UCRL-11225

University of California

Ernest O. Lawrence Radiation Laboratory

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

QUASI-PARTICLE INTERACTIONS IN EVEN-EVEN NUCLEI

Berkeley, California

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UCRL-11225

Non-Standard--for Nuclear Physics Journal

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

QUASI-PARTICLE INTERACTIONS IN EVEN-EVEN NUCLEI

Mannque Rho

January 27, 1964

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

ERRATA

TO: All recipients of UCRL-11225

FROM: Technical Information Division

SUBJECT: UCRL-11225, "Quasi-Particle Interactions In Even-Even Nuclei," Mannque Rho.

Page	Line	As it is	Should read
5	9	$u_{a_1} \equiv u_{a_1}$	$u_{\alpha_1} \equiv u_{a_1}$
11	1	Added $\langle \hat{0} \| C \| \hat{0} \rangle$ at end of first line of Eq. (19).	
11	8	$+\sum_{\gamma_1\gamma_2}$	$+\sum_{\gamma_1\gamma_2 \neq \alpha_1\alpha_2}$
13	4	$C(j_a j_b; m_a m_b M)$	$C(j_a j_b J; m_a m_b M)$
15	8	$\sum_{ab}'' \frac{B_2\ (ab)}{E_a + E_b - \omega} = \lambda$	$\sum_{ab}'' \frac{B_2^2\ (ab)}{E_a + E_b - \omega} = \lambda^{-1}$
16	18	$\zeta v_a v_b) \eta_\alpha^+ \eta_\beta^+$	$\zeta v_a v_b) \eta_\alpha^+ \eta_\beta$
17	5	$\|\Phi J_\Lambda\rangle$	$\|\Phi_{J\Lambda}\rangle$
17	5	$\theta(\kappa)$	$\theta(M)$
17	10	$\|\delta_{\kappa,J}\delta_{\mu,M}$	$\|\delta_{\kappa,J}\delta_{\mu,-M}$
17	17	(40) holds . . .	(40) always holds.
18	1	Omitted first line up to "To see this, . . . "	

ERRATA - Continued

Page	Line	As it is	Should read
18	8	$\sum_{ab}'' \frac{eB_2^2\,(ab)}{(E_a + E_b - \omega_{21})^2}$	$\sum_{ab}'' \frac{eB_2^2\,(ab)}{E_a + E_b - \omega_{21}}$,
18	9	$\left\{\sum_{ab}'' \left\lvert \frac{eB_2^2\,(ab)}{E_a + E_b - \omega_{21}} \right\rvert\right\}$	$e\lambda^{-1}$
18	10	Omitted the phrase, "provided $E_a + E_b - \omega_{21} > 0$"	
23	4	j_p	j_k
27	5,12	$N_{abcdJ'}$	$\overline{N}_{abcdJ'}$
27	7	Inserted "and $\overline{N} \equiv (2J'+1)N$" after "previously"	
28	14	Omitted $(2J'+1)$	
29	3	$(\delta_{J'2})/2$	$(\delta_{J'2})/5$
29	16	(66c)	(66e)
30	3,11	$-\frac{1}{\sqrt{5}}$	$-\frac{1}{5\sqrt{5}}$
30	4	Inserted "$\xi_{f_3 f_4}$" after "$(f_1 f_2)$"	
30	11	$\left\{\sum_{p_1 p_2}'' \frac{B_2^2\,(p_1 p_2)}{E_{p_1} + E_{p_2} - \omega_{21}}\right\}^{-\frac{1}{2}}$	$\left\{\sum_{p_1 p_2}'' \frac{B_2^2\,(p_1 p_2)}{(E_{p_1} + E_{p_2} - \omega_{21})^2}\right\}^{-\frac{1}{2}}$
30	12	Omitted $(1+\delta_{s_1 s_2})^{-1}$	
49	3	". . . magnitude that the. . ."	". . .magnitude than the. ."
52	18,19	"In this case, we assume $\overline{N}_{abcdJ'} = [(1+\delta_{cd})(1+\delta_{ab})]^{-1/2}$."	

QUASI-PARTICLE INTERACTIONS IN EVEN-EVEN NUCLEI

Mannque Rho

Lawrence Radiation Laboratory
University of California
Berkeley, California

January 27, 1964

ABSTRACT

The residual terms arising from Bogoliubov-Valatin transformation are studied and their roles in even-even spherical nuclei are examined by means of simple perturbation theory. It is argued that but for the uncertainty in the number of particles, the quasi-particle formalism simplifies the energy matrix even for complicated systems. The simplified equations are examined and certain conditions are found necessary in order to be consistent with experiments. The approximate formulas are applied to some 82-neutron nuclei, and the results show qualitative agreement with experiments.

QUASI-PARTICLE INTERACTIONS IN EVEN-EVEN NUCLEI

Mannque Rho

Lawrence Radiation Laboratory
University of California
Berkeley, California

January 27, 1964

1. Introduction

In this paper we study the roles played by the residual terms arising from the Bogoliubov-Valatin transformation in the low-energy properties of even-even spherical nuclei. In the lowest order, the application of the superconductivity theory of Bardeen et al.[1]) (or alternatively Bogoliubov's version[2]) seems to already have considerable success in explaining some of the properties of finite nuclei[3]). One notable success is, of course, the gap phenomenon which is observed in both the superconductivity system and even-even nuclei[4]).

Besides the energy-gap phenomenon, there are also other features in finite nuclei that need to be explained on an equal footing. The observation of low-lying "collective" states is not explained by the pair-correlation pictures. In the lowest-order superconductivity theory, it costs more energy to break a pair than to scatter it from one shell to the other near the Fermi surface in the $J = 0$ state. Thus, the existence of a level below the energy gap with $J \neq 0$ implies that the neglected interaction between the quasi particles may play a role that the pairing interaction cannot account for. The easiest and perhaps the best method of dealing with the residual interactions between the quasi particles is the random-phase approximation (RPA)[5]). However, unlike the electron-gas

problem, for which the RPA is known to be valid in the weak coupling limit[6]), the validity of the RPA in finite nuclei still is an open question. Here we do not do an RPA calculation; we shall instead take a somewhat naive approach, and examine by perturbation method the residual terms in connection with the low-energy properties of even-even nuclei.

The starting point is the BCS wave function, which we assume to be sufficiently good (the validity of BCS wave functions is examined elsewhere[7]), and treat the excitations in the conventional shell-model method. When ground-state correlation is considered to be important, then higher-lying configurations with the same quantum numbers as the ground state are taken into account by perturbation theory. The force used is a realistic one, being close to that used in the conventional shell-model calculation; however, we consider some of the parameters adjustable so that theoretical prediction is consistent with experiments. We show that one particular set of parameters is preferred over the other on the basis of the experimental trends and on the structure of the equations we get.

Whenever a qualitative discussion is desired, we make a somewhat drastic assumption and take a separable potential [such as the quadrupole-quadrupole (Q-Q) force[8])]; this approximation serves also to aid our understanding of the origin of success of such a simple force in a nuclear-structure problem.

First, we give a brief summary of well-known results on the BCS equations, which can be found in the papers of Belyaev and Baranger (Sec. 2). In Sec. 3, a general discussion of the problem is given so that the approximations we make are understood from the beginning. Then in Sec. 4 we derive relevant formulas for the quasi-particle spectra and a

calculation of physically measurable quantities (such as energy level, electromagnetic transition properties, etc.). In Sec. 5, the effect of the ground-state correlation is examined by means of the lowest-order perturbation theory, and in the last section (Sec. 6), we illustrate our previous discussions with numerical computations done on some of the single closed-shell, 82-neutron nuclei. This region has not yet been studied theoretically except in the simplified Q-Q model[9]), and though experimental results are scarce, it is one of the simplest systems available.

2. Hamiltonian

The discussions given in this section are well known, but we repeat some of the equations in order to define notation that is used throughout.

In the conventional fermion operators, the Hamiltonian is given by

$$H = \sum_{\alpha_1} \epsilon_{\alpha_1} c^+_{\alpha_1} c_{\alpha_1} + \frac{1}{4} \sum_{\alpha_1\alpha_2\alpha_3\alpha_4} (\alpha_1\alpha_2|V|\alpha_3\alpha_4)\, c^+_{\alpha_1} c^+_{\alpha_2} c_{\alpha_4} c_{\alpha_3} \tag{1}$$

where the operator c's obey the usual anticommutation rules. After performing the Bogoliubov-Valatin transformation[2]) of the form

$$c^+_\alpha = u_\alpha \eta^+_\alpha + v_\alpha s_\alpha \eta_{-\alpha} \text{ , with } u^2_\alpha + (u_\alpha s_\alpha)^2 = 1 \tag{2}$$

on the Hamiltonian Eq. (1) and eliminating the "dangerous diagrams" to the lowest order, one obtains

$$H' - U_0 = H_0(\eta) + H_I(\eta) \equiv H(\eta) \tag{3}$$

where U_0 = a constant term independent of η operators,

$$H_0(\eta) = \sum_\alpha E_\alpha \eta^+_\alpha \eta_\alpha , \tag{4a}$$

$$E_{\alpha_1} = \left[(\zeta_{\alpha_1} - \lambda)^2 + \Delta^2_{\alpha_1} \right]^{1/2} , \tag{4b}$$

$$\Delta_{\alpha_1} = -\frac{1}{2} \sum_{\alpha_2} s_{\alpha_1} s_{\alpha_2} (\alpha_1 - \alpha_1 | \bar{V} | \alpha_2 - \alpha_2) u_{\alpha_2} v_{\alpha_2} , \tag{4c}$$

$$\zeta_{\alpha_1} = \epsilon_{\alpha_1} - \mu_{\alpha_1} \equiv \epsilon_{\alpha_1} + \sum_{\alpha_2} (\alpha_1 \alpha_2 | \bar{V} | \alpha_1 \alpha_2) V^2_{\alpha_2} , \tag{4d}$$

$$H_I(\eta) = \frac{1}{4} \sum_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} (\alpha_1 \alpha_2 | \bar{V} | \alpha_3 \alpha_4) N(c^+_{\alpha_1} c^+_{\alpha_2} c_{\alpha_4} c_{\alpha_3}) \tag{4e}$$

where N is a normal-ordering operator,

and $$(\alpha_1 \alpha_2 | \bar{V} | \alpha_3 \alpha_4) \equiv \langle \alpha_1 \alpha_2 | V | \alpha_3 \alpha_4 \rangle - \langle \alpha_1 \alpha_2 | V | \alpha_4 \alpha_3 \rangle .$$

The following definitions and notations, which were also used above, will be useful in following the equations we will write down.

(a) Greek indices denote a complete set of quantum numbers labeling a state, while corresponding Roman indices denote all except the magnetic quantum number. Thus in the j-j coupling scheme we have $\alpha_1 \equiv (n_{a_1} \ell_{a_1} j_{a_1} m_{a_1})$, $a_1 \equiv (n_{a_1} \ell_{a_1} j_{a_1})$ or just j_{a_1}. In the deformed-shell model, we need only Greek indices, since a state is labeled by the projection of total angular momentum.

(b) The matrix element of the potential term V should be distinguished in the following way:

$$(\alpha_1\alpha_2|\bar{V}|\alpha_3\alpha_4) \equiv \int [\phi_{\alpha_1}(\underset{\sim}{r}_1)\phi_{\alpha_2}(\underset{\sim}{r}_2)\, V(\underset{\sim}{r}_1, \underset{\sim}{r}_2)\, \{\phi_{\alpha_3}(\underset{\sim}{r}_1)\phi_{\alpha_4}(\underset{\sim}{r}_2) - \phi_{\alpha_3}(\underset{\sim}{r}_2)\phi_{\alpha_4}(\underset{\sim}{r}_1)\}\,]\; d\underset{\sim}{r}_1 \quad d\underset{\sim}{r}_2$$

and $\langle\alpha_1\alpha_2|V|\alpha_3\alpha_4\rangle \equiv \int \phi_{\alpha_1}(\underset{\sim}{r}_1)\phi_{\alpha_2}(\underset{\sim}{r}_2)\, V\,(\underset{\sim}{r}_1, \underset{\sim}{r}_2)\phi_{\alpha_3}(\underset{\sim}{r}_1)\phi_{\alpha_4}(\underset{\sim}{r}_2)\; d\underset{\sim}{r}_1 \quad d\underset{\sim}{r}_2$.

This distinction applies also to the coupled matrix element,

(c) $s_{\alpha_1} \equiv (-)^{j_{a_1} - m_{a_1}}$,

$v_{\alpha_1} \equiv v_{a_1}$,

$u_{a_1} \equiv u_{a_1}$ in spherical representation,

and $s_{\alpha_1} \equiv +1$ in the deformed field.

(d) The total angular momentum is labeled J.

In spherical representation, it is more convenient to write Eqs. (4b) and (4c) in the form ($\Omega \equiv j + \frac{1}{2}$)

$$\Delta_a = -\frac{1}{2}\sum_b (\Omega_b/\Omega_a)^{1/2}\; u_b v_b (aa0|\bar{V}|bb0), \tag{5}$$

$$\mu_a = -\sum_{bJ} \frac{\Omega_J}{\Omega_a}\; v_b^2\; (abJ|\bar{V}|abJ), \tag{6}$$

where in accordance with the definition of paragraph (b) above for the coupled scheme,

$$(abJ|\bar{V}|cdJ) \equiv \langle abJ|V|cdJ\rangle - \theta(cdJ)\ \langle abJ|V|dcJ\rangle\ , \tag{7}$$

with $\theta(cdJ) = (-)^{j_c+j_d+J}$.

The interaction term H_I is given by three terms,

$$H_I = H_{22} + H_{40} + H_{31}\ ,$$

where
$$H_{22} = \frac{1}{4} \sum_{\alpha_1\alpha_2\alpha_3\alpha_4} (\alpha_1\alpha_2|Z_1|\alpha_3\alpha_4)\ \eta^+_{\alpha_1}\ \eta^+_{\alpha_2}\ \eta_{\alpha_4}\ \eta_{\alpha_3}\ , \tag{8}$$

$$H_{40} = \frac{1}{4} \sum_{\alpha_1\alpha_2\alpha_3\alpha_4} (\alpha_1\alpha_2\alpha_3\alpha_4|Z_2|0)\ \eta^+_{\alpha_1}\ \eta^+_{\alpha_2}\ \eta^+_{\alpha_4}\ \eta^+_{\alpha_3} + \text{Hermitian conjugate}, \tag{9}$$

$$H_{31} = \frac{1}{2} \sum_{\alpha_1\alpha_2\alpha_3\alpha_4} (\alpha_1\alpha_2\alpha_4|Z_3|\alpha_3)\ \eta^+_{\alpha_1}\ \eta^+_{\alpha_2}\ \eta^+_{\alpha_4}\ \eta_{\alpha_3} + \text{Hermitian conjugate}, \tag{10}$$

with the matrices Z_1, Z_2, and Z_3 defined as

$$(\alpha_1\alpha_2|Z_1|\alpha_3\alpha_4) = (\alpha_1\alpha_2|V|\alpha_3\alpha_4)(u_{a_1}\ u_{a_2}\ u_{a_3}\ u_{a_4} + v_{a_1}\ v_{a_2}\ v_{a_3}\ v_{a_4}) \tag{8a}$$

$$+ 4\ (\alpha_1\text{-}\alpha_3|\bar{V}|\alpha_4\text{-}\alpha_2)\ s_{\alpha_2}\ s_{\alpha_3}\ u_{a_1}\ u_{a_4}\ v_{a_2}\ v_{a_3}\ ,$$

$$(\alpha_1\alpha_2\alpha_3\alpha_4|Z_2|0) = (\alpha_1\alpha_2|\bar{V}|\text{-}\alpha_3\text{-}\alpha_4)\ s_{\alpha_3}\ s_{\alpha_4}\ u_{a_1}\ u_{a_2}\ v_{a_3}\ v_{a_4}\ , \tag{9a}$$

and

$$(\alpha_1\alpha_2\alpha_4|Z_3|\alpha_3) = -(\alpha_1\alpha_2|\bar{V}|-\alpha_3\alpha_4)\, s_{\alpha_3}(u_{a_1}u_{a_2}v_{a_3}u_{a_4} - v_{a_1}v_{a_2}u_{a_3}v_{a_4}) . \tag{10a}$$

We see readily that

$$(\alpha_1\alpha_2|Z_1|\alpha_3\alpha_4) - (\alpha_3\alpha_4|Z_1|\alpha_1\alpha_2) = (\alpha_1\alpha_2|Z_1|\alpha_3\alpha_4) - (\alpha_2\alpha_1|Z_1|\alpha_4\alpha_3) \tag{11a}$$

$$= 4(\alpha_1-\alpha_3|\bar{V}|\alpha_4-\alpha_2)\, s_{\alpha_2}s_{\alpha_3}\,(u_{a_1}u_{a_4}\,v_{a_2}\,v_{a_3} - u_{a_2}\,u_{a_3}\,v_{a_1}\,v_{a_4}).$$

Hence we have $(\alpha_1\alpha_2|Z_1|\alpha_3\alpha_4) - (\alpha_4\alpha_3|Z_1|\alpha_2\alpha_1) = 0,$ (11b)

and

$$(\alpha_1\alpha_2\alpha_3\alpha_4|Z_2|0) = (\alpha_2\alpha_1\alpha_4\alpha_3|Z_2|0) = -(\alpha_2\alpha_1\alpha_3\alpha_4|Z_2|0) = -(\alpha_1\alpha_2\alpha_4\alpha_3|Z_2|0), \tag{11c}$$

and

$$(\alpha_1\alpha_2\alpha_4|Z_3|\alpha_3) = -(\alpha_2\alpha_1\alpha_4|Z_3|\alpha_3). \tag{11d}$$

These relations are useful in computing matrix elements of the operators.

Let us now describe the physical significance of the operators given in (8) through (10), though it is rather obvious from the structure. Here H_{22} is the familiar scattering term which "couples" two-quasi-particle states; since a quasi-particle is a linear combination of a particle and hole, we have two modes of scattering—particle-particle type and particle-hole type. The term H_{40} couples the vacuum to four-quasi-particle states, two-quasi-particle states to six-quasi-particle states, etc. Notice that this term will be important for mainly two-particle-two-hole configurations. We expect H_{31} to be significant for odd-A nuclei, since it may couple a single-quasi-

particle state to a three-quasi-particle state, with an energy denominator of twice the single-quasi-particle excitation energy. For our purpose, we neglect this term entirely; the reason for dropping this term is as follows: If one constructs an equation of motion by taking the commutator $[H,A^+]$ and $[H,A]$, where A^+ and A are two quasi-particle-creation and destruction operators, the commutator taken with H_{31} is uncoupled from two-quasi-particle modes, and hence is unimportant in the system with an even number of particles.

3. General Considerations

Our aim here is to construct an equation to find the solution of

$$H(\eta)\ \Phi_{\alpha\Lambda} = \omega_{\alpha\Lambda}\ \Phi_{\alpha\Lambda}\ . \tag{12}$$

The symbol α denotes a set of all quantum numbers labeling the state, and Λ serves as an extra label to distinguish one state from the other within the set α. [For example, since $H(\eta)$ is rotation-invariant, α labels the angular momentum of the state JM, and within the states of JM, $\Lambda = 1,2, \cdots, n$ span the space.] If one expands Φ in terms of eigenstates of $H_0(\eta)$, i.e.,

$$\Phi_{\alpha\Lambda} = \sum_{[\beta]} C_{\alpha\Lambda}\ [\beta]\ \phi_\alpha\ [\beta] \tag{13}$$

one obtains the usual eigenvalue equation

$$\sum_{[\gamma]} (\phi_\alpha[\gamma]\ |H\ (\eta)|\phi\ [\beta])\ C_{\alpha\Lambda}[\gamma] = \omega_{\alpha\Lambda}\ C_{\alpha\Lambda}[\beta]\ . \tag{14}$$

The symbol $[\beta]$ denotes a complete set of quantum numbers, since Eq. (13) is a linear combination of various numbers of quasi-particles. As an example, consider an excited state in even-A nuclei. For this case, Eq. (13) may read

$$\Phi_{\alpha\Lambda} = \sum_n \sum_{\beta_1\beta_2\cdots\beta_n} C_{\alpha\Lambda}(\beta_1\beta_2\cdots\beta_n)\phi_\alpha(\beta_1\beta_2\cdots\beta_n) \quad , \tag{15}$$

where n runs over $n = 2, 6, \cdots, m$, where $m \leqslant N$, N being the number of protons or neutrons (or both) that are effectively taken into account. The sum over $\beta_1\beta_2\cdots\beta_n$ takes into account all the effective single-particle levels considered in the problem. In practice, for the nuclei with many particles, the bookkeeping, and also the normalization, gets tremendously complicated; thus the RPA certainly does have the advantage—or, probably, is the only means of practical calculations. In particular, an RPA amplitude is defined by $\langle\Phi_0|A_\alpha(\Omega_1\Omega_2)|\Phi_{\alpha\Lambda}\rangle$, where Φ_0 is the exact ground state [an eigenstate of $H(\eta)$] and $\Phi_{\alpha\Lambda}$ is an excited eigenstate of $H(\eta)$ with the quantum number α and a label Λ, and A_α is an operator which destroys two quasi particles with the total quantum number α. Then we have

$$\langle\Phi_0|A_\alpha(\Omega_1\Omega_2)|\Phi_{\alpha\Lambda}\rangle = \sum_{[\beta][\gamma]} C_0[\beta]\, C_{\alpha\Lambda}[\gamma]\, \langle\phi_0[\beta]|A_\alpha(\Omega_1\Omega_2)|\phi_\alpha[\gamma]\rangle. \tag{16}$$

The last factor is just the matrix element of $A_\alpha(\Omega_1\Omega_2)$ between nonperturbed states, and can be calculated once the representation of ϕ's is given. In m-scheme, it is just

$$\langle\phi_0[\beta]|A_\alpha|\phi_\alpha[\gamma]\rangle = \langle\eta_{\beta_{n-2}}\eta_{\beta_{n-3}}\cdots\eta_{\beta_1}\eta_{\Omega_1}\eta_{\Omega_2}\eta^+_{\gamma_1}\eta^+_{\gamma_2}\cdots\eta^+_{\gamma_n}\rangle_0, \quad n=2,4,6\ldots .$$

The relation (16) thus shows that the RPA amplitude consists of components with various quasi-particle numbers. Suppose the ground state is the quasi-particle vacuum. Then Eq. (16) shows that

$$\langle\hat{0}|A_\alpha(\Omega_1\Omega_2)|\Phi_{\alpha\Lambda}\rangle = C_{\alpha\Lambda}\ (\Omega_1\Omega_2), \tag{17}$$

which is just the two-quasi-particle amplitude. Thus the ground-state correlation is the main feature of the RPA, as is well known.

Returning to the matrix equation (14), we see that the complete matrix will consist of various blocks connecting states that differ by four quasi particles. Now if one considers a spherical nucleus, the quasi-particle energy E is on the average about 1 MeV; hence, the second block already involves an excitation of around 4 MeV beyond the lowest block. Since only the low-energy properties are of particular interest here, it is in practice a good approximation to drop all the other blocks and keep the lowest one only. This is in fact born out by actual numerical calculation done by Ariveu et al.[10]). One advantage of the quasi-particle method in spherical nuclei is precisely this fact: for low-energy spectra, one needs to consider only the two-quasi-particle configurations, thus reducing the dimension of the matrix to a tractable size, unlike the usual shell-model calculation.

If one desires, one could also write a set of coupled equations corresponding to Eq. (14). Then for the ground state, one has[11])

$$(\omega - E_0)\langle\hat{0}|C|\hat{0}\rangle = \sum_{\alpha_1\cdots\alpha_4} \langle\hat{0}|H|\phi(\alpha_1\cdots\alpha_4)\rangle\ \langle\phi(\alpha_1\cdots\alpha_4)|C|\hat{0}\rangle, \tag{18}$$

$$(\omega - \sum_{\kappa=\alpha_1\cdots\alpha_4} E_\kappa)\langle\phi(\alpha_1\cdots\alpha_4)|C|\hat{0}\rangle = \langle\phi(\alpha_1\cdots\alpha_4)|H|\hat{0}\rangle$$

$$+ \sum_{\beta_1\cdots\beta_4} \langle\phi(\alpha_1\cdots\alpha_4)|H|\phi(\beta_1\cdots\beta_4)\rangle\langle\phi(\beta_1\cdots\beta_4)|C|\hat{0}\rangle$$

$$+ \sum_{\delta_1\cdots\delta_8} \langle\phi(\alpha_1\cdots\alpha_4)|H|\phi(\delta_1\cdots\delta_8)\rangle\langle\phi(\delta_1\cdots\delta_8)|C|\hat{0}\rangle \tag{19}$$

$$\vdots\ ,$$

and for an excited state

$$(\omega - E_{\alpha_1} - E_{\alpha_2})\langle\phi(\alpha_1\alpha_2)|C|\phi(\alpha_1\alpha_2)\rangle = \sum_{\beta_1\beta_2} \langle\phi(\alpha_1\alpha_2)|H|\phi(\beta_1\beta_2)\rangle\langle\phi(\beta_1\beta_2)|C|\phi(\alpha_1\alpha_2)\rangle$$

$$+ \sum_{\beta_1\cdots\beta_6} \langle\phi(\alpha_1\alpha_2)|H|\phi(\beta_1\cdots\beta_6)\rangle\langle\phi(\beta_1\cdots\beta_6)|C|\phi(\alpha_1\alpha_2)\rangle\ , \tag{20}$$

$$(\omega - E_{\beta_1} - E_{\beta_2})\langle\phi(\beta_1\beta_2)|C|\phi(\alpha_1\alpha_2)\rangle = \langle\phi(\beta_1\beta_2)|H|\phi(\alpha_1\alpha_2)\rangle\langle\phi(\alpha_1\alpha_2)|C|\phi(\alpha_1\alpha_2)\rangle$$

$$+ \sum_{\gamma_1\gamma_2} \langle\phi(\beta_1\beta_2)|H|\phi(\gamma_1\gamma_2)\rangle\langle\phi(\gamma_1\gamma_2)|C|\phi(\alpha_1\alpha_2)\rangle$$

$$+ \sum_{\delta_1\cdots\delta_6} \langle\phi(\beta_1\beta_2)|H|\phi(\delta_1\cdots\delta_6)\rangle\ \langle\phi(\delta_1\cdots\delta_6)|C|\phi(\alpha_1\alpha_2)\rangle \tag{21}$$

$$\vdots$$

In Eqs. (18) through (21), we have suppressed the subscript in the unperturbed quasi-particle wave function ϕ (if in a J scheme, one should put the total angular momentum of the state). Notice that Eqs. (18) through (21) generate the Wigner-Brillouin perturbation series.

4. Excited States

4.1 EIGENVALUE EQUATION

In this section, we discuss the excitation spectra of even-even spherical nuclei. We assume the Hamiltonian in the form

$$H = H_0 + H_{22} \tag{22}$$

Then

$$\Phi_{J\Lambda} = \sum_{ab}'' C_{J\Lambda}(ab)\, \phi_J(ab), \tag{23a}$$

with the double prime on the sum denoting the restriction $a \geqslant b$.

$$H_0\, \Phi_J(ab) = (E_a + E_b)\, \Phi_J(ab) \tag{23b}$$

$$H\Phi_{J\Lambda} = \omega_{J\Lambda}\, \Phi_{J\Lambda}$$

$$\underset{\sim}{J}^2 \begin{Bmatrix} \phi_J(ab) \\ \Phi_{J\Lambda} \end{Bmatrix} = J\,(J+1) \begin{Bmatrix} \phi_J(ab) \\ \Phi_{J\Lambda} \end{Bmatrix} \tag{23c}$$

$$\langle \Phi_{J\Lambda} | \Phi_{J'\Lambda'} \rangle = \delta_{\Lambda\Lambda'}\, \delta_{JJ'} \tag{23d}$$

$$\langle\phi_J(ab)|\phi_{J'}(a'b')\rangle = \frac{\delta_{aa'}\,\delta_{bb'} + \delta_{ab'}\,\delta_{ba'}}{\left[(1+\delta_{ab})(1+\delta_{a'b'})\right]^{1/2}}\,\delta_{JJ'} \tag{23e}$$

From now on, we shall take the phase convention taken by Edmonds[12]); the phase of the two-quasi-particle wave function is dictated by the definition

$$\phi_J(ab) \equiv (1+\delta_{ab})^{-1/2}\sum_{m_a m_b} C(j_a j_b; m_a m_b M)\eta^+_\beta\,\eta^+_\alpha|0\rangle. \tag{24}$$

Then, using Eqs. (8), (8a), and (11a), we easily obtain[13])

$$\begin{aligned}\langle\phi_J(ab)|H_{22}|\phi_J(cd)\rangle = [(1+\delta_{ab})(1+\delta_{cd})]^{-1/2}\,[&\nu(abcd)(abJ|\bar{V}|cdJ)\\ &+ \chi(acbd)(abJ|\bar{U}|cdJ) - \theta(cdJ)\,\chi\,(adbc)(abJ|\bar{U}|dcJ)],\end{aligned} \tag{25}$$

where

$$\begin{aligned}\nu\,(abcd) &= u_a u_b u_c u_d + v_a v_b v_c v_d\,,\\ \chi\,(acbd) &= u_a u_c v_b v_d + v_a v_c u_b u_d\,,\end{aligned} \tag{26a}$$

and

$$(abJ|\bar{U}|cdJ) \equiv -\sum_{J'}(2J'+1)W(adbc;J'J)\,(daJ'|\bar{V}|bcJ'). \tag{26b}$$

The matrix equation now reads

$$C_{J\Lambda}(ab) = (\omega_{J\Lambda} - E_a - E_b)^{-1}\sum_{cd}{}'' \langle\phi_J(ab)|H_{22}|\phi_J(cd)\rangle\,C_{J\Lambda}(cd). \tag{27}$$

An explicit form of the matrix element (25) may be obtained for a given nuclear force[21]. If one considers only a central force and one kind of particle, then the matrix element can be written

$$\langle \phi_J(ab) \mid H_{22} | \phi_J(cd) \rangle = f_{abJ} f_{cdJ} R_J(abcd) + \sum_k h_k(abcdJ), \tag{28}$$

where the first term corresponds to a term with a separable angular part and the second to a nonseparable term that contains two terms. It should be noted that in the first term of Eq. (28) the rank of tensor k is replaced by J, and hence if one wants to compute the J state, the first term contributes only through the $k = J$ component. As it stands, even the first term in Eq. (28) is not completely separable because of the presence of the radial term R. In a schematic work, it is usually taken to be also separable[9] i.e.,

$$R_J(abcd) = R_J(ab) R_J(cd) \ . \tag{29}$$

It turns out that if one uses the quadrupole-quadrupole (Q-Q) force used in many schematic studies, we may neglect the second term in Eq. (28) as it is small compared with the first.

If we write

$$V(r_1, r_2) = - \lambda \, r_1^2 r_1^2 \, Y_2 \cdot Y_2 \ , \tag{30}$$

then

$$\langle \phi_J(ab) | H_{22} | \phi_J(cd) \rangle \approx - \lambda \, B_2(ab) \, B_2(cd) \ , \tag{31}$$

where

$$B_2(ab) = \theta(j_b+\tfrac{1}{2}) \left[\frac{(2j_a+1)(2j_b+1)}{20\pi\ (1+\delta_{ab})}\right]^{1/2} C(j_a j_b 2; \tfrac{1}{2} - \tfrac{1}{2}\, 0)(u_a v_b + v_a u_b) R_2(ab) \tag{32}$$

and

$$R_2(ab) = \int_0^\infty r^4 R_a(r) R_b(r) dr. \tag{33}$$

With (31) and (27), we obtain the well-known wave function[14])

$$C_{2\Lambda}(ab) = \frac{B_2(ab)}{E_a+E_b-\omega_{2\Lambda}} \left[\sum_{cd}'' \frac{B_2^2\ (cd)}{(E_c+E_d-\omega_{2\Lambda})^2}\right]^{-1/2} \tag{34}$$

where the eigenvalue ω is obtained from

$$\sum_{ab}'' \frac{B_2\ (ab)}{E_a + E_b - \omega} = \lambda. \tag{35}$$

We do not indulge in any detailed discussion of Eq. (34) and (35), since they have been extensively studied; let it suffice to say that the solutions (34) and (35) are not suited for very low-lying states; i.e., when $\frac{\omega}{E} << 1$. In such cases, one is required to use the random-phase approximation or the adiabatic approximation given by Kisslinger and Sorensen[9]). However, we will use (34) and (35) to compare two-quasi-particle interactions in the Q-Q force with those interactions in a more general force.

4.2 MATRIX ELEMENT OF SINGLE-PARTICLE OPERATORS

Now, assuming we have obtained wave functions through Eq. (27) for a certain potential, we demonstrate some results that have experimental significance. What we show is rather heuristic and qualitative, but will be of help in understanding the results of numerical computations given later. The experimental situation goes as follows: (a) in spherical nuclei, the lowest 2+ state is collective and lies lower than the energy gap (2Δ), and the E2 transition from the first 2+ to the ground state is greatly enhanced in comparison with the single-particle estimate; (b) the crossover transition from the second 2+ to the ground state is very much smaller than (a); (c) the E2 transition from the first 4+ to the first 2+ transition is sometimes observed to be largely retarded in comparison with the single-particle estimate. We shall in the following disucssion denote such reduced transition probabilities by B(E2;a), B(E2;b), and B(E2;c) respectively. We denote the single-particle estimate as $B(E2)_{sp}$.

An arbitrary single-particle operator in quasi-particle second-quantized form is given by

$$\hat{T} = \sum_{\alpha\beta} \langle \alpha | T_{\kappa\mu} | \beta \rangle [(u_a u_b - \zeta v_a v_b)\eta_\alpha^+ \eta_\beta^+ + s_\beta u_a v_b \eta_\alpha^+ \eta_{-\beta}^+ + s_\alpha u_a v_a \eta_{-\alpha} \eta_\beta], \qquad (36)$$

where $T_{\kappa\mu}$ is a tensor of rank κ, $\mu = -\kappa, -\kappa + 1, \cdots, \kappa$; and $\zeta = +1$ (or -1) if T is even (or odd) under time-reversal operation.

4.2.1 Transition Between the Unperturbed Ground State and an Excited State

It is obvious that in this case, we have

$$\hat{T} = \sum_{\alpha\beta} \langle\alpha|T_{\kappa\mu}|\beta\rangle [s_\beta u_a v_b \eta^+_\alpha \eta^+_{-\beta} + s_\alpha u_b v_a \eta_{-\alpha}\eta_\beta] \ . \tag{37}$$

Now, by using (23), (24), and (37), one can easily obtain

$$\langle\hat{0}|\hat{T}|\Phi J_\Lambda\rangle = - \frac{\theta(\kappa)}{[2\kappa+1]^{1/2}} \sum_{ab}'' \langle a\|T_\kappa\|b\rangle (u_a v_b + v_a u_b)\ C_{J\Lambda}(ab)\ \delta_{\kappa J}\ \delta_{\mu -M} \ . \tag{38}$$

In deriving Eq. (38), we have assumed

$$\langle a\|T_\kappa\|b\rangle = - \theta(ab\kappa)\langle b\|T_\kappa\|a\rangle \ . \tag{39}$$

We are interested in the absolute square of the matrix element; thus, by triangular inequality, we have

$$|\langle\hat{0}|\hat{T}|\Phi_{J\Lambda}\rangle| \leqslant \frac{1}{[2J+1]^{1/2}} \sum_{ab}'' |\langle a\|T_\kappa\|b\rangle\ \xi_{ab}\ C_{J\Lambda}\ (ab)|\delta_{\kappa,J}\delta_{\mu,M} \ , \tag{40}$$

where we have put for convenience $\xi_{ab} = (u_a u_b + v_a u_b)$.

The dominant component for the first 2+ (i.e., $J = 2$, $\Lambda = 1$) does not contribute sufficient strength to the matrix element to fit the experimental data of $2+ \rightarrow 0+$ transition; therefore, all the components of the wave function should contribute coherently to build up the needed strength. Thus we desire to find the condition such that the equality in (40) holds. One can easily show in Q-Q force that the equality in (40) holds provided the

lowest 2+ state energy is smaller than E_a+E_b for any a and b. To see this, put $\kappa=2$,

$$T_{2\mu} = Y_{2\mu} , \tag{41}$$

$$\hat{T} = \mathcal{M}(E2,\mu) ; \tag{42}$$

then we see

$$\langle a\|T_2\|b\rangle \quad \xi_{ab} = eB_2 \ (ab) . \tag{43}$$

Now substituting (34) and (43) into (38), we have

$$\begin{aligned} M \equiv \langle\hat{0}|\hat{T}|\Phi_{21}\rangle &= -\frac{\theta(M)}{\sqrt{5}} \left\{ \sum_{ab}'' \frac{eB_2^2 \ (ab)}{(E_a+E_b-\omega_{21})^2} \right\} \left\{ \sum_{cd}'' \frac{B_2^2 \ (cd)}{(E_c+E_d-\omega_{21})^2} \right\}^{-1/2} \\ &= -\frac{\theta(M)}{\sqrt{5}} \left\{ \sum_{ab}'' \left| \frac{eB_2^2 \ (ab)}{E_a + E_b - \omega_{21}} \right| \right\} \left\{ \sum_{cd}'' \frac{B_2^2 \ (cd)}{(E_c + E_d - \omega_{21})^2} \right\}^{-1/2} , \end{aligned} \tag{44}$$

provided $E_a + E_b - \omega_{21} > 0$ for all a and b, which shows the equality. Let us now examine the case of a more realistic force. Taking the Morse-Feshbach phase convention[15]) for harmonic-oscillator radial integrals, and examining the Clebsh-Gordan coefficient, one can obtain

$$\langle a\|T_2\|b\rangle = (-)^{\eta} |\langle a\|T_2\|b\rangle| , \tag{45}$$

where

$$\eta = \frac{N_a-\ell_a}{2} - \frac{N_b-\ell_b}{2} + \delta(j_a,j_b-1) + \delta(j_a,j_b), \tag{46}$$

and where N and ℓ are the oscillator quantum and orbital-angular momentum of the state, respectively. Since we have

$$\Delta N = 0, \pm 2, \tag{47}$$

we take N's out of η, and noticing

$$\Delta \ell = 0, \pm 2, \tag{48}$$

we have the possibilities as given in Table 1.

Table 1. The signs of 2+ → 0+ transition matrix elements.

$(-)^{\eta}$	$\Delta\ell$	$\delta(j_a, j_b-1)$	$\delta(j_a, j_b)$	Configuration Examples
-1	0	0	1	$(2d^2_{5/2}), (1g^2_{7/2})$, etc.
+1	0	0	0	$(2d_{5/2}\ 2d_{3/2})$
-1	0	1	0	$(2d_{3/2}\ 2d_{5/2})$
+1	±2	1	0	$(2d_{5/2}\ 1g_{7/2})$
-1	+2	0	0	$(1g_{7/2}\ 2d_{5/2}), (1g_{7/2}\ 2d_{3/2})$

The last row entry corresponds to a case of $j_a = j_b + 1$. Thus, if we choose $j_a > j_b$ (except for the case where $\Delta\ell$, $j_a = j_b + 1$, in which case we choose $j_a = j_b - 1$) we always get the same sign. Then, once such a representation is chosen, the coherence of the matrix element is guaranteed if

$$C_{21}(ab) = \epsilon|C_{21}(ab)| \text{ for all } a \text{ and } b, \tag{49}$$

where ϵ is a constant phase factor. The condition for this can be easily deduced; i.e., if $E_a + E_b - \omega_{21} > 0$ for all a and b, a sufficient condition is to have all negative (attractive) off-diagonal elements in the H_{22} matrix. Thus we have to choose force parameters that give correct energy spectra as well as all attractive off-diagonal terms, still consistent with other experimental results. Notice this condition is not so obvious from the matrix-element formula itself, since it is a linear combination of particle-particle and particle-hole matrix elements.

Assume we have obtained the lowest eigensolution satisfying Eq. (49); for the same parameters, the next 2+ state would have a wave function that possesses incoherent components, since this has to be orthogonal to the lowest. Aside from different coefficients, the interference effect in the components alone would reduce the matrix element from the previous case, because of inevitable cancellation, which is expected to be considerable. The magnitude of reduction, of course, depends upon parameters involved, but this at least shows $B(E2;b) < B(E2;a)$.

4.2.2. Transitions Between Two-Quasi-Particle States

The operator connecting two-quasi-particle states is given from (36):

$$\hat{T} = \sum_{\alpha\beta} \langle\alpha|T_{\kappa\mu}|\beta\rangle \, (u_a u_b - \zeta v_a v_b) \, \eta_\alpha^+ \, \eta_\beta . \tag{50}$$

The matrix element between states $J_1\Lambda_1$ and $J\Lambda$ is then given by

$$\langle\Phi_{J_1\Lambda_1}|\hat{T}|\Phi_{J\Lambda}\rangle = \sum_{\substack{\alpha\beta \\ k_1>q_1 \\ k>q}} \langle\alpha|T_{\kappa\mu}|\beta\rangle \, (u_a u_b - \zeta v_a v_b) \, C_{J_1\Lambda_1}(k_1q_1) \, C_{J\Lambda}(kq)$$

$$\times \langle\phi_{J_1}(k_1q_1)|\eta_\alpha^+ \, \eta_\beta|\phi_J(kq)\rangle . \tag{51}$$

Using Eq. (24), we obtain, after straightforward algebra,

$$|\langle\Phi_{J_1\Lambda_1}|\hat{T}|\Phi_{J\Lambda}\rangle| = | \sum_{(k_1q_1)\in J_1}'' \sum_{(k,q)\in J}'' \theta(j_{k_1}j_q) \, C_{J_1\Lambda_1}(k_1q_1) \, C_{J\Lambda}(kq)$$

$$\times P^{kq}_{k_1q_1} \left\{ \langle k\| T_\kappa \|k_1\rangle \, W(JJ_1k_1k;\kappa q) \, (u_k u_{k_1} - \zeta v_k v_{k_1}) \, \delta_{q_1q} \right\} |\delta_{\mu,-(M+M_1)}, \tag{52}$$

where we have introduced a symbol $P^{kq}_{k_1q_1}$ which is defined to act on a function f as

$$P^{kq}_{k_1q_1} \, f(kk_1qq_1) = f(kk_1qq_1) + f(kq_1qk_1) + f(qk_1kq_1) + f(qq_1kk_1). \tag{53}$$

In deriving Eq. (52), we have again used Eq. (39). For electric-quadrupole transition, we have $\zeta = +1$, and $T_{\kappa\mu}$ was given before.

Examining Eq. (52), we see that

(a) the Racah coefficient is usually much less than unity;

(b) in general, we have $|C_J C_J\ (uu-vv)| \ll 1$;

(c) cancellation among the components occurs.

Therefore we may expect the quantity in (52) to be in some cases considerably smaller than the single-particle strength. Of special interest in this matrix element is the E2 transition between a 4+ and a 2+ state. For example, in Ce^{140}, experimental findings show $B(E2;c)/B(E2)_{sp} \approx 1/17$, indicating a large retardation compared with B(E2;a)[16]). Since the cancellation effect is very sensitive to the coefficients of the wave functions, the B(E2;c) is expected to be strongly dependent upon the parameters of the force used. Therefore, if too drastic a change occurs in B(E2) for a small variation of the parameters, it should not be taken too seriously.

4.2.3. Magnetic Moment of an Excited State

To supplement the energy spectra and transition rates computed above, it is also useful to compute the magnetic moment of a two-quasi-particle state to see how good the wave function is. Experimentally, not many nuclei have been explored for their magnetic moments of excited states. But in the region that we will consider later, the moment of the 4+ state in Ce^{140} has recently been measured.

Within two-quasi-particle subspace, the magnetic-moment operator is given by

$$\mu_{op} = \sum_{\alpha\beta} \langle\alpha|\underset{\sim}{\mu}|\beta\rangle\ (u_a u_b + v_a v_b)\ \eta_\alpha^+ \eta_\beta \tag{54}$$

The positive sign in the u and v factor is, as is well known, due to the odd-time-reversal property (i.e., $\zeta = -1$). Now, using

$$\underset{\sim}{\mu} = \sum_{\nu} g_{\nu} \underset{\sim}{j}_{\nu} , \tag{55}$$

$$\underset{\sim}{J} = \underset{\sim}{j}_{p} + \underset{\sim}{j}_{q} \tag{55}$$

$$\underset{\sim}{j}_{k} \cdot \underset{\sim}{J} = \frac{1}{2} (\underset{\sim}{J}^{2} + \underset{\sim}{j}_{k}^{2} - \underset{\sim}{j}_{q}^{2}) \tag{56}$$

and Eq. (23) for the wave function, we get

$$\mu_J = g_J J = \frac{1}{(2J+1)} \sum_{kq} [C_{J\Lambda}(kq)]^2 \left\{ (g_k+g_q) J(J+1)+(g_k-g_q)[j_k(j_k+1)-j_q(j_q+1)] \right\} \tag{57}$$

where g_i's are magnetic g factors for the orbital denoted by i (i.e., $g_i = g_{j_i}$).

Notice that the u and v factors do not appear here, since $u_k^2 + v_k^2 = 1$ for all k. Thus, except for the different coefficients of the wave function, the equation is exactly the same as the ordinary shell-model result.

5. Lowest-Order Ground State Correlation

So far all the calculations have been made under the assumption that the ground state is uncorrelated; that is to say, the ground state is a pure quasi-particle vacuum. Such an assumption is a convenient one in many algebraic manipulations, and furthermore may be a good one in spherical nuclei. But in many cases, the ground-state correlation turns out to be

very important[17]). One notable example is the enhanced E2 transition from a "collective" 2+ state to 0+ ground state in even-even nuclei.

In this section, we take the lowest-order perturbation terms, and consider configuration mixing of higher-lying states with the vacuum, and study specifically what effects they have on the E2 transition from 2+ to 0+ state. Because of the large number of configurations that could be mixed in (constructed from a set of single-quasi-particle states) a complete treatment (of even the lowest order) is a rather tedious procedure. We shall therefore take only a few configurations in our numberical calculation and see the qualitative effect.

In Eq. (19), take only the lowest-order term, getting the first-order correction to the wave function of the ground state:

$$\langle\phi(\alpha_1\cdots\alpha_4)|C|\hat{0}\rangle \approx \frac{\langle\phi(\alpha_1\cdots\alpha_4)|H_I|\hat{0}\rangle}{\omega - \sum_{K=\alpha_1\cdots\alpha_4} E_K} \tag{58}$$

Thus we have

$$|\Phi_0\rangle \approx |\hat{0}\rangle + \sum_{\alpha_1\alpha_2\alpha_3\alpha_4} \langle\phi(\alpha_1\cdots\alpha_4)|C|\hat{0}\rangle|\phi(\alpha_1\cdots\alpha_4)\rangle \tag{59}$$

In the following, we make a further approximation by setting

$$\omega \simeq 0 \ . \tag{60}$$

Then, of course, Eq. (59) becomes the first-order Rayleigh-Schrödinger expansion for the wave function. Now if we were to include the second term of (20), with the approximation of (58), this term would correspond to

interactions between the four-quasi-particle states. Such a term might be very important, but for reasons of simplicity, we shall neglect it in the computation.

In order to compute (58), we need to construct four-quasi-particle wave functions, with total angular momentum $J = 0$. It is well known in shell-model theory that for a state with more than two particles in one j-shell, the straightforward method of constructing a wave function by angular momentum coupling successively and then antisymmetrizing runs into difficulty, for then we obtain redundant states (i.e., wave functions with different intermediate J' are not necessarily orthogonal to each other), and the true state might be a linear combination of them[18]).

The group theory, however, tells us how many states of given angular momentum J are present. For example, consider four particles in the j_a shell. We know that for $j_a \leqslant 7/2$, there exists only one $J = 0$ state. Therefore, if we construct an antisymmetrized and normalized wave function as $\Psi[(j_a^2)_J(j_a^2)_{J'}; 0]$, where the first two and the remaining two are coupled to J' respectively and finally J' is coupled with J' to give $J = 0$, with antisymmetrization being ensured by use of second quantized operators, then one would expect the $J = 0$ state to be described by

$$\Psi_{J=0} = \sum_{J'} F_{J'} \Psi[(j_a^2)_{J'} (j_a^2)_{J'}; 0], \tag{61}$$

provided $j_a \leqslant 7/2$.

One can show by actual computations that $F_J = \frac{1}{\Lambda}$, where Λ = number of J' 's that are allowed by the angular-momentum coupling rule (i.e., $2j_a \geqslant J' \geqslant 0$).

Or, we have

$$\Psi_{J=0} = \frac{1}{\Lambda} \sum_{J'} \Psi[(j_a^2)_{J'}(j_a^2)_{J'};\ 0\ ] = \Psi[(j_a)^2_{J'}\ (j_a^2)_{J'};\ 0] \quad (62)$$

for any allowed J'. The most convenient one is $J' = 0$, which we may use in actual calculations. In the same way, one can construct a correct state for states with three particles in the a shell, and one particle in the b shell, etc. For $j_a \geqslant 7/2$, one could find sets of functions satisfying Eq. (62); then each different set (totality corresponding to the number of $J = 0$ states) may be orthogonalized by the Schmidt process. For example, $j_a = 11/2$ has two states with $J = 0$. In our numerical work, we take only $J' = 0$ (seniority zero) assuming the seniority four states to be negligible.

From the foregoing discussion, it is clear that we can simply write the four-quasi-particle wave function as (where $N_{abcdJ'}$ is the normalization constant)

$$\phi_{0J'}(abcd) = N_{abcdJ'} \sum_{m_a m_b m_c m_d} \sum_{M'} (-)^{M'} C(j_a j_b J'\ ;\ m_a m_b M')\ C(j_c j_d J'; m_c m_d M')$$

$$\times\ \eta_\alpha^+ \eta_\beta^+ \eta_\gamma^+ \eta_\delta^+ \ |\ 0\ \rangle \quad (63)$$

which also defines our phase convention [we have multiplied by $(-)^{J'}$]. We will simply understand that whenever more than two particles occupy one j-shell, we take precautions not to add redundant states as described above, and in the rest of this section, we do not make any distinctions.

Let us now compute the matrix element of H_I between the vacuum and a four-quasi-particle state. By use of Eq. (63) and the relations given by (9a) and (11c), it is straightforward to obtain

$$\langle \phi_{0J'}(abcd) \mid H_I \mid \hat{0} \rangle = \langle \phi_{0J'}(abcd) \mid H_{40} \mid \hat{0} \rangle$$

$$= N_{abcdJ'} \, [-\chi(abcd) \, (abJ|\bar{V}|cdJ) + \chi \, (adbc) \, (abJ|\bar{U}|cdJ)$$

$$-\theta(cdJ)\chi(acbd) \, (abJ|\bar{U}|dcJ)] \, , \tag{64}$$

where χ is as defined previously. Now by comparing Eqs. (64) and (25), we can write

$$\langle \phi_{0\,J'}(abcd) \, |H_{40}| \, \hat{0} \rangle = \langle \phi_J(ab) \mid H'_{22} \mid \phi_J(cd) \rangle \tag{65}$$

where H'_{22} may be obtained by comparing Eqs. (8) and (8a); we do not need its explicit form here; let it suffice to say that H'_{22} is in a similar form to H_{22} with an additional factor $N_{abcdJ'} \, [(1+\delta_{ab})(1+\delta_{cd})]^{\frac{1}{2}}$ to take into account the different normalization. What is important is that H'_{22} replaces the coupling of four-quasi-particle states to ground state (vacuum) by an effective two-body scattering term. Thus, the next higher order approximation to Eq. (59) would be a replacement of ϕ on the right-hand side of Eq. (65) by Φ, and the noninteracting quasi-particle energies by $\omega_{J\Lambda}$ and $\omega_{J'\Lambda'}$. This is equivalent to a "two-phonon" approximation. It should be noticed, however, that for $J' = 0$, such a procedure is dangerous, since the two-quasi-particle state with $J' = 0$ has a spurious component due

to the fluctuation of a number of particles. If we assume $J' = 2$ is the only important component for the $2+ \rightarrow 0+$ (ground-state) transition, then we can see that higher order corrections could be easily computed within two-quasi-particle configurations by repeated use of H'_{22}.

In the following, we confine ourselves to the lowest order approximation for simplicity. Of particular interest is the effect of the ground-state correlation on B(E2;a), i.e., the first $2+ \rightarrow 0+$ transition; hence, we shall discuss only this point, though it is also of importance for the ground-state energy.

The operator which connects two-quasi-particle states to four-quasi-particle states was already given in Eq. (37). Taking a matrix element of $\hat{T}$ between $\Phi_{J\Lambda}(ab)$ and $\phi_{0J'}(pqrs)$ as given by Eq. (63), we get after an easy algebra

$$\langle \Phi_{0J'}, (pqrs) | \hat{T} | \Phi_{2\Lambda} \rangle = - \frac{1}{\sqrt{5}} (2J'+1) N_{pqrsJ'} \delta(M, -\mu)\theta(M) \tag{66a}$$

$$\times \sum_{ab}'' \sum_{f_1 f_2 f_3 f_4} \Delta(f_1 f_2 f_3 f_4; pqrs) \gamma_{ab}(f_1 f_2) \xi_{f_3 f_4} \langle f_3 \| T_2 \| f_4 \rangle \beta(f_1 f_2 f_3 f_4)$$

where, for compactness, we have used the following relations:

$$\Delta(f_1 f_2 f_3 f_4; pqrs) \equiv \delta_{f_1 p}\delta_{f_2 q}\delta_{f_3 r}\delta_{f_4 s} + \delta_{f_1 r}\delta_{f_2 s}\delta_{f_3 p}\delta_{f_4 q} + \delta_{f_1 p}\delta_{f_2 r}\delta_{f_3 q}\delta_{f_4 s} \tag{66b}$$

$$+ \delta_{f_1 q}\delta_{f_2 s}\delta_{f_3 p}\delta_{f_4 r} + \delta_{f_1 q}\delta_{f_2 r}\delta_{f_3 p}\delta_{f_4 s} + \delta_{f_1 p}\delta_{f_2 s}\delta_{f_3 q}\delta_{f_4 r},$$

$$\gamma_{ab}(f_1f_2) \equiv [C_{2\Lambda}(f_1f_2)\ \delta_{bf_1}\delta_{af_2} - \theta(f_1f_2)\ C_{2\Lambda}(f_2f_1)\ \delta_{af_1}\delta_{bf_2}](1+\delta_{f_1f_2})^{-\frac{1}{2}} \tag{66c}$$

$\mathcal{B}$ has the properties:

$$\mathcal{B}\,(pqrs) = \mathcal{B}\,(rspq) = (\delta_{J'2})/2 \tag{66d}$$

$$\mathcal{B}\,(prqs) = \mathcal{B}\,(qspr) = -\theta(ps)\ W\ (pqrs;J'2), \tag{66e}$$

$$\mathcal{B}\,(qrps) = \mathcal{B}\,(psqr) = -\theta(ps)\ W\ (pqsr;J'2); \tag{66f}$$

and ξ was defined in Eq. (40).

Notice that the two-quasi-particle coefficients should be taken in accordance with the representation we have chosen for diagonalization of the H_{22} matrix. Thus, if we took (f_1f_2) for $f_1 \neq f_2$, then $C(f_2f_1) = 0$.

Now suppose we take the "two-phonon" approximation (that is, let a pair of fermion operators obey Bose statistics); then the matrix element (66a) involves an evaluation of the following vacuum expectation value: (let $A_{\beta_1\beta_2}$ come from the operator T)

$$\langle A_{\alpha_1\alpha_2} A_{\beta_1\beta_2} A^+_{\gamma_1\gamma_2} A^+_{\delta_1\delta_2}\rangle_0 \propto \langle A_{\beta_1\beta_2} A^+_{\delta_1\delta_2}\rangle_0\ \delta_{\alpha_1\gamma_1}\delta_{\alpha_2\gamma_2} + \langle A_{\beta_1\beta_2} A^+_{\gamma_1\gamma_2}\rangle_0 \delta_{\alpha_1\delta_1}\delta_{\alpha_2\delta_2} \tag{67}$$

In this case, we get a simplified equation from (66a) by setting Eqs. (66c) and (66f) to zero. Then only $J' = 2$ contributes to the transition probability. We can then easily discuss what the second-order contribution looks like[19]).

We have

$$\sum_{pqrs} \langle\phi_{0J'}(pqrs)|C|\hat{0}\rangle \langle\phi_{0J'}(pqrs)|\hat{T}|\Phi_{2\Lambda}\rangle$$

$$\equiv \Delta M = -\frac{1}{\sqrt{5}}\,\delta(M,-\mu)\theta(M)\sum_{pqrs}[(1+\delta_{pq})(1+\delta_{rs})]^{-1/2}\langle\phi_{0J'}(pqrs)|C|\hat{0}\rangle$$

$$\times \sum_{ab}{}'' \sum_{f_1f_2f_3f_4} \Delta'(f_1f_2f_3f_4;\ pqrs)\ \gamma_{ab}(f_1f_2)\langle f_3\|T\|f_4\rangle\ \delta_{J'2}\ , \tag{68}$$

where Δ' contains only the first two terms of Eq. (66b). A similar argument to that in the previous section holds, and hence, as long as $\langle\phi_{0J'}(pqrs)|C|\hat{0}\rangle > 0$, the above quantity contributes coherently. In turn, this inequality always holds, provided we drop the $(abJ|\bar{V}|cdJ)$ term in Eq. (64). We have already asserted that this could be done for the Q-Q force, for which Eq. (68) reduces to

$$\Delta M = -\frac{2}{\sqrt{5}}\ \lambda e\ \theta(M)\delta\ (M,-\mu)\left\{\sum_{p_1p_2}{}''\frac{B_2^2\ (p_1p_2)}{E_{p_1}+E_{p_2}-\omega_{21}}\right\}^{-\frac{1}{2}}$$

$$\times \sum_{s_1s_2s_3s_4}{}''(1+\delta_{s_1s_2})^{-1}\left(\frac{B_2^2\ (s_1s_2)}{E_{s_1}+E_{s_2}-\omega_{21}}\right)\times\left(\frac{B_2^2\ (s_3s_4)}{E_{s_1}+E_{s_2}+E_{s_3}+E_{s_4}}\right) \tag{69}$$

which clearly has the same sign but is much smaller than Eq. (44). In the actual numerical computation, we do not make the two-phonon approximation and hence no such conclusive remark could be made. But except for pure four-quasi-particle configurations, where the $\overline{V}$ matrix element is usually dominant, other configurations seem to enhance the matrix element. Of course a pure four-quasi-particle state lies lowest and hence the cancellation effect causes the contribution to be in general rather small. This effect can be seen qualitatively from Eqs. (66a) and (64).

6. Applications

In this section, we apply the formulas discussed in the preceding sections to the practical computations of some single-closed-shell nuclei. Calculations of nuclear spectra with a general force have been done by Arvieu et al. for Sn and Pb regions[10], but the 82-neutron nuclei do not seem to have been investigated except by Kisslinger and Sorensen (KS)[9], who tried their schematic model. We study this region. Our treatment goes beyond the KS study in three aspects:

(a) In pairing-correlation calculations, we do not take a constant matrix element; instead we introduce general pairing-type matrix elements with a finite range.

(b) All components of force are taken into account, whereas KS uses quadrupole-quadrupole force.

(c) The pairing and quasi-particle interaction strengths are not two independent variables. We assume that either the same force is operative in both cases or one is related to the other, the difference being due to neglect of forces which act in a different way for different J (for example, tensor force).

In the numerical calculations, we make the following approximations:

(a) The neutron closed shell is completely inert, and hence the behavior of the neutron is completely neglected. The proton closed shell (50) is also inert, and hence only the protons outside the 50-proton-closed shell are taken into account.

(b) We neglect the Coulomb, tensor, and spin-orbit forces, and thus the force consists of the simple central force.

In the following subsections, we describe the experimental and theoretical aspects, give plausible justifications for taking a particular set of parameters, and suggest possible improvements. Rather than a quantitative agreement, a precise interpretation of the solutions of the equations considered in the previous sections is emphasized.

The outline of calculations is as follows:

(a) We solve the BCS equations (see Sec. 2) with a general force and a set of single-particle energies (defined by ϵ in Sec. 3). This then gives the BCS wave functions (i.e., u, v, Δ, etc.), and also independent quasiparticle energies.

(b) Next, we diagonalize the H_{22} matrix as given by Eqs. (27) and (25) with the wave functions defined by Eq. (24). The eigenvalues then give the excitation spectrum for various J. (We do not give the results for J=0, since they are mainly spurious.)

(c) We use the wave functions obtained in step (b) to calculate single-particle matrix elements (see Sec. 4 for the relevant equations) and compare the results with experimental B(E2) values and magnetic moments, etc.

(d) Then we take into account the ground-state correlations by first order; in so doing, we choose only a few configurations to see how large they contribute to the B(E2) for $2+ \rightarrow 0+$ (ground) transition.

6.1. SINGLE-PARTICLE LEVELS (ϵ)

In the absence of better knowledge, we choose single-particle levels (ϵ_α) that are essentially the same as the KS values. Only a small variation was made in our calculation so that an approximate fit could be made with the known levels in odd-A nuclei in this region. We take $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $1h_{11/2}$, $1h_{9/2}$, and $3s_{1/2}$ with ϵ_α's as given in Table 2. Set I of

Table 2 corresponds to that used by Tamura and Udagawa[20]) with the spacing between $g_{7/2}$ and $d_{5/2}$ exactly the same as the KS value of 1 MeV; Set III has an increased level spacing between $d_{5/2}$ and $h_{11/2}$ and a lower spacing between $d_{5/2}$ and $g_{7/2}$. This choice turns out to give better quasi-particle energy spacing between the ground state and the first excited state in La^{139} and Pr^{141}. Set II is changed slightly from Set III. The separation of 1 MeV between $d_{5/2}$ and $g_{7/2}$ seems to be better than anything smaller, on the ground of theoretical level density in the multiplets arising from these two single-particle states.

6.2. RANGE OF FORCE

We take the interaction between nucleons to be of a Gaussian form, and thus the radial dependence is given by

$$f(r) = e^{-\beta r^2},$$

where $\beta^{-1/2}$ in Fermis corresponds to the range of the interaction. The values we have used are $\beta^{-1/2} = 1.755$ for Sets II and III and 1.732 for Set I. These values have been extensively used in shell-model calculations.

In the interaction matrix element, the radial part is dependent on the product $B = \frac{\nu}{\beta}$, where ν is defined below. For the radial wave function, we choose the harmonic-oscillator wave function. It is believed that the harmonic-oscillator wave function is a fairly good approximation for finite nuclei except perhaps for heavy nuclei.

Table 2. (a) Unperturbed single-particle energies (ϵ_α) used for computations of quasi-particle energies and wave functions; (b) Force constants, and ranges used with (a).

		Set I	Set II	Set III
		ϵ_α	ϵ_α	ϵ_α
a	$g_{7/2}$	0	0	0
	$d_{5/2}$	1.00	1.00	0.75
	$d_{3/2}$	2.88	2.90	2.90
	$s_{1/2}$	3.20	3.40	3.40
	$h_{11/2}$	2.18	2.60	2.60
	$h_{9/2}$	∞	5.40	5.40
b	V_o	-32.9XMeV	-32.4y MeV	-32.4y MeV
	$\beta^{-1/2}$	1.732F	1.755F	1.755F

X = 1.0, 1.15, 1.20

y = 1.0, 1.05, 1.1

The radial wave function has the form

$$R_{n\ell}(r) = N_{n\ell} \; e^{-\frac{\nu}{2} r^2} \; r^{\ell} \; L^{\ell+1/2}_{n+\ell+1/2} \; (\nu r^2),$$

where L is the associated Laguerre polynomial, and ν, which is needed for the computations, is determined from the harmonic-oscillator spacing given by

$$\hbar\omega = \frac{\hbar^2}{m} \nu \approx 41 \; A^{-1/3} \text{ MeV}.$$

6.3. FORCE STRENGTH

In the absence of n-p interactions, only singlet-even and triplet-odd parts of the central force contribute. Thus, our force is

$$V = (V_{SE}P_{SE} + V_{TO}P_{TO}) \; f(r_{12}) = V_{SE}(P_{SE} + tP_{TO}) \; f(r_{12}),$$

where V_{SE} and V_{TO} are singlet-even and triplet-odd force strength respectively, P's are projection operators and t is triplet-to-singlet force ratio. In principle, one could use a force more general than above. A sophisticated shell-model calculation would involve a force more close to a free nucleon-nucleon force[21].

We take the depth of the potential close to $V_0 = -33$ for the range ≈ 1.73F. We have determined V_0 by computing odd-even mass differences in BCS approximations. Theoretical mass difference is given approximately by $P \approx 2 \; E_{min}$, where E_{min} is the lowest odd-A quasi-particle energy. One could improve the estimate by using a four-point formula given by[23],

Table 3. Gap solutions (v_α, E_α, λ, Δ_α) calculated by using Set I of Table 2, $V_0 = -1.15 \times 32.9$ MeV, $\beta^{-1/2} = 1.732$F, $\frac{v_{TO}}{v_{SE}} = 0$.

A	λ		$g_{7/2}$	$d_{5/2}$	$d_{3/2}$	$s_{1/2}$	$h_{11/2}$
		Δ_α	0.706	0.587	0.586	0.490	0.600
134	-0.522	v_α	0.450	0.183	0.085	0.065	0.109
		E_α	0.878	1.631	3.452	3.755	2.768
		Δ_α	0.926	0.826	0.825	0.692	0.794
136	-0.216	v_α	0.622	0.294	0.130	0.100	0.159
		E_α	0.950	1.470	3.204	3.486	2.524
		Δ_α	1.047	1.006	1.005	0.846	0.908
138	0.094	v_α	0.738	0.407	0.172	0.133	0.204
		E_α	1.051	1.354	2.962	3.220	2.275
		Δ_α	1.116	1.145	1.144	0.972	0.980
140	0.411	v_α	0.820	0.521	0.215	0.167	0.250
		E_α	1.189	1.288	2.721	2.954	2.022
		Δ_α	1.161	1.248	1.246	1.075	1.031
142	0.738	v_α	0.876	0.630	0.260	0.204	0.305
		E_α	1.375	1.275	2.479	2.687	1.773
		Δ_α	1.204	1.321	1.319	1.164	1.079
144	1.067	v_α	0.912	0.725	0.309	0.247	0.376
		E_α	1.609	1.323	2.242	2.430	1.550
		Δ_α	1.251	1.372	1.370	1.242	1.129
146	1.381	v_α	0.933	0.796	0.362	0.295	0.460
		E_α	1.863	1.424	2.030	2.203	1.383

6.5. TWO-QUASI-PARTICLE SPECTRA

The zero-order wave functions are those given by the BCS prodecures; proper angular-momentum coupling and anitsymmetrization by means of second quantization are as given before. Equation (27) is then solved by the IBM 7094. We include all the two-quasi-particle configurations except those involving $s_{1/2}$ and $h_{9/2}$; i.e., $(g^2_{7/2})$, $(g_{7/2}d_{5/2})$, $(d^2_{5/2})$, $(d^2_{3/2})$, $(h^2_{1/2})$, $(g_{7/2}d_{3/2})$, $(d_{5/2}d_{3/2})$. We believe that the neglect of $s_{1/2}$ and $h_{9/2}$ does not cause a serious error in the results owing to the fact that they are rather high lying.

To determine the force parameters, we have varied t for a fixed V_{SE} which was determined in the gap solutions. We have also attempted to vary V_{SE} and V_{TO} at the same time in both the gap equations and the quasi-particle interactions, but it was found to be difficult to fit simultaneously odd-even mass difference and energy spacings in the two-quasi-particle spectra. At present, we take V_{SE} as given by odd-even mass differences. Then the result of such variation in Ce^{140} is given in Fig. 6. This graph shows the values of V_{SE} and t at which 2+ and 4+ in Ce^{140} are fitted, with u and v values held fixed for one particular V_{SE}. At about $t \approx 0.3$ and $V_{SE} \approx -1.6 \times 32.9$ MeV the agreement is the best, and then for $t > 0.8$ the second-best agreement is obtained. However, as we shall discuss later, the B(E2) values and magnetic g factor seem to rule out the first one. Even though the whole procedure is artificial (more experimental studies will help), the determination of t at the same value of V_{SE} as used in gap solutions is meaningful. As is well known, a repulsive odd-force is necessary for low-energy nuclear properties, and the usual shell-model calculations also include such a component. Here we shall try to give a plausible explanation why t in the quasi-particle interaction turns out to be larger than that in the pairing interaction in our calculations.

(a) The study by Kim[21] shows that inclusion of a tensor force [in our case, the tensor-triplet-odd (TTO) part]affects mostly the states with $J = 0$, whereas it modifies very little the states with $J > 0$ in a system of one kind of particles.

(b) The inclusion of TTO in the analysis of free-nucleon-scattering phase shifts with the Brueckner-Gammel-Thaler (BGT) force turns out to be equivalent to taking $t > 0$, say $t \approx -0.4$, although this is strictly true for only $J = 0$ (for $J > 0$, this may not hold)[29].

(c) Since, as we have shown before, only $J = 0$ states are involved in pairing interaction, we should perhaps compute the gap solutions with $t \approx -0.4$. This was not a proper choice of t because of the discrepancy on odd-even mass differences. Let us suppose that for some reason (say, a renormalization of force strength), we have $t \approx 0$ in the pairing interaction.

(d) We are primarily interested in the states with $J > 0$ arising from the quasi-particle interactions. Then, according to the statement (a), we may neglect TTO.

Now, if we assume that the renormalization acts in the same way (though perhaps not exactly in the same amount) in the pairing and quasi-particle interactions, then the statements (a) through (d) imply that it is reasonable to use a larger t for calculations of two-quasi-particle spectra than for the pairing interactions. With this much justification, we shall from now on take $t \approx 1$ for all quasi-particle interactions, and the results are given in Figs. 7, 8, and 9 and some wave functions in Table 4.

The 2+ states are of special interest for microscopic or macroscopic theories because of their collective nature. One can see from Figs. 8 and 9 that both sets of ϵ_α's give "satisfactory" results if we choose t between 0.6 and 1.4. One can see in the results that the lowest 2+ in Nd^{142}

lies higher than that in Ce^{140}. This seems to disagree with the experiments. The reason why the theory fails there is as follows: though the zero-order quasi-particle description identifies the ground state in Nd^{142} to be $(d^2_{5/2})$, the interaction among them brings down $(g^2_{7/2})$ below $(d^2_{5/2})$. Since the zero-order quasi-particle energy of $(g^2_{7/2})$ in Nd^{142} is higher than that in Ce^{140}, it is expected that the 2+ in Nd^{142} is higher than that in Ce^{140}. The remedy is obvious: one should either take a smaller separation of $d_{5/2}$-$g_{7/2}$ or renormalize the force strength for Nd^{142}.

Also the reason why Sets I and III give a completely different picture at Gd^{146} is that the 2+ with Set I has the dominant component of $(h^2_{11/2})$, while Set III has that of $(d^2_{5/2})$. This is of course due to the choice of $d_{5/2}$-$h_{11/2}$ separation. In Set I, the diagonal element of $(h^2_{11/2})$ is large enough to give a depression that is observed in our result. Experimental data[30] on Eu^{147} and Eu^{149} which has 63 protons put the $h_{11/2}$ level about 500 keV above the ground state $d_{5/2}$. The first excited state is $g_{7/2}$, making us suspect that $|\epsilon h_{11/2}-\epsilon d_{5/2}|>|\epsilon d_{5/2}-\epsilon g_{7/2}|$. Though this statement is not strictly correct—for, in these nuclei, one has to take into account also the proton-neutron interaction—it seems likely that one has to choose the spacing somewhat larger than we have in Set I. This, of course, is the motivation for including Set III.

We believe that the higher-lying states such as 3+ and 4+ can be fairly well described by H_{22} alone. There is only one experimentally known 4+ level in this region (4+ in Ce^{140}), and it seems that the theoretical spacing between 2+ and 4+ in this nucleus is consistent with experiment. The experimental 3+ level, which also is observed only in Ce^{140}, is lower than the Set I result (about 150 keV) and higher than the Set III result (about 70 keV). The only low-lying 3+ state is of the configuration $(d_{5/2}g_{7/2})_{3+}$,

UCRL-11225

Table 4. Eigenvalues and eigenvectors for 2+ and 4+ states computed with $V_0 = -1.15 \times 32.9$ MeV, $\beta^{-1/2} = 1.732$F, Set I

	A	$\frac{v_{TO}}{v_{SE}}$	$\omega(\alpha)$	$d^2_{5/2}$	$g^2_{7/2}$	$d^2_{3/2}$	$h^2_{11/2}$	$d_{5/2}g_{7/2}$	$d_{5/2}d_{3/2}$	$g_{7/2}d_{3/2}$
J = 2+	136	0.8	1.308	0.183	0.973	0.036	0.017	-0.066	-0.037	0.110
			2.316	0.210	-0.110	0.012	-0.061	-0.968	-0.011	0.046
			2.432	0.949	-0.169	0.034	-0.101	0.235	0.023	0.063
		1.0	1.243	0.189	0.967	0.039	0.038	-0.097	-0.056	0.118
			2.256	0.331	-0.165	0.016	-0.101	-0.919	-0.016	0.089
			2.400	0.912	-0.142	0.022	-0.098	0.369	-0.001	0.033
	138	1.0	1.397	0.305	0.931	0.046	0.076	-0.111	-0.076	0.121
			2.287	0.914	-0.318	0.027	-0.154	-0.196	-0.018	0.038
			2.413	0.230	0.042	0.000	0.035	0.971	0.010	-0.027
	140	1.0	1.558	0.424	0.871	0.056	0.148	-0.111	-0.103	0.126
			2.307	0.889	0.072	-0.003	0.081	0.990	0.011	-0.008
			2.535	0.099	-0.234	0.009	0.956	-0.069	-0.097	0.083
	142	1.0	1.741	0.534	0.758	0.074	0.290	-0.101	-0.144	0.140
			2.384	0.808	-0.410	0.025	-0.421	-0.020	-0.020	-0.036
			2.669	0.138	-0.094	-0.000	0.309	0.936	-0.004	0.013
		1.2	1.614	0.496	0.753	0.078	0.337	-0.121	-0.175	0.148
			2.350	0.806	-0.336	0.020	-0.477	-0.068	-0.061	-0.039
			2.644	0.221	-0.177	0.001	0.377	0.881	-0.023	-0.010
J = 4+	136	0.8	1.713	0.074	0.992	--	0.000	-0.087	-0.043	0.040
			2.310	0.082	-0.096		-0.016	-0.990	-0.052	0.015
			2.645	0.990	-0.070		-0.051	0.093	-0.057	0.032
		1.0	1.687	0.090	0.982		0.006	-0.141	-0.068	0.047
			2.297	0.163	-0.161		-0.036	-0.970	-0.062	0.041
			2.595	0.970	-0.072		-0.056	0.186	-0.125	0.283
	138	1.0	1.889	0.159	0.963	--	0.017	-0.192	-0.089	0.045
			2.359	0.359	-0.246		-0.044	-0.894	-0.084	0.035
			2.478	0.908	-0.099		-0.049	0.398	-0.094	0.016
	140	1.0	2.106	0.297	0.908	--	0.034	-0.264	-0.122	0.051
			2.428	0.826	-0.393		-0.074	-0.383	-0.098	0.025
			2.501	0.459	0.107		-0.013	0.881	-0.004	-0.008
	142	1.0	2.342	0.647	0.657	--	0.061	-0.336	-0.169	0.062
			2.544	0.746	-0.620		-0.147	0.195	0.003	-0.012
			2.659	0.089	0.385		0.045	0.917	0.027	-0.008
		1.2	2.275	0.608	0.658		0.073	-0.371	-0.220	0.076
			2.530	0.755	-0.611		-0.179	0.150	-0.055	-0.012
			2.679	0.141	0.377		0.073	0.912	-0.008	-0.029

and the other configurations which might perturb the lower 3+ state are $(d_{5/2}d_{3/2})_{3+}$ and $(d_{3/2}g_{7/2})_{3+}$. But the last two states are very high in energy, and thus one would expect the lowest 3+ to be an almost pure $(d_{5/2}g_{7/2})_{3+}$ two-quasi-particle state. This is borne out by the result. The wave function turns out to be (for Set I, t = 1.0)

$$|3+\rangle = 0.99992|(d_{5/2}g_{7/2})_{3+}\rangle + 0.00421|(d_{5/2}d_{3/2})_{3+}\rangle + 0.01172|(d_{3/2}g_{7/2})_{3+}\rangle.$$

Now, since this state is almost pure, the fit with experiment may be obtained by adjusting parameters used for pairing solutions. This could perhaps be put to advantage in determining some of the parameters, but in practice we found that if we fitted theory to the experiment for the 3+ state, the spacing between 4+ and 2+ was upset.

6.6. B(E2) FOR 4+ → 2+ TRANSITIONS

It has been observed experimentally that in Ce^{140}, the electric-quadrupole transition from the 2.08-MeV 4+ state to the 1.6-MeV 2+ state is very retarded; in fact we have $\frac{B(E2)_{sp}}{B(E2)_{exp}} \approx 17$, where $B(E2)_{sp}$ is the single-particle B(E2) unit defined by

$$B(E2)_{sp} = 3 \times 10^{-5}\ A^{4/3}\ e^2\ 10^{-48}\ cm^4.$$

As we have discussed before, theoretically we expect this transition to be considerably slower. The calculation results are shown in Table 5 for the choice of parameters as given by Set I. In all nuclei, one observes, the transitions are markedly retarded. The only possible explanation is that there is a large cancellation in the transition-matrix elements, as was discussed already.

Our value of $\frac{B(E2)_{sp}}{B(E2)_{th}} = 28$ seems to be considerably larger than the experimental value 17 in Ce^{140}, but at least it shows the correct behavior of large retardation. One should note, however, that B(E2;c) (for 4+ → 2+ transition) is extremely sensitive to wave functions when a large calcellation occurs, and hence a too large value of the ratio (such as Ba^{138}) should not be taken too seriously.

Although no experimental results are available in this region, we have also computed B(E2) for 2'+ → 4+ transitions, where 2'+ means the second 2+ state. This is also shown in Table 5. The results B(E2; 2'+ → 4+) are comparable in magnitude of retardation to the B(E2; 4+ → 2+)

6.7. B(E2) FOR 2+ → 0+ TRANSITIONS

This quantity is of special interest, since experimentally the first 2+ → 0+ ground-state transition even in spherical nuclei is enhanced by more than ten times the single-particle estimate, and a reasonable theory should be able to display this characteristic. We have shown previously that in order to obtain such a large enhancement, the matrix elements have to be mostly coherent, and hence the coefficients C_{21} (ab) have to be mostly of the same sign for the choice of configuration we discussed before. Actual numerical computation shows that such a condition fails to hold for small positive values of t or negative t. Already at $t \approx 0.5$, the cancellation occurs appreciably and the theoretical value becomes too small compared with experiment; this shortcoming is believed to be remedied neither by higher-order terms, since they are in general fairly small, nor by increasing V_{SE}, since it is only an overall multiplicative factor. Thus within the region of agreement with other data, t close to 1 seems to be the most favorable value. As can be seen from the wave functions, at this value of t, the coefficients C_{21} (ab) are all of the same sign in the phase convention adopted before.

Table 5 contains the transition probabilities computed with the same parameters as used in two-quasi-particle excitation spectra (i.e., t = 1.0). Still, the computed B(E2) is not quite sufficient to fit the experimental results. One can easily guess that this insufficiency will be more marked the lower-lying the 2+ state is. This is because a very low-lying solution may not be obtained from our treatment.

To see what can be done to lessen this defect, we have introduced independent four-quasi-particle states into the ground state. The four-quasi-particle states taken into account are $(g^4_{7/2})_{\nu=0}$, $(d_{5/2}g^3_{7/2})_{\nu=3}$, $[(d^2_{5/2})_J(g^2_{7/2})_J]_0$, $(d^4_{5/2})_{\nu=0}$, $(h^4_{11/2})_{\nu=0}$, $[(h^2_{11/2})_J(g^2_{7/2})_J]_0$, $[(h^2_{11/2})_J(d^2_{3/2})_J]_0$, $[(h^2_{11/2})_J(d^2_{5/2})_J]_0$, $[(d^2_{3/2})_J(g^2_{7/2})_J]_0$, and $[(d^2_{3/2})_J(d^2_{5/2})_J]_0$. Here ν denotes the seniority quantum number and J goes over all allowed values.

The trend of the B(E2) is rather obvious; as Z is increased, the contributions from both the H_{22} term and the H_{40} term increase. If one examines the matrix element of H_{40} between the ground state and a four-quasi-particle state, one can see that the particle-particle and particle-hole interactions compete, i.e., they have opposite signs. Since in a pure configuration (i.e., $[j_a{}^4]$), we have G>>F, it is clear that such a pure configuration yields a repulsive matrix element in $\langle 4|H_{40}|0\rangle$. On the other hand, configurations of the form $[(j_a{}^2)_J\ (j_b{}^2)_J]$ have a large F terms, and hence mainly enhance the transition probability. It is thus a reasonable conclusion that a model calculation by KS amounts to taking only the second type of matrix elements, including the pure configurations in the form $[(j_a{}^2)_J(j_a{}^2)_J]$, with J = 2 (it is easy to see that these are all of a coherent sign).

Table 5. B(E2) values for $2+ \rightarrow 0+$ and $4+ \rightarrow 2+$ transitions in units of 10^{-48} cm^4 e^2. Wave functions used in the calculation correspond to those obtained with $V_0 = -32.9\chi$ MeV for $\chi = 1.15$ and 1.20, $\beta^{-\frac{1}{2}} = 1.732$ F, $\frac{V_{TO}}{V_{SE}} = 1.0$, and Set I ($e_{eff} = 2e$).

		2+ → 0+ transitions						4+ → 2+ transitions			
	χ	First 2 + → 0 +				Second 2 + → 0 +		4 + → First 2 +			Second 2+ → 4 +
		B(E2)				B(E2)					
		H_{22} [a]	$H_{22}+H_{40}$ [b]	$\frac{B(E2)cal}{B(E2)sp}$ [b]	$\frac{B(E2)exp}{B(E2)sp}$	H_{22} [a]	$\frac{B(E2)cal}{B(E2)sp}$	B(E2)	$\frac{B(E2)sp}{B(E2)cal}$	$\frac{B(E2)sp}{B(E2)cal}$	$\frac{B(E2)sp}{B(E2)cal}$
$_{52}Te^{134}$	1.15	0.101	0.101	4.9				0.993	2.0		56
	1.20	0.103	0.103	5.0				1.05	1.8		
$_{54}Xe^{136}$	1.15	0.178	0.169	8.1		0.0122	0.58	0.203	10.0		415
	1.20	0.180	0.174	8.3				0.233	9.0		
$_{56}Ba^{138}$	1.15	0.228	0.231	10.8	14	0.0103	0.48	0.29×10^{-4}	7.2×10^{4}		49
	1.20	0.237	0.240	11.2				0.29×10^{-2}	7.2×10^{2}		
$_{58}Ce^{140}$	1.15	0.271	0.284	13.1	17	0.0038	0.18	0.079	28	17	14
	1.20	0.283	0.300	13.9				0.052	42		
$_{60}Nd^{142}$	1.15	0.331	0.358	16.1	15	3.02×10	0.014	0.084	26		20
	1.20	0.353	0.385	17.3				0.077	29		
$_{62}Sm^{144}$	1.15	0.417	0.458	20.2		0.0007	0.025	0.16×10^{-3}	1.5×10^{4}		17
	1.20	0.436	0.481	21.1				0.026	86		
$_{64}Gd^{146}$	1.15	0.431	0.469	20.3		0.0365	1.58	1.003	2.3		6.6
	1.20	0.453	0.495	21.5				0.879	2.6		

a: the effect of the H_{22} term on the B(E2 ; 2 + → 0 +)

b: the effect of the H_{22} + H_{40} on the B(E2 ; 2 + → 0 +)

Now, in the nuclei of lower Z, the only energetically favored states are the first four configurations listed above. Among these, $[g_{7/2}{}^4]_0$ with seniority zero gives the largest contribution to the transition-matrix element, but with an opposite sign to the H_{22} contributions. Thus the four-quasi-particle states either lower the B(E2) or make no significant difference.

However, as more particles are added, all the states become gradually more important, and the net effect is to increase the B(E2) values, since there are more configurations of the form $[(j_a{}^2)_J(j_b{}^2)_J]$ than pure ones.

Comparison with experimental results (though not abundant enough to be conclusive) shows that the theoretical trend fails to be consistent with the experimental one. Since the four-quasi-particle effect is fairly small in general, the main theoretical trend is dictated by the two-quasi-particle interactions (through H_{22}). We have already observed a similar discrepancy between theory and experiment in the two-quasi-particle spectra, and as pointed out in Sec. 6.5, a renormalization of the force constant seems to be present in both the energy spectra and the electric-quadrupole transitions. Though not tabulated here, a calculation with a higher t shows the lowering of the B(E2) for the nuclei with $Z > 58$. The monotonic increase of B(E2) was also observed by KS[9]; their B(E2) results are, however, larger than the experimental results by almost a factor of two, whereas ours are lower by about 19% (in Ce^{140}, for example). In our case, the interaction between four quasi-particles may also be significant; this point needs further study if the importance of four-quasi-particle states in the ground state is to be understood more rigorously.

As a side interest, we have also computed B(E2) for the second $2+ \rightarrow 0+$ ground-state transition. It is known experimentally that this transition is highly retarded compared with the first $2+ \rightarrow 0+$ transition. Our results

(Table 5) seem to be consistent with experimental results. Kisslinger and Sorensen computed these also by an extended RPA, and one finds that their results are frequently lower by an order of magnitude that the experimental values[31]. We have not much to compare with theirs except Ce^{140}. They find $B(E2)_{2'+ \rightarrow 0+}$ in Ce^{140} to be ≈ 0.0001, while our results give ≈ 0.004.

6.8. MAGNETIC g FACTOR OF 4+ STATE

There seem to be no experimental data for magnetic g factors of excited states in this region, except for the 4+ level in Ce^{140}, which was recently measured. However, we believe it is worthwhile to calculate this quantity also as an additional test of the wave functions of 4+ states. The results are shown in Fig. 10. In computing the g factor of the 4+ state, the values of g_j for $d_{5/2}$ and $g_{7/2}$ are taken from neighboring odd-A nuclei (i.e., g factors are 0.79, 1.76, and 1.17 for $(g^2_{7/2})$, $(d^2_{5/2})$ and $(d_{5/2}g_{7/2})$ respectively[32]) and those for $d_{3/2}$ and $h_{11/2}$ are estimated with

$$g_\ell = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \text{ and } g_s = \begin{Bmatrix} 5.585 \\ -3.826 \end{Bmatrix}$$

where the upper entry corresponds to the proton and the lower to the neutron. Since the configurations involving $d_{3/2}$ and $h_{11/2}$ have very small amplitudes in Ce^{140}, the contributions from them are negligible. Thus the g factor can also tell us something about the degree of mixing among the configurations $|g^2_{7/2}\rangle$, $|d_{5/2}g_{7/2}\rangle$ and $|d^2_{5/2}\rangle$. Our result, g = 0.92 for Ce^{140} is somewhat lower than the experimental value of 1.11 measured by Bodenstedt et al.[33]. According to this, it seems that the 4+ level is either the $(d_{5/2}g_{7/2})$ state, or a mixed configuration with the dominant configuration being the $(d_{5/2}g_{7/2})$ state. Our calculation shows, on the other hand, that $(g^2_{7/2})$ is more likely

to be the dominant state with a mixture of $(d^2_{5/2})$ and $(d_{5/2}g_{7/2})$ (i.e., $|4+\rangle \cong 0.30|d^2_{5/2}\rangle + 0.91|g^2_{7/2}\rangle - 0.26|d_{5/2}g_{7/2}\rangle$).

It is interesting to note that, as Fig. 11 shows, if one takes the wave functions for $t < 0$ as one does in the conventional shell-model calculations, then agreement gets better; in fact at $t \approx -0.5$, a fair agreement, within experimental error, can be found if contributions from all the components of the wave function are taken into account. But the value $t \approx -0.5$ is ruled out, since energy spectra cannot be fitted by such t and also theoretical B(E2) values fail to fit the experiments. Notice that $t \approx 0.3$ seems to give the lowest g factor in disagreement with experiment.

ACKNOWLEDGEMENTS

The author is grateful to Prof. John O. Rasmussen for guidance in completing this paper, and to Dr. Y. E. Kim for helpful discussions on the algebra. This work was done under the auspices of the U. S. Atomic Energy Commission.

FOOTNOTES AND REFERENCES

1. J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108 (1959) 1175.

2. N.N. Bogoliubov, Nuovo Cimento 7 (1958) 794; J. G. Valatin, Nuovo Cimento 7 (1958) 843; N. N. Bogoliubov, V. V. Tolmachev, and D. V. Shirkov, A new method in the theory of superconductivity (Consultants Bureau, Inc., New York, 1959).

3. S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd 31, No. 11 (1959).

4. A. Bohr, B. Mottelson, and D. Pines, Phys. Rev. 110 (1958) 936.

5. M. Baranger, Phys. Rev. 120 (1960) 957; T. Marumori, Progr. Theoret. Phys. (Kyoto) 24 (1960) 331; R. Arvieu and M. Vénéroni, Compt. rend. 250 (1960) 992. These papers appeared almost concurrently with similar contents. For convenience, we shall refer only to Baranger.

6. See, for an extensive discussion on this point, D. B. DuBois, Ann. Phys. 7 (1959) 174.

7. M. Rho and J. O. Rasmussen, to be published in Nuclear Physics.

8. J. P. Elliott, Proc. Roy. Soc. (London) A245 (1958) 562.

9. L. S. Kisslinger and R. A.Sorensen, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 32, No. 9 (1960), hereafter referred to by KS.

10. Robert Arvieu, Description des noyaux sphériques dans un modèle de quasi-particles en interaction. Application au isotopes de l'etan (Thesis), University of Paris, 1962; R. Arvieu, E. Baranger, M. Vénéroni, M. Baranger, and V. Gillet, Phys. Letters 4 (1963) 119; R. Arvieu and M. Vénéroni, Phys. Letters 5 (1963) 142.

11. This coupled system of equations is equivalent to that of R. K. Nesbet, Phys. Rev. 109 (1958) 1632.

12. A. R. Edmonds, Angular momentum in quantum mechanics (Princeton University Press, Princeton, 1957).

13. For notational convenience, we shall use in Sec. IV the definition $G(abcdJ) \equiv (abJ|\bar{V}|cdJ)$, and $F(abcdJ) \equiv (abJ|\bar{U}|cdJ)$ which differ from Baranger (reference 5) in the factor of -2 (which should be multiplied into Baranger's formulas to get ours).

14. See, for example, G. E. Brown, J. A. Evans, and D. J. Thouless, Nucl. Phys. 24 (1961) 1, and references given there.

15. P. M. Morse and H. Feshbach, Methods of theoretical physics (McGraw-Hill Book Company, Inc., New York, 1953).

16. W. M. Currie, Nucl. Phys. 32 (1962) 574.

17. Vincent Gillet, Théorie des spectres de noyaux à couches complètes (Thesis), University of Paris, 1962.

18. A. de-Shalit and I. Talmi, Nuclear shell theory (Academic Press, New York and London, 1963).

19. In this case, it is a good approximation to take $N_{abcdJ'} = [(2J'+1)(1 + \delta_{cd})(1 + \delta_{ab})]^{-1/2}$.

20. T. Tamura and T. Udagawa, Progr. Theoret. Phys. (Kyoto) 26 (1961) 947.

21. Yeong E. Kim, I: Tensor-Force Effects of Isomeric States in Odd-Odd Yttrium Nuclei (Thesis), Lawrence Radiation Laboratory Report UCRL-10865 (June 1963); Y.E. Kim and J. O. Rasmussen, Nucl. Phys. 47 (1963) 184.

22. L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. 31 (1962) 18.

23. Some authors prefer to use a three-point formula given by $P(Z,N) = 2E(Z,N)-E(Z-1,N)-E(Z+1,N)$. However, as pointed out by Nilsson and Prior (reference 24), the four-point formula seems to have less fluctuation. One should note that the differences between the two formulas are not negligible.

24. S. G. Nilsson and O. Prior, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 32, No. 16 (1960).

25. This appears to be contridictory to what we obtain in the calculation of two-quasi-particle spectra where $t \approx 1$ seems to be favored. Plausible argument is given in Subsection 6.5.

26. R. Macfarlane, J. O. Rasmussen, and M. Rho, Theoretical Calculations of Properties of Rare Earth Alpha Emitters, submitted to Phys. Rev.

27. V. G. Soloviev, Kgl. Danske Videnskab. Selskav, Mat.-Fys. Medd. 33, No. 11 (1961); also S. Wahlborn, Nucl. Phys. 37 (1962) 554.

28. Hans J. Mang, K. Dietrich, and J. Pradal, Conservation of Particle Number in the Nuclear Pairing Model, Lawrence Radiation Laboratory Report UCRL-11083 (November 1963), submitted to Phys. Rev.

29. J. O. Rasmussen, (Lawrence Radiation Laboratory), private communication.

30. Landolt-Börnstein, Vol. I; Energy Levels of Light Nuclei: A = 5 to A = 257 (Springer-Verlag, Berlin, Göttingen and Heidelberg, 1961).

31. L. S. Kisslinger and R. A. Sorensen, Office of Naval Research Report No. 11 (Carnegie Institute of Technology, Pittsburgh, 1963).

32. D. A. Shirley, (Lawrence Radiation Laboratory), private communication.

33. The best value of Ce^{140} $g_{J=4}$ factor is obtained by H. J. Körner, E. Gerdau, C. Gunther, K. Auerbach, G. Mielken, and E. Bodenstedt, Z. Physik 173 (1963) 203; also see R. Levy and D. A. Shirley, Phys. Letters 3 (1963) 256; and N. Kaplan, S. Ofer, and B. Rosner, Phys. Letters 3 (1963) 291.

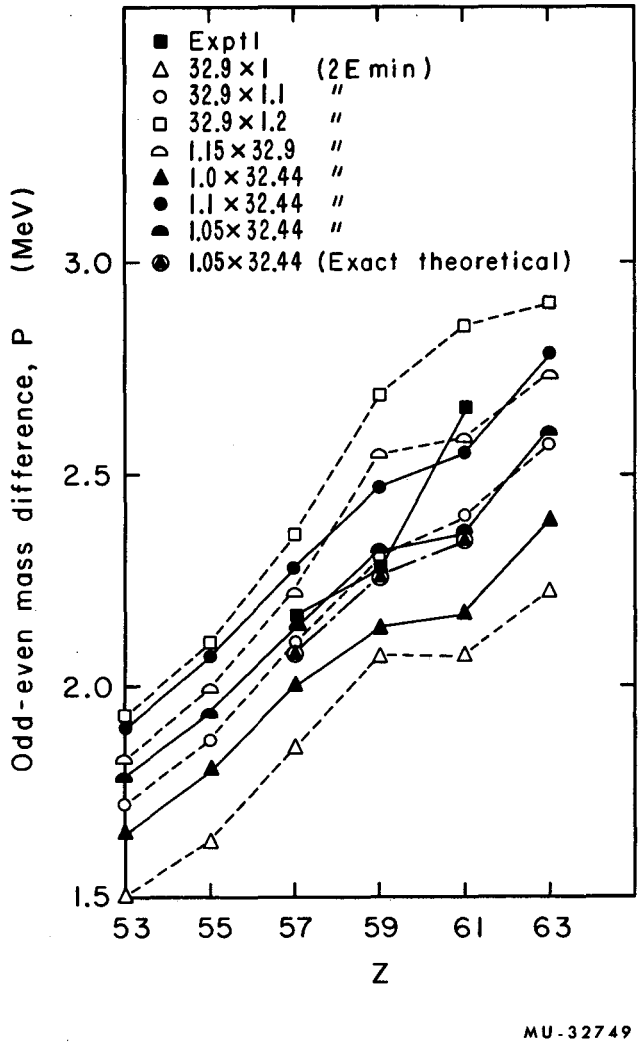


Fig. 1. Odd-even mass differences in 82-neutron nuclei. Theoretical values are obtained by $2E_{min}$.

- - - Set I;

——— Set III;

■—■ experimental points (estimated from the four-point equation with total binding energy);

▲ theoretical values obtained by using the complete formula of BCS ground-state energy, including self-energy term (y=1.05, Set III). Notice that the approximation $2E_{min}$ is in excellent agreement with the exact theoretical value.

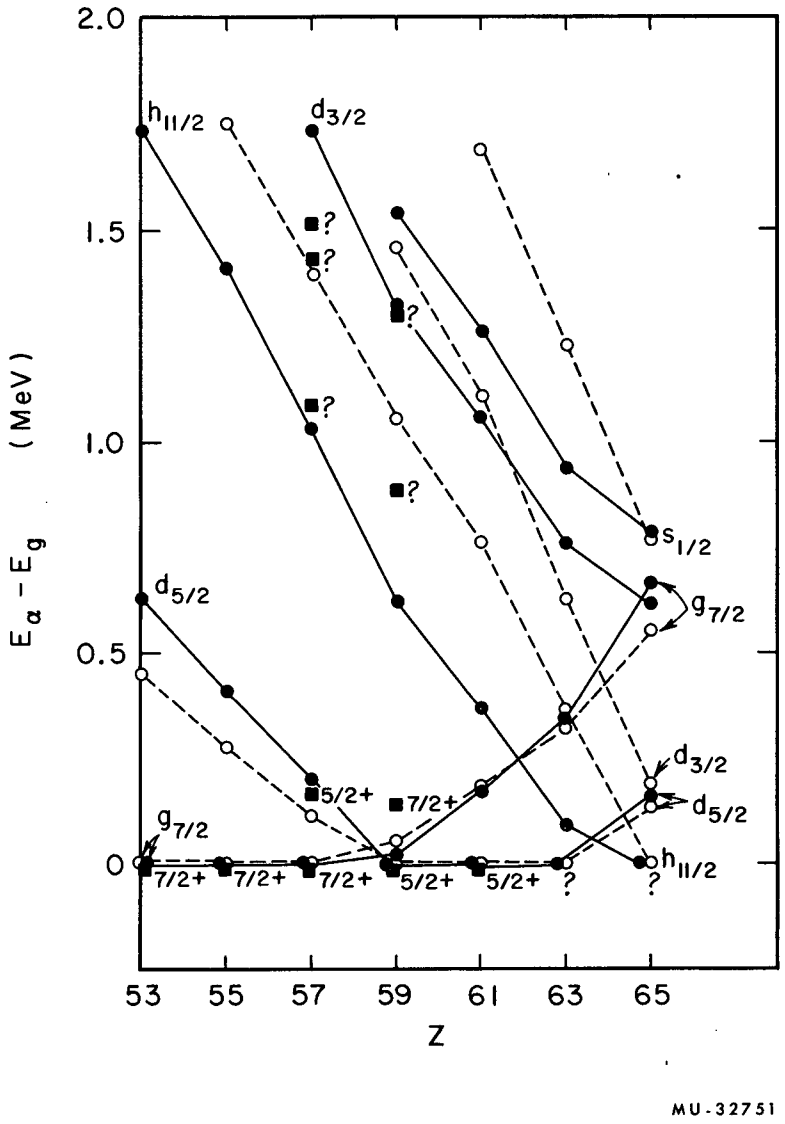


Fig. 2. Theoretical single-quasi-particle energies of 82-neutron odd-A nuclei, given in relative scale (relative to the lowest level).

● ———— Set I (V_0= -1.15 × 32.9 MeV)

○ - - - - Set III (V_0= -1.05 × 32.44 MeV)

■ experiment

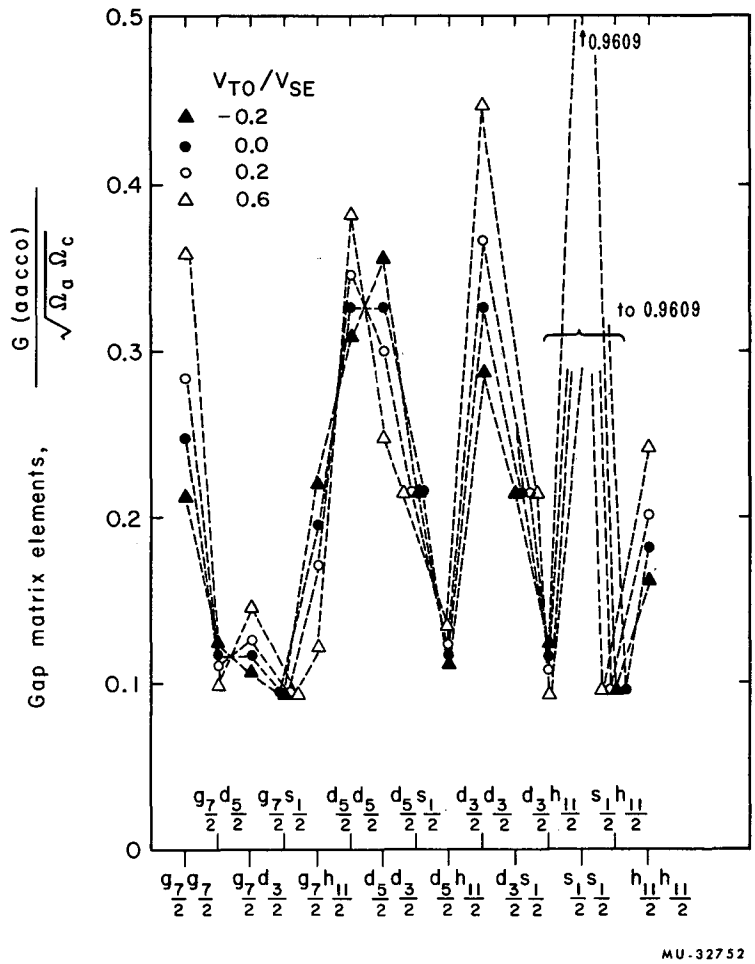


Fig. 3. Gap matrix elements for various triplet-to-singlet force ratios, computed with $V_0 = -32.44$ MeV, $\beta^{-1/2} = 1.755$ F, $\nu^{-1/2} = 2.288$ F.

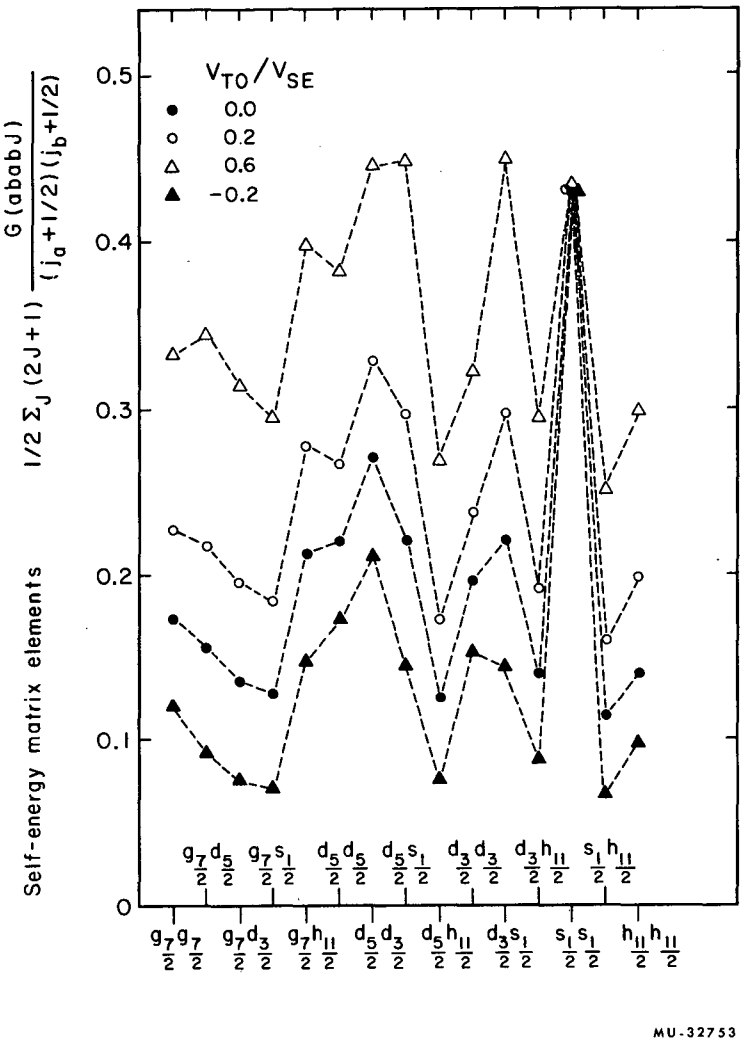


Fig. 4. Self-energy matrix elements for various triplet-to-singlet force ratios, computed with V_0 = -32.44 MeV, $\beta^{-1/2}$ = 1.755 F, $\nu^{-1/2}$ = 2.288 F.

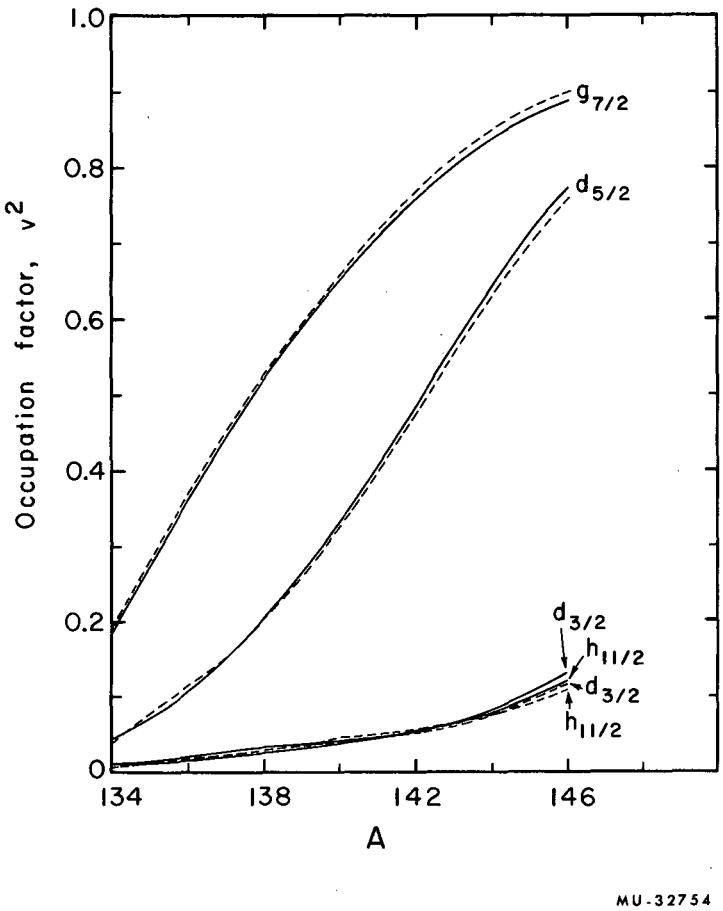


Fig. 5. Effect of self-energy terms on the occupation factor v^2 in 82-neutron nuclei, computed with y = 1.05 in Set III. --- v^2 with self-energy terms, ——— v^2 without self-energy terms.

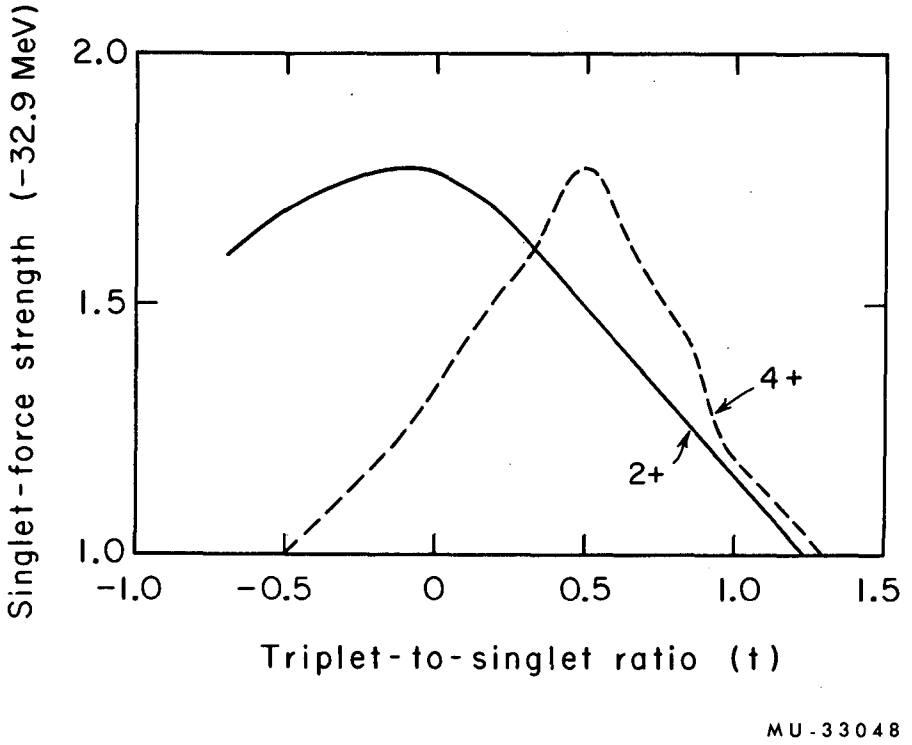


Fig. 6. "Equi-energy" diagram for Ce^{140}. Drawn are f(x,t) = constant where x = singlet-force strength, and t = triplet-to-singlet ratio; the figure gives the motion of x and t values which give the correct location of 2+ (1.60 MeV) and 4+ (2.08 MeV) levels. Thus, the closest point of approach of the two lines gives the correct 2+ - to - 4+ level spacing.

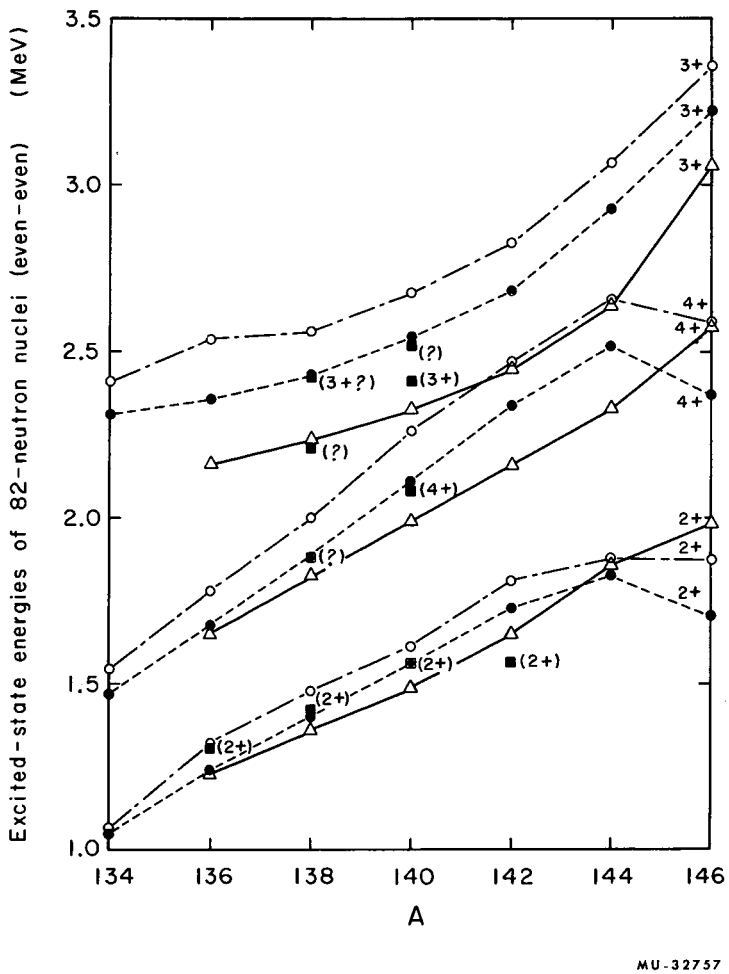


Fig. 7. Theoretical two-quasi-particle spectra for 82-neutron even-A nuclei computed with all seven configurations, t = 1.0. Experimental points are given in solid squares, with known spins in parenthesis. If spins are unknown, or tentatively assigned, question marks are put in the parenthesis.

● ---- -1.15 × 32.9 (Set I) O---- -1.20 × 32.9 (Set I)

△ ---- -1.05 × 32.4 (Set III)

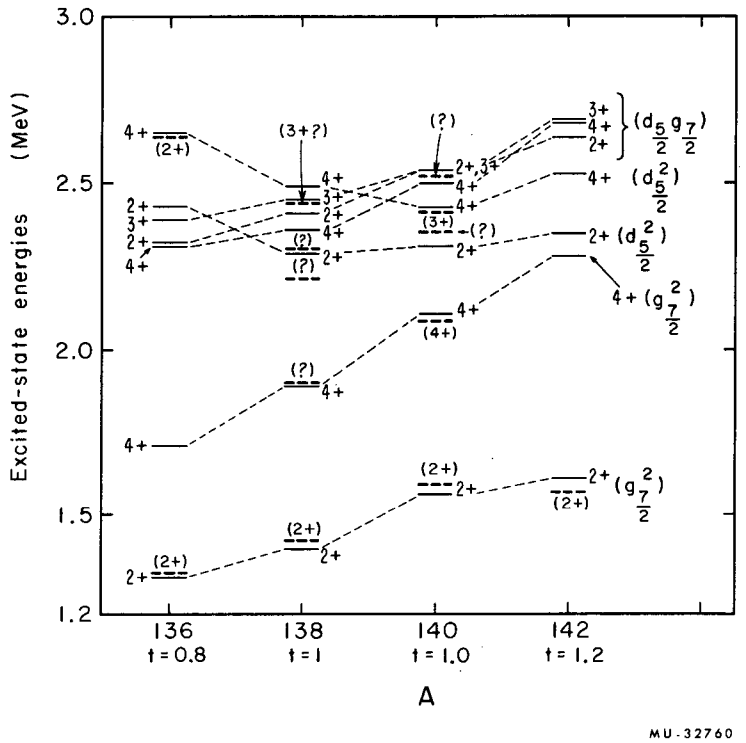


Fig. 8. Theoretical 2+, 3+, and 4+ states in A = 136 to A = 142 with slight changes in t. Differences in t show the renormalization of force as a function of number of particles outside the core.

x = 1.15 with Set I.

——— J = theoretical;

- - - (J) = experimental;

— — — (?) = experimental with spins unidentified or tentative.

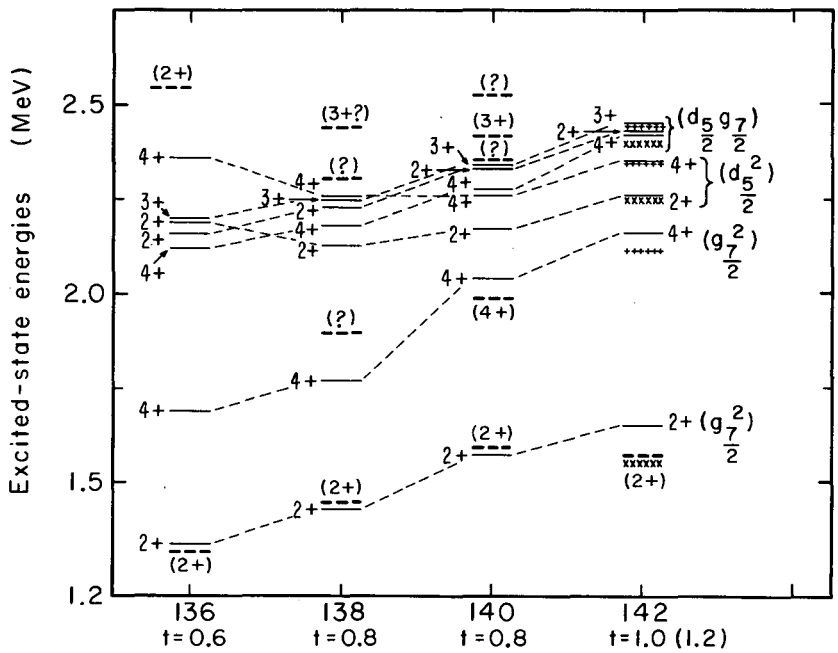


Fig. 9. Theoretical 2+, 3+, and 4+ states in A = 136 to A = 142 with slight changes in t. Differences in t show the renormalization of force as a function of number of particles outside the core.

y = 1.05 with Set III.

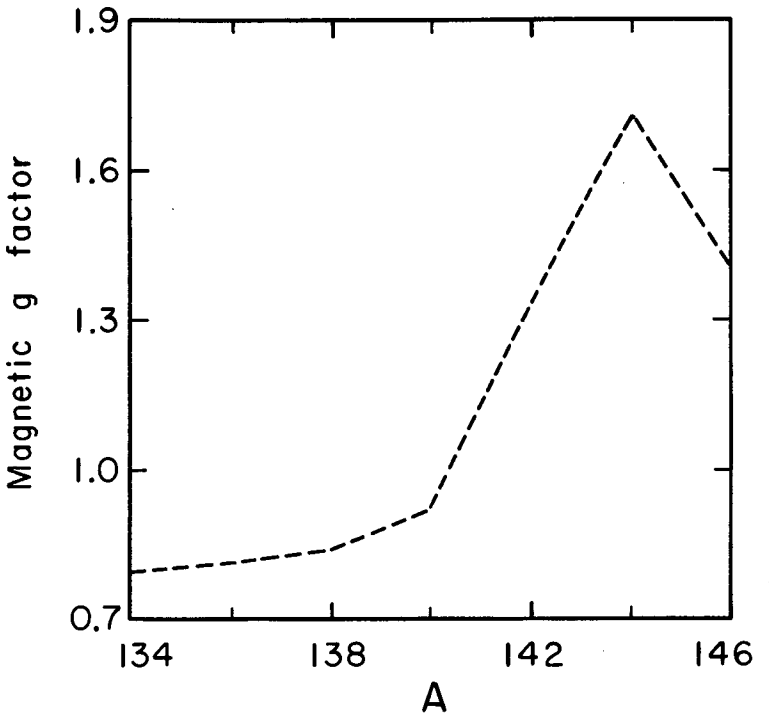
——— J = theoretical;

- - - (J) = experimental;

xxxx theoretical 2+ for t in parenthesis;

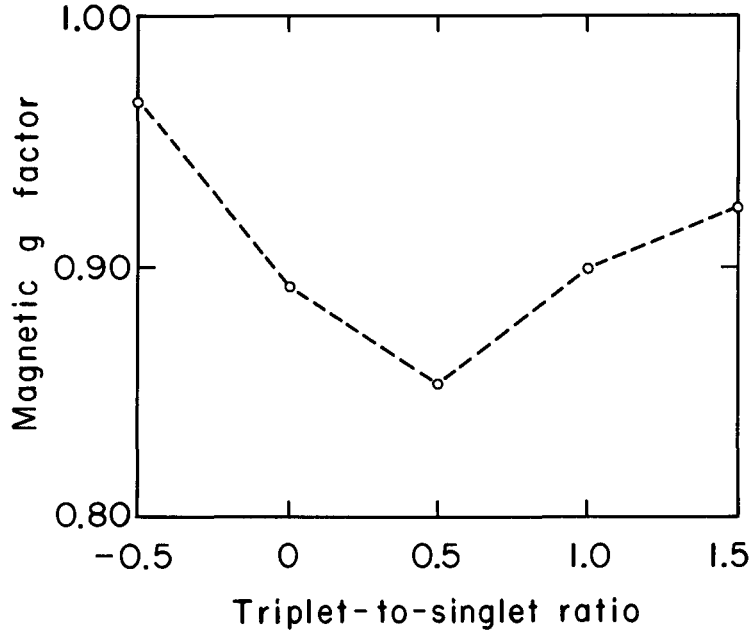
++++ theoretical 2+ for t in parenthesis;

- - - (?) experimental with spins unidentified or tentative.



MU-32762

Fig. 10. Magnetic g factor of 4+ states in 82-neutron, even-A nuclei computed with wave functions obtained with x = 1.15 (Set I) and t = 1.0; g factors for noninteracting configurations $(d_{5/2}{}^2)$, $(d_{5/2}\ g_{7/2})$, and $(g_{7/2}{}^2)$ are taken from empirical results of neighboring odd-A nuclei, the rest are taken from Schmidt values.



MU-32763

Fig. 11. Variation of magnetic g factor for Ce^{140} as a function of t. Only $(d_{5/2}{}^2)$, $(d_{5/2}\ g_{7/2})$ and $(g_{7/2}{}^2)$ components calculated with x = 1.15 (Set I) are taken into account.

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.