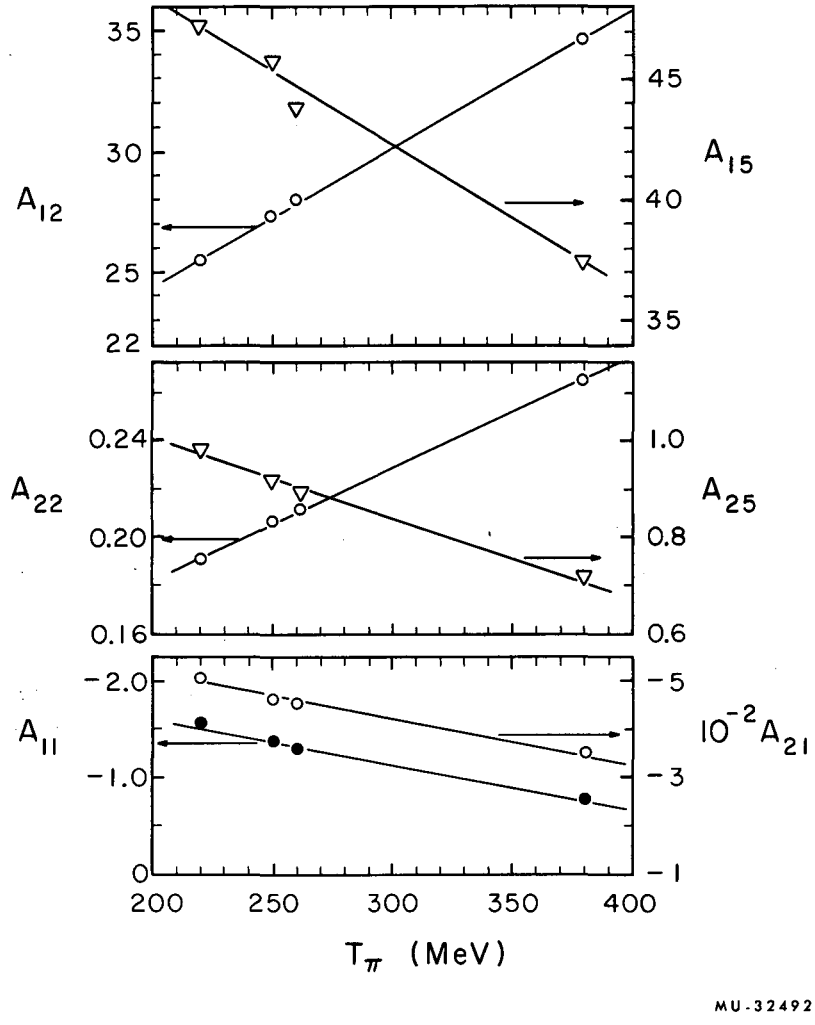

Fig. 7. A_{ij} as a function of kinetic energy, T.

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