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Submitted to Physical Review Letters

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

UCRL-11248

AEC Contract No. W-7405-eng-48

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F. S. Stephens, N. Lark, and R. M. Diamond

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It is well established that nuclei in the region $150 < A < 190$ have sizeable prolate-spheroidal deformations, and consequently have well-defined rotational bands built on the various intrinsic levels. In the present work rather high-lying members of the ground-state rotational bands of a number of even-even nuclei in this region have been studied following heavy ion reactions of the type, $Ho^{165}(B^{11},4n)Hf^{172}$. Odd-proton-number targets (Tb^{159}, Ho^{165}, and Tm^{169}) and projectiles (B^{11}, N^{14}, and F^{19}) have been used to produce Yb^{166}, $Hf^{166,168,170,172}$, and $W^{172,174,176}$. The bombarding energy has been adjusted in each case to give predominantly the particular even-even nucleus desired. We have studied the conversion electron and gamma-ray spectra from such targets during the 3 msec beam burst of the Hilac in order to observe the deexcitation of the final nucleus to its ground state.[1]

The rotational transitions connecting states having spins up to about 12 were normally very well-defined, and could be identified without question. Some of the transitions between higher-spin states were equally well-defined, but others were included among several unassigned transitions of comparable energy and intensity. Obviously, these latter assignments are not completely certain, and for the present discussion we have included only transitions whose association with the ground state rotational band is highly probable. A thorough discussion of assignments will be presented, together with the detailed experimental data, in a more complete publication.

From the transition energies, $E(I \rightarrow I-2)$, we define the rotational constant, A_I, as follows:

$$A_I = \frac{\hbar^2}{2\mathfrak{J}_I} = \frac{E(I \rightarrow I-2)}{4I-2} \qquad (1)$$

where $\mathfrak{J}_I$ represents the moment of inertia appropriate to the transition. In Fig. 1 log A_I has been plotted versus I for the nuclei studied. At low spins, the general features of this plot are well known: (1) a regular decrease in A_I with increasing spin; and (2) smaller slopes (more perfect rotors) associated with lower A_I values. The similarity in rotational properties of all these isotopes at higher spins is very pronounced. The points in all cases (except possibly Hf^{166} for which there is no information above spin 12) are, or become with increasing spin, quite linear with a common limiting slope of 8 or 9% decrease in A_I per state. Furthermore the absolute A_I values are converging into two groups. Thus, for the $14 \rightarrow 12$ transition, five of the seven cases for which there is information have A_{14} values within 2% of 11.05 keV and the other two cases both have A_{14} values of 10.17 keV, within our limits of error (0.3%). It cannot be ruled out that at still higher spins these groups will diverge again; however, our most tentative data at the highest spins rather suggests that they may converge to a single group.

The two nuclei with low A_I values (W^{172} and Hf^{170}) both have 98 neutrons, and a cursory examination of the (α,xn) data taken by the Amsterdam group[2] suggests that Yb^{168} and Er^{166}, with 98 neutrons, behave similarly. Furthermore, from a plot of first excited state energies against neutron number in the rare-earth region, it appears that the energies for nuclei having 98 neutrons are low. Thus from the lowest to the highest spins observed, such nuclei seem to have rotational constants 5-10% lower than others in the region studied here (94-102 neutrons inclusive). A possible explanation is that this is due to a reduction of the pairing correlations due to the energy gap in the Nilsson diagram between the levels 5/2- [523] (98 neutrons) and 7/2+ [633]. This effect seems to be reproduced in Nilsson and Prior's calculations of the moments of inertia based on the pairing model.[3]

There are a number of models with which these rotational energies can be compared. We have not attempted a thorough examination of all such models partly because sizeable computations are required in some cases, and partly because some general observations have suggested to us certain essential features required in such a model. These observations are: (1) the very nearly identical behavior of the moments of inertia observed at high spin values for the nuclei studied (with the notable exception discussed in the preceding paragraph) suggests that a very general property of rotating nuclei must be involved; (2) the evidence accumulating from the study of vibrational states in deformed nuclei indicates that the β-band is admixed into the ground state rotational band some 10 times more heavily than is the γ-band, and the deviations of the ground state band from a perfect rotor can be largely accounted for at low spins by a perturbation treatment of such β-ground mixing;[4,5] and (3) the average change in moment of inertia with spin observed in this study is about a factor of two, which indicates that an attempt to explain this should avoid using perturbation theory. In a model, therefore, we must look primarily for a non-adiabatic treatment of the coupling between β-vibrations and rotation. Within the framework of the hydrodynamic model developed by Bohr and Mottelson,[6] the solution of this problem has been given by Davydov and Chaban (DC) in conjunction with their treatment of asymmetric rotors.[7] We have used their treatment, but in accordance with observation (2) above we have set γ (asymmetry parameter) equal to zero and, therefore, have looked only at the effects calculated due to non-rigidity with respect to β deformation. As mentioned by DC this amounts in the ground state band to accounting for the centrifugal expansion of the nucleus, and hence it seems a priori most consistent with observation (1) above.

A comparison of the Hf^{170} data with the DC solution is shown in Fig. 2. The ordinate here is the ratio, A_{I+2}/A_I, which is related to the slope of Fig. 1, and is used primarily because it gives a plot which is very sensitive to the transition energies.[8] Our error of ±0.25% is indicated on one of the points (the ratio

of two transition energies can be measured more accurately than either one absolutely). For comparison, the effect of the usual $I^2(I+1)^2$ correction term is shown, fitted to the 2+ and 4+ energies, and also that of a three parameter expression involving the third term in the series, $I^3(I+1)^3$, fitted to the 2+, 4+, and 6+ energies. The DC calculation as used here ($\gamma \equiv 0$) has two parameters, one of which ($\hbar\omega_0$) cancels out in the ratio of two energies as plotted, so that there is only one adjustable parameter (μ). Similarly, the plots of the two comparison expressions have one and two adjustable parameters, respectively. This case, Hf^{170}, represents one of the best fits of our eight nuclei to the DC model, but the others are not qualitatively different. The average deviation of the experimental points from the calculated curve for Hf^{170} is 0.26%; for seven of the eight cases (49 points) it is 0.62%. The remaining case, Hf^{166}, will be discussed later. We consider the agreement to be surprisingly good (and for level rather than transition energies the percent deviations would be much smaller). There are, however, systematic deviations. For all cases there is a tendency for the points to fall below the theoretical curves at the highest spins. Another way to express this is to say that the common limiting slope in Fig. 1 is 8 or 9% per state, whereas the DC common limiting slope is about 5-6% per state (the calculations should be extended above spin 20 to be sure of this number). The other systematic deviation only occurs when the first excited state is above roughly 105 keV. Then the first points fall below the calculation and progressively more so as the energy of the first excited state increases. Thus, for Hf^{166} (first excited state 159 keV) the first point is experimentally about 5% low, and there is no real fit to the calculated curve. However, it does not seem to us reasonable to expect such a simple model to account for all deviations from the perfect rotor, particularly near the edge of the region of deformed nuclei (high first excited state energy).

Since our use of the DC calculation, with $\gamma \equiv 0$, accounts for the rotational spacings quite impressively, we were interested to see whether the energy of the

β-vibrational band could be correctly predicted using the same restriction. Table I summarizes the data for all cases we could find where both the β-energy and the 2+ and 4+ ground state band members are accurately known. The ratio of the β-band energy (0+ state) to the first excited state energy is calculated from the ground state band members and compared with the experimentally observed ratio. In all cases the DC ratios are high, and the last column gives the percentage error in the calculation. For the rare earths seven cases lie within the range +8 ±5%. In the one exception, Hf^{178}, the 0+ levels have been assigned as predominantly two-quasi-particle states,[9] and this, if correct, would mean the real β-band might lie higher, as predicted. In the heavy elements, all cases fall within the range +20 ±10%. We feel the absolute agreement is not bad, and the higher internal consistency for each group is very encouraging.

In summary we can say that ground state rotational band members for several nuclei have been identified to spin 14, and in a few cases, very probably to spin 16 or 18, and their energies measured to about ±0.3%. These energies provide a good test of various models for rotational bands, and impressive agreement is observed with a simplified ($\gamma \equiv 0$) Davydov-Chaban calculation involving the centrifugal stretching of the nucleus along the symmetry axis. The closely related β-vibrational band energies are given to an accuracy of about 20% with higher relative precision within each of the two groups of deformed nuclei surveyed (rare-earths and actinides). We consider this quite surprisingly good in as much as only two parameters are involved, and no corrections have yet been included for the γ (non-axial) vibrations, or any other type of perturbation.

ACKNOWLEDGMENTS

We are very much indebted to Dr. Klaus Dietrich for many helpful discussions clarifying theoretical points. We would also like to thank Dr. Paul Day for sending us the results of numerical calculations prior to publication and Dr. Jacob Burde for help with some of the experiments. The interest and encouragement of Prof. I Perlman is gratefully acknowledged

Table I. Beta-Vibrational Energies

Nuclide	$E(2\rightarrow 0)$ (keV)	$E(4\rightarrow 2)$ (keV)	$E(0'\rightarrow 0)$ (keV)	$\frac{E(0'\rightarrow 0)}{E(2\rightarrow 0)}_{calc}$	$\frac{E(0'\rightarrow 0)}{E(2\rightarrow 0)}_{Obs}$	$\Delta(\%)$
Pu^{240}	42.87	98.9	858[a]	24.9	20.0	+20
Pu^{238}	44.11	101.9	943.1[b]	26.5	21.38	+19
U^{238}	$\bar{\mu} = 0.180^{\dagger}$		993[c]	29.0	22.2	+23
U^{234}	43.49	99.8	811.6[d]	20.6	18.66	+10
U^{232}	47.6	109.0	692.9[e]	19.5	14.6	+25
Th^{232}	$\bar{\mu} = 0.232^{\dagger}$		730[c]	17.3	14.7	+15
Th^{230}	53.15	120.8	634[f]	16.3	11.9	+27
Os^{188}	155.03	322.94	{1086[g], 1766}	7.2	{7.01, 11.4}	{+3, -58}
‡(Hf^{178}	93.17	213.42	{1197[h], 1440}	19.6	{12.9, 15.5}	{+34, +21})
Er^{166}	80.57	265.1	1460.3[g]	19.5	18.12	+7
‡(Gd^{158}	79.5	182.4	1427[i]	20.5	17.9	+13)
Gd^{156}	88.97	199.19	1040[j]	13.0	11.7	+10
Gd^{154}	123.07	248.08	680.6[k]	6.0	5.53	+8
Sm^{152}	121.79	244.84	685.0[g]	6.0	5.62	+7
‡(Nd^{150}	131	259	687[j]	5.5	5.2	+5)

† Precise 2+ and 4+ energies are not available for Th^{232} and U^{238}, but many band members are known. Therefore an average μ value is determined for the band as a whole.

‡ Parentheses indicate assignments of less certainty or data of poorer quality than usual.

[a] M.E. Bunker, B.J. Dropesky, J.D. Knight, S.W. Starner, and B. Warren, Phys.Rev. 116, 143 (1959).

[b] R.G. Albridge and J.M. Hollander, Nucl. Phys. 21, 438 (1960); J. Borggreen, O. B. Nielsen, and H. Nordby, Nucl. Phys. 29, 515 (1962).

Table I. (continued)

[c] F. S. Stephens, B. Elbek, and R. M. Diamond, paper E-8, Proceedings of the Third Conference on Reactions Between Complex Nuclei, eds. A. Ghiorso, R. M. Diamond, H. E. Conzett, (University of California Press, Los Angeles and Berkeley, 1963).

[d] C. S. Gallagher and T. D. Thomas, Nucl. Phys. 14, 1 (1959).

[e] S. Bjørnholm, F. Boehm, A. B. Knutsen, and O. B. Nielsen, Nucl. Phys. 42, 469 (1963).

[f] F. Asaro and I. Perlman, UCRL-9566, Feb. 1961, p.50; I. Perlman, F. Asaro, B. G. Harvey, and F. S. Stephens, Bull. Am. Phys. Soc. Ser. 2, 2, 394 (1957).

[g] I. Marklund, B. van Nooijen, and Z. Grabowski, Nucl. Phys. 15, 533 (1960).

[h] C. J. Gallagher, H. L. Nielsen, and O. B. Nielsen, Phys. Rev. 122, 1590 (1961).

[i] L. V. Groshev, A. M. Demidov, V. A. Ivanov, V. N. Lutsenko, and V. I. Pelekhov, Doklady Akademi Nauk USSR, 141, 59 (1961); Soviet Physics Doklady (in translation) 6, 979 (1962).

[j] Y. Yoshizawa, B. Elbek, B. Herskind, and M. C. Olesen, paper E-6, Proceedings of the Third Conference on Reactions Between Complex Nuclei, eds. A. Ghiorso, R. M. Diamond, H. E. Conzett, (University of California Press, Los Angeles and Berkeley, 1963).

[k] B. Harmatz, T. H. Handley, and J. W. Mihelich, Phys. Rev. 123, 1758 (1961).

FOOTNOTES AND REFERENCES

*This work was supported by the U.S. Atomic Energy Commission.

1. Studies of this nature have been published recently/ a) for 50 MeV alpha-particles by H. Morinaga and P. C. Gugelot, Nuc. Phys. 46, 210 (1963), and b) for 12 MeV protons by G. B. Hansen, B. Elbek, K. A. Hagemann, and W. F. Hornyak, Nucl. Phys. 47, 529 (1963).

2. H. Morinaga, K. Kotajima, and D. Vinciguerra, private communication, November 1963.

3. S. G. Nilsson and O. Prior, Mat. Fys. Medd. Dan. Vid. Selsk. 32, No. 16 (1961).

4. F. S. Stephens, B. Elbek, and R. M. Diamond, Paper E-8, Proceedings of the Third Conference on Reactions Between Complex Nuclei, eds. A. Ghiorso, R. M. Diamond, H. E. Conzett, (University of California Press, Los Angeles and Berkeley, 1963).

5. J. S. Greenberg, D. A. Bromley, G. C. Seaman, and E. V. Bishop, paper E-7, Proceedings of the Third Conference on Reactions Between Complex Nuclei, eds. A. Ghiorso, R M. Diamond, H. E. Conzett, (University of California Press, Los Angeles and Berkeley, 1963); Phys. Rev. Letters 11, 211 (1963).

6. a) A. Bohr, Mat. Fys. Medd. Dan. Vid. Selsk. 26, No. 14 (1952).
   b) A. Bohr and B. R. Mottelson, Mat. Fys. Medd. Dan. Vid. Selsk. 27, No. 16 (1953).

7. A. S. Davydov and A. A. Chaban, Nuc. Phys. 20, 499 (1960).

8. Such a plot is related to the plot of $\Delta\mathfrak{J}/\mathfrak{J}$ given in reference 1a.

9. C. J. Gallagher and V. G. Soloviev, Mat. Fys. Skr. Dan. Vid. Selsk. 2, No. 2 (1962).

FIGURE CAPTIONS

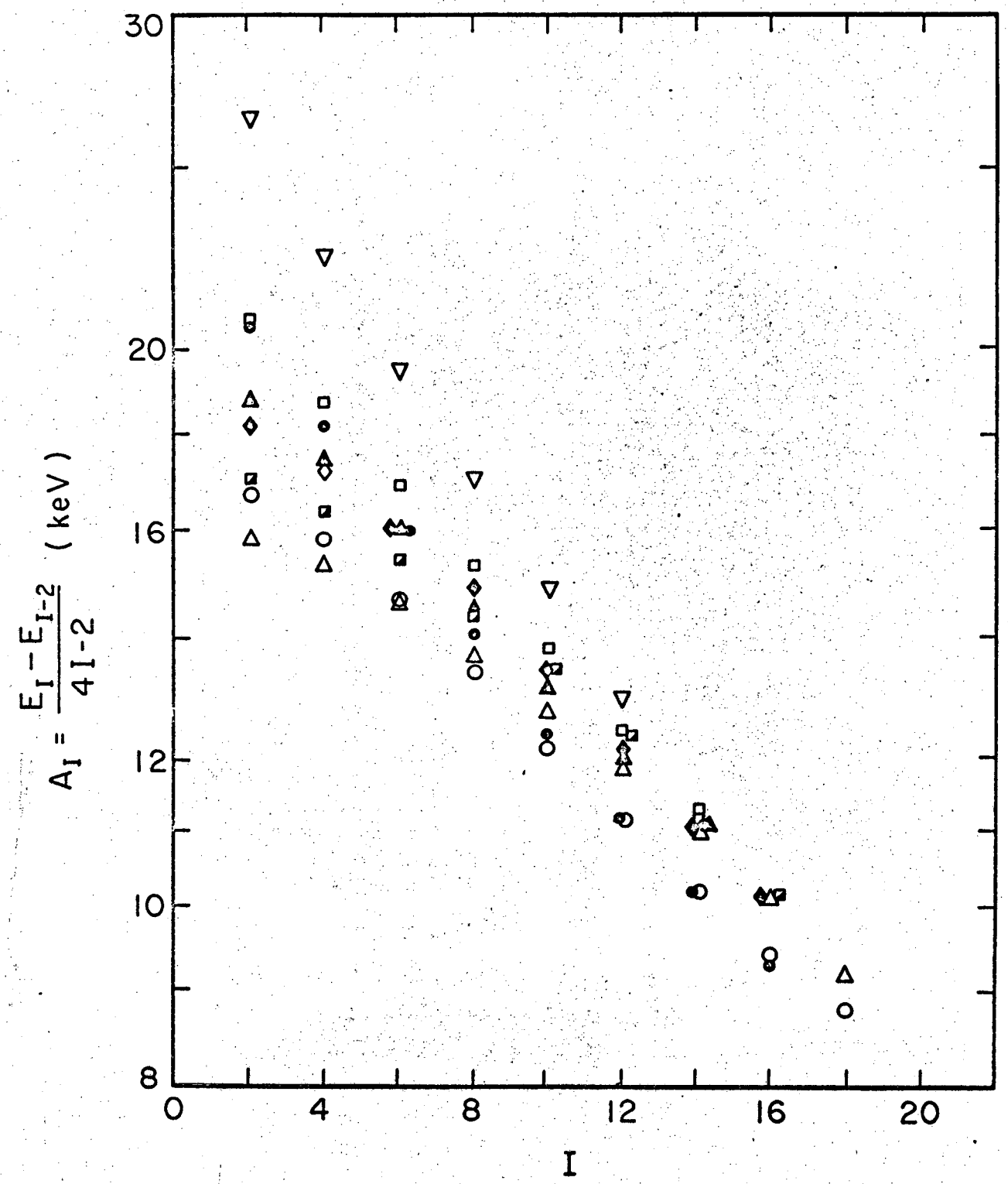
Fig. 1. Rotational constants (appropriate to the transitions) plotted against spin, I. The points are coded as follows: closed symbols, tungsten; open symbols, hafnium; half-closed symbols, ytterbium: neutron number 94, ▽; 96, □; 98, ○; 100, △; 102, ◇.

Fig. 2. Ratios of successive rotational constants, A_{I+2}/A_I, plotted against spin, I. The circles are the experimental data for Hf^{170}, and the curves are:

—— the non-adiabatic Davydov-Chaban calculation with $\gamma = 0$;

--- $AI(I+1) + BI^2(I+1)^2$ fitted to the 2+ and 4+ energies; and

-·-· the previous expression plus the term $CI^3(I+1)^3$ fitted to the 2+, 4+, and 6+ energies.



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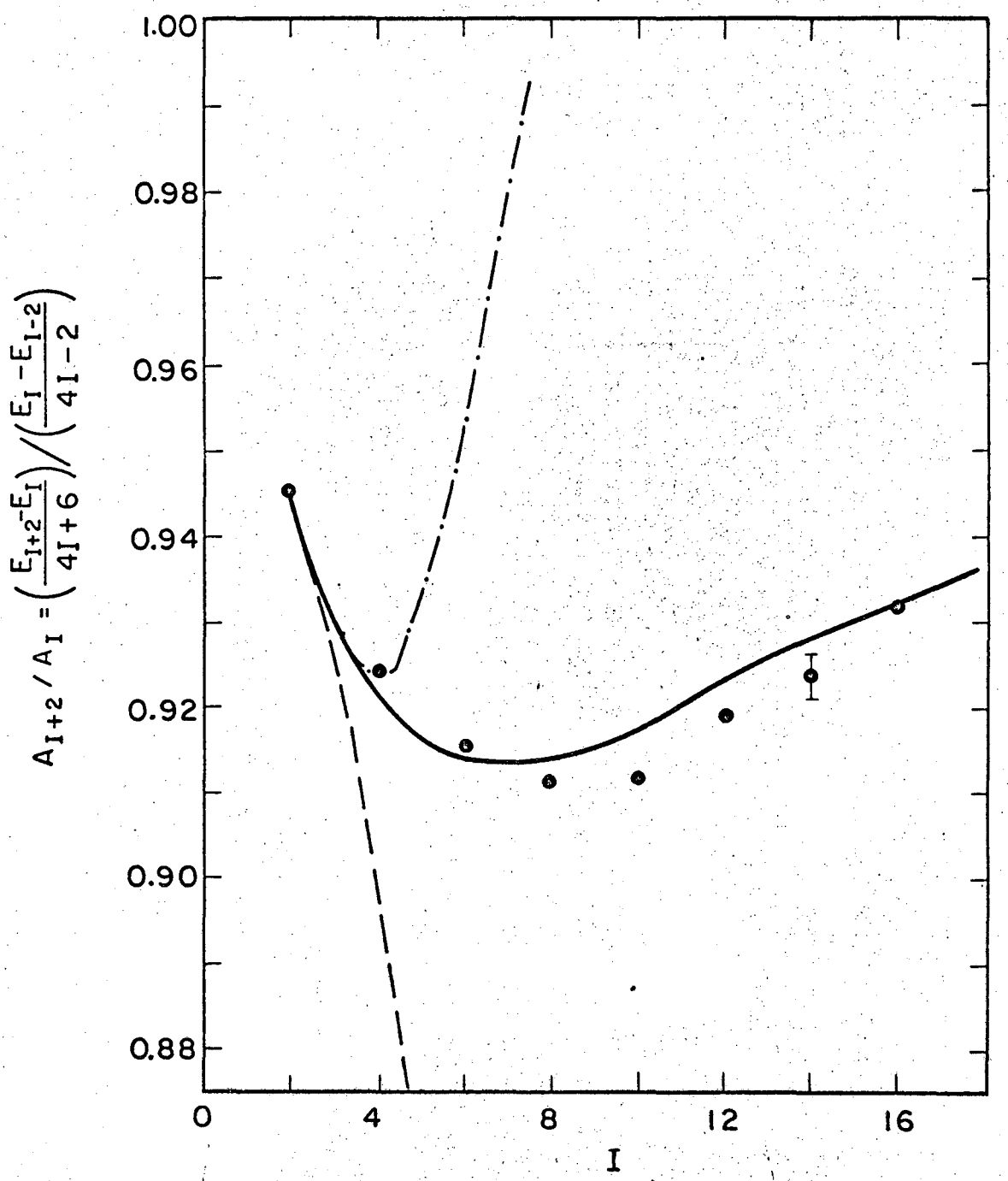
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I
$A_{I+2}/A_I = \left(\frac{E_{I+2}-E_I}{4I+6}\right) \Big/ \left(\frac{E_I-E_{I-2}}{4I-2}\right)$

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