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UNITS FOR ATOMIC AND SUBATOMIC PHYSICS

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## ABSTRACT

The consequences are examined of setting $\hbar = 1$ and $c = 1$, while retaining the electron-volt. The system of units thus defined is simple and practical for atomic and subatomic physics. A convention is also suggested which permits any quantity to be expressed by a number near unity.

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## I. Introduction

In physics we already have too many systems of units. A new one is justified only if it simplifies or has peculiar advantages.

For atomic and subatomic physics we still employ distance and time units which have their origins in the dimensions of the earth and its angular velocity. They are quite suitable for the building of a bridge or for the operation of an airline. But atomic and subatomic physics is the domain of quantum mechanics and relativity for which natural units are $\hbar$ and c. The quantum of charge is also a fundamental unit. These three natural units are not independent, however, because $e^2/\hbar c$ is a dimensionless number, the fine-structure constant. Practicality also must be considered in the selection of units. Because all electrical instruments and accelerators are calibrated in conventional units, a very great revolution would be required to overturn the electron-volt as the unit of energy. We have no subversive plans.

We do show how to simplify what we now have by an appropriate use of units. We also introduce a convention for expressing cross sections, reaction rates, masses, etc., as numbers without remote decimal points. The system actually teaches physics. It should lead one to a better appreciation of the uncertainty principle, and to meaningful relations between cross sections, rates of processes, decay widths, etc. With it one

is put into the world of atomic dynamics with scales suited to the measured quantities.

II. Outline of Proposal

Atomic Units (AU) are developed as follows:

1. The quantum of charge is related to the unit of energy by introducing the electron-volt in the usual way. This is hardly a revolutionary suggestion.

2. In addition, the unit of velocity is that of light, and the unit of action is $\hbar$. This is not novel either; many theorists drop $\hbar$ and c from their formulas.

How can the combination of these two simple proposals provide units for all conceivable atomic physics? It is because the result of a calculation or measurement will be a mass, a time, a cross section, a rate, a momentum, etc.--one already knows the physical character of the quantity, and it is redundant to distinguish by a different unit a decay time or a wave length.

In high energy physics we are even now familiar with the consequences of setting c = 1. Energies, masses and momenta all are expressed in BeV (or GeV). We know the proton mass is 0.938 BeV, and we no longer think of it in such awkward units as the cgs. or MKS. A momentum of one BeV is in the practical range of particle momenta for strong interaction physics. We also have the Pythagorean theorem of relativity, which is exact at all velocities:

$$E^2 = M^2 + P^2.$$

Here P is the particle momentum, M is its mass, and E is its energy. The particle velocity, $\beta$, is P/E.

While in particle physics the BeV is a unit of a suitable size, in nuclear physics the MeV is convenient, in atomic physics the KeV, and in

chemical physics the eV--each a factor of $10^3$ smaller than the preceeding one. We can extend the scale upward to the TeV( $= 10^{12}$eV), or higher, and downward indefinitely to measure any quantity of any magnitude. Prefix letters of mnemonic value do not seem to be at hand for this range, however. Until appropriate and easily remembered letters are suggested it may be just as well to write $10^{-3}$eV or $10^{-6}$eV.

On setting $\hbar = 1$ we carry the process farther and find no difficulties. Then $e^2$ is the fine structure constant, $\alpha(\approx 1/137)$. In high energy physics one $(\text{BeV})^{-1}$, or simply one VeB is a typical length: $1.97 \times 10^{-14}$ cm. One $(\text{VeB})^2$ is a typical cross section: $3.88 \times 10^{-28}$ cm$^2$. But the calculation of the rate of a process or the width of a state now will also come out in BeV. The unit of time for particle physics is therefore also one VeB. Converted to seconds this is $6.58 \times 10^{-25}$ sec--short enough to be a unit for strong interaction decay time.

A person working in a particular field will seldom need more than one unit to express all times, lengths, rates, line widths, etc. Quantum-mechanically conjugate quantities; energy and time, momentum and distance have reciprocal units. The width of a state and its decay rate are the same thing in this system.

Do the units lead to equal simplifications outside of high energy physics? Suppose we consider the energy term value in the hydrogen atom

$$E_n = \frac{me^4}{2n^2\hbar^2} .$$

In atomic units this becomes

$$E_n = \frac{\alpha^2 m}{2n^2} .$$

Now $\alpha \approx 1/137$, n is an integer, and m is the electron mass in energy units (511006eV), so that $E_1 = 13.6$ eV for n = 1.

The Bohr radius $a_o = \frac{\hbar^2}{me^2}$ becomes $\frac{137}{m} = 0.268$ VeK (in order of magnitude unity) when expressed in this appropriate unit of length. The Compton wave length of an electron is 1/m or 1.96 VeM. The proton Compton wave length is 1.07 VeB, while the Thompson cross section is 32.1 $(\text{VeM})^2$.

## III. Conversion of Units

Suppose one wishes to convert atomic units to or from another system. If in that system (for example the cgs-esu system) one knows the velocity of light, c, ($\approx 3 \times 10^{10}$cm/sec), the electronic charge, e, ($\approx 4.8 \times 10^{-10}$ esu) and also knows the fine structure constant, $\alpha$, ($\approx 1/137$) he has no other physical constants to memorize. The most recent values for these numbers are[1]

$$c = 2.997925 \pm 0.000003 \times 10^{10} \text{ cm/sec}$$
$$e = 4.80298 \pm 0.00020 \times 10^{-10} \text{ esu}$$
$$\alpha^{-1} = 137.0388 \pm 0.0010$$

To calculate in AU one merely replaces c and $\hbar$ in the conventional formula by unity and $e^2$ by $\alpha$. One is then left with some numerical factors and with energy or $(\text{energy})^{-1}$ factors. These are all to be expressed in eV, KeV, MeV, BeV or TeV, as appropriate.

On the other hand, perhaps one has a number in AU and wishes to convert it to cgs units. He can always do it on multiplying by a factor containing c, $\alpha$, and e. For quantities of the following physical dimensions the factors are:

| | | | | |
|---|---|---|---|---|
| length | : | $ec/10^8\alpha$ | $= 1.9732 \times 10^{-5}$ | cm/Ve |
| time | : | $e/10^8\alpha$ | $= 6.5819 \times 10^{-16}$ | sec/Ve |
| energy | : | $10^8 e/c$ | $= 1.6021 \times 10^{-12}$ | ergs/eV |
| charge | : | $e\alpha^{-1/2}$ | $= 5.6225 \times 10^{-9}$ | esu |

For a velocity the factor is of course c. In AU the expectation value of the K-electron velocity is $\approx z\alpha$. For angular momentum or action the factor is $e^2/\alpha c(=\hbar)$. The Ve, which equals $ec/10^8\alpha$ cm, is the distance between two electrons at which their potential energy is $\alpha$ electron-volts. It is also the wave length, $\lambda\!\!\!^-$, of a 1 eV photon. It is, moreover, about the smallest distance resolvable with an optical microscope. One cm equals 50679 Ve. The Ve is also the time for a one eV photon to go a distance of one wave length, $\lambda\!\!\!^-$, (really one radian-length).

Of course, this system would perhaps be better from the theoretical point of view were the unit of energy something atomic, such as the electron mass or the proton mass. Then, however, the system would have lost its practicality, which derives from the fact that a practical electrical unit, the volt, enters centrally in the system. We cannot eliminate it from our instrumentation or our accelerators.

## IV. Possible Extensions

For macroscopic mechanics atomic units are unsuitable, but for electromagnetism they have a number of interesting features. Some of these we shall mention, although the system is not seriously proposed for electrical engineering.

Since $e^2 = \alpha$ the unit of charge is that carried by $\alpha^{-1/2}(\approx 11.7)$ electrons.

Unit current is one unit of charge per unit time ($6.5819 \times 10^{-16}$ sec). It is therefore about 2.84 milliamperes. For bending beams by laboratory magnetic fields we recall that in cgs units one electron stat-volt ($c/10^8$ volts) is equivalent to 1 gauss-cm. The size of e affects the magnitude of one eV, the unit in which we express all magnitudes. The magnetic field, B, now is meaningful only in combination with e.[2] The unit of eB in AU is $(eV)^2$.

$$\text{One } (eV)^2 = 50679 \left(\frac{10^8}{c}\right)\cdot \text{electron gausses}$$

$$\approx 169 \text{ electron-gausses},$$

A field of 169 gausses is a reasonable unit for atomic physics. The maximum magnetic field from a typical magnet may be about 100 $(eV)^2$.

As an example, suppose we calculate the energy E of the Bohr magneton,

conventionally $e\hbar/2mc$, in a field of 100 $(eV)^2$:

$$E = 100/2m = 100/2(511006) \approx 10^{-4} eV.$$

The angular frequency of precession is g times larger.

The unit of eE is also $(eV)^2$, that is eV/Ve. This unit is equivalent to 50.7 kilovolts per cm, which is a reasonable number for analyzing fields.

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[1] E. Richard Cohen and J. W. M. DuMond "Report of the Commission on Nuclidic Masses and Related Atomic Constants of the International Union of Pure and Applied Physics" June 24, 1963.

[2] Since e is effectively dimensionless in Atomic Units, B can be expressed in $(\text{Volts})^2$ but we need and want only one unit--the eV.

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