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ROTATIONAL STATES PRODUCED IN HEAVY-ION NUCLEAR REACTIONS

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F. S. Stephens, N. L. Lark, and R. M. Diamond

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Abstract

The de-excitation of the ground state rotational band in nine deformed even-even nuclei has been observed following heavy-ion nuclear reactions. The transitions from states up to spin 16 (on the average) were observed and their energies were measured with an accuracy of ±0.3%. The rotational spacings thus identified have been compared with several calculations, and much the best agreement is obtained with a simplified calculation by Davydov and Chaban taking into account the effects of centrifugal stretching of the nucleus.

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1. Introduction

The study of the gamma-ray cascade de-exciting the final nucleus produced in a nuclear reaction yields data which are useful in considering two types of problems. These are: 1) the mechanism of the reaction to produce the final product; and 2) the systematics of the energy levels in the product nucleus. In particular, on the first account, the nature of the states observed and their population can, at least in principle, indicate the excitation energy and the angular momentum distribution imparted to the final nucleus, as well as the total yield of that particular nucleus. For the second problem, this method of reaching the excited states of a nucleus might well complement the more usual spectroscopic studies involving radioactive decay and Coulomb excitation. Specifically, one would expect to be able to excite high-energy states and especially high-angular momentum states much easier and more systematically in the nuclear reaction than by the other mechanisms.

The work of Morinaga and Gugelot[1]), using NaI(Tl) gamma-ray detectors to observe the excited deformed even-even nuclei produced in $(\alpha,4n)$ reactions showed that the ground state rotational band in such nuclei could be identified up to about the 10+ level. At about the same time, Hansen, Elbek, Hagemann, and Hornyak[2]) also produced excited deformed even-even nuclei in the rare earths using (p,2n) reactions. This group studied the de-excitation cascades with a conversion electron spectrometer and demonstrated clearly its desireability, due largely to the better resolution obtained. However, with the 15 MeV protons available, they did not populate levels as high in the ground state rotational bands as did Morinaga and Gugelot.

In the present work a number of even-even nuclei of ytterbium, hafnium, and tungsten have been studied following heavy-ion reactions of the type $Ho^{165}(B^{11},4n)Hf^{172}$. A variety of odd-Z targets and projectiles have been used, with the bombarding energy adjusted in each case to give predominantly the

particular nucleus desired. Both the conversion electron and gamma-ray spectra have been observed from such nuclei. As in previous work of this type the principal transitions observed belong to the ground state rotational band; surprisingly little else could be identified.

2. Experimental Procedure

The Lawrence Radiation Laboratory Hilac was the source of the beams of B^{11}, N^{14}, and F^{19} projectiles used in the present study. Currents of ~0.1 μamp on a target area of 2-3 mm by 10 mm were normally employed. The beam energy was measured using a solid state counter calibrated against the full energy beam (10.4 MeV per nucleon), and is expected to be accurate to about 2%.

An electron spectrometer of the single wedge-gap type developed by Kofoed-Hansen, Lindhard, and Nielsen[3]) was used to observe conversion electrons from the de-excitation cascade of the excited nuclei produced in the irradiation. This spectrometer and its semi-automatic operation has been described previously[4]). The only change in its operation is that a 180° flip coil has been added to measure directly the field in the spectrometer at any time during the run and so permit a more direct determination of the momentum of a conversion line. The spectrometer was operated such that the full-width of the conversion electron peaks at half-maximum was usually between 0.5 and 1.0%, and the energies could be determined to an accuracy of ±0.3%.

Self-supporting metallic foils of Tb^{159}, Ho^{165}, and Tm^{169}, 1-3 mg/cm^2 thick and inclined usually at 15° to the beam, were used as targets, and the beam was stopped in a shielded Faraday cup about a meter beyond the target. The spectra of the prompt cascades were taken during the 3 msec beam bursts, but the targets were also observed in the intervals between pulses (15 pulses/sec) in order to look for metastable states and to observe the radioactive decay to

the daughter products. The spectra for each nucleus were usually taken at three projectile energies (centered around the expected optimum energy) 5-10 MeV apart in order to obtain limited excitation functions of the various lines. This had a two-fold purpose: to enable a grouping of the conversion lines by nuclide, and to determine the energy for obtaining the cleanest spectrum of that particular even-even nucleus.

In some cases gamma-ray spectra were also taken, using both NaI(Tl) and Ge (solid-state) detectors. The background was higher relative to the peaks in these spectra than in the electron spectra; and, even worse, this background continuum proved to be largely in coincidence with the peaks. These measurements, therefore, were not extensively pursued and will not be further discussed.

3. Results

Examples of the conversion electron spectra obtained near the peak of their respective excitation functions are shown in figs. 1-3 for $Yb^{164,166}$, $Hf^{166,168,170,172}$, and $W^{172,174,176}$. It should be noted that the yields in the different spectra cannot be intercompared as the targets were of different thickness. Those transitions which we have assigned to the ground-state rotational bands are indicated on the spectra and summarized in table 1.

In a particular spectrum, the assignment by element of the lower energy transitions could be made from the K-L conversion line spacing. Mass assignments were made on the basis of the change in bombarding energy necessary to go from the maximum of the excitation function of one even-even nucleus to that of the next lighter one—about 30 MeV per pair of neutrons out. The close similarity in bombarding energy to produce a given reaction in any of these three targets (as well as those of other nearby elements that have been studied), coupled with

the results of a number of cases where a given product could be made via more than one reaction (the use of another projectile-target system), leaves absolutely no doubt as to the mass assignments.

It was found that at the optimum bombarding energy almost all of the intense low energy transitions shown in figs. 1-3 could be assigned either to Coulomb excitation in the target or to the ground state rotational band of the final even-even product nucleus. The latter assignments were made because 1) their energies approximated those of a symmetric rotor,

$$E(I \rightarrow I-2) = \frac{\hbar^2}{2\mathfrak{J}} [I(I+1)-(I-2)(I-1)] = \frac{\hbar^2}{2\mathfrak{J}} [4I-2]; \qquad (1)$$

2) their intensities were high, and decreased (assuming E2 transitions) in a very regular way with increasing energy (spin); and 3) the lower energy transitions could be shown to be E2 from their K/L intensity ratio. Direct spin measurements are largely absent, but in a subsequent work[5]) three cases were studied where the first four of these prompt transitions also occurred in low intensity following the beam pulse; that is, held up by an isomeric state. In these three cases, the spins 2+, 4+, 6+, and 8+ were established by angular distribution measurements.

Those transitions that fit the above requirements and are of outstanding intensity in the spectrum are given an "A" classification in table 1. In a few cases, such as Hf^{170}, essentially only such transitions are observed in the spectrum. More usually, though, a few apparently extraneous transitions of a moderately low intensity occur at the higher energies; sometimes, as in W^{176}, there is apparently an entire band of additional transitions. As the intensities of the ground state band transitions decrease with increasing spin, they become at some point no more intense than the extraneous transitions, and it often becomes difficult to decide which of the lines are the members of the

ground state band. We have attempted to do this based on the above requirements and on the detailed nature of the excitation functions of the lines. The more certain of these transitions (80% expected to be correct) are labelled "B" in the tables, and those less certain or any that are appreciably weaker than the strongest of the unassigned transitions are given a "C" classification (50% expected to be correct). This last classification does not mean any doubt that the conversion line exists, only that there is some doubt whether it is a member of the ground state rotational band.

At the most favorable bombarding energy for the particular nuclide being studied, there is little interference from the neighboring odd-mass nuclides. Partly this is because the peaks of the excitation functions for the odd-mass nuclides are 15 MeV away. Partly it is because these nuclei have so many different pathways to the ground state from the excited levels reached in the evaporation of the final neutron. That is, the ground state rotational band in the odd-mass nuclei is not the unique channel to ground it is in the even-even nuclei which have a 1-1.5 MeV gap in particle states.

4. Discussion

In this section we will first mention briefly several points related to the nuclear reaction mechanism, and then go on to a discussion of the rotational spacings observed. As has been stated, one of the most surprising conclusions of a survey of the spectra is that the ground state rotational cascade from about spin 14-18 down provides almost all the intense transitions observed. There is little evidence for transitions from or within the vibrational bands or bands based on two quasi-particle states. This is due in part (but not entirely) to the decrease in the sensitivity of observation of the higher energy transitions with the decrease in conversion coefficients. Another experimental

observation is that the entire cascade is usually fast. In two of the nuclei studied we have determined[6]) that there is no appreciable intensity feeding the second excited state with a half-life longer than $\sim 3 \times 10^{-10}$ sec.

These observations about the nuclear reaction can be accounted for in at least two essentially different ways. If the semi-classical description of the collision is correct, the nuclei have been left, after the emission of the last neutron, with about 10 MeV of excitation energy and ~40 units of angular momentum. The only plausible explanation, in this case, is that this energy and angular momentum must be carried off in a large variety of gamma-ray cascades of which the only common feature is the ground state rotational band. Thus, any particular non-ground-state-band transition must be very weak. There are, in fact, a considerable number of unassigned low-intensity lines observed over the entire range of energies examined and there is also a background continuum in coincidence with the rotational cascade which might be at least partially composed of such transitions. Also, where isomers have been observed, it is in intensities of only 1-5% of the prompt cascade, again consistent with the idea of many de-excitation pathways to the ground state band.

The other explanation is that the product nuclei (after neutron emission) do not have much more energy and angular momentum than is represented by the observed entrance into the ground state rotational cascade; namely 18-20ħ and ~4 MeV. One possible reason for this is that those nuclei having much larger angular momenta may preferentially emit a particle heavier than a neutron[7]). The increase in binding energy of neutrons relative to charged particles as one goes out to the more neutron-deficient nuclei makes the emission of charged particles much more likely in these nuclei[8]). It may be preferentially the high-angular-momentum states that emit the charged (especially heavier) particles due to the lack of final states for neutron emission. Also, the collisions of the heavy-ion projectile and target nucleus which would bring in

much more angular momentum on the semi-classical calculation occur at the surfact of the nucleus, and instead of leading to compound nucleus formation, many such collisions may instead contribute to multinucleon transfer reactions and other direct interactions. Weak evidence along these lines is furnished by the fact that producing a given nucleus by B^{11}, N^{14}, or F^{19} beams (of different energies) on the appropriate targets gave similar conversion electron spectra, i.e., the relative intensities of the various lines did not differ greatly. This suggests (as proposed above) that even though much more angular momentum should be brought (classically) into the compound nucleus by the heavier projectile, those collisions leading to the desired product have a smaller momentum transfer which can as well be accomplished by a somewhat lighter projectile. Furthermore, the higher backgrounds which invariably accompanied the heavier projectiles (especially F^{19}), as well as the lighter projectiles at higher energies, also suggest that reactions other than the desired (HI,xn) ones may be occuring. Probably the real situation involves both of the explanations outlined briefly here.

Considerably more progress has been made in the other aspect of this work to be discussed, namely understanding the rotational energies observed. This progress has been previously summarized in two short publications[9,10]. In the first we showed briefly the systematic behavior of the data, and interpreted it in terms of centrifugal stretching of the nuclei. Remarkably good agreement was obtained between the data and a calculation of centrifugal stretching by Davydov and Chaban (DC)[11]. We, however, restricted their calculation to axially-symmetric shapes. Furthermore, based on the ground state spacings, this calculation was shown to give the energy of the closely-related beta vibrational band to an accuracy of 10-20%. (Subsequent examination[12]) suggests that the absolute B(E2) values between the ground state and the beta band[13]) are also very nearly correctly given.) In the second publication we showed that the DC

quantum mechanical calculation could be replaced for the nuclei considered by a simple semi-classical treatment. This treatment permitted variation of the β-dependence of the two energy terms involved, the rotational kinetic energy and the potential energy. It was shown that potentials more realistic than the parabola assumed by DC could explain qualitatively the small systematic deviations between the data and the DC calculation, although no quantitative calculations with these potentials have been carried out yet. It was also pointed out that this semi-classical treatment of centrifugal stretching gave reasonable results in the limit of spherical nuclei (harmonic oscillator spacings), and hence might be useful in the transition region between deformed (rotational) and spherical (vibrational) nuclei. In this section we will not repeat these arguments, but will present some of the data analyses on which they are based.

We will consider here only the present data on rotational spacings. Other information, particularly concerning the beta-vibrational band and the various transition probabilities, is essential to confirm the cause of these spacings as centrifugal stretching of rotating nuclei, but no such information is available for these particular nuclei. (Our first letter[9]), summarized some of the information available for other nuclei.) The measured spacings will be compared with the calculations of DC and to a lesser extent with other schemes.

There are three parameters in the DC calculation: the asymmetry parameter, γ; the β-vibrational band energy, $\hbar\omega_0$ (an overall scale parameter); and a parameter measuring the non-adiabaticity, μ. The first of these, γ, we have set equal to zero (requiring axially-symmetric shapes), although the effect of varying γ somewhat is shown later. The scale parameter, $\hbar\omega_0$, cancels out if a ratio of two energies is taken, and most of our comparisons are therefore of this type. Thus, in the figures there is generally only one adjustable parameter, μ (the ratio of the rms zero-point vibration amplitude in the ground state

to the equilibrium deformation in that state). An overall comparison of the data with the DC calculation is shown in fig. 4. Here, the more or less horizontal lines are the theoretical ratios of the energy of the state having spin I to the energy of the 2+ state plotted as a function of the parameter, μ. The nine more or less vertical lines are the data for the nine nuclei studied, where each line is made to cross each theoretical line at exactly the measured ratio. Thus an exactly vertical line would represent a perfect fit to the calculation. Most of the experimental lines are quite close to vertical if one considers that the horizontal scale is rather expanded (numerical comparisons will be given later). Also the very systematic nature of the deviations of the data from the theory is evident. From spin 8 to 12, all the lines are very nearly vertical. Above spin 12 all significant deviations are in the direction of higher μ values with increasing spin, and the best rotors (low μ values) deviate as much as, if not more than, the other nuclei. Below spin 8, all the departures from the theory are toward lower μ values as the spin increases. Only nuclei having μ values above about 0.3 behave in this way. It has been suggested[10]) that both of these systematic deviations from the DC theory can be largely corrected by using potential energy expressions derived from the mass formula of Swiatecki[14]).

A more sensitive way to examine the data is demonstrated in figs. 5 to 9. In these plots we work only with the transition energies, and remove the general I(I+1) energy dependence of the levels by defining the transition rotational constant, A_I as follows:

$$A_I = \frac{\hbar^2}{2\mathfrak{J}_I} = \frac{\Delta E(I \rightarrow I-2)}{4I-2} .$$

Then, in order to eliminate the scale parameter, $\hbar\omega_o$, a ratio of adjacent rotational constants is taken; so that the ordinate in figs. 5 to 9 is A_{I+2}/A_I.

These ratios are plotted against the intermediate spin, I. These plots are very sensitive to the transition energies since they show on an expanded scale only deviations from the I(I+1) expression. Our experimental error in the ratio of the rotational constants, ±0.25%, is indicated on one of the points in each figure. It can be seen that the agreement between the experimental points and those calculated from the simplified DC expression (drawn as a solid curve) is remarkably good. The agreement is better than we were able to obtain with any other model or expression we tried. For example, in fig. 5 for Hf^{170} are shown plots of the ratios as calculated from the addition of the usual $BI^2(I+1)^2$ term and also as calculated using this term and an additional $CI^3(I+1)^3$ term. The constants for these two expressions were obtained by fitting to the 2+ and 4+ and to the 2+, 4+, and 6+ levels respectively; and it can be seen that even with the addition of both terms (making a two parameter expression for the ratios) the fit is quite poor. Such a power series expansion would require almost as many terms as there are points to be fitted if it is to be used for extrapolation; for interpolation, it is true, better fits could be obtained with fewer parameters. Figures 5 and 6 show the effect of adding the non-axiality parameter, γ, to the simplified DC treatment for Hf^{172} and Yb^{164}. Varying γ from 0 to 10-12° (with a corresponding change in μ) makes little difference in the plots; about half of them are improved slightly, the other half are unchanged or made slightly worse. In particular, the systematic deviations from the theory (mentioned earlier and quite apparent in these figures as points falling below the DC curves) are not at all improved by allowing non-axial shapes. Figure 6, for Yb^{166}, compares the DC fit with the best one obtained from the purely asymmetric rotor model of Davydov and Filipov[15]). The DC curves are clearly better. Finally, figs. 6 and 9, for Yb^{166} and W^{172}, show the results of using the rotation-vibration calculations of Faessler and Greiner[16]). These are the best fits we have found to the experimental data except for the DC

curves. It is interesting that the shape of these Faessler-Greiner curves depends very little on whether it is the β or the γ band admixed into the ground band. Thus in the DC treatment the total neglect of effects due to γ is probably rather accurately compensated for (in the ground state spacings) by overestimating slightly the value of μ. The remarkably good fits given by this simplified DC calculation are thus somewhat easier to understand.

As a last means of comparing the data with the DC model, we list in table 2 the level energies as measured and as calculated using the μ values from figs. 5 to 9 and adjusting $\hbar\omega_0$ for the best fits. This method is not a very sensitive one, but has the advantage of greater simplicity and familiarity. In constructing table 2, we used only transitions which have been given an A classification in table 1. This is primarily because we did not want to affect the entire fit for a particular band with a transition which might be misassigned. In addition, we did not want to obscure the really excellent fits usually possible up to spin 14 or so by including the admittedly more poorly fit data above these spins. The five nuclei whose first excited state energies lie below 115 keV have an overall rms % deviation of 0.21%. This is hardly outside our limits of error; however, the systematic nature of the deviations makes it seem likely that this does represent a real difference from the theory. Of the four nuclei whose first excited state is over 115 keV, W^{172} is fit almost as well as the previous five, and the considerably poorer fits for the other three, $Hf^{166,168}$ and Yb^{164}, are due almost entirely in each case to the first excited state energy. Both this systematic deviation, and the other one occuring at the highest spins are in the direction to be explained by better potential energy curves, as previously mentioned.

In summary, it appears to us that the rotational spacings observed are significantly better accounted for by the DC treatment of centrifugal stretching, than by any other means. On an absolute scale the fit is quite good, and

the small systematic deviations that are observed can be, at least qualitatively, accounted for by using more realistic potential energy curves than parabolas. Perhaps more important is the reasonable agreement of the energies and B(E2) values of the closely-related beta vibrational band to those calculated using the ground-state band "fits." As far as the presently available data have been analyzed, it seems that a simple and reasonably consistent picture can be constructed of a rotating nucleus, free to stretch and vibrate along its principal axis. Nothing is assumed or implied in this picture about the γ-vibrational band except, indirectly, that it is not the principal cause of the deviations of the ground state band from a perfect rotor. There is considerable evidence[17]) that those γ-bands identified are not very strongly coupled to the ground state band, (of order 10% of the β band), and this presumably means that as the nucleus stretches, there are no large changes in the value of γ. Such effects on the ground-state spacings as come from the γ-band and higher-order corrections are undoubtedly compensated for in our "fits" by adjustments (presumably small) of the beta-ground coupling.

The treatment of centrifugal stretching which we have discussed is strictly phenomenological; the potentials (ideally) are taken from the observed behavior of the nuclear masses, and the rotational kinetic energies are assumed to have the empirically observed β dependence. It is, of course, possible to try to calculate these quantities: the ground state moments of inertia of deformed nuclei have already been rather successfully calculated[18,19]). Such calculations, however, are complex, and extending them to higher spins is clearly outside the scope of the present investigation. Our objective has been rather to try to account phenomenologically for the collective properties observed; and hope such a description can be made sufficiently convincing to warrant a detailed microscopic treatment of the problem.

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Table 1

Ground-state rotational band transitions*

Transition	Yb^{164}	Yb^{166}	Hf^{166}	Hf^{168}	Hf^{170}	Hf^{172}	W^{172}	W^{174}	W^{176}
2 → 0	122.5 A	101.8 A	158.7 A	123.9 A	100.0 A	94.5 A	122.9 A	111.9 A	108.7 A
4 → 2	261.5 A	227.9 A	312.0 A	261.1 A	220.6 A	213.4 A	254.0 A	243.1 A	239.8 A
6 → 4	374.0 A	337.4 A	426.9 A	371.1 A	320.5 A	319.1 A	350.3 A	349.2 A	350.9 A
8 → 6	461.3 A	430.0 A	509.5 A	456.1 A	400.2 A	408.8 A	419.3 A	432.5 A	440.6 A
10 → 8	528.4 A	506.8 A	564.0 A	522.0 A	462.0 A	483.6 A	469.6 A	498.0 A	508.2 A
12 → 10	574.5 B	567.8 A	593.8 A	569.4 A	510.0 A	543.1 A	512.5 A	551.2 A	557.3 A
14 → 12	606 C	602.7 C	613.4 C	606.5 C	550.3 A	588.6 A	548.5 A	594.0 B	595.1 B
16 → 14		627.8 C			583.7 A	621.9 B	576.2 B		624.2 C
18 → 16					614.3 B	641.8 B	596.8 C		
20 → 18					652 C				

*The accuracy of these transitions is expected to be ± 0.3%. An additional significant figure has been kept in most of the cases, but this can only be useful in comparing transitions of comparable energy in the same nucleus.

Table 2

Experimental rotational energies compared with the DC calculation*

Isotope		2+	4+	6+	8+	10+	12+	14+	16+	rms % dev.
W^{176}	Exp	108.7	348.5	699.4	1140	1648	2206			0.20
	Calc	108.7	349.1	698.8	1137	1646	2213			
W^{174}	Exp	111.9	355.0	704.2	1137	1635	2186			0.16
	Calc	111.8	355.8	705.3	1137	1633	2181			
W^{172}	Exp	122.9	376.9	727.2	1147	1616	2129	2677		0.27
	Calc	122.6	378.7	729.3	1147	1617	2126	2668		
Hf^{172}	Exp	94.5	307.9	627.0	1036	1519	2063	2651		0.31
	Calc	94.4	307.5	625.5	1033	1517	2066	2669		
Hf^{170}	Exp	100.0	320.6	641.1	1041	1503	2013	2564	3147	0.22
	Calc	100.0	320.5	639.8	1038	1500	2012	2568	3160	
Hf^{168}	Exp	123.9	385.0	756.1	1212	1734	2304			0.79
	Calc	121.7	384.9	758.6	1216	1740	2315			
Hf^{166}	Exp	158.7	470.7	897.6	1407	1971	2565			1.45
	Calc	153.7	471.3	902.2	1413	1983	2600			
Yb^{166}	Exp	101.8	329.7	667.1	1097	1604	2172			0.12
	Calc	101.9	330.2	667.7	1097	1601	2169			
Yb^{164}	Exp	122.5	384.0	758.0	1219	1748				0.52
	Calc	121.2	384.7	760.3	1222	1752				

*There are two adjustable parameters, μ and $\hbar\omega_o$ for each nucleus.

Figure Captions

Fig. 1. Electron spectra of Yb^{164} and Yb^{166}. The reactions and beam energies are indicated on the spectra.

Fig. 2. Electron spectra of Hf^{166}, Hf^{168}, Hf^{170}, and Hf^{172}. The reactions and beam energies are indicated on the spectra.

Fig. 3. Electron spectra of W^{172}, W^{174}, and W^{176}. The reactions and beam energies are indicated on the spectra.

Fig. 4. Comparison of the observed rotational energies with the DC calculation. The approximately horizontal lines are the calculated energy ratios of the state having spin I to the first excited (spin 2) state, as a function of the parameter μ. The approximately vertical lines connect the measured ratios for each nucleus. An exactly vertical line would thus represent perfect agreement with the calculation.

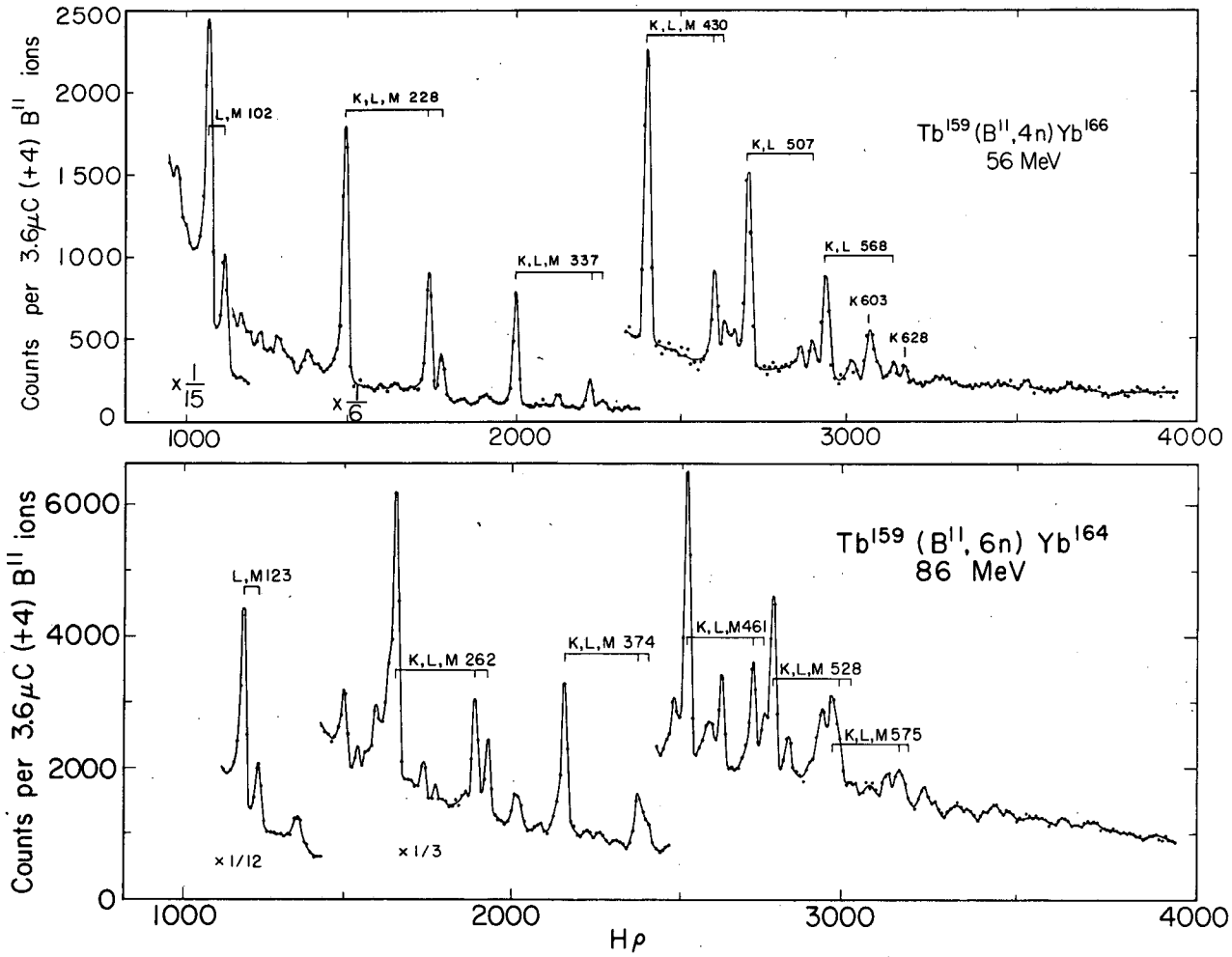
Fig. 5. The points represent the experimental ratios of succesive rotational constants, A_{I+2}/A_I, plotted against the intermediate spin, I. In the left-hand section for Hf^{172}, the solid line is the DC calculation for $\gamma = 0$, and the dashed line shows the effect of $\gamma = 10°$ with suitable readjustment of μ to fit the first point. In the right-hand section for Hf^{170}, the solid line is the DC calculation for $\gamma = 0$. The dashed line is the power series expression, $E = A\ I(I+1) + B\ I^2(I+1)^2$, where the ratio B/A is adjusted to fit the first point (labeled PS). The dash-dot curve is the above power-series plus the term, $C\ I^3(I+1)^3$, where B/A and C/A are adjusted to fit the first two points (also labeled PS).

Fig. 6. The points represent the experimental ratios of succesive rotational constants, A_{I+2}/A_I, plotted against the intermediate spin, I. The solid lines in each section represent the DC calculation for $\gamma = 0$. In the left-hand section for Yb^{166}, the dashed line is the Davydov-Fillipov calculation with $\gamma = 16.5°$ (labeled DF). The dash-dot curve is the Faessler-Greiner calculation with EB = 50 and EG = 30 (labeled FG). In the right-hand section for Yb^{164} the dashed line is the DC calculation for $\gamma = 10°$ and μ readjusted to give general agreement with the data.

Fig. 7. The points represent the experimental ratios of succesive rotational constants, A_{I+2}/A_I, plotted against the intermediate spin, I. The solid lines are the DC calculations with $\gamma = 0$ and μ as indicated.

Fig. 8. The points represent the experimental ratios of succesive rotational constants, A_{I+2}/A_I, plotted against the intermediate spin, I. The solid lines are the DC calculations with $\gamma = 0$ and μ as indicated. Note the change in ordinate scale from Figs. 5,6, and 7.

Fig. 9. The points represent the experimental ratios of succesive rotational constants, A_{I+2}/A_I, plotted against the intermediate spin, I. The solid line is the DC calculation for $\gamma = 0$ and μ as indicated. The dashed curve is the Faessler-Greiner calculation for EB = 20, EG = 20 (labeled FG). Note the change in ordinate scale from Figs. 5,6, and 7.



MUB-3674

Fig. 1

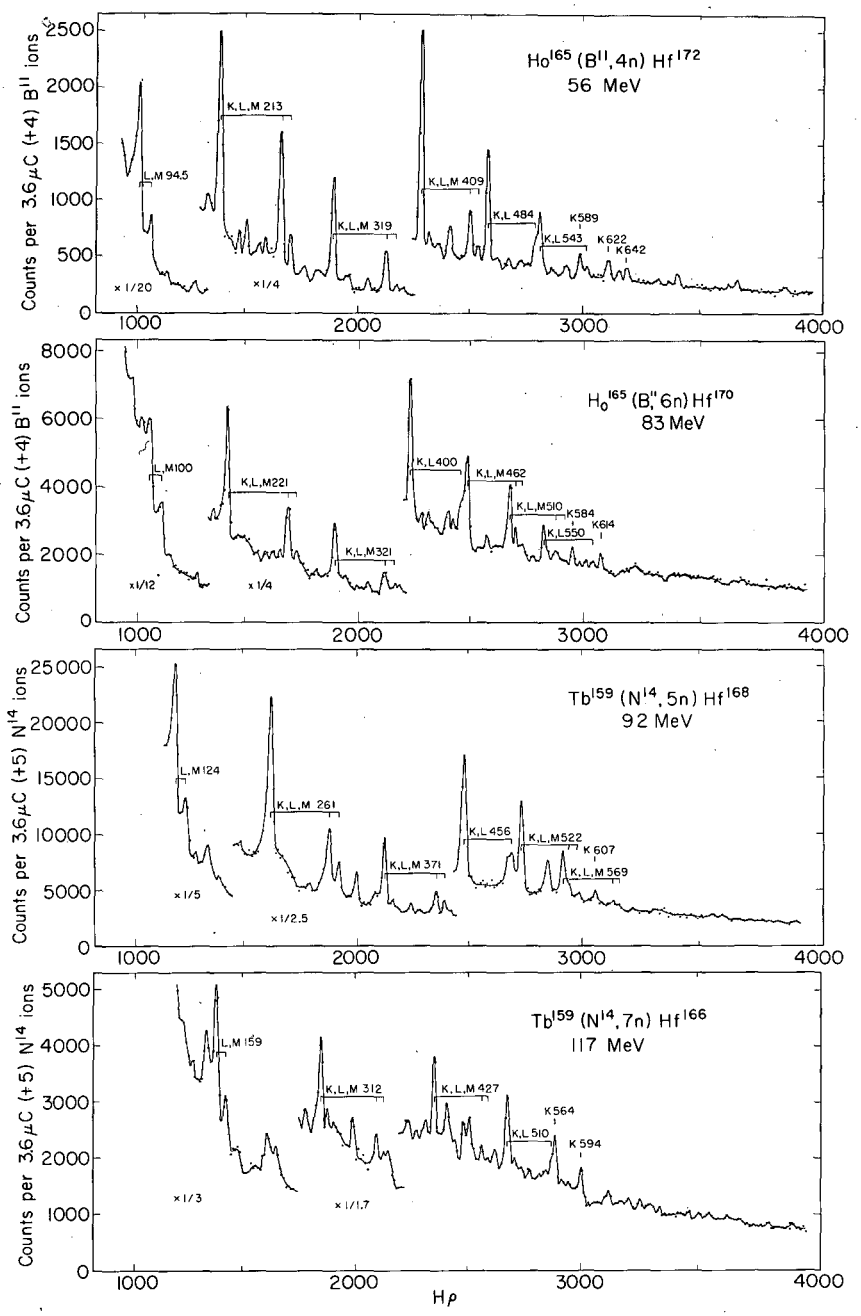
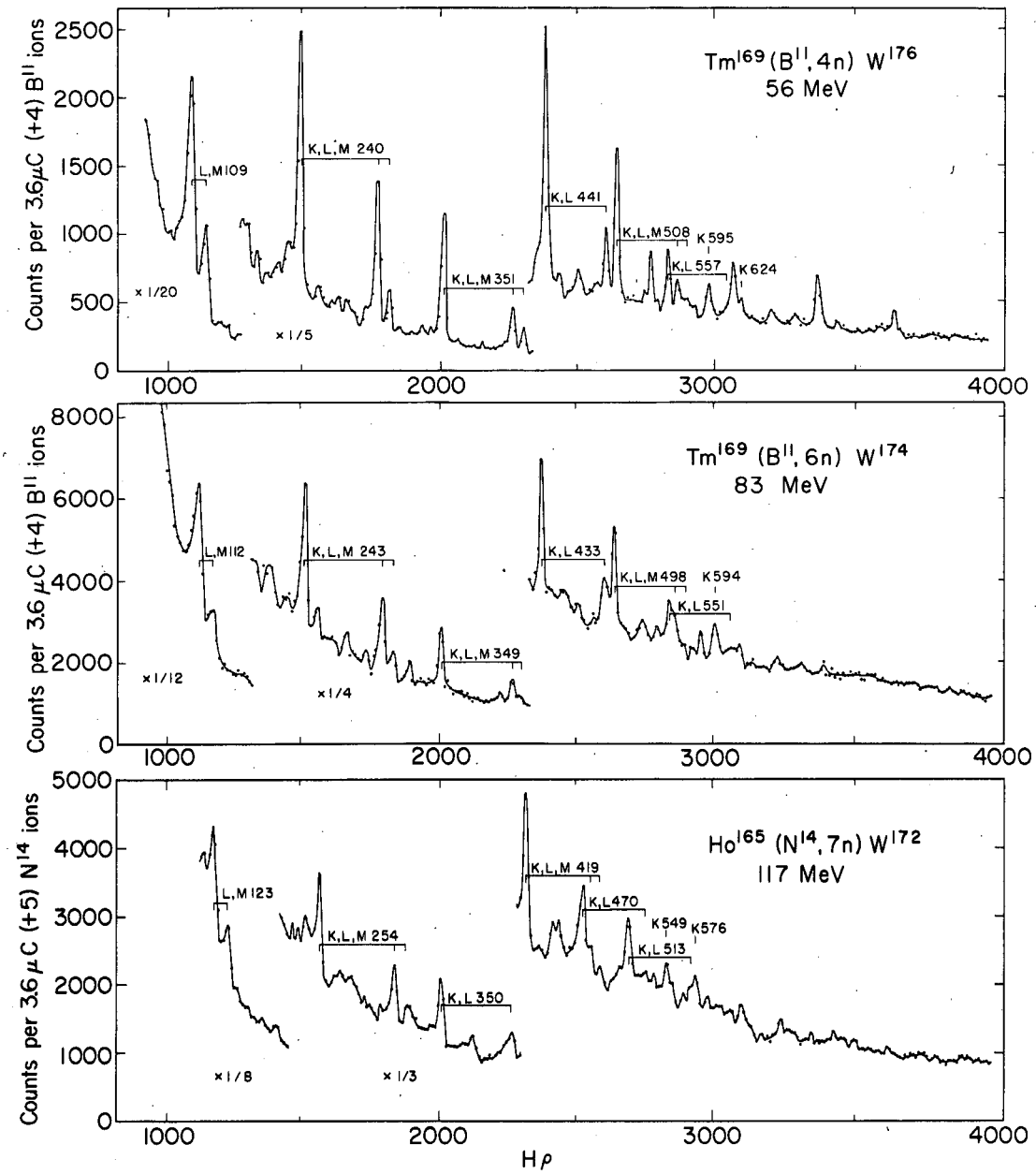

Counts per 3.6μC (+4) B11 ions
Ho165 (B11,4n) Hf172
56 MeV
L,M 94.5
K,L,M 213
K,L,M 319
K,L,M 409
K,L 484
K589
K,L543
K622
K642
x 1/20
x1/4
Counts per 3.6μC (+4) B11 ions
H0165 (B11,6n) Hf170
83 MeV
L,M100
K,L,M221
K,L,M321
K,L400
K,L,M462
K,L,M510
K584
K614
K,L550
x1/12
x 1/4
Counts per 3.6μC (+5) N14 ions
Tb159 (N14,5n) Hf168
92 MeV
L,M124
K,L,M 261
K,L,M 371
K,L 456
K,L,M522
K 607
K,L,M 569
x 1/5
x1/2.5
Counts per 3.6μC (+5) N14 ions
Tb159 (N14,7n) Hf166
117 MeV
L,M 159
K,L,M 312
K,L,M 427
K,L 510
K564
K 594
x 1/3
x 1/1.7
Hρ
MUB-3763

Fig. 2



MUB-3675

Fig. 3

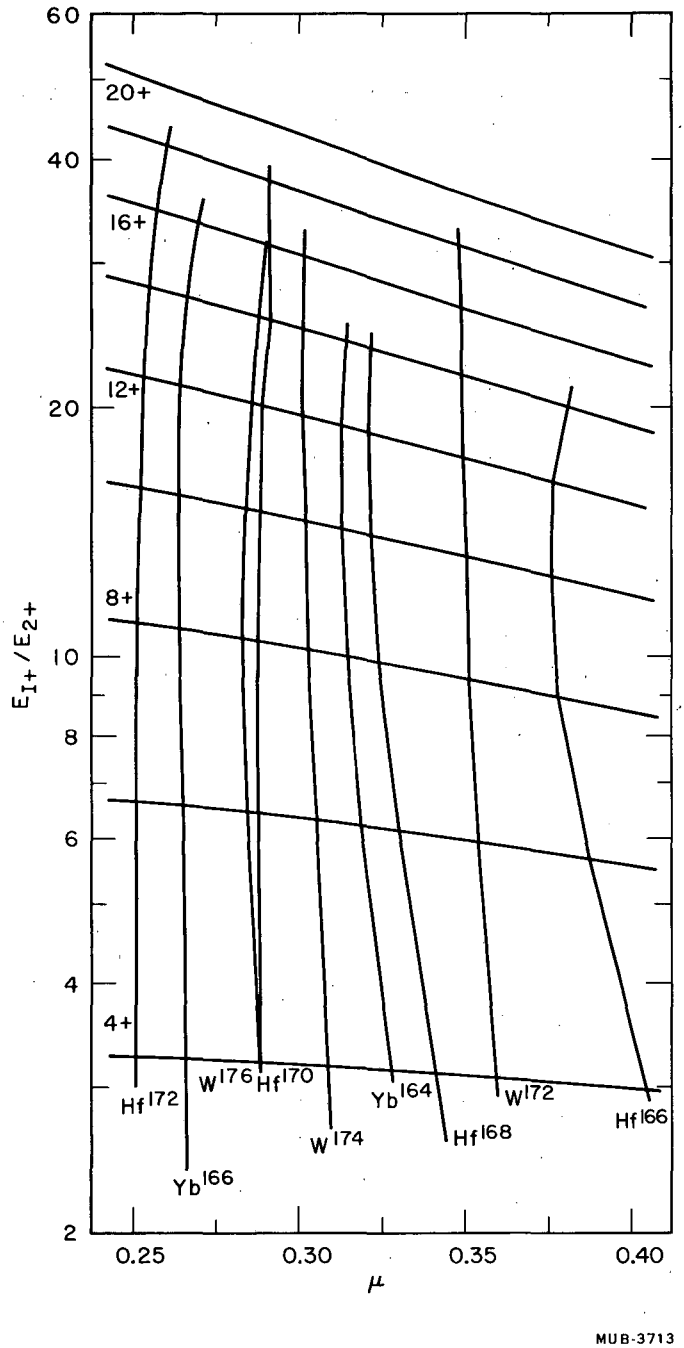

60
40
20
10
8
6
4
2
20+
16+
12+
8+
4+
E_{I+}/E_{2+}
Hf172
Yb166
W176
Hf170
W174
Yb164
Hf168
W172
Hf166
0.25
0.30
0.35
0.40
μ
MUB-3713

Fig. 4

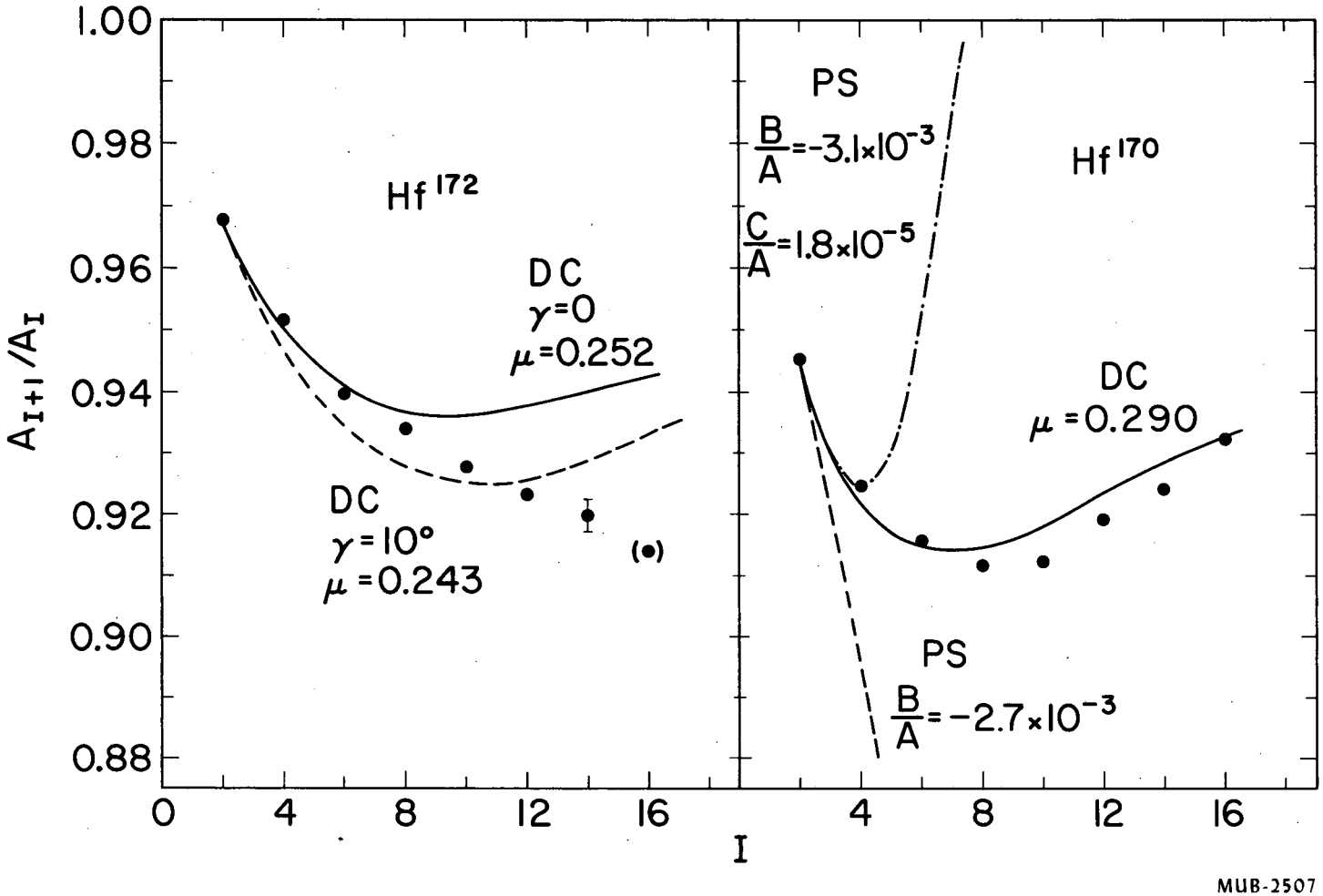

1.00
0.98
0.96
0.94
0.92
0.90
0.88
A_{I+1}/A_I
Hf 172
DC
γ=0
μ=0.252
DC
γ=10°
μ=0.243
(•)
PS
B/A=-3.1×10^-3
C/A=1.8×10^-5
Hf 170
DC
μ=0.290
PS
B/A=-2.7×10^-3
0
4
8
12
16
4
8
12
16
I
MUB-2507

Fig. 5

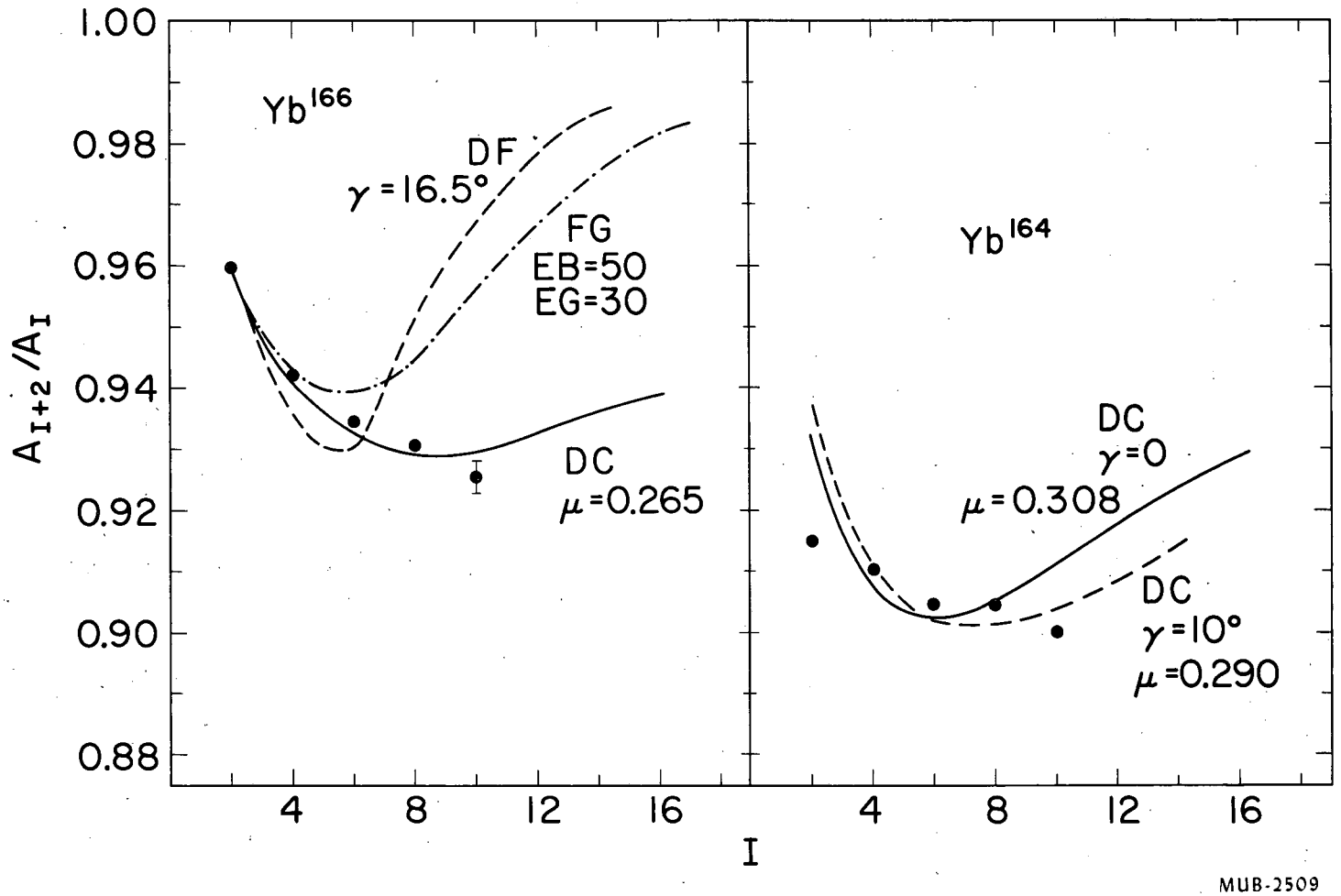

1.00
0.98
0.96
0.94
0.92
0.90
0.88
A_{I+2}/A_I
Yb^{166}
DF
$\gamma = 16.5°$
FG
EB=50
EG=30
DC
$\mu = 0.265$
Yb^{164}
DC
$\gamma = 0$
$\mu = 0.308$
DC
$\gamma = 10°$
$\mu = 0.290$
4
8
12
16
I
MUB-2509

Fig. 6

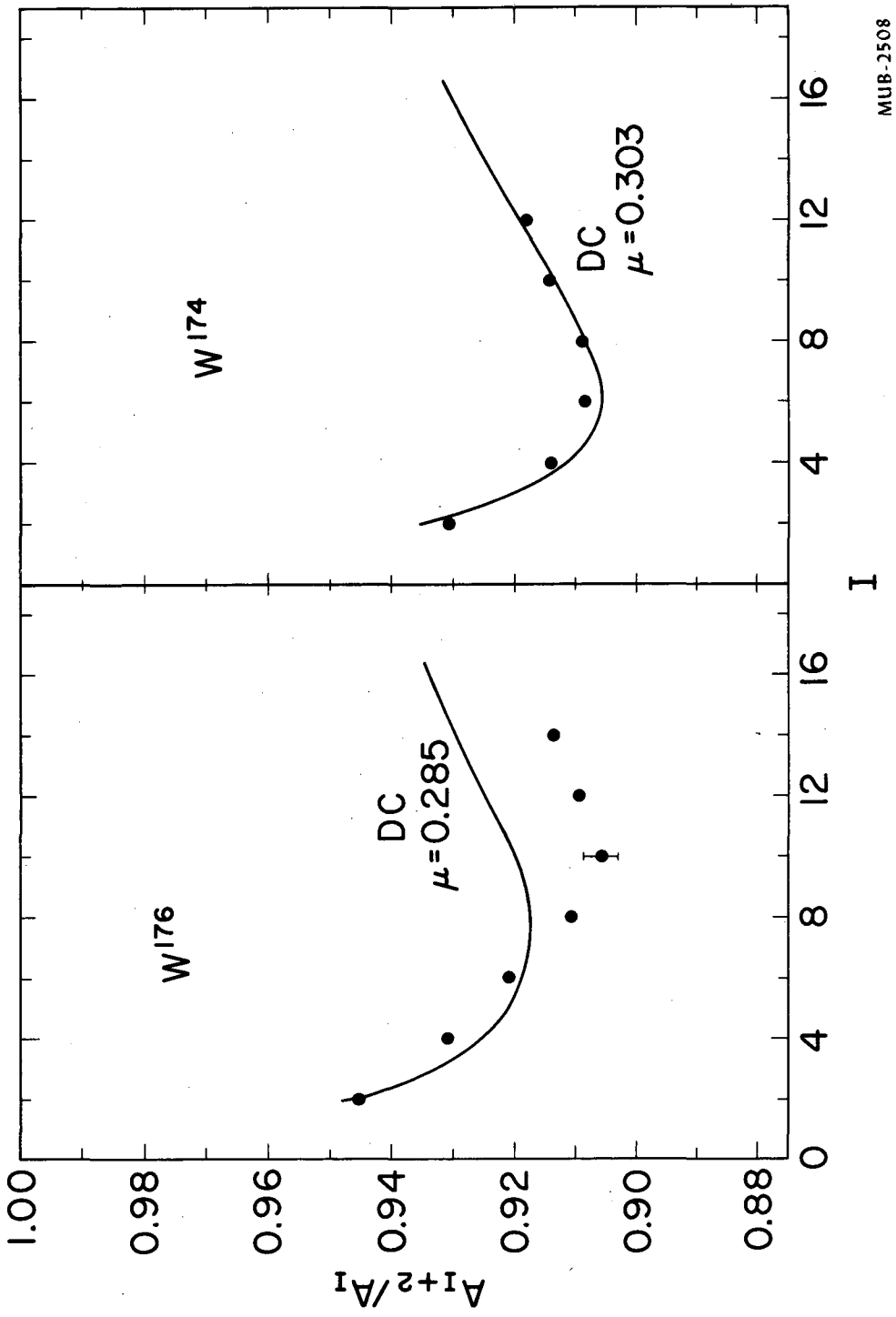

W176
W174
DC
μ=0.285
DC
μ=0.303
A I+2/A I
I
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0
4
8
12
16
MUB-2508

Fig. 7

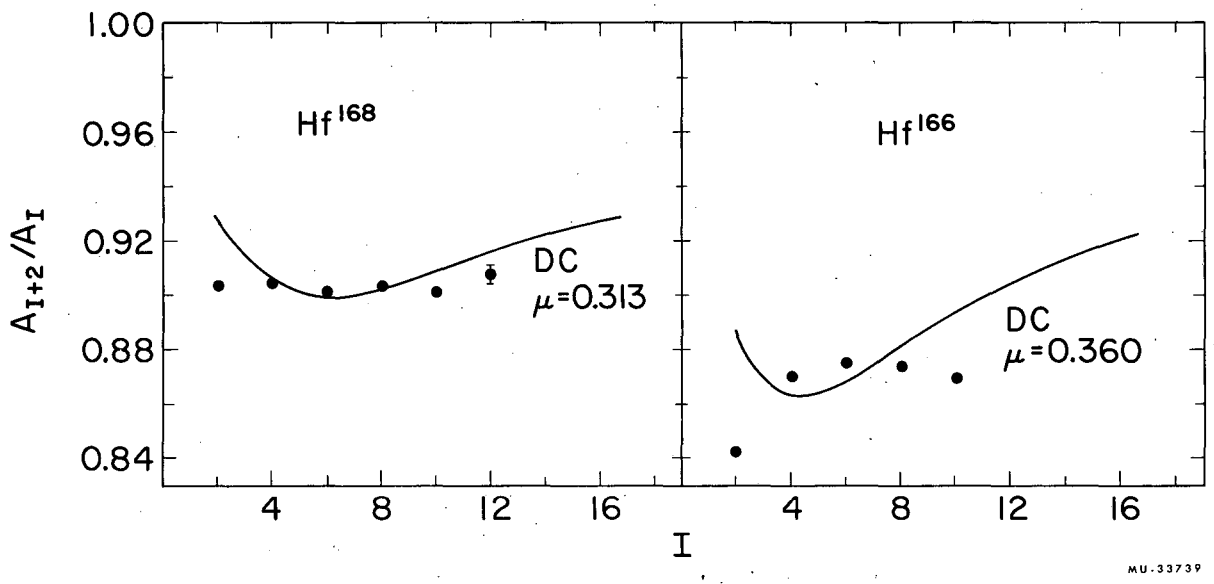

1.00
0.96
0.92
0.88
0.84
A_{I+2}/A_I
Hf^{168}
DC
μ=0.313
Hf^{166}
DC
μ=0.360
4
8
12
16
I
MU-33739

Fig. 8

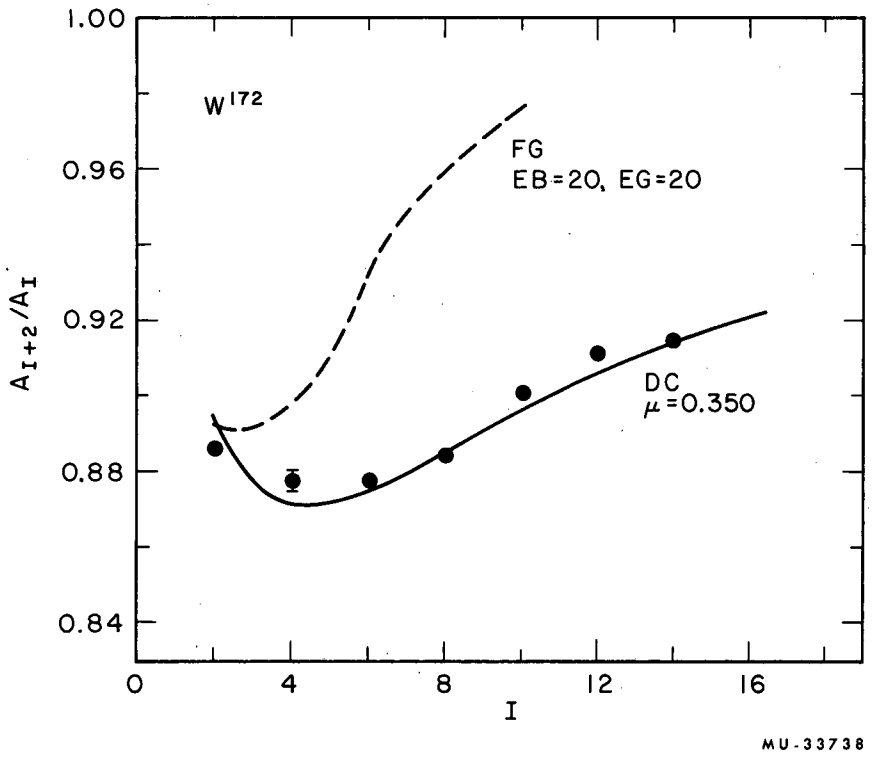

1.00
0.96
0.92
0.88
0.84
A_{I+2}/A_I
W^{172}
FG
EB=20, EG=20
DC
μ=0.350
0
4
8
12
16
I
MU-33738

Fig. 9

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