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CENTRIFUGAL STRETCHING OF NUCLEI

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The object of this note is to show in a simple way the physical content of the close agrement between the Davydov-Chaban (DC) calculations and the experimental rotational energies described in Ref. 1 and, furthermore, to suggest a qualitative explanation for the remaining deviations. We shall show that the ideal of a spinning nucleus being stretched out under the influence of the centrifugal force, when incorporated in an elementary classical treatment, is capable of reproducing the quantum-mechanical DC calculations[2] and thus of accounting, with remarkable accuracy, for the rotational spectra.

Let us assume, as suggested by Bohr[3] (and DC), that the energy of an axially symmetric nucleus consists of a potential energy, quadratic in the deviation of a deformation parameter β from its equilibrium value β_o, and a kinetic energy of rotation equal to $\hbar^2 I(I+1)$ divided by twice the moment of inertia of the nucleus (a function of β):

$$E = P.E. + K.E. = \frac{1}{2}C\,(\beta-\beta_o)^2 + \frac{\hbar^2}{2\mathfrak{J}(\beta)}\,I(I+1). \qquad (1)$$

If we assume that $\mathfrak{J}$ is proportional to β^2 ($\mathfrak{J} = 3B\beta^2$, where B is a constant) we have a simplified version of the problem that is solved quantum mechanically by DC. The two terms of Eq. (1) are represented graphically in Fig. 1 for several values of I. The minimum in the sum of the P.E. and K.E. for a given I represents the equilibrium deformation of a nucleus with a given angular momentum. It is clear from Fig. 1 that the minimum will move to higher values of β as I increases. To obtain a quantitative relationship expressing this centrifugal stretching we differentiate Eq. (1) with respect to β, equate the result to zero thus obtaining the equilibrium value of β, and then substitute this β_{eq}

into Eq. (1) to obtain the classical result for the energy E as a function of angular momentum I. A comparison with the DC calculation for the energy spacings shows that, with a suitable choice of B and C, the classical formula reproduce the quantum mechanical result to better than 1/2% over the entire region of spins and nuclei discussed in Ref. 1.

It is not immediately clear why the classical formula does so well, since the zero-point vibrational energy, included by DC but not in the classical approximation, varies appreciably with I. (The result is that a given set of experimental levels E_I requires somewhat different parameters B and C if one uses the classical or quantum mechanical formulae.) It is not difficult, however, to make an estimate of the zero-point vibrational energies. The second derivative of Eq. (1) with respect to β, evaluated at β_{eq}, gives the curvature of the effective potential for any I. These curvatures, if used to define new parabolic potentials, lead at once to an estimate of the zero-point vibrational energy as a function of I. Adding this energy to the classical formula based on Eq. (1) gives exactly the DC quantum mechanical solution (DC Eq. 2.11) so long as the DC quantum number ν is integral—and this turns out to be an excellent approximation[4] for all but the lowest spins of the worst rotors discussed in Ref. 1.

The foregoing is, then, a simple, semi-classical derivation of those features of the DC calculation that are essential to the explanation of the energy spectra of Ref. 1. The simplicity of the semi-classical treatment enables us to go further than DC and to test the effect of different input assumptions, in particular the result of varying the forms of the potential and kinetic energies in their dependence on β.

Examining first the kinetic energy term, one might question the hydrodynamic argument for using moments of inertia proportional to β^2, since, in absolute magnitude, the hydrodynamic moments are wrong by a factor of six or so. Fortunately at least a partial test of the β^2 relationship is possible. The moments of inertia derived from the first (2+) rotational energies of nuclei in

the region $150 < A < 180$ have been plotted in Fig. 2 against their respective ground state equilibrium deformations (β_o), as determined from the appropriate reduced E2 transition probabilities, $B(E2;0+ \rightarrow 2+)$. The data are primarily those of Elbek et al.,[5] who have presented and discussed such plots. The two heavy lines correspond to the rigid rotor and the hydrodynamic estimates for the moment of inertia. The dashed line through the experimental points is a parabola and shows that the points do happen to be approximately representable by a quadratic dependence of $\mathcal{J}$ on β. Furthermore, Grodzins[6] has shown that this dependence holds over the entire periodic table if $\mathcal{J}$ varies with mass number approximately as $A^{7/3}$ instead of the expected $A^{5/3}$. This suggests that if the mass parameter B is given this same smooth variation with A, the kinetic energy term in Eq. (1), with $\mathcal{J} = 3B\beta^2$, may be useful for all even-even nuclei, irrespective of their shape.

Although the quadratic dependence of $\mathcal{J}$ on β seems grossly adequate over this limited region of deformation, it may be somewhat different for the higher spins considered here, and must eventually fail at higher deformations. We have therefore examined the effect on the calculated energy spacings of assuming $\mathcal{J} \propto \beta^3$ and $\mathcal{J} \propto (\beta-\alpha)$, where α is an adjustable parameter. The resulting energy spacings differ characteristically from those obtained with $\mathcal{J} \propto \beta^2$, but for a suitable choice of the available parameters these differences can be made less than 1-2% over the entire range of spins of interest here. Thus a precise β^2 dependence of $\mathcal{J}$ is not essential for agreement with experiment. A different dependence which presents certain advantages would be:

$$\mathcal{J}(\beta) = \mathcal{J}_{rigid}(\beta)\,[1-e^{-(\beta/\beta_1)^2}]\,,$$

where β_1 is a parameter. With this choice, the moment of inertia is approximately parabolic for moderate deformations and tends to the rigid body value for large deformations.

As regards the potential-energy part of Eq. (1), deviations from a parabolic dependence on $(\beta-\beta_o)$ may be expected for large deviations of β from its equilibrium value. The nature of these deviations is suggested by the functional form of the deformation energy used in Ref. 7 in connection with an interpretation of nuclear masses and deformations. The essential features of this potential are a broad (approximate) parabola in β, representing the restoring potential associated with the nuclear surface tension (modified by the electrostatic repulsion) and a central gaussian bump of relatively short range, representing shell effects. This bump is negative for nuclei close to magic numbers, stabilizing the spherical shapes, and positive for nuclei between closed shells, producing stable deformations. Several typical cases are illustrated in Fig. 1. An important feature is that for large values of β the potential-energy curves of neighboring nuclei are all essentially identical. The immediate prediction is that for rotational states of sufficiently high spin one might expect all nuclei to be forced over against this common part of the potential, with the result that all neighboring nuclei should have similar rotational spacings at high spin. Just such similar behavior at high spins was, in fact, one of the striking features observed for the nine nuclei studied in Ref. 1.

A second feature of the "mass-formula potential" of Ref. 7 is that nuclei approaching the region where ground-state deformations disappear (bad rotors) are characterized by a small gaussian bump; the potential energy tends therefore to be more nearly quadratic in β. Under these conditions the DC equations describe quadrupole surface vibrations of a spherically symmetric nucleus and one knows[3] the solution is essentially a harmonic-oscillator spectrum. With this potential, the semi-classical treatment outlined earlier in the present note also gives a series of very nearly equally spaced levels (spaced as $\sqrt{I(I+1)}$ except for the the 0+ state, where the zero-point vibrational energy has to be rather carefully considered). One might hope, therefore, that this simplified treatment is

capable of tracing in a continuous way the transition of the "rotational" states into the highest-spin member of the "vibrational" multiplets. If the observations of Grodzins really imply that, apart from a smooth variation with mass number, the kinetic-energy term in Eq. (1) is the same for rotors and vibrators, it would follow that the character of the spectrum is determined essentially by the shape of the potential. The potentials suggested by the mass formula of Ref. 7 would then appear to provide a possible choice for unifying the phenomenological description of rotors and vibrators. Thus when the gaussian bump is large enough to produce a minimum in the potential energy with sufficient curvature to hold β approximately constant with increasing I, one has recognizable rotational spectra; but when the curvature is small (the bump is small), β changes appreciably with increasing spin, obscuring the characteristic I(I+1) rotational energy dependence.

We now turn to a qualitative comparison of the energies given by the mass-formula potentials with those given by the DC parabolic potentials. It appears that both of the systematic deviations of the experimental data[1] from the DC solutions are in the direction expected on the basis of the mass-formula potentials. The better rotors (low first excited states) were fitted very well by DC up to spin 10 or 12, but thereafter usually had smaller level spacings than expected. This can be explained by the decreasing curvature of the potential for large β, which the mass formula predicts for good rotors as they are being driven over against the broad surface-tension parabola. In fact, if the surface-tension parabola were reached while the kinetic energy was still proportional to β^{-2}, one would expect harmonic oscillator spacings as discussed in the previous paragraph. This limit is not reached in any of our experiments, and may never be reached, because the kinetic-energy term will eventually deviate from β^{-2} as $\mathcal{J}$ approaches $\mathcal{J}_{rigid}$. A calculation ignoring zero-point energies has shown that a reasonable mass-formula potential together with a β^{-2} kinetic energy term does reproduce the smaller experimental spacings at high spin without affecting the excellent fit at lower spins.

The other systematic deviation of the experimental data from DC occurs only in nuclei near the "vibrational" region (high first excited state energies). Here the lowest spacings, particularly the 2+ to 0+ one, are larger than expected from a DC calculation fitted to the intermediate spin states. Again the data deviate in the direction of the harmonic oscillator; and this, as we have noted before, is what the mass-formula potential predicts when the gaussian bump is small.

Perhaps the best test of these ideas would be the measurement of several E2 transition probabilities within the same rotational band. The increasing deformations expected at higher spins should produce larger E2 transition probabilities than otherwise expected. The existing data[8,9,10] indicate that this is the case, but are not accurate enough to afford a quantitative test. Such measurements should be quite feasible in very poor rotors where the expected effects are large (see Ref. 8).

In summary, it seems that the DC calculations can be reproduced by an elementary semi-classical treatment, which brings out the simplicity of the underlying physical effect (centrifugal stretching) and which permits an examination of the forms of the potential and kinetic energy expressions assumed in the model. Although the DC kinetic energy expression seems adequate over the region of deformation for which there are data, the mass-formula potential of Ref. 7 would appear to be a significant improvement over the potential assumed by DC. The indications are that using the mass-formula potential and the simple semi-classical treatment, it may be possible to follow the "rotational" sequence of levels into the region of good "vibrators." For this general approach to be valid, it is essential that the potentials and kinetic energies of a particular nucleus be smooth functions of the deformation, and it would appear that this is the case, at least for the nuclei studied. The more complicated problem of calculating the required functions explicitly from the detailed behavior of the nucleons, remains to be solved.

FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. The heavy solid curve is the parabolic potential-energy term in Eq. (1), and the heavy dashed curves are the corresponding kinetic energy term for three values of I. These curves represent those required to fit the W^{174} data (right-hand ordinate scale), except that for comparison purposes β_o is plotted at 0.25 instead of the measured 0.26 ± 0.05. The lighter curves are the mass-formula potentials for W^{174}, Pt^{194}, and Hg^{200} in descending order (left-hand ordinate scale). The W^{174} potential as determined from the nuclear masses,[7] is given by: $E = 32.8\beta^2 - 12.0\beta^3 + 5.25e^{-23.1\beta^2}$ MeV; and the extremely close agreement with the experimental parabola near the minimum is undoubtedly somewhat fortuitous.

Fig. 2. The solid circles are the experimental moments of inertia for rotational nuclei (in units of the rigid sphere) plotted against the measured β_o values. The triangular points are for vibrators and β_o presumably represents an rms value of β. The heavy solid lines are the rigid rotor and hydrodynamic estimates, and the dashed line is a parabola passed through the experimental data. The open circles are the measured moments of inertia for W^{174} up to spin 14 plotted against the DC calculated β values. The β_o values of W^{174} has been roughly measured to be 0.26 ± 0.05.

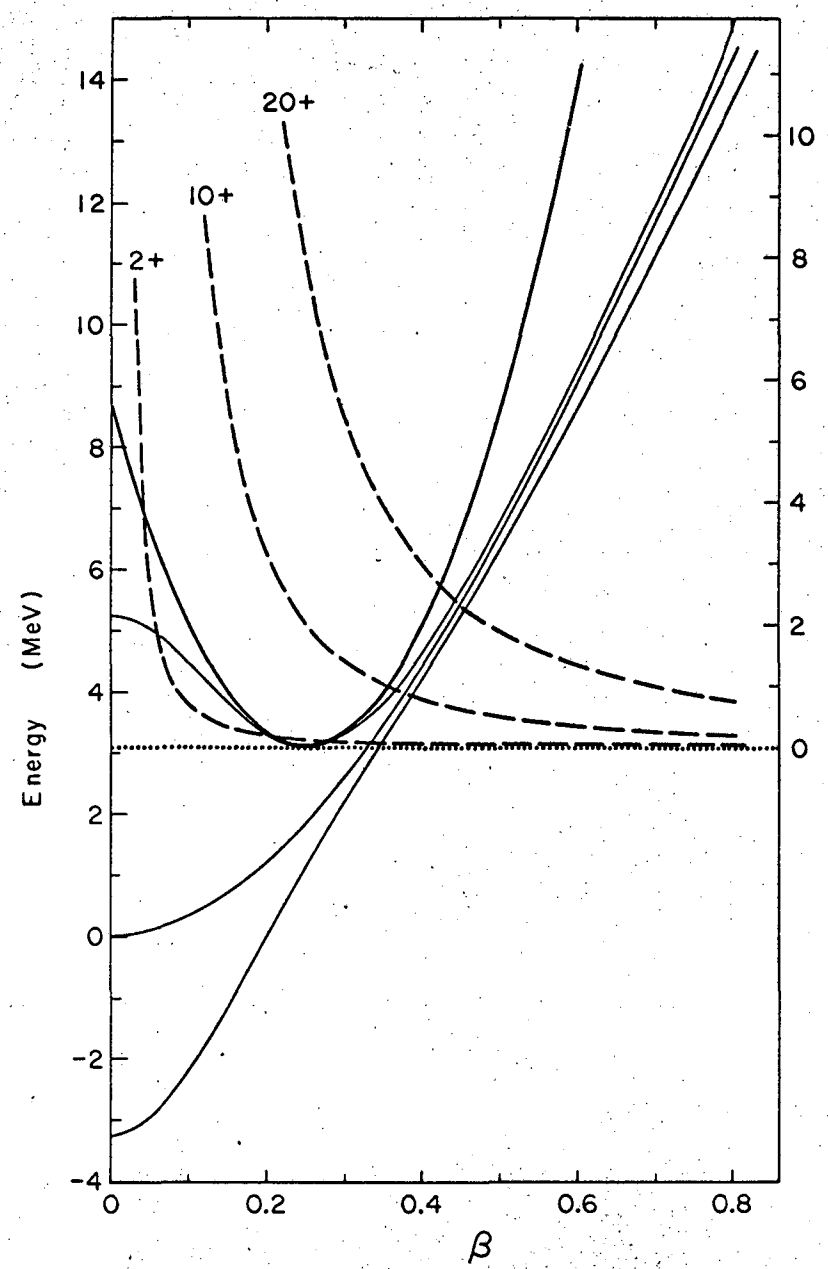

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Fig. 1

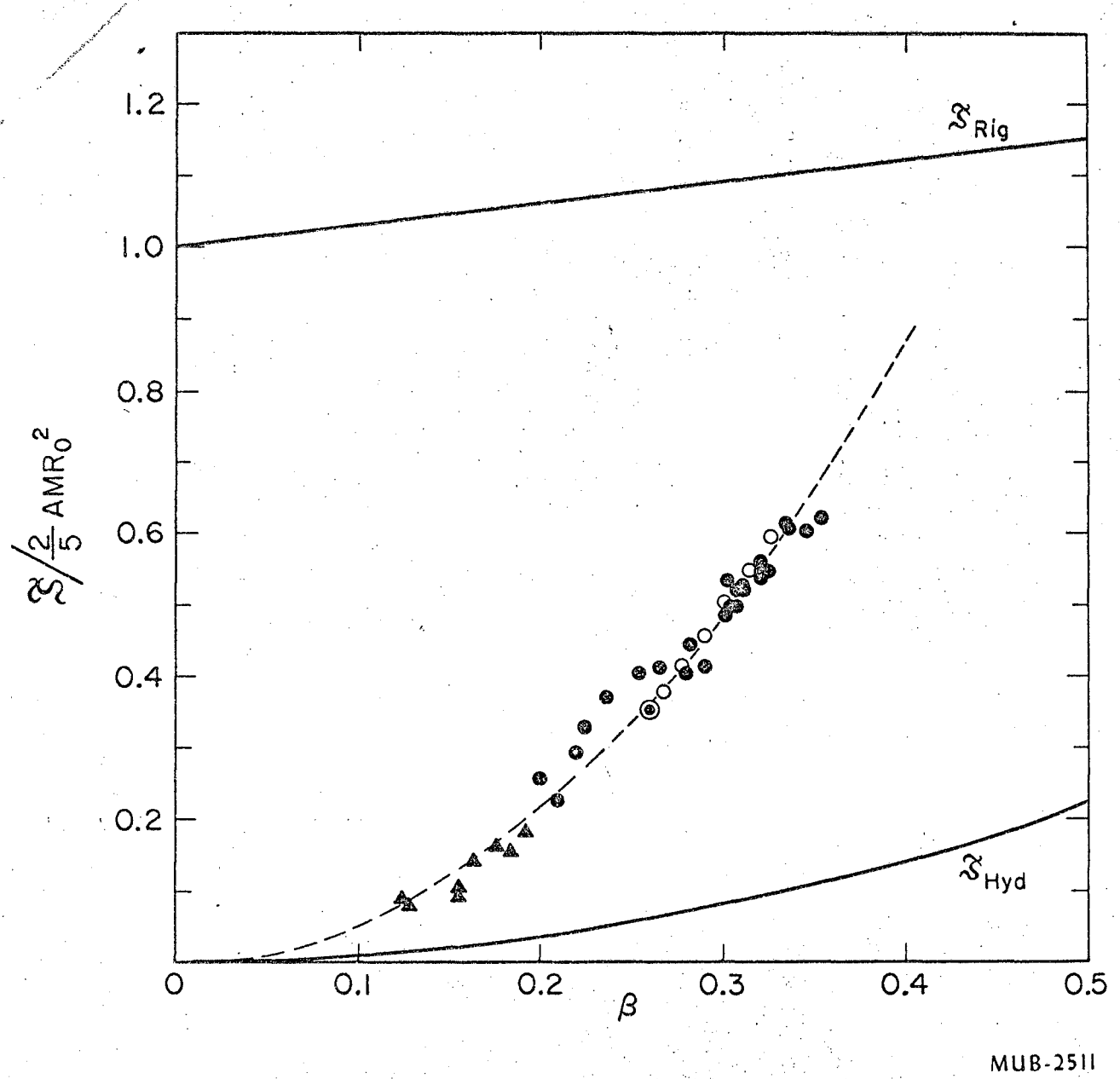
1.2
1.0
0.8
0.6
0.4
0.2
0
0.1
0.2
0.3
0.4
0.5
β
$\mathfrak{I}_{Rig}$
$\mathfrak{I}_{Hyd}$
$\mathfrak{I}/\frac{2}{5}AMR_0^2$
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Fig. 2

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