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A MEASUREMENT OF $d\sigma(0)/d\Omega$ FOR THE REACTION $\pi^- + p \rightarrow \pi^0 + n$ AT 900 MeV IN THE 72-INCH HYDROGEN BUBBLE CHAMBER

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ABSTRACT

The pion-nucleon dispersion relations, based on the principle of microscopic causality, permit the calculation of the real parts of the forward amplitudes for π^--p and π^+-p scattering at a fixed energy, if the corresponding total cross sections are known as a function of energy. Assuming charge independence to hold, several authors have calculated in addition, the forward differential cross section, $d\sigma^{ce}(0)/d\Omega$, for charge exchange ($\pi^- + p \rightarrow \pi^0 + n$). Experimental values for the latter, therefore, can help to determine the validity of the principle of microscopic causality.

The paper describes a low-statistics measurement of $d\sigma^{ce}(0)/d\Omega$ at 1.03 BeV/c in the 72-inch hydrogen bubble chamber. The π^- beam comes from the Bevatron, and the secondary neutrons are detected when subsequent elastic scatters $n + p \rightarrow n + p$ yield visible stopping protons. The real events were separated from the spurious by a Boolean Algebraic method that reduced the set of all events to a sum of a small number of physically significant sets. The residual background fraction was estimated at 10% by a computer simulation technique using the known beam distribution and a pure sample of background events. The bubble chamber, so used, detects about 5% of the secondary neutrons emitted in the charge-exchange reaction.

There were 12 experimental events in three intervals lying in the range $0.84 \leq \cos\theta_{\pi^0}^{c.m.} \leq 0.96$. A zero-constraint, least-squares fit of the distribution yielded the estimate $d\sigma^{ce}(0)/d\Omega = (3.65 \pm 1.93)$mb/sr. This result is consistent with the predictions based on the dispersion relations.

From the dispersion relations[1] for pion-nucleon scattering, the optical theorem, and charge independence, a prediction emerges about the forward differential cross section $(d\sigma_E^{ce}(0)/d\Omega)$ for elastic charge-exchange scattering $(\pi^- + p \rightarrow \pi^0 + n)$ at a fixed energy E. This prediction states that $d\sigma_E^{ce}(0)/d\Omega$ can be calculated if $\sigma^{\pm}(E')$, the total cross sections for $\pi^{\pm}$-p scattering, are known for all energies E'. The calculation has been performed by several authors,[2] and the comparison of their predictions with experimental results provides a test of the joint validity of the principles of microscopic causality and charge independence. This paper describes a low statistics measurement of $d\sigma^{ce}(0)/d\Omega$ at 900 MeV performed in the 72-inch Berkeley H_2 bubble chamber.

The reaction $\pi^- + p \rightarrow \pi^0 + n$ can be studied by detecting the π^0 (as in spark chambers or heavy-liquid bubble chambers[3]) or, as we do, by detecting the neutron.[4] The efficiency of our method for detecting neutrons depends on the high n-p scattering cross section for slow neutrons and the reasonably high fraction of such scatterings that lead to visible stopping protons.

Figure 1 shows a π^- beam particle (produced originally in the Bevatron) that has undergone charge exchange (at point P). The neutron scattered subsequently at point R, while one of the γ rays resulting from the π^0 decay converted (at point C) to an electron pair. The proton track starting at point S is spurious (that is, it is not associated with the track), but the full computer analysis is needed to deduce this. Since both proton recoils, R and S, stop inside the chamber, we can determine their momenta accurately and thereby deduce all the important kinematic quantities for the particles taking part in

the reaction. Pictures of events like the one in Fig. 1 pass through the standard hydrogen bubble chamber analysis sequence described by Rosenfeld and Humphrey.[5]

The central problem of the analysis is the extraction, as efficiently as possible, of the valid events of the sample, contaminated by as small as possible an admixture of background. Further, this extraction must be based on experimental quantities that are reliably known.

We rely on measurements of short proton tracks, which measurements become less and less reliable as the tracks become shorter and less favorably situated. Further, the population of stray proton tracks, resulting mostly from collisions of protons with neutrons produced directly in the Bevatron, becomes denser as the tracks become shorter. To deal with these weaknesses, we restrict ourselves to a sample of events that does not exhibit them. The restrictions necessary to achieve reliability and reduced background are given in the next paragraph. The result is a system that detects approximately 4 to 10% of neutrons in events in which the π^0 was emitted forward in the center-of-mass system (c. m.), and includes no more than an estimated 10% background. The restrictions adopted are given below.

1. The projected length of the proton track on the plane of the film must exceed 0.7 cm, and the magnitude of the dip of the proton track relative to the same plane may not exceed 60 deg, irrespective of length.

2. The square of the calculated missing mass must not differ from its ideal value, $m^2(\pi^0)$, by more than 1.65 times its own error. This error, too, may not exceed 5000 $(\mathrm{MeV}/c^2)^2$ (an approximate error of 16.0 MeV/c^2 in the missing mass itself.)

3. The measured curvature of the supposed proton track must be positive.

A measured event that originated from an acceptable beam track, in a frame whose good quality is certified by two scanners and that contains 25 or fewer beam tracks, is accepted as valid if it satisfies the three criteria (1) through 3) above.

Our experiment yielded 12 accpetable events in the region $0.84 \leq \cos\theta_{\pi^0}^{c.m.} \leq 0.96$ from $(1.09 \pm 0.03) \times 10^7$ cm of beam track (2.55 ± 0.07 μb/event) Of these 12 events, 0.51 ± 0.26 are expected to be background. This is a Monte Carlo estimate based on the assumption that the background is composed entirely of stray protons caused by the elastic scattering of primary neutrons formed in the Bevatron target. The calculation of this estimate starts with a sample of proton tracks known to be background. Each of these failed to fit the basic hypothesis relative to the nearby beam track apparently associated with it. Leaving the recoil track unchanged, we translate the beam track by a transverse distance, chosen Monte Carlo fashion according to the known beam distribution. This generates a new event from a spurious recoil. The fraction of such randomly generated events satisfying our acceptance criteria was our estimate of the background fraction.

We verify the sufficiency of our criteria by performing a Boolean Algebraic analysis of the behavior of the experimental sample, relative to the given criteria. The result is that the experimental sample splits into seven logical classes, one class of valid and six of spurious events.[6] The experimental sample contains all events that appear valid to a scanner. The analysis referred to takes the set of combinations of variables occurring experimentally and reduces it to the most economical expression, eliminating as many redundant variables as possible. In particular, there are $2^8 = 256$ possible combinations

relative to the eight fundamental binary variables used to characterize our events; only 20 of these occur in our sample. The information about our experiment that is contained in the knowledge of which combinations occur is what we seek. The mere listing of the 20 combinations is not instructive. What we seek is the pattern of their occurrence. That is, we seek a rule, as simple as possible, that expresses the membership of the set. This rule will contain all the essential physical information expressible in terms of the original variables. Because our variables can each take on only two values, a convenient means already exists for extracting the essential rule. The method, initiated by Quine,[7] is called "the reduction of a Boolean truth function to minimal form." We now give its results.

First we list the criteria used to characterize validity (Table I). Each criterion X_i is either satisfied ($X_i = 0$) or not satisfied ($X_i = 1$) for each event. Because each X can have only two values, it is called a binary variable. Criteria 2, 3, 4, and 7 refer to the recoil-proton track. The parameter L_p is the length of proton track projected on the plane of the top glass of the chamber (which is also the film plane of the cameras). The χ^2 parameter measures how well the measured tracks fit our hypothesis when all measured quantities are varied within their quoted experimental errors. The missing mass is the mass of the particle U in the reaction $\pi^- + p \rightarrow n + U$ that is needed to balance energy momentum with the unfitted tracks measured; it should correlate very closely with χ^2 as an index of the validity of the event. No valid events were found in which the missing particle, U, was a particle other than π^0.

The set of the 20 experimental combinations of eight variables each, when expressed most economically, is composed of seven subsets. In five of these, reference to one redundant variable has been removed. In the two cases remaining, two and three redundant variables have been suppressed. No further

redundant variables remain (this is the meaning of "economical" in the sense we have been using).

The essential feature of this method is this. For a sample of events (a) large enough that the fluctuations in populations of individual classes can be ignored and (b) based on a well-chosen set of physical criteria, the classes appearing in the minimal expression correspond to the largest possible sets of physically similar events. In our case we assume the real events to be physically similar to one another and the background events of each kind to resemble each other. If in our reduction, the class containing the real events also contains an appreciable number of background events, we assume the background events to be physically similar to the real events in the same class. We would conclude, in that case, that the experimental criteria do not adequately separate real from background events.

In the present case, five of the seven final classes are characterized by a high value of the relative deviation of the missing mass. This marks them as classes of background events. There is one class in which RD is a redundant variable. In that class both dip error and error in missing mass are high; we can call this a class of poorly measured events. We discard these events and apply an appropriate correction to the remaining class, the one containing the real events. In this last class there are 11 events that pass all criteria and 4 that fail only the criterion of low missing-mass error. This result sets an upper limit of four events on the total of our background due to events whose errors are high. We choose to ignore these and introduce appropriate corrections. Our real events could be confused only with those having large experimental errors. All other possibly confusing events fall into logical groups other than the one containing the real events. More details of the calculation are given in reference 6.

In the original search of the film for interesting events, scanners note any "zero-prong" formations (a beam[8] track ending suddenly inside the chamber), including those in which associated productions have apparently taken place, as evidenced by a nearby decay pattern. In the neighborhood of such zero prongs, scanners seek recoil-proton tracks longer than 0.1 in. in the two best stereo views projected on the scanning table. This corresponds to a spatial projection of about 0.38 cm, which compares with our insistence on 0.7 cm as the minimum projection for an acceptable event. No actual events acceptable to our other criteria, however, were rejected by adopting the 0.7-cm requirement. The correction for all events thus lost is made by the computer program HOUND.[9]

The film (13 rolls were accepted out of some few dozen sampled extensively) was scanned independently by two scanners who, when working well, scanned 40 frames/hour at an average efficiency of 0.76±0.01. Were the minimum-length cutoff to be raised, both rate and efficiency could be expected to rise rapidly. On a high-statistics experiment this course would be indicated and justified by the low yield of reliable valid events of short projection that were found with less restrictive criteria. No event was considered whose neutron path lay obviously within a 60-degree cone about the beam track. Such neutrons would have relatively high momentum transferred to them and could not, therefore, be associated with π^0's emitted in a forward direction. This restriction accelerated the scanning and explains the narrow, near-forward spectrum of our π^0's.

The distribution, with appropriate corrections, is tabulated in Table II. Figure 2 shows a plot of the angular distribution $d\sigma^{ce}(\cos\theta)/d\Omega$ with the best least-squares fit to the curve $d\sigma^{ce}(\cos\theta)/d\Omega = b_0 + b_1(\cos\theta - 1) + b_2(\cos\theta - 1)^2$ where (b_0, b_1, b_2) 0 = (3.65±1.93, 60.5±38.0, 265±181) mb/sr.

The indicated value, (3.7±1.9) mb/sr, for the forward differential cross section compares with an approximate range of 2 1/2 to 3 3/4 mb/sr for the predictions calculated at 900 MeV from the dispersion relations.[2] The more recent calculations are closer to the higher figure. No disagreement is indicated.

More interestingly, the dispersion relation predicts a rapid fall of $d\sigma^{ce}(0)/d\Omega$ with increase in energy in this region. The heavy-liquid bubble chamber measurement performed at Harvard[3] at 960 MeV gave the result (1.34 ± 0.39) mb/sr, which is consistent with this prediction.

The authors would like to thank Professor Luis W. Alvarez, the leader of the group in which the research described was performed. The project was a part of the Associated Production Experiment and we wish to thank the members of that group for their cooperation and particularly its leader, Professor Frank S. Crawford, Jr., for his helpful discussions and criticism.

FOOTNOTES AND REFERENCES

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Table I. The binary criteria used to characterize each event.

Criterion No., i	X_i relates to the phrase		X_i = (0 , 1) if
0	Forward	$\cos\theta^{c.m.}$	is (≥ , <) 0.7
1	Chi square (χ^2)	χ^2	is (≤ , >) 2.706
2	Dip error ($\delta\lambda$)	$\delta\lambda$	is (≤ , >) 2.5 deg
3	Projected length (L_p)	L_p	is (≥ , <) 0.7 cm
4	Dip [relative to top glass] (λ)	$\lvert\lambda\rvert$	is (≤ , >) 60 deg
5	Missing mass error (δ MM)	δMM	is (≤ , >) 16.0 MeV/c^2
6	Relative deviation of the missing mass RD = [MM-M (π^0)]/δ MM	RD	is (≤ , >) 1.65
7	Curvature (K)	K	is (≥ , <) 0 c/MeV

Table II. Distribution of experimental events with successive corrections.

$\cos\theta^{c.m.}$	N	N'	N''	N'''	Partial cross section (μb)
0.84					
	$2\pm\sqrt{2}$	$2.51^{+1.75}_{-1.50}$	$3.54^{+1.75}_{-1.52}$	$37.018^{+26.0}_{-22.6}$	$94.4^{+66.4}_{-64.4}$
0.88					
	$2\pm\sqrt{2}$	$2\pm\sqrt{2}$	2.82 ± 1.41	24.743 ± 17.8	63.1 ± 45.4
0.92					
	$8\pm\sqrt{8}$	$8\pm\sqrt{8}$	11.3 ± 2.82	96.004 ± 36.2	244.8 ± 92.5
0.96					

N Number of experimental events in the interval.

N' = N corrected for casualties to low missing-mass-error criterion.

N'' = N' corrected for background, scanning efficiency, and casualties to the missing-mass-deviation criterion.

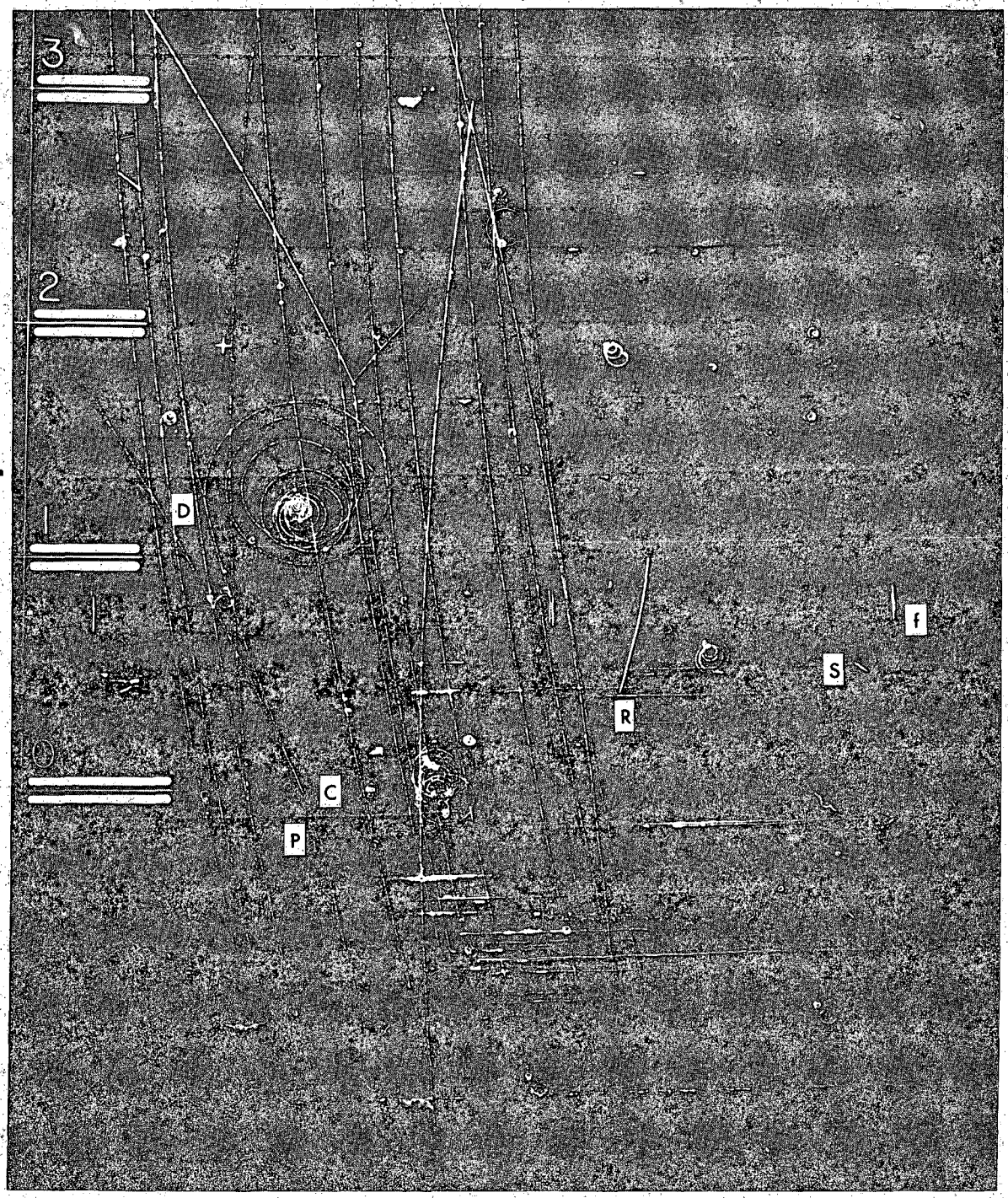
N''' = N'' corrected for geometric, visibility, and escape probabilities.

N''' = Estimated number of actual events, detected and undetected, that occurred in our experiment in the given histogram interval.

FIGURE LEGENDS

Fig. 1. Reproduction of a photograph of an experimental event showing zero-prong, recoil, and associated conversion pair.

Fig. 2. Differential cross section for reaction ($\pi^- + p \rightarrow \pi^0 + n$) showing uncertainty in extrapolated forward value under least-squares fit.



ZN-4084

Fig. 1.

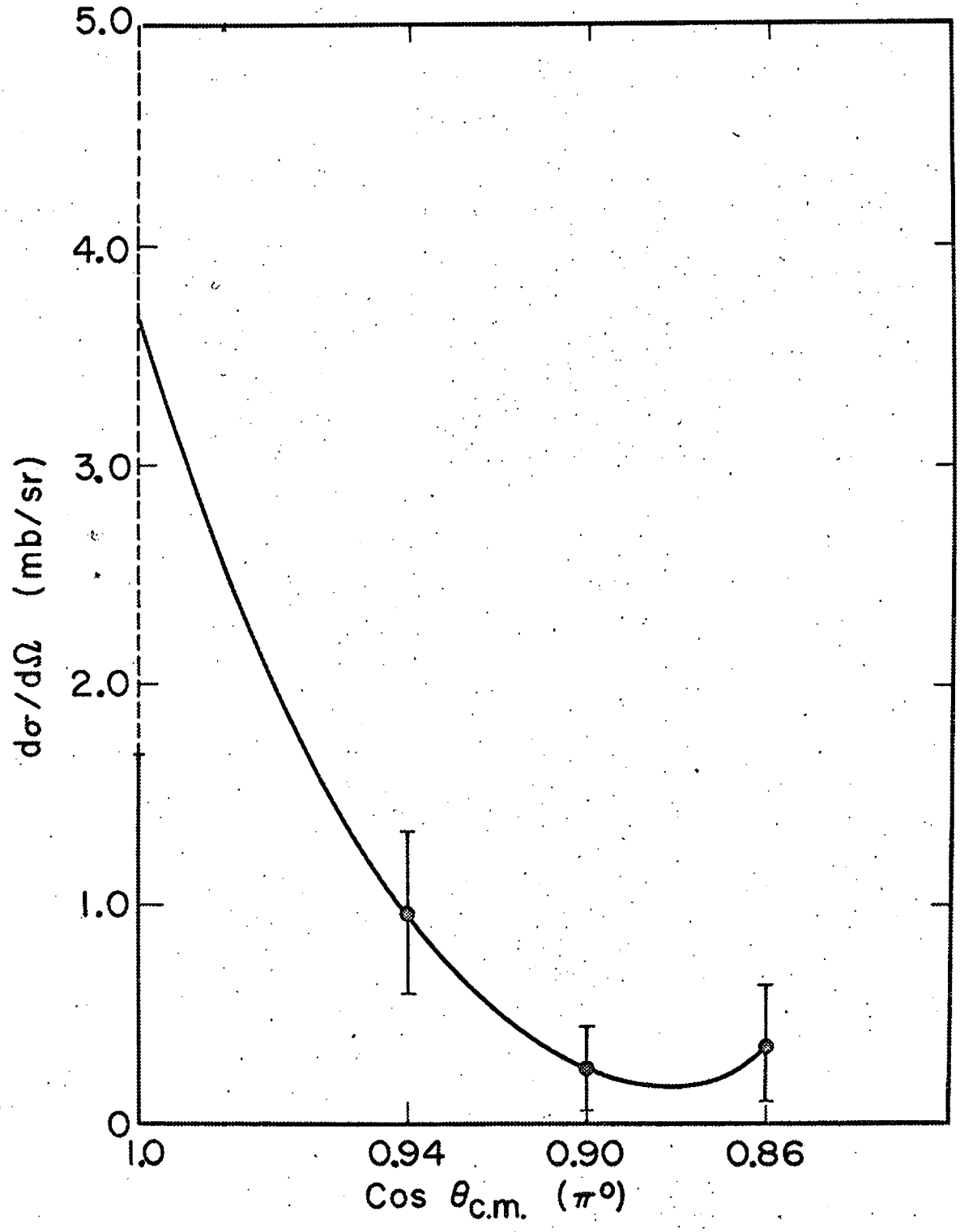


Fig. 2

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